Efficient entanglement distribution for quantum communication against collective noise

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We present an efficient faithful entanglement distribution scheme for quantum communication against collective noise. Its setup is composed of polarizing beam splitters, half wave plates, polarization independent wavelength division multiplexers, and frequency shifters. An arbitrary qubit error on the polarization state of each qubit caused by the noisy channel can be rejected, without resorting to additional qubits, fast polarization modulators, and nondestructive quantum nondemolition detectors. Its success probability is 100%, which is independent of the noise parameters, and it can be applied directly in all one-way quantum communication protocols based on entanglement.

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Quantum entanglement is an important resource in quantum information processing and transmission, such as quantum computation [1] and quantum key distribution (QKD) [2]. In optical QKD protocols, the two remote parties, say the sender Alice and the receiver Bob, exploit entangled photon pairs to set up their quantum channel for creating a sequence of random bits as their private key [3–6], in particular in a long-distance quantum communication. However, entangled photon systems will inevitably interact with their environments, which will make them decoherence. For instance, the thermal fluctuation, vibration, and the imperfection of the fiber will affect the polarization of photons transmitted in an optical fiber. For accomplishing the task of quantum communication, people have proposed some interesting methods to overcome the effect of the channel noise, such as phase coding [7], quantum error correct code (QEC) [1], entanglement purification [8–10], error rejection [11–13], decoherence-free subspace (DFS) [14–16], and so on. In QKD protocols with phase coding [7], the fluctuation of optical fiber is passively compensated with the Faraday orthoconjugation effect via a two-way quantum communication. However, the use of two-way quantum communication makes the system vulnerable to the Trojan horse attack. Entanglement purification will consume exponentially the quantum resource for obtaining a subset of maximally entangled photon pairs from a set of less entangled systems [8, 9] or it will resort to hyperentanglement which is the entanglement in several degrees of freedom of photons [10], such as polarization, frequency, spatial mode, and so on. Although QEC and DFS methods can be used to suppress the effect of channel noise effectively, they are sensitive to channel losses and need much resource [17].

Usually, the fluctuation in an optical fiber is slow in time, so that the alteration of the polarization is considered to be the same over the sequence of photons [13]. This feature provides people a good way to design quantum error-rejection protocols. For instance, Yamamoto *et al.* [13] proposed an error-rejecting scheme for the faithful transmission of a single-photon polarization state over a collective-noise channel, resorting to a reference single photon in a fixed state in 2005. Its success probability is in principle 1/16 without two-qubit operations [18]. Subsequently, Kalamidas [11] proposed two schemes to reject and correct arbitrary qubit errors without additional particles, but fast polarization modulators. In 2007, Li *et al.* [12] also proposed a faithful qubit transmission scheme against a collective noise without ancillary qubits. Its success probability is 50% with only linear optical elements in a passive way. These error-rejection protocols can also be used for the distribution of entangled photon pairs between two remote parities. However, their success probability is very low.

In fact, the polarization of photons is, on the one hand, sensitive to channel noise. The frequency of photons suffers less from channel noise as its alteration requires a nonlinear interaction between photon and an optical fiber, which takes place with a negligible probability. The previous experiments showed that the polarization entanglement is quite unsuitable for transmission over distances of more than a few kilometers in an optical fiber [2]. For example, Naik *et al.* [19] observed the quantum bit error rate (QBER) increase to 33% in the experimental implementation of the six-state protocol [20, 21] over only a few meters. For frequency coding, for example, the Besancon group performed a key distribution over a 20-km single-mode optical-fiber spool and they recorded an QBER contribution of approximately 4%. They estimated that 2% could be attributed to the transmission of the central frequency by the Fabry-Perot cavity [22]. On the other hand, the measurement on the polarization states of photons is easier than that on the frequency states, which is important in quantum communication protocols in which the parties resort to at least two bases for checking eavesdropping, such as the Bennett-Brassard-Mermin 1992 (BBM92) QKD protocol [4] and the Hillery-Bužk-Berthiaume (HBB) quantum secret sharing (QSS) protocol [23]. The alternation of basis of polarization equals a Hadamard operation and it can

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be completed with a $\lambda/4$ plate, but it is difficult for frequency.

In this paper, we present a new setup for faithful entanglement distribution against collective noise, without resorting to additional qubits, fast polarization modulators, and nondestructive quantum nondemolition detectors based on nonlinear media. This setup is composed of polarizing beam splitters, half wave plates, polarization independent wavelength division multiplexers, and frequency shifters, which is feasible with current technology. An arbitrary qubit error on the polarization state of each qubit caused by the noisy channel can be rejected with the success probability 100%, which is independent of the noise parameters. This setup has good applications in all one-way quantum communication protocols based on entanglement. We will discuss its applications in the famous BBM92 QKD scheme and the HBB QSS protocol.

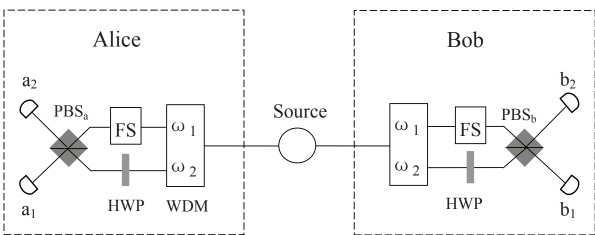

FIG. 1: The principle of our entanglement distribution scheme over a collective noise. Two photons a and b entangled in the frequency degree of freedom are transmitted to Alice and Bob, respectively. Subsequently, Alice and Bob transfer the frequency entanglement to the polarization entanglement for getting a standard entangled state. WDM represent a polarization independent wavelength division multiplexer which will lead the photons to different spatial modes according to their frequencies. FS is a frequency shifter which is used to complete the frequency shift from ω_1 to ω_2. HWP represents a half-wave plate which is used to accomplish the transformation $|H\rangle \leftrightarrow |V\rangle$. PBS represents a polarizing beam splitter.

The setup for implementation of the present faithful entanglement distribution scheme between two parties is shown in Fig.1. Suppose that there is an entangled quantum source placed at a point between the two parties in quantum communication, say Alice and Bob. The source emits an entangled photon pair ab in the state $|\Psi\rangle_{ab}$ each time. Here

$$|\Psi\rangle_{ab} = \frac{1}{\sqrt{2}}|H\rangle_a|H\rangle_b(|\omega_1\rangle|\omega_2\rangle + |\omega_2\rangle|\omega_1\rangle)_{ab}, \qquad (1)$$

where $|H\rangle$ and $|V\rangle$ present the horizontal polarization and the vertical polarization of photons, respectively. $|\omega_1\rangle$ and $|\omega_2\rangle$ are two different frequency modes of photons. The subscripts a and b mean that the two photons transmitted are distributed to Alice and Bob, respectively. There ate two channels for the two parties. One is connecting the quantum entangled source and Alice, and the other is used to link the source with Bob. Suppose that the collective noise in these two channels have the same form but different noise parameters, and the alternation of the polarization caused by the noisy channels can be described as:

$$\begin{aligned} |H\rangle_a &\xrightarrow{noise_a} \alpha|H\rangle + \beta|V\rangle, \\ |H\rangle_b &\xrightarrow{noise_b} \delta|H\rangle + \gamma|V\rangle, \end{aligned} \qquad (2)$$

where

$$|\alpha|^2 + |\beta|^2 = 1, \qquad |\delta|^2 + |\gamma|^2 = 1. \qquad (3)$$

After being transmitted through the optical-fiber channels, each photon of the entangled pair is influenced by the collective noise in its channel. That is, the evolution of the entangled state through the noisy channels can be written as:

$$\begin{aligned} |\Psi\rangle_{ab} &= \frac{1}{\sqrt{2}}(|H\rangle_{\omega_1}|H\rangle_{\omega_2} + |H\rangle_{\omega_2}|H\rangle_{\omega_1})_{ab} \\ &\xrightarrow{noise} \frac{1}{\sqrt{2}}\{(\alpha|H\rangle_{\omega_1} + \beta|V\rangle_{\omega_1})(\delta|H\rangle_{\omega_2} + \gamma|V\rangle_{\omega_2}) \\ &\quad + (\alpha|H\rangle_{\omega_2} + \beta|V\rangle_{\omega_2})(\delta|H\rangle_{\omega_1} + \gamma|V\rangle_{\omega_1})\} \\ &= \frac{1}{\sqrt{2}}\{\alpha\delta(|H\rangle_{\omega_1}|H\rangle_{\omega_2} + |H\rangle_{\omega_2}|H\rangle_{\omega_1}) \\ &\quad + \alpha\gamma(|H\rangle_{\omega_1}|V\rangle_{\omega_2} + |H\rangle_{\omega_2}|V\rangle_{\omega_1}) \\ &\quad + \beta\delta(|V\rangle_{\omega_1}|H\rangle_{\omega_2} + |V\rangle_{\omega_2}|H\rangle_{\omega_1}) \\ &\quad + \beta\gamma(|V\rangle_{\omega_1}|V\rangle_{\omega_2} + |V\rangle_{\omega_2}|V\rangle_{\omega_1})\}. \end{aligned} \qquad (4)$$

After the noisy channel, each of the photons will pass through a a polarization independent wavelength division multiplexer (WDM) to guide photons to different spatial modes, according to their frequencies. That is, the photon with the frequency ω_1 is led to the upper spatial mode and the photon with the frequency ω_2 is led to the lower spatial mode. This device can also be substituted by frequency beam splitter [24] or fiber bragg grating [25, 26]. After the WDMs, Alice and Bob adjusts photons transmitted through the upper spatial modes from the frequency state ω_1 to ω_2, so as to make the quantum states in the two outports of each WDM have the same frequency. Also, they transform the polarization states of the photons transmitted through the lower spatial modes. Obviously, the state of the system after the two PBSs (i.e., PBS_a and PBS_b) becomes

$$\begin{aligned} \xrightarrow{PBSs} \frac{1}{\sqrt{2}}\{&\alpha\delta(|H\rangle|V\rangle + |V\rangle|H\rangle)_{a_1b_1} \\ &+ \alpha\gamma(|H\rangle|H\rangle + |V\rangle|V\rangle)_{a_1b_2} \\ &+ \beta\delta(|V\rangle|V\rangle + |H\rangle|H\rangle)_{a_2b_1} \\ &+ \beta\gamma(|V\rangle|H\rangle + |H\rangle|V\rangle)_{a_2b_2}\}. \end{aligned} \qquad (5)$$

Here a_1 and b_2 are two output ports of PBS_a, and b_1 and b_2 are those of PBS_b. In essence, these two PBSs filter the polarization states of photons with different spatial modes. That is, Alice and Bob can determine the polarization states of their photons according to their spatial modes by postselection. There are four combination modes of the output ports for the two entangled photons ab. That is, they emit from the output ports a_1b_1, a_1b_2, a_2b_1, or a_2b_2 and they are in the frequency-entangled states $|\psi^+\rangle_{a_1b_1} = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle)_{a_1b_1}$, $|\phi^+\rangle_{a_1b_2} = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)_{a_1b_2}$, $|\phi^+\rangle_{a_2b_1} = \frac{1}{\sqrt{2}}(|V\rangle|V\rangle + |H\rangle|H\rangle)_{a_2b_1}$, and $|\psi^+\rangle_{a_2b_2} = \frac{1}{\sqrt{2}}(|V\rangle|H\rangle + |H\rangle|V\rangle)_{a_2b_2}$, respectively. The total success probability for this entanglement distribution scheme is $|\alpha\delta|^2 + |\alpha\gamma|^2 + |\beta\delta|^2 + |\beta\gamma|^2 = 1$ in principle, independent of the noise parameters.

By far, we have discussed our entanglement distribution scheme for a collective in which the noise affects the polarization states of the photons but not entangles with them. In detail, the polarization states of the photons transmitted are some pure states. In general, the polarization states of the photons after the transmission over a noisy channel are mixed ones. Suppose the two photons transmitted are in the mixed state as follows

$$|\Psi\rangle_{ab} \xrightarrow{noise} |\Psi'\rangle_{ab} = \rho_p \cdot \rho_f \tag{6}$$

where $\rho_p = F_1|HH\rangle\langle HH| + F_2|HV\rangle\langle HV| + F_3|VH\rangle\langle VH| + F_4|VV\rangle\langle VV|$ and $\rho_f = \frac{1}{2}(|\omega_1\rangle|\omega_2\rangle + |\omega_2\rangle|\omega_1\rangle)_{ab}(\langle\omega_1|\langle\omega_2| + \langle\omega_2|\langle\omega_1|)$. This state can be viewed as a probabilistic mixture of four pure states: with a probability of F_1, F_2, F_3, or F_4, the photon pair in the state $\frac{1}{\sqrt{2}}|H\rangle_a|H\rangle_b(|\omega_1\rangle|\omega_2\rangle + |\omega_2\rangle|\omega_1\rangle)_{ab}$, $\frac{1}{\sqrt{2}}|H\rangle_a|V\rangle_b(|\omega_1\rangle|\omega_2\rangle + |\omega_2\rangle|\omega_1\rangle)_{ab}$, $\frac{1}{\sqrt{2}}|V\rangle_a|H\rangle_b(|\omega_1\rangle|\omega_2\rangle + |\omega_2\rangle|\omega_1\rangle)_{ab}$ or $\frac{1}{\sqrt{2}}|V\rangle_a|V\rangle_b(|\omega_1\rangle|\omega_2\rangle + |\omega_2\rangle|\omega_1\rangle)_{ab}$, respectively. With the setup shown in Fig. 1, Alice and Bob can get the entangled state $|\psi^+\rangle_{a_1b_1}$, $|\phi^+\rangle_{a_1b_2}$, $|\phi^+\rangle_{a_2b_1}$, and $|\psi^+\rangle_{a_2b_2}$ from the output ports a_1b_1, a_1b_2, a_2b_1, and a_2b_2 respectively, the same as the case with a collective noise. In a word, this entanglement distribution scheme works for an arbitrary noise on the polarization states of photons.

The present scheme can also be used to transmit multi-qubit entangled states. Suppose a multi-qubit system is originally in the state $|\Psi\rangle_N = \frac{1}{\sqrt{2}}|H\rangle_1\cdots|H\rangle_j\cdots|H\rangle_N(|\omega_1\rangle_1\cdots|\omega_1\rangle_j\cdots|\omega_2\rangle_N + |\omega_2\rangle_1\cdots|\omega_2\rangle_j\cdots|\omega_1\rangle_N)$. Here the subscript j ($j = 1, 2, \cdots, N$) represents the j-th qubit which is distributed to the j-th party in quantum communication. Let us assume that the effect of channel noise on each qubit can be described as $|H\rangle_i \xrightarrow{noise} \alpha_j|H\rangle + \beta_j|V\rangle$, where $|\alpha_j|^2 + |\beta_j|^2 = 1$. After the transmission over the collective-noise channels, the N-qubit system is in the state

$$\begin{aligned}\frac{1}{\sqrt{2}}\{&(\alpha_1\cdots\alpha_j\cdots\alpha_N|H\rangle_1\cdots|H\rangle_j\cdots|H\rangle_N + \cdots\\ &+\alpha_1\cdots\beta_j\cdots\alpha_N|H\rangle_1\cdots|V\rangle_j\cdots|H\rangle_N\\ &+\cdots + \beta_1\cdots\beta_j\cdots\beta_N|V\rangle_1\cdots|V\rangle_j\cdots|V\rangle_N)\\ \cdot(&|\omega_1\rangle_1\cdots|\omega_1\rangle_j\cdots|\omega_2\rangle_N + |\omega_2\rangle_1\cdots|\omega_2\rangle_j\cdots|\omega_1\rangle_N)\}.\end{aligned}$$

With the similar setups shown in Fig.1, the entangled photon systems with different polarization states will emit from different spatial modes and the N parties can obtain an N-qubit entangled state $\frac{1}{\sqrt{2}}(|H\rangle_1\cdots|H\rangle_j\cdots|H\rangle_N + |V\rangle_1\cdots|V\rangle_j\cdots|V\rangle_N)$ by converting the frequency-entangled state $\frac{1}{\sqrt{2}}(|\omega_1\rangle_1\cdots|\omega_1\rangle_j\cdots|\omega_2\rangle_N + |\omega_2\rangle_1\cdots|\omega_2\rangle_j\cdots|\omega_1\rangle_N)$ to a polarization-entangled one.

The present entanglement distribution scheme has several important characters. First, an arbitrary error on the polarization state of each qubit caused by the noisy channel can be rejected with the success probability of 100%. The significant hypothesis is that the channel noise is frequency-independent. Secondly, it works without resorting to additional qubits [13], fast polarization modulators [11], and nondestructive quantum nondemolition detectors (QND) based on nonlinear media [17]. Thirdly, its success probability is independent of the noise parameters and the two parties in quantum communication can obtain a perfect entangled state by postselection in a deterministic way, not a probabilistic one [12].

Obviously, this scheme has some good applications in almost all one-way quantum communication protocols based on entanglement for rejecting the errors caused by the collective noise in channel. For example, the BBM92 QKD protocol [4] can be carried out perfectly through a noisy channel by using this entanglement distribution scheme. In this time, the two parties Alice and Bob first distribute a polarization-entangled state $\frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b + |V\rangle_a|V\rangle_b)$, resorting to the frequency-entangled state $\frac{1}{\sqrt{2}}|H\rangle_a|H\rangle_b(|\omega_1\rangle|\omega_2\rangle + |\omega_2\rangle|\omega_1\rangle)_{ab}$, and then they can choose one of the two nonorthogonal bases, $Z = \{|H\rangle, |V\rangle\}$ basis and $X = \{\frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)\}$ basis, to measure the quantum state of the two photons emit from the spatial modes a_1b_1, a_1b_2, a_2b_1, or a_2b_2. That is, Alice and Bob measure their photons from each outport with one of the two bases, the same as the original BBM92 QKD protocol. After measurement, they compare their bases for each entangled photon pair and keep the outcomes when they choose the correlated bases. Also, this entanglement distribution scheme can be used directly in the HBB QSS protocol [23] in which the three parties first share an entangled three-photon GHZ state $\frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b|H\rangle_c + |V\rangle_a|V\rangle_b|V\rangle_c)$ and then each party measures his photon polarization state with one of the two bases Z and $Y = \{\frac{1}{\sqrt{2}}(|H\rangle \pm i|V\rangle)\}$. They can create a private key by the correlation among the three photons with the suitable bases.

The main part of this scheme is the entanglement transfer between the frequency degree of freedom and the polarization degree of freedom. On the one hand, the frequency difference between the two photons should be set to a small value which could be discriminated by the WDM efficiently. On the other hand, the frequency of the two photons must be reset to the same value by FS, which is essential to get standard Bell states. The FS can be implemented by several means with current technique, such as sum-frequency generation process [27] whose internal conversion efficiency is 99% [28].

In summary, we have proposed a faithful entanglement distribution scheme against a collective channel noise, without resorting to additional qubits, fast polarization modulators, and nondestructive quantum nondemolition detectors based on nonlinear media. Its setup is composed of PBSs, HWPs, WDMs, and FSs, which is feasible with current technology. An arbitrary qubit error on the polarization state of each qubit caused by the noisy channel can be rejected with the success probability 100%. Moreover, its success probability is independent of the noise parameters. This setup has good applications in all one-way quantum communication protocols based on entanglement. We have discussed its applications in the famous BBM92 QKD scheme and the HBB QSS protocol.

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