A Simplified Approach to modeling the credit-risk of CMO

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Abstract

The credit crisis of 2007 and 2008 has thrown much focus on the models used to price mortgage backed securities. Many institutions have relied heavily on the credit ratings provided by credit agency. The relationships between management of credit agencies and debt issuers may have resulted in conflict of interest when pricing these securities which has lead to incorrect risk assumptions and value expectations from institutional buyers. Despite the existence of sophisticated models, institutional buyers have relied on these ratings when considering the risks involved with these products. Institutional investors interested in non-agency MBS are particularly vulnerable due to both the credit risks as well as prepayment risks. This paper describes a simple simulation model that model non-agency MBS and CMO. The simulation model builds on existing models for agency MBS. It incorporates credit risks of mortgage buyers using existing models used in capital requirements as specified by the Basel II Accord.

Keywords: Simulation, Mortgage Backed Securities, credit risk

1 Introduction

Mortgage backed securities (MBS) are those investment vehicles that are backed by both residential and commercial mortgages. These securities are created from a pool of mortgages and portions of these are sold. Prior to MBS, the mortgage originating banks kept the loans on the books and hence, were unable to finance more mortgages unless new source of funding was found. With the advent of MBS, the mortgage originating banks could package the mortgage loans as MBS and sell the securities to investors. The proceeds from the sale of MBS can be used to finance more mortgage loans, hence adding liquidity to the mortgage industry. This process lead to various players taking part in the process; the consumer who takes a loan, the originator who markets the loans and provides credit and receives guarantee fees which are generally portions of the interest paid by the consumers, the investment banker

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who packages the loans into MBS and markets it to the institutional buyers and finally the investors or institutional buyers who are seeking new investment vehicle to invest in. Generally, there is a third party who manages these loans and receive part of the interest paid by the consumer for servicing the MBS. The third party's role varies from collecting and paying out the due to the various MBS holders, as well as managing delinquencies and collections.

The earliest form of of MBS were pass-through MBS. Pass-through MBS were created by packaging pools of mortgages into one security and then dividing up this securities into smallers MBS securities. Each portion of the MBS received a proportional fraction of the principal as well as interest received after paying the third party for servicing the laons and guarantee fees paid to the issuer. Agency MBS are those pass-throughs issued by agencies such Ginnie Mae, Fannie Mae or Freddie Mac. Ginnie Mae pass-throughs are backed by the full faith and credit of the US government and are generally viewed as free from default risk. Freddie Mae and Fannie Mae issued pass-throughs are guaranteed by the respective issuer and viewed as having low default risk. However, non-agency MBS, those that are issued by private labels do not have implicit or explicit guarantees from the US government and carry the default risk associated with mortgages. All MBS carry pre-payment risk.

Pre-payment risk is the risk associated with early payment by the consumer. This generally occurs when interest rates are decreasing and the consumer has facility to refinance the loans. When early payment occurs, investors are now in possession of money that they would have to invest at a lower interest rate. There is a cost involved with prepaying a mortgage loan and not all consumers take up this option. On the hand, there is always some proportion of consumers who prepay. They may refinance due to relocation, divorce, etc. Pricing of MBS were especially difficult due to prepayment option available to the consumers.

As the investing product needs of the buyers became more complex, the MBS were then used to create Collateralized Mortgage Obligation (CMO). This provided for greater liquidity to the market and there was higher demand for risk specific CMOs. The CMOs were created as follows; a pool of mortgage loans was created and divided up into smaller CMO similar to pass-through MBS but rules were specified on how the payment from the consumers would be paid out. The higher tranches were given higher priority in payment while the lower tranches had lower priority. An example of priority rules is one where all principal payment is paid into the highest tranche while interest payment is proportionally divided up for each tranche. The principal payment is first paid into the highest tranche until it is retired, i.e. its principal has been paid back. After the highest tranche eis retired, the next highest tranche receives all principal payment and this process continues. The interest payment on the other hand is divided proportionally based on the current principal outstanding and paid into each tranche. Since principal is paid according to a predetermined priority, the lower tranches absorb default risk disproportionately. Further types of MBS have been offered in the market, such as principal only repayments (PO) or interest only (IO) tranches.

According to the Securities Industry and Financial Markets Association, agency MBS and CMO outstandings in 2007 were US$5947 billion. The outstanding has doubled from 1999. This increase has provided significant liquidity to the mortgage market, releasing

funds to mortgage underwriters to issue further credit to consumers. MBS has allowed for the transfer of risk from the issuers books to other institutions. Furthermore, they have provided investment vehicles based on the risk needs of various institutions. The mortgage securitization industry was highly lucrative while the home prices were increasing and interest rates were low as refinancing option was readily available to the consumers. Due to the high stakes involved with these securities, mortgage underwriters became lax about who they offered the mortgages. Consumers with high credit risks were offered mortgage loans and these loans were then securitized and sold off to buyers thereby transferring the risks to other institutions and in the process, a large profit was realized by the originators.

In 2007-2008, increasing interest rates and lower home prices made refinancing option harder for the consumer. This resulted in the credit crisis, where many mortgages defaulted. This, in turn, led to many MBS and CMOs defaulting resulting in institutions reluctant to further invest in these securities. This in turn led to less funds available to consumers to buy homes. This also affected the consumer driven US economy with consumers tightening their belt. Many institutional buyers did not understand the risks involved with pricing the various tranches.

Another important player in the industry has been the credit rating agencies. Once a mortgage pool is securitized, rating agencies rate the different tranches. Due to the difficulty in pricing these complex securities, institutional buyers relied heavily on the ratings to price the securities. Many investors such as pension funds are restricted to investing in triple-A securities [3]. Monolines that insure bonds also heavily relied on the ratings. The ratings were thought to be stable and hence, there was confidence in the ratings and they were heavily utilized. There was little transparency in how the tranches were rated and the rating agencies performed no due diligence on the quality of the mortgage loans backing the securities [3]. The rating agencies profited substantially from MBS and CMOs as they not only rate these securities at the beginning but throughout the life of the security, resulting in both the buyers and the sellers relying heavily on the ratings. There may have also been conflict of interest. Rating agencies were paid to provide credit ratings while MBS issuers chose those agencies that were favorable in their rating. The rating industry is not well regulated. Since prices were not observable [3], institutional buyers had to mathematically price these securities.

With globalization, many governments were concerned with the adverse domino effect of bank failure. The Bank for International Settlement (BIS) was proposed methods to limit bank failures through capital adequacy requirements, especially those banks that cut across geographical boundaries. The Basel Accord proposed by BIS provides a method for calculating the economic capitalization at a bank. It calculates the amount that a bank needs to keep in its reserve for unexpected defaults. The reserve is meant to cover unexpected defaults with a 99.9% confidence. The initial Basel Accord stipulated that 10% of outstandings should be kept in reserve to cover unexpected defaults. Subsequently a one factor model was developed by the Bank for International Settlement that improved on the flat 10% stipulation. This formula is used for all consumer credit products with different coefficient of correlation for different consumer credit products. The coefficient of correlation models the correlation

between consumers' asset defaults. The input into the Basel formula is the account holders' expected probability of default. One of the difficulty in pricing the CMO is the credit risk default models. We propose using the Basel II Accord formula as it was built for consumer credit products and the basis for CMOs are consumer credit mortgages.

The objective of this paper is to describe a simple simulation model that institutional buyers could use to price non-Agency CMOs. Sophisticated simulation models have been constructed to model agency-MBS (see [6],[1]). Li [7] used copulas to model credit risks that can be readily incorporated into a simulation model. We propose a different method to model credit risks of mortgage borrowers and then incorporate this into the simulation model. While this is not a sophisticated model, we hope it will lead to further extension of this model and the reliance on credit agency ratings to price these securities will diminish.

In section 2, we describe the prepayment model, interest rate model and credit risk model used in the simulation. The prepayment model and interest rate models described in section 2 are a summary of already existing models. The credit risk model is the one-factor model from the Basel II Accord that we incorporate into the CMO pricing. In section 3, we describe the simulation process and we illustrate our model using a numerical example in section 4 and finally, conclude in section 5.

2   Mathematical models

Various mathematical models used in the simulation models are described in this section. We summarize the prepayment, interest and credit default models before incorporating all three models into the simulation model in section 3.

2.1   Prepayment Model

Conditional Prepayment Rate or CPR method have been widely used and expanded in the industry. Public Security Association's (PSA) standard method assumes prepayment rate of 0.2% in the first month with the rate growing by 0.2% every month until the 30th month, after which it remains at 6% for the remainder of the 360 months. The rates are annualized, hence it needs to be converted to monthly rates as follows,

$$SMM(t) \;\; = \;\; 1 - (1 - CPR(t))^{\frac{1}{12}} \,,$$

where CPR(t) is the conditional prepayment rate and SMM(t) is the single monthly mortality rate.

This model was then extended by Richard and Roll from Goldman Sachs [10]. CPR is influenced by four factors. If the prevailing interest rates on offer in the market is lower than the interest rates charged to the consumer for the home loan then there would be a higher incentive to refinance. An older mortgage account will likely to refinance than a younger account. This is known as the seasoning factor and follows the same rationale as the CPR curve modeled by the PSA. The third factor is the monthly multiplier which models the monthly seasonality in prepayment across a calendar year. The fourth factor is pool burnout

factor. This models the assumption that an account that has repaid a higher portion of the principal is less likely to prepay. Richard and Roll's CPR is constructed as follows,

$$CPR(t) \ = \ RI(t)AGE(t)MM(t)BM(t)$$

where RI(t), AGE(t), MM(t) and BM(t) are refinancing incentive, seasoning factor, monthly multiplier and pool burnout factor.

Refinancing incentive is modeled as follows

$$RI(t) \ = \ 0.28 + 0.14\left(tan^{-1}(-8.571 + 430(WAC - R(t,T)))\right),$$

where WAC is the weighted average coupon rate and $R(t,T)$, the long term rate. The long term rate is modeled as follows,

$$R(t,T) \ = \ \frac{-lnA(t,T) + B(t,T)r(t)}{T - t}.$$

The factors $A(t,T)$ and $B(t,T)$ are computed using the following,

$$A(t,T) \ = \ \left[\frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + 0.28)(e^{r(T-t)} - 1) + 2\gamma}\right]^{0.0784}$$

$$B(t,T) = \ = \ \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + 0.28)(e^{r(T-t)} - 1) + 2\gamma}$$

with $\gamma = (0.28^2 + 2\sigma^2)^{\frac{1}{2}}$. The short term interest rate $r(t)$ is modeled in the next section. The seasoning factor increases by a factor of $\frac{1}{30}$ monthly from the first month to the 30th month and is constant from month 30 for the life of the account, i.e.,

$$Age(t) \ = \ min(1, \frac{t}{30}).$$

The monthly multiplier, $MM(t) = [0.94, 0.76, 0.74, 0.95, 0.98, 0.92, 0.98, 1.1, 1.18, 1.22, 1.23, 0.98]$ and the pool burnout factor is,

$$BM(t) \ = \ 0.3 + 0.7\frac{B(t)}{B(0)},$$

where $B(t)$ is the balance at beginning of time t and $B(0)$ is the initial mortgage balance.

2.2 Interest Rate Model

Vasicek [11] proposed an interest rate model, which was widely used but could result in negative interest rate. Cox, Ingersoll and Ross [2] improved the model so that it will not general negative interest rate. We follow Huang [6] and use this model due to its simplicity. The CIR model is,

$$\delta r \ = \ a(b - r)\delta t + \sigma r^{0.5}\delta z,$$

where a is the instantaneous drift speed, b the long term equilibrium interest rate, σ the instantaneous standard deviation and δz is a Wiener process.

2.3 Credit Risk Model

We, initially construct an expected probability of default curve. This curve describes the expected probability of default during each month in the cycle of the mortgage loan. The time dependant probability of default. $p_D(t)$ can be constructed using historical default data. While, institutional buyers of CMOs may have access to historical probability of default across time, it is difficult to predict defaults far into the future for new accounts. When consumers are evaluated on their mortgage applications, credit checks are generally conducted and a credit score is provided. The credit score predicts the probability of a consumer defaults within the following two years. We may then forecast the probability of default for consumers by adjusting the historical default curve by the credit rating estimated probability of default. However, this new curve predicts the expected probability of default for consumers but not the variance in the default.

Basel II Accord formula provides a framework to construct the distribution for the probability of default. The formula used in Basel II Accord specifies the economic capital requirement for a portfolio such that the sudden capital requirement for unexpected defaults will be covered with a given confidence. This confidence was set at 99.9% by the Bank of International Settlement. The formula for the economic capitalization is,

$$F(x) = N\left(\sqrt{\frac{1}{1-\rho}}N^{-1}(p_D(x)) + \sqrt{\frac{\rho}{1-\rho}}N^{-1}(99.9\%)\right), \tag{1}$$

where F(x) is the unexpected probability of default for a customer with credit score x, N is the cumulative distribution function for the standard normal distribution, ρ is the correlation of coefficient, $p_D(x)$ is the probability of default for a customer with score x. We reconstruct the derivation below from Perli [9].

Suppose each customer's value, V_i, is drive by a single macroeconomic common factor Y and by an idiosyncratic noise component ϵ_i,

$$V_i = \sqrt{\rho}Y + sqrt1 - \rho\epsilon_i$$

where $Y, \epsilon_i \sim N(0,1)$ and ρ is the common correlation for all assets. Customer i defaults when the value V_i goes below K_i. Probability of default given a realization of y of Y is,

$$p(y) = p(V_i < K_i|Y = y) = p(\sqrt{\rho}Y + \sqrt{1-\rho}\epsilon_i < K_i|Y = y) = p(\sqrt{\rho}y + \sqrt{1-\rho}\epsilon_i < K_i).$$

Hence,

$$p(y) = p(\epsilon_i < \frac{K_i - \sqrt{\rho}y}{\sqrt{1-\rho}}) = N(\frac{K_i - \sqrt{\rho}y}{\sqrt{1-\rho}}).$$

Given a realization y of Y, the assets values are independent and hence, defaults are independent. We may then determine the probability of n defaults from N customers using the binomial formula

$$p(n) = \int_{-\infty}^{+\infty} \binom{n}{N} (p(y))^n (1-p(y))^{N-n} f(y)dy,$$

where $f(y)$ is the distribution of y.

Let X be the fraction of the accounts that default. As N tends to infinity, the fraction of accounts that default will tend to the default probability, i.e. as $N \to \infty$, $X \to p(y)$.

$$p(X < x) = \int_{-\infty}^{+\infty} p(p(y) \leq x | Y = y) f(y) dy = \int_{-\infty}^{+\infty} 1_{p(y) \leq x} f(y) dy$$

$$p(X < x) = \int_{-y^*}^{+\infty} f(y) dy = 1 - N(-y^*) = N(y^*),$$

where $p(-y^*) = x$. Hence,

$$y^* = \frac{1}{\sqrt{\rho}} \left(\sqrt{1 - \rho} N^{-1}(x) - K \right)$$

. Therefore,

$$p(X < x) = F(x) = N\left(\frac{1}{\sqrt{\rho}} \left(\sqrt{1 - \rho} N^{-1}(x) - K \right)\right),$$

where x is the default probability. We may now sample from the above distribution for the probability of default using the inverse transform method [4] [5]. Let $F(x) = U$ where U is the uniform distribution between 0 and 1, i.e.

$$U = N\left(\frac{1}{\sqrt{\rho}} \left(\sqrt{1 - \rho} N^{-1}(x) - K \right)\right).$$

Therefore,

$$x = N\left(\frac{1}{\sqrt{1 - \rho}} \left(\sqrt{\rho} N^{-1}(U) + N^{-1}(p) \right)\right).$$

In the next section, we describe the simulation model that incorporate the prepayment model, the interest rate model and the credit risk models to price a CMO.

3   Simulation Model

In this section, we describe the simulation model incorporating the prepayment, interest rate and credit risk models. The simulation was built using Matlab 2006 and verified using a model built on excel 2003. We simulate the cash flow for each period t.

We assume that the number of initial accounts in the mortgage portfolio is large. Hence, we simulate the cash flows as a continuous number of accounts for simplicity.

There is a balance of $B(t)$ at the beginning of cash flow period t. A portion of this balance will default during the month. Suppose U is a uniform random variable, then x is the portion of the balance defaulting during the month, i.e.

$$x = N\left(\frac{1}{\sqrt{1 - \rho}} \left(\sqrt{\rho} N^{-1}(U) + N^{-1}(p) \right)\right).$$

Let
$$B'(t) = [1 - x] B(t),$$

where $B'(t)$ is the balance at time t after defaults. We then calculate the scheduled mortgage payment $MP(t)$ at time t based on interest rate or coupon rate r.

$$MP(t) = B'(t)\frac{r/12}{1 - (1 + \frac{r}{12})^{-(WAM)}}$$

where WAM is the weighted average maturity of mortgage. The scheduled mortgage payment consists of two types of payment; interest payment $IP(t)$ and principal payment $PP(t)$. Interest payment can be calculated as follows,

$$IP(t) = B'(t)\frac{r}{12}.$$

Hence, the principal payment is

$$PP(t) = MP(t) - IP(t)$$

. Some portion of the remaining balance would be prepayed based on the single month mortality rate determined from the conditional prepayment rate. Therefore,

$$SMM(t) = 1 - (1 - CPR(t))^{\frac{1}{12}},$$
$$PP(t) = (B'(t) - SP(t))(SMM(t)).$$

The cash flow $CF(t)$ at time t may now be determined as the sum of the scheduled principal payment, interest payment and prepayment, i.e.

$$CF(t) = SP(t) + IP(t) + PP(t).$$

We may then determine the balance at the beginning of the following period as follows,

$$B(t + 1) = B(t) - SP(t) - PP(t).$$

The scheduled principal payment, interest payment and the prepayment from each period are distributed to the various tranches based on the priority rules. A simplified view of the simulation is described in the figure 1.

We now illustrate the simulation with a numerical example.

4   Numerical Example

We illustrate the simulation described in section 2 and 3 with a numerical example. Suppose a portfolio manager wants to determine the fair price of a CMO. The initial balance of the CMO is \$1000 with the following rules:
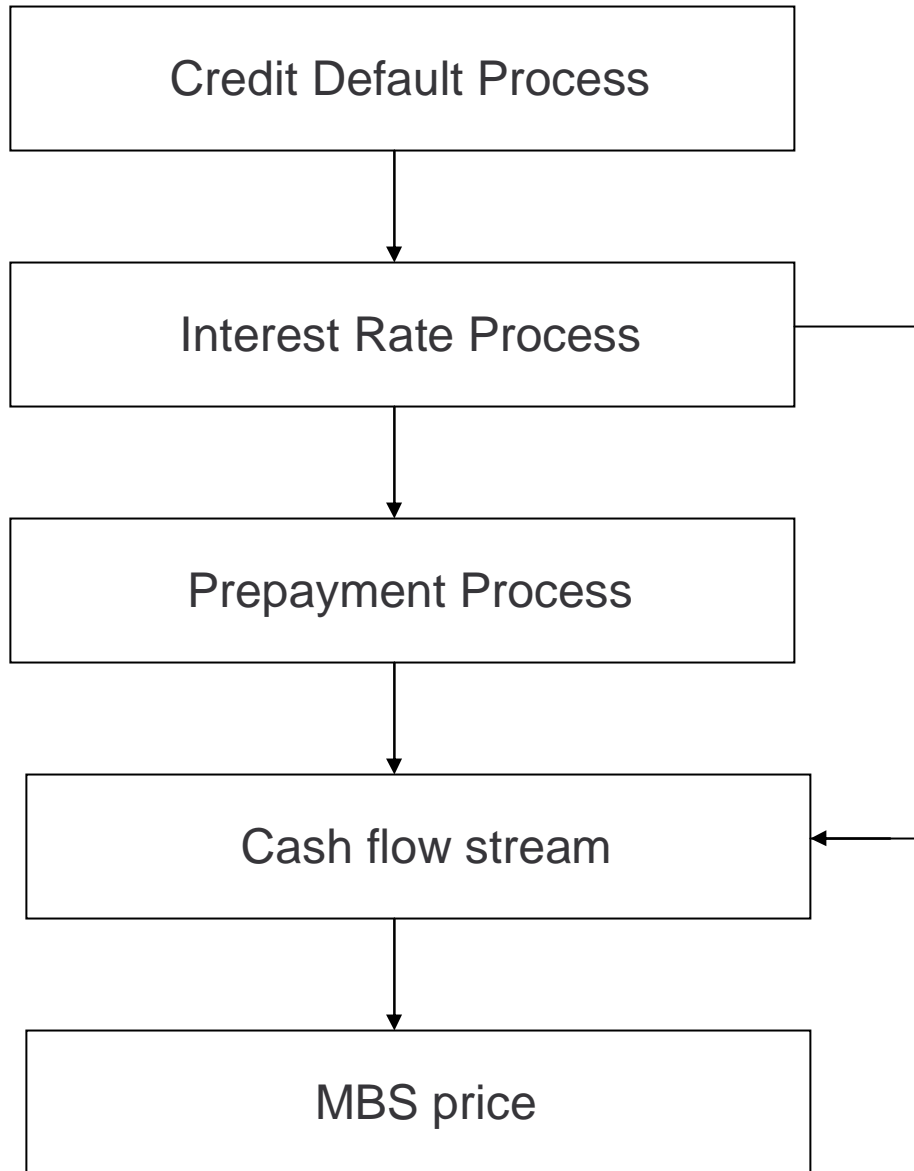
Figure 1: The simulation algorithm.

- The initial balance of the three trances (A, B and C) are $500, $300 and $200,

- All principal payments are paid to tranche A initially,

- Once tranche A is retired, the principal is paid to tranche B,

- Once tranche B is retired, the principal is paid to trance C,

- The interest payments are paid proportionately based on the balance at the beginning of the time period.

The following parameters are also used in the simulation,

Table 1: Parameter values for numerical experiments.

Parameter	ρ	Portfolio default rate	r_f	WAC
Value	0.15	0.05	0.05	0.08

Since the cost of running each iteration was low, we ran 10000 iterations. The results are as follows,

Table 2: Numerical results.

	Tranche A	Tranche B	Tranche C	Total Portfolio
Mean	573.36	403.67	187.64	1164.67
Standard Deviation	2.29	1.32	28.61	25.09

The high variance in tranche C was largely driven by the uncertainty in the performance of the portfolio with respect to default risk. Tranche A and B are protected largely from the fluctuation in the default probability. The distribution of the results for tranche A, B and C are illustrated in figure 2, 3 and 4 respectively.

We, then ran the simulation as specified by Morokoff [8] using the copula function. In order to compare the impact of the two credit risk models, variance reduction technique was used. We used common random numbers for both method in determining prepayment. The following results were seen,

Table 3: Numerical results using the copula function.

	Tranche A	Tranche B	Tranche C	Total Portfolio
Mean	573.75	396.12	168.58	1138.45
Standard Deviation	4.99	23.46	65.89	75.98

The variance in the results using the copula function were greater than those from our method. We tested the difference in mean between the two tests and we rejected the null hypothesis that the means are equal with a confidence of 99%.

5   Conclusion

Using the distribution developed by Vasicek [12], we developed a simulation algorithm to determine the fair price of a Collateralized Mortgage Obligation. Due to the complexity of modeling CMOs and due to the information asymmetry that exists between buyers and originators, institutional buyers rely heavily on credit agency ratings. We propose a new method to simulate the fair price of CMOs. Since prices are unobservable, we were unable to compare the fair price determined by our method with the prices that exist for CMOs. However, we did compare our result to the fair price as determined by the copula method. We used variance reduction technique in order to ensure a pure comparison of two different credit risk simulation models and saw that the prices were significantly different at 99% confidence level. We look forward to testing our methodology on CMO products sold in the market and compare the simulation results to performance data.
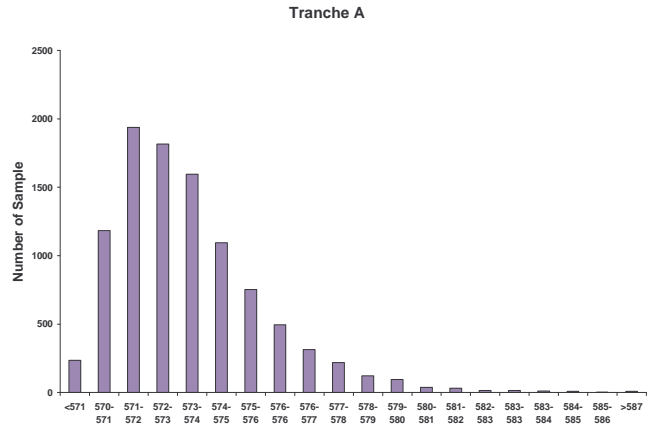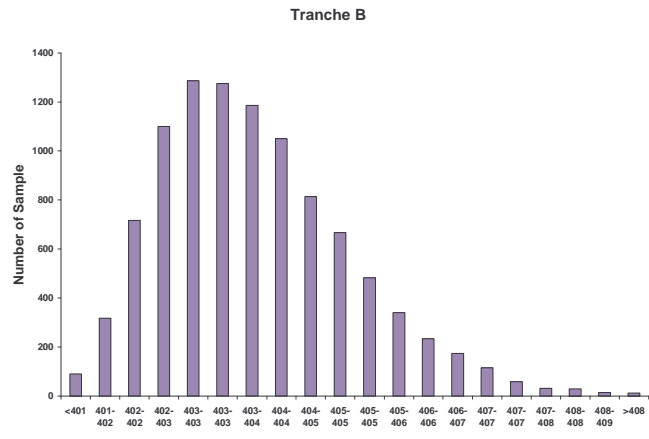
Figure 2: Distribution of the results for Tranche A.

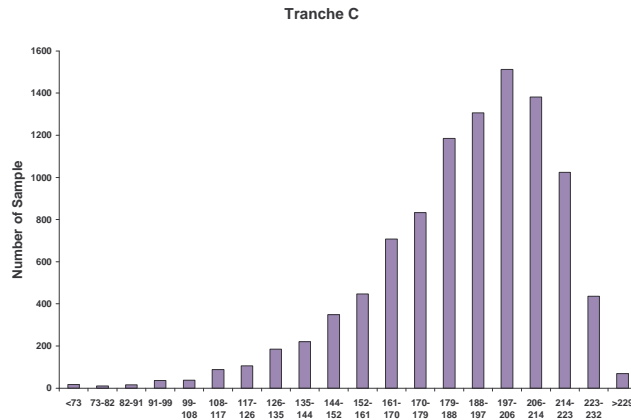

Figure 3: Distribution of the results for Tranche B.

Figure 4: Distribution of the results for Tranche C.

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