Spin Squeezing, Negative Correlations, and Concurrence in the Quantum Kicked Top Model

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We study spin squeezing, negative correlations, and concurrence in the quantum kicked top model. We prove that the spin squeezing and negative correlations are equivalent for spin systems with only symmetric Dicke states populated. We numerically analyze spin squeezing parameters and concurrence in this model, and find that the maximal spin squeezing direction, which refers to the minimal pairwise correlation direction, is strongly influenced by quantum chaos. Entanglement (spin squeezing) sudden death and sudden birth occur alternatively for the periodic and quasi-periodic cases, while only entanglement (spin squeezing) sudden death is found for the chaotic case.

I. INTRODUCTION

Quantum entanglement is one of the central concepts in quantum information theory, and can be viewed as a physical resource for quantum information [1–3]. Characterizing entanglement is still an open question for many-body system, however, for two-qubit case, the problem is solved by using concurrence which is defined by Wootters [4, 5]. In Refs. [2, 3, 6–15], it was found that spin squeezing has a close relation with entanglement, and spin squeezing parameters can be used as entanglement witnesses. The definitions of spin squeezing parameters are not unique, and the most popular parameters are ξ^2_{KU}, given by Kitagawa and Ueda [6], and ξ^2_W, given by Wineland *et al* [11]. The parameter ξ^2_{KU} was shown to be related to pairwise correlations and concurrence [6–10], while the parameter ξ^2_W is related to many-body entanglement [10, 11]. There is another spin squeezing parameter proposed by Sørensen *et al* [13] as a measure of many-body entanglement which can be considered as a generalization of parameter ξ^2_W [10]. Recently, a new spin squeezing parameter ξ^2_T was proposed, and it was found that if $\xi^2_T < 1$, the system is entangled [7, 12].

Reference [9] found that spin squeezing parameter ξ^2_{KU} has close relation with pairwise correlation. Here, we consider the following pairwise correlation

$$\mathcal{C}_{i\vec{n},j\vec{n}} = \langle \sigma_{i\vec{n}}\sigma_{j\vec{n}} \rangle - \langle \sigma_{i\vec{n}} \rangle \langle \sigma_{j\vec{n}} \rangle, \qquad (1)$$

in the $\vec{n}$ direction, where $\sigma_{i\vec{n}} = \vec{\sigma}_i \cdot \vec{n}$, with $\vec{\sigma}_i$ being the Pauli matrix of the i-th qubit and $\vec{n}$ being the normalized direction vector. The above definition can be rewritten as

$$\mathcal{C}_{i\vec{n},j\vec{n}} = \vec{n}\mathbb{C}\vec{n}^T, \qquad (2)$$

where $\mathbb{C}$ is the pairwise correlation matrix whose elements are given by

$$\mathbb{C}_{i\alpha,j\beta} = \langle \sigma_{i\alpha}\sigma_{j\beta} \rangle - \langle \sigma_{i\alpha} \rangle \langle \sigma_{j\beta} \rangle, \quad \alpha, \beta = x, y, z. \qquad (3)$$

In Ref. [9], it was found that, if the system has exchange symmetry, the pairwise correlation and the spin squeezing parameter ξ^2_{KU} have the following relation,

$$\mathcal{C}_{\vec{n}_\perp,\vec{n}_\perp} = \frac{\xi^2_{KU} - 1}{N - 1}, \qquad (4)$$

where indices i, j were omitted due to exchange symmetry, $\vec{n}_\perp$ is the direction which is perpendicular to the mean spin direction $\vec{n}$, and N is the number of qubits. In this paper, we find that the parameter ξ^2_T is related to the minimal pairwise correlation for systems with only symmetric Dicke states populated [17, 18]. The relation is given by

$$\mathcal{C}_{\min} = \min_{\vec{n}} \mathcal{C}_{\vec{n},\vec{n}} = \frac{\xi^2_T - 1}{N - 1}, \qquad (5)$$

where the minimization is over arbitrary direction $\vec{n}$. Thus, for systems with only symmetric Dicke states populated, negative correlations ($\mathcal{C}_{\min} < 0$) are equivalent to the spin squeezing ($\xi^2_T < 1$).

Many works have been devoted to understand spin squeezing, quantum entanglement, and quantum chaos [13, 16, 19–29]. Here, quantum chaos [30–32] mainly focuses on the researches of quantum characteristics of a quantum system whose classical correspondence exhibits chaos. In Refs. [28, 29], the authors used the parameter ξ^2_{KU} as an efficient signature of quantum chaos. They also studied the relations between ξ^2_{KU} and concurrence, and found that ξ^2_{KU} does not match concurrence well. In this paper, we use the new spin squeezing parameter ξ^2_T to characterize quantum chaos, and study the dynamical evolutions of concurrence and spin squeezing parameters ξ^2_{KU} and ξ^2_T. The quantum kicked top (QKT) model [24, 25, 28, 33–35] is a typical model

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that exhibits quantum chaos, and its chaotic behaviors were demonstrated experimentally by using linear entropy [36]. In this paper we find that the concurrence decreases abruptly and non-smoothly to zero in a finite time in the QKT model, and this phenomenon is called entanglement sudden death (ESD) [37], which has been widely studied both theoretically [38–50] and experimentally [42, 46] in decoherence dynamics. Similar to the definition of ESD, there is entanglement sudden birth (ESB) [51–55], which is a sudden feature in the temporal creation of entanglement in a dissipative evolution of interacting qubits. In Ref. [7], the authors found that spin squeezing sudden death (SSSD) may occur due to decoherence. In this paper, we want to study these sudden features of entanglement and spin squeezing in the QKT model.

This paper is organized as follows. In Sec. II, we first introduce the definitions of spin squeezing parameters, negative correlations, and concurrence, then give the relationship between the spin squeezing parameter ξ_T^2 and the minimal pairwise correlation $\mathcal{C}_{\min}$. In Sec. III, we introduce the QKT model and its classical correspondence. In Sec. IV, we analyze the quantum chaos of the QKT model by means of the dynamics of spin squeezing and entanglement. In Sec. V, we study the influences of chaos on ESD, ESB, SSSD and spin squeezing sudden birth (SSSB). The conclusions are given in Sec. VI.

II. SPIN SQUEEZING, NEGATIVE CORRELATIONS, AND CONCURRENCE

A. Definitions of Spin squeezing parameters

The spin squeezing parameters are useful tools to detect the quantum entanglement [6, 11–15]. Here, we consider an ensemble of N spin-1/2 particles described by the collective angular momentum operators

$$J_\alpha = \frac{1}{2}\sum_{k=1}^N \sigma_{k\alpha}, \quad \alpha = x, y, z. \quad (6)$$

The Dicke states can be expressed as $J_+^n |1\rangle^{\otimes N}$, where $|1\rangle$ is the spin down state and $J_\pm = J_x \pm iJ_y$. We mainly study the following two types of spin squeezing parameters ξ_{KU}^2 and ξ_T^2. The parameter ξ_{KU}^2 is defined as [6]

$$\xi_{KU}^2 = \frac{4\left(\triangle J_{\vec{n}_\perp}\right)_{\min}^2}{N}, \quad (7)$$

where $\vec{n}_\perp$ denotes the direction which is perpendicular to the mean spin direction $\vec{n} = \langle \vec{J}\rangle/|\vec{J}|$, and $\left(\triangle J_{\vec{n}_\perp}\right)_{\min}^2$ is the minimal value of the variance $\left(\triangle J\right)^2$ in the $\vec{n}_\perp$-direction. The spin squeezing parameter ξ_{KU}^2 can be

written in an explicit form as [16]

$$\xi_{KU}^2 = \frac{2}{N}\left[\langle J_{\vec{n}_1}^2 + J_{\vec{n}_2}^2\rangle \right.$$
$$\left. -\sqrt{(\langle J_{\vec{n}_1}^2 - J_{\vec{n}_2}^2\rangle)^2 + \langle [J_{\vec{n}_1}, J_{\vec{n}_2}]_+\rangle^2}\right], \quad (8)$$

where $\vec{n}_1$ and $\vec{n}_2$ are two orthogonal directions which are perpendicular to the mean spin direction, and $[A, B]_+ = AB + BA$.

The spin squeezing parameter ξ_T^2 is defined as [7, 12]

$$\xi_T^2 = \frac{\lambda_{\min}}{\langle \vec{J}^2\rangle - \frac{N}{2}}, \quad (9)$$

where $\lambda_{\min}$ is the minimal eigenvalue of the real symmetric matrix

$$\Gamma = (N - 1)\gamma + \mathbf{C}, \quad (10)$$

where $\mathbf{C}$ is the correlation matrix of which the matrix elements are

$$\mathbf{C}_{\alpha\beta} = \frac{1}{2}\langle J_\alpha J_\beta + J_\beta J_\alpha\rangle, \quad \alpha, \beta = x, y, z, \quad (11)$$

The covariance matrix γ is given as

$$\gamma_{\alpha\beta} = \mathbf{C}_{\alpha\beta} - \langle J_\alpha\rangle \langle J_\beta\rangle. \quad (12)$$

Below, for the sake of simplicity, we assume the mean spin direction $\vec{n}$ is along the z-axis. And then we can write the matrix Γ in an explicit form as

$$\Gamma = \begin{pmatrix} \Gamma_{\vec{n}_\perp} & \vec{B}^T \\ \vec{B} & \Gamma_{\vec{n}} \end{pmatrix}, \quad (13)$$

where $\Gamma_{\vec{n}_\perp}$ is a 2×2 matrix

$$\Gamma_{\vec{n}_\perp} = \begin{pmatrix} N\langle J_x^2\rangle & \frac{N}{2}\langle [J_x, J_y]_+\rangle \\ \frac{N}{2}\langle [J_x, J_y]_+\rangle & N\langle J_y^2\rangle \end{pmatrix}, \quad (14)$$

and

$$\Gamma_{\vec{n}} = N\left(\triangle J_z\right)^2 + \langle J_z\rangle^2, \quad (15)$$

$$\vec{B} = \left(\frac{N}{2}\langle [J_z, J_x]_+\rangle, \frac{N}{2}\langle [J_z, J_y]_+\rangle\right). \quad (16)$$

Note that $\vec{B}$ is a 1×2 vector. According to Eq. (8), the parameter ξ_{KU}^2 is equal to the minimal eigenvalue of the matrix

$$\tilde{\Gamma} = \frac{4}{N^2}\Gamma_{\vec{n}_\perp} = \frac{4}{N^2}\begin{pmatrix} N\langle J_x^2\rangle & \frac{N}{2}\langle [J_x, J_y]_+\rangle \\ \frac{N}{2}\langle [J_x, J_y]_+\rangle & N\langle J_y^2\rangle \end{pmatrix}. \quad (17)$$

In analogy to the relation between ξ_{KU}^2 and $\Gamma_{\vec{n}_\perp}$, we define a spin squeezing parameter

$$\xi_{\vec{n}}^2 = \frac{4}{N^2}\Gamma_{\vec{n}} = \frac{4}{N^2}\left[N\left(\triangle J_z\right)^2 + \langle J_z\rangle^2\right], \quad (18)$$

which characterizes the spin squeezing along the mean spin direction. Here we consider the case of $\langle \vec{J}^2\rangle = \frac{N}{2}\left(\frac{N}{2} + 1\right)$. According to the Rayleigh-Ritz theorem [56], the minimal eigenvalue of Γ is less or equal to that of $\Gamma_{\vec{n}_\perp}$, thus we have $\xi_T^2 \leq \xi_{KU}^2$.

B. Relations between spin squeezing and the minimal pairwise correlation

Now, we discuss the relation between the spin squeezing parameter ξ_T^2 and the minimal pairwise correlation $\mathcal{C}_{\min}$. Here, we consider the system with only symmetric Dicke states populated, which has exchange symmetry and $\langle \vec{J}^2 \rangle = \frac{N}{2}\left(\frac{N}{2}+1\right)$. In this case, we have the following relations,

$$\langle J_\alpha^2 \rangle = \frac{N}{4} + \frac{N(N-1)}{4}\langle \sigma_{1\alpha}\sigma_{2\alpha}\rangle, \qquad (19)$$

$$\langle J_-^2 \rangle = N(N-1)\langle \sigma_{1-}\sigma_{2-}\rangle, \qquad (20)$$

$$\langle [J_\alpha, J_\beta]_+ \rangle = \frac{N(N-1)}{4}\langle [\sigma_{1\alpha}, \sigma_{2\beta}]_+ \rangle, \ (\alpha \neq \beta). \qquad (21)$$

Thus, with the above relations, the matrix Γ, as shown in Eq. (10), can be rewritten as

$$\begin{aligned}\Gamma_{\alpha,\beta} &= \frac{N}{2}\langle [J_\alpha, J_\beta]_+ \rangle - (N-1)\langle J_\alpha\rangle\langle J_\beta\rangle \\ &= \frac{N^2(N-1)}{4}\left(\langle \sigma_\alpha\sigma_\beta\rangle - \langle\sigma_\alpha\rangle\langle\sigma_\beta\rangle\right) \\ &= \frac{N^2(N-1)}{4}\mathbb{C}_{\alpha,\beta}, \quad (\alpha \neq \beta); \end{aligned} \qquad (22)$$

$$\begin{aligned}\Gamma_{\alpha,\alpha} &= N\langle J_a^2\rangle - (N-1)\langle J_\alpha\rangle^2 \\ &= \frac{N^2(N-1)}{4}\left(\langle\sigma_{1\alpha}\sigma_{2\alpha}\rangle - \langle\sigma_\alpha\rangle^2\right) - \frac{N^2}{4} \\ &= \frac{N^2(N-1)}{4}\mathbb{C}_{\alpha,\alpha} - \frac{N^2}{4}. \end{aligned} \qquad (23)$$

The relation between the matrix Γ and the pairwise correlation matrix $\mathbb{C}$ can be written as

$$\mathbb{C} = \frac{4\Gamma}{N^2(N-1)} - \frac{\mathbb{I}}{(N-1)}. \qquad (24)$$

where $\mathbb{I}$ is a 3×3 identity matrix. Thus, the matrix $\mathbb{C}$ and Γ can be diagonalized with the same unitary transformation. So

$$\mathcal{C}_{\min} = \frac{4\lambda_{\min}}{N^2(N-1)} - \frac{1}{N-1}, \qquad (25)$$

where $\mathcal{C}_{\min}$ and $\lambda_{\min}$ are the minimal eigenvalues of the matrix $\mathbb{C}$ and Γ, respectively. Here the spin squeezing parameter ξ_T^2 can be written as $\xi_T^2 = 4\lambda_{\min}/N^2$ since $\langle\vec{J}^2\rangle = \frac{N}{2}\left(\frac{N}{2}+1\right)$. The relation between the minimal pairwise correlation and the parameter ξ_T^2 can be written as

$$\mathcal{C}_{\min} = \min_{\vec{n}}\mathcal{C}_{\vec{n},\vec{n}} = \frac{\xi_T^2 - 1}{(N-1)}. \qquad (26)$$

Therefore, negative correlation ($\mathcal{C}_{\min} < 0$) is equivalent to spin squeezing ($\xi_T^2 < 1$). From Refs. [7, 12], $\xi_T^2 <$

1 is also a criterion of entanglement, so it reveals that there are close relations among negative correlation, spin squeezing, and entanglement. If the minimal pairwise correlation is in the plane which is perpendicular to the mean spin direction, the above relation will degenerate to Eq. (4).

C. Relations between spin squeezing and concurrence

Here, we briefly introduce the relations between spin squeezing and concurrence. The entanglement between a pair of spin-1/2 particles is quantified by the concurrence C [4, 5], which is defined as

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \qquad (27)$$

where the quantities λ_i are the square roots of the eigenvalues of the matrix $\rho_{12}(\sigma_{1y}\otimes\sigma_{2y})\rho_{12}^*(\sigma_{1y}\otimes\sigma_{2y})$ in descending order, ρ_{12}^* is the complex conjugate of ρ_{12}. For spin states with parity, it was found that [16], when $\xi_{KU}^2 < 1$, the relation between ξ_{KU}^2 and C is

$$\xi_{KU}^2 = 1 - (N-1)C. \qquad (28)$$

In Refs. [8], the authors found that for systems with parity and exchange symmetry, when concurrence $C > 0$, the spin squeezing parameter $\xi_T^2 < 1$, and vice versa.

III. QUANTUM KICKED TOP

Now we introduce the QKT model. Consider an ensemble of N spin-1/2 particles, the QKT Hamiltonian reads [33–35]

$$H = \frac{\kappa}{2j\tau}J_z^2 + pJ_y\sum_{n=-\infty}^{\infty}\delta(t-n\tau), \qquad (29)$$

where $j = N/2$, and J_α ($\alpha = x, y, z$) are angular momentum operators that obey the commutation relations $[J_\alpha, J_\beta] = i\hbar\varepsilon_{\alpha\beta\gamma}J_\gamma$, where $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Civita symbol. The first term of Eq. (29) describes a nonlinear precession around the z-axis with strength κ, and the second term describes the kicks around the y-axis with strength p, separated by a period τ. In the below, we set $p = \pi/2$ and $\tau = 1$, and the magnitude $\langle\vec{J}^2\rangle = j(j+1)$ is a constant of the motion.

Now, we study the classical corresponding of the QKT. The evolutions of the expectation values of the angular momentum operators are

$$\langle J_a\rangle_{n+1} = \langle U^\dagger J_\alpha U\rangle_n, \qquad (30)$$

where U is the Floquet operator describing the unitary evolution for each kick,

$$U = \exp\left(-\frac{i\kappa}{2j}J_z^2\right)\exp\left(-ipJ_y\right). \qquad (31)$$

To study the quantum chaos of the QKT, we should first analyze its classical corresponding, which is obtained in the classical limit, i.e. $j \to \infty$. For convenience, we use the following three quantities

$$X = \frac{\langle J_x \rangle}{j}, \quad Y = \frac{\langle J_y \rangle}{j}, \quad Z = \frac{\langle J_z \rangle}{j}, \qquad (32)$$

when $j \to \infty$, these three variables become

$$(X, Y, Z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\phi), \qquad (33)$$

where θ is the polar angle and ϕ is the azimuthal angle. Therefore, (X, Y, Z) represents a point on the Bloch sphere with radius $r = 1$. In the classical limit, we can factorize the products of the mean values of the angular momentum operators as

$$\frac{\langle J_x J_y \rangle}{j^2} = XY. \qquad (34)$$

By substituting Eqs. (31), (32), and (34) into Eq. (30), we can derive the classical equations of motions as [34]

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{n+1} = \begin{bmatrix} Z\cos(\kappa X) + Y\sin(\kappa X) \\ -Z\sin(\kappa X) + Y\cos(\kappa X) \\ -X \end{bmatrix}_n. \qquad (35)$$

Therefore, in the classical limit, the dynamics of the QKT is governed by the above equation, and by using Eq. (33), we plot the stroboscopic dynamics of the classical variables θ and ϕ in Fig. 1. Each point represents a state of (X, Y, Z) in the phase space. In this plot, we choose $\kappa = 3$, and thus there are periodic, quasi-periodic, and chaotic regions in the phase space.

The quantum evolution of the QKT is studied when j is finite, and the chaos is indicated by the stroboscopic plot shown in Fig. 1. Thus, to make connection between quantum and classical evolutions, the initial state should be chosen as coherent spin state (CSS) [33–35], which can be viewed as a most classical state

$$|\theta, \phi\rangle = R(\theta, \phi)|j, j\rangle = R(\theta, \phi)|1\rangle^{\otimes N}, \qquad (36)$$

where $|j, j\rangle$ is the eigenstate of J_z with the eigenvalue j, and the rotation operator is defined as

$$R(\theta, \phi) = \exp\{i\theta[J_x\sin\phi - J_y\cos\phi]\}, \qquad (37)$$

where $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. The expectation values of the angular momentums on this state are given by

$$\langle \mathbf{J} \rangle = (\langle J_x \rangle, \langle J_y \rangle, \langle J_z \rangle) = j(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \qquad (38)$$

which is the same as Eq. (33). This is the reason that we choose CSS to be the initial state. Since CSS can be regarded as a classical state, and it is a product state, thus there is no correlations between qubits. Substituting Eq. (6) into Eq. (37), we can see that the operator R can be written as a direct product of N operators acting
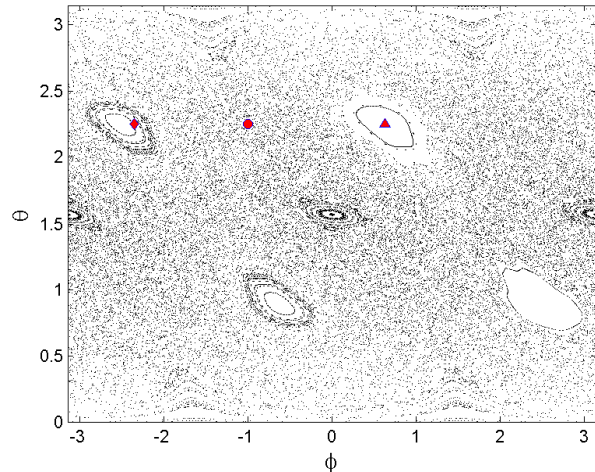

FIG. 1: (Color online) Stroboscopic phase-space map of the classical kicked top with $\kappa = 3$ and $p = \pi/2$. we choose 200 random initial states and each evolves 200 kicks. A triangle at $(\theta, \phi) = (2.25, 0.63)$ is at the fixed point, namely in the periodic region. A diamond at $(\theta, \phi) = (2.25, -2.35)$ is in the quasi-periodic region. A circle at $(\theta, \phi) = (2.25, -1)$ is in the chaotic region.

independently on each qubit. And note that $|j, j\rangle$ is a product state, and thus the CSS is also a product state. In the classical limit, the initial classical state is indeed a CSS, and during the evolution, the state is still classical. However, in the quantum case, the factorization (34) is not valid, since quantum correlations are created during the evolution, and thus the state is no longer a CSS. Such quantum correlations are characterized below by spin squeezing and concurrence.

IV. DYNAMICS OF SPIN SQUEEZING AND CONCURRENCE IN THE QKT MODEL

At first, we use spin squeezing to characterize quantum correlations and quantum chaos. According to the discussions of Sec. II, the spin squeezing parameter ξ_T^2 characterizes the minimal pairwise correlation, the parameter ξ_{KU}^2 characterizes the minimal pairwise correlation in the plane which is perpendicular to the mean spin direction $\vec{n}$, and the parameter $\xi_{\vec{n}}^2$ is the pairwise correlation along $\vec{n}$-direction. The numerical results of quantum evolutions of spin squeezing parameters ξ_T^2, ξ_{KU}^2, and $\xi_{\vec{n}}^2$ are given in Fig. 2, and we find that the maximal spin squeezing directions are strongly influenced by quantum chaos.

Here, we analyze these three spin squeezing parameters in the periodic region, as shown in Fig. 2(a). It can be seen that the spin squeezing parameter ξ_{KU}^2 is much smaller than $\xi_{\vec{n}}^2$, but there are very slight differences between ξ_{KU}^2 and ξ_T^2, thus the maximal spin squeezing, which refer to the minimal pairwise correlation, is around
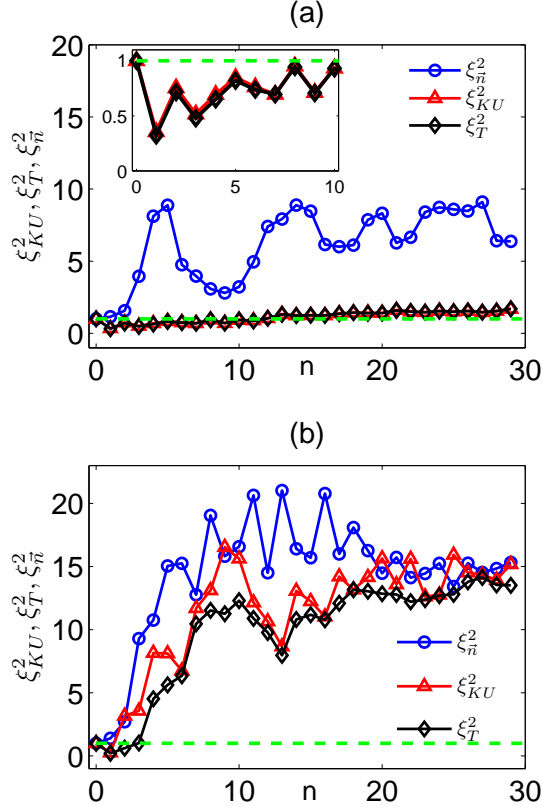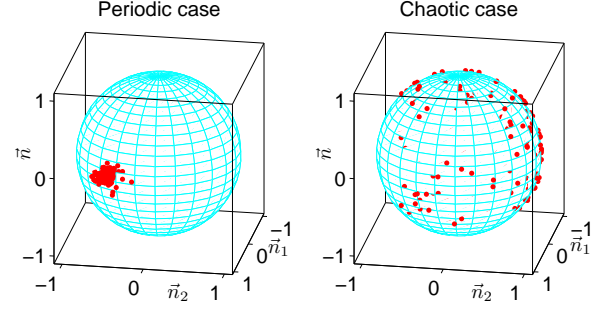
FIG. 3: (Color online) Directions of the maximal spin squeezing. The dots on the uint sphere represent the end points of the directions of maximal spin squeezing. And the axis label $\vec{n}$ denotes the mean spin direction, $\vec{n}_1$ and $\vec{n}_2$ are two orthogonal directions which are perpendicular to $\vec{n}$. In the left plot, the initial state is in the periodic region, with $(\theta, \phi) = (2.25, 0.63)$. And in the right plot, the initial state is in the chaotic region, with $(\theta, \phi) = (2.25, -1)$.

spectively. They are calculated as

$$
\theta_0 = \arccos \left(\langle J_z \rangle / |\vec{J}| \right),
$$

$$
\phi_0 = \begin{cases} \arccos \left(\frac{\langle J_x \rangle}{|\vec{J}| \sin \theta_0} \right) & \text{if } \langle J_y \rangle > 0, \\ 2\pi - \arccos \left(\frac{\langle J_x \rangle}{|\vec{J}| \sin \theta_0} \right) & \text{if } \langle J_y \rangle \le 0. \end{cases} \tag{40}
$$

where $|\vec{J}| = \sqrt{\langle J_x \rangle^2 + \langle J_y \rangle^2 + \langle J_z \rangle^2}$. The other two axes in Fig. 3 are chosen as

$$
\vec{n}_1 = \left(-\cos \theta_0 \cos \phi_0, -\cos \theta_0 \sin \phi_0, \sin \theta_0 \right),
$$
$$
\vec{n}_2 = \left(-\sin \phi_0, \cos \phi_0, 0 \right). \tag{41}
$$

From Fig. 3, it can be easily found that the directions of the maximal spin squeezing are around the $\vec{n}_\perp$-direction in the periodic case. But in the chaotic case, the directions are not concentrated in a certain direction. Thus the directions of maximal spin squeezing are strongly influenced by quantum chaos.

Then, we use concurrence to character the quantum chaos, and give comparisons among the spin squeezing parameters ξ_{KU}^2, ξ_T^2, and concurrence. From Ref. [16], when $\xi_{KU}^2 < 1$, the relation between ξ_{KU}^2 and C is shown as Eq. (28) for states with a fixed parity, and when $\xi_{KU}^2 \ge 1$, the relation does not hold. In the QKT model, there is not a simple relation between spin squeezing and concurrence as the states here are more general than those with a parity. In order to make a direct comparison between spin squeezing parameter and concurrence, here, we introduce a new quantity to describe the concurrence, the form is

$$
\xi_C^2 = 1 - (N - 1) C_1, \tag{42}
$$

where

$$
C_1 = \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4,
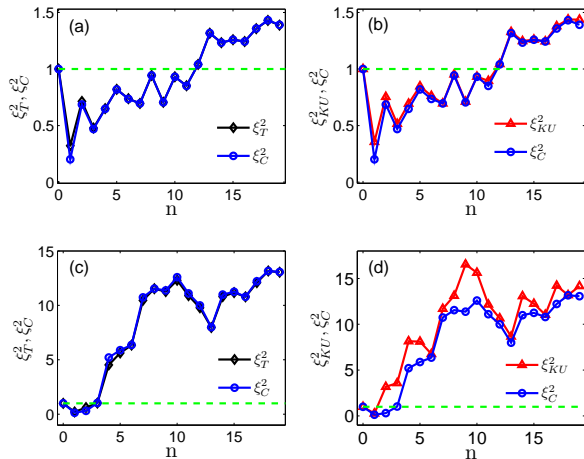$$

FIG. 2: (Color online) Dynamical evolutions of the parameters ξ_T^2, ξ_{KU}^2, and $\xi_{\vec{n}}^2$ with different initial states. We choose $N = 50$. In the periodic region (a), $(\theta, \phi) = (2.25, 0.63)$, ξ_{KU}^2 and ξ_T^2 are almost the same, while $\xi_{\vec{n}}^2$ is very large. This implies that, the maximal spin squeezing is around the $\vec{n}_\perp$-direction. However, in the chaotic region (b), $(\theta, \phi) = (2.25, -1)$, the differences between ξ_{KU}^2 and ξ_T^2 are obvious, that means, the maximal spin squeezing is not restricted to the $\vec{n}_\perp$-direction. The dashed line corresponds to 1.

the $\vec{n}_\perp$-direction.

In the chaotic case, as shown in Fig. 2 (b), we can find that the parameter ξ_T^2 is much smaller than both ξ_{KU}^2 and $\xi_{\vec{n}}^2$ at some time, that means the maximal spin squeezing is along neither the $\vec{n}_\perp$-direction nor the $\vec{n}$-direction. Therefore, the directions of maximal spin squeezing are around the $\vec{n}_\perp$-direction in the periodic case, while they are not restricted to the $\vec{n}_\perp$-direction in the chaotic case.

The directions of the maximal spin squeezing are calculated in Fig. 3. The $\vec{n}$-axis in Fig. 3 is the mean spin direction [28]. It can be written in the spherical coordinate as

$$
\vec{n} = \left(\sin \theta_0 \cos \phi_0, \sin \theta_0 \sin \phi_0, \cos \theta_0 \right), \tag{39}
$$

where θ_0 and ϕ_0 are the polar and azimuthal angles, re-

FIG. 4: (Color online) Evolutions of ξ_T^2, ξ_{KU}^2 and ξ_C^2 with different initial states, and we choose $N = 50$. In (a) and (b), the initial state is in the periodic region, at $(\theta, \phi) = (2.25, 0.63)$. The values of ξ_T^2 and ξ_C^2 are almost the same except for the first two kicks, but there are small differences between ξ_{KU}^2 and ξ_C^2. In (c) and (d), we choose $(\theta, \phi) = (2.25, -1)$, which is centered in the chaotic region. There are small differences between ξ_T^2 and ξ_C^2, but the differences between ξ_{KU}^2 and ξ_C^2 are large. The dashed line corresponds to 1.

which can be got from the Eq. (27) without the max function, and when $\xi_C^2 < 1$, $C_1 > 0$, namely the state is pairwise entangled. The numerical results of the dynamical evolutions of ξ_T^2, ξ_{KU}^2, and ξ_C^2 in the QKT model are given in Fig. 4.

In the periodic case, as shown in Fig. 4(a) and (b), we can see that, at the first two kicks, there are small differences between ξ_T^2 (ξ_{KU}^2) and ξ_C^2. After the third kick, the parameters ξ_T^2 and ξ_C^2 are nearly coincide, while there are also small differences between ξ_{KU}^2 and ξ_C^2. We also note that when $\xi_C^2 < 1$, both ξ_T^2 and ξ_{KU}^2 are smaller than 1, so both the two spin squeezing parameters can well describe the pairwise entanglement.

In the chaotic case, as shown in Fig. 4(c) and (d), we can find that there are very small differences between ξ_T^2 and ξ_C^2, but the differences between ξ_{KU}^2 and ξ_C^2 are large. We also observe that when $\xi_C^2 < 1$, $\xi_T^2 < 1$, and vice versa. It means the spin squeezing parameter ξ_T^2 may indicate the pairwise entanglement. At the second and third kicks, the parameter $\xi_C^2 < 1$, $\xi_{KU}^2 > 1$, namely, the spin squeezing parameter ξ_{KU}^2 is not a good quantity to describe the pairwise entanglement in this chaotic case.

V. SUDDEN DEATH OF ENTANGLEMENT AND SPIN SQUEEZING

Recently, entanglement sudden death and sudden birth have received a lot of attentions. In this paper, we find
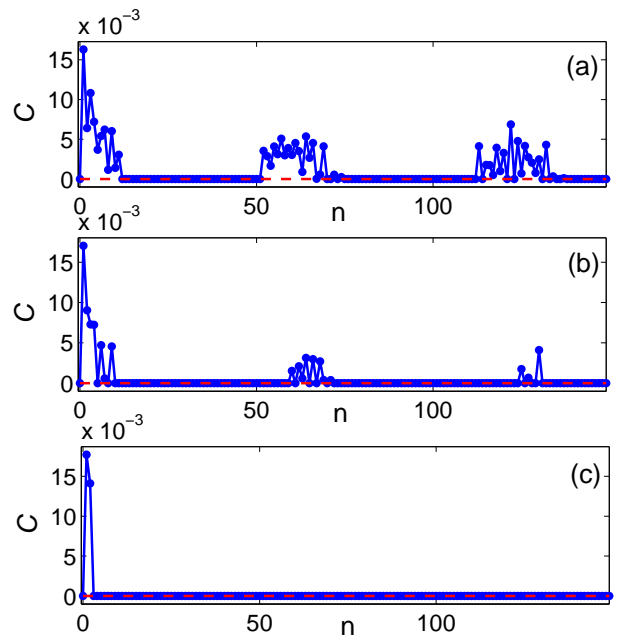

FIG. 5: (Color online) Dynamical evolutions of concurrence in the QKT model for (a) the periodic case $(\theta, \phi) = (2.25, 0.63)$, (b) the quasi-periodic case $(\theta, \phi) = (2.25, -2.35)$, and (c) the chaotic case $(\theta, \phi) = (2.25, -1)$. Here, we choose $N = 50$.

that both ESD and ESB occur in the QKT model. Here we study the influences of quantum chaos on ESD and ESB. We choose the initial states are in the periodic, quasi-periodic, and chaotic regions, and the dynamics of concurrence are shown in Fig. 5. Since the initial state is a CSS, there is no entanglement at first, after the first kick, the pairwise entanglement $(C > 0)$ is produced. As time evolves, the concurrence decreases to zero, and remains for a period of time. As shown in Fig. 5 (c), we can see that, when the initial state is in the chaotic region, there is no entanglement again. However, we observe ESB when the initial states are in the periodic and quasi-periodic regions, as shown in Fig. 5 (a) and (b). The whole length of the time intervals for zero entanglement in the periodic case is shorter than that in the quasi-periodic case.

Similar to concurrence, we also find SSSD and SSSB in the QKT model. In Ref. [7], the authors introduced a quantity to describe the spin squeezing, the form is

$$\zeta^2 = \max\left\{0, 1 - \xi_T^2\right\}, \qquad (43)$$

therefore, if there is no spin squeezing, i.e. $\xi_T^2 > 1$, we have $\zeta^2 = 0$. The numerical results of ζ^2 in the periodic, quasi-periodic, and chaotic cases are illustrated in Fig. 6. At first, we consider the periodic case, as shown in Fig. 6 (a), there is spin squeezing $(\zeta^2 > 0)$ at the first kick, and it quickly decreases to zero. As time evolves, ζ^2 becomes larger than 0. Both SSSD and SSSB appear multiple times under this condition. In the quasi-periodic case, as shown in Fig. 6(b), both SSSD and SSSB appear alternatively, but the whole length of time intervals for
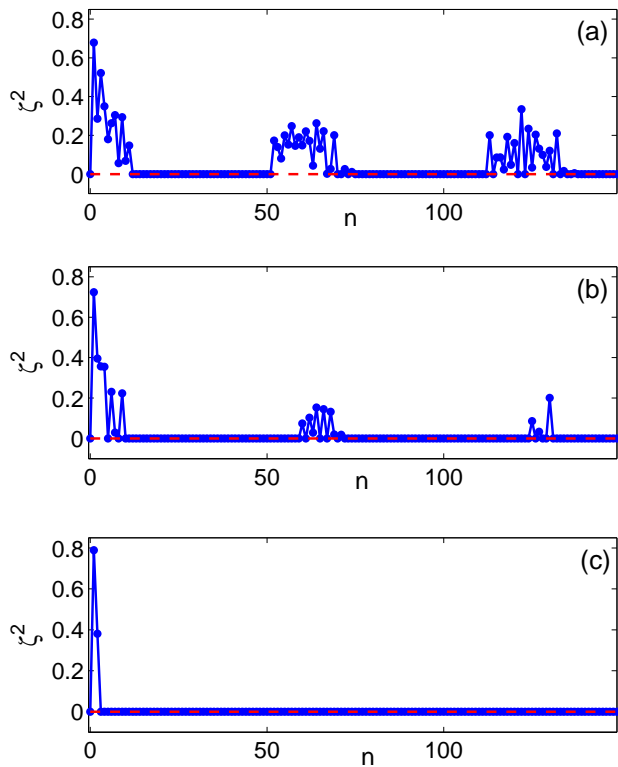
FIG. 6: (Color online) Dynamical evolutions of spin squeezing in the QKT model with (a) the periodic case $(\theta, \phi) = (2.25, 0.63)$, (b) the quasi-periodic case $(\theta, \phi) = (2.25, -2.35)$, and (c) the chaotic case $(\theta, \phi) = (2.25, -1)$. Here, we choose $N = 50$.

zero spin squeezing parameter is longer than that in the periodic case. At last, we consider the chaotic case, as shown in Fig. 6(c), the spin squeezing vanishes after a few kicks, and then there is no spin squeezing again.

From the above discussions, we find that the quantum chaos greatly affects the dynamics of spin squeezing and entanglement. When the initial states are in the periodic and quasi-periodic regions, both ESD (SSSD) and ESB (SSSB) appear alternatively, and when the initial states are in the chaotic region, there is only ESD (SSSD).

VI. CONCLUSIONS

In summary, we first prove that negative correlations are equivalent to spin squeezing for systems with only symmetric Dicke states populated. Then we study the effects of quantum chaos on spin squeezing and entanglement in the QKT model. Using the spin squeezing parameter ξ_T^2, we find that, in the periodic case the directions of the maximal spin squeezing are around the $\vec{n}_\perp$-direction, while in the chaotic case, they deviate from the $\vec{n}_\perp$-direction and behave irregularly. Then, we study the dynamics of spin squeezing parameters ξ_{KU}^2, ξ_T^2, and concurrence, and find that ξ_T^2 is a good quantity to characterize the pairwise entanglement. At last, we study the influences of quantum chaos on ESD (SSSD) and ESB (SSSB) in the QKT model. We find that both ESD (SSSD) and ESB (SSSB) occur alternatively in the periodic and quasi-periodic cases, but there is only ESD (SSSD) in the chaotic case. We believe that the behaviors of spin squeezing in the QKT model may also be found in other models that exhibit quantum chaos, e.g. the Dicke model.

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