Landauer Transport Model for Hawking Radiation from a Black Hole

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We investigate the Hawking radiation energy and entropy flow rates from a black hole viewed as a one-dimensional (1D) Landauer transport process. The conformal symmetry in the near-horizon region leads directly to radiation rates that are identical to those of a single 1D quantum channel connected to a thermal reservoir at the Hawking temperature. The particle statistics independence of the 1D energy and entropy currents is applied to a black hole radiating into vacuum as well as one near thermal equilibrium with its environment. The Hawking radiation entropy production ratio is also examined.

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Introduction.— The rate of entropy production by a black hole due to Hawking radiation was first considered by Zurek [1], where it was found that the radiation emitted by a Schwarzschild black hole into vacuum, neglecting backscatter due to the radial potential barrier [2], produced 4/3 times the entropy correspondingly lost by the black hole. Implicit in this calculation is the assumption that a black hole should radiate as a three-dimensional (3D) thermal body obeying the Stefan-Boltzmann law. However, there has been an increasing body of evidence suggesting that black hole emission is instead a 1D radiative process. One indicator is the well-known near-horizon approximation under which the four-dimensional (4D) Schwarzschild metric of a black hole can be formally reduced to a (1+1)-dimensional Rindler space possessing infinite-dimensional conformal symmetry [3]. The ability to calculate the stress-energy tensor using conformal symmetry is the basis for standard derivations of the Hawking flux [4, 5]. More recently it has been suggested that this conformal symmetry is responsible for the Hawking effect [6] as it has been shown that this symmetry alone is enough to determine both the Hawking thermal spectrum [7, 8] and radiation flux [9]; the Hawking radiation is an inherently (1+1)-dimensional process. This near-horizon conformal symmetry also reproduces the Bekenstein-Hawking form of the black hole entropy [10], thus connecting to the other familiar dimensional reduction in black hole physics, namely the holographic principle [11].

The first to focus on the entropic and information implications of a 1D evaporation process was Bekenstein [12] who showed that the entropy flow rate from a black hole is of the same form as that of a 1D quantum channel [13], thus constraining the information flow from a black hole. This same 1D channel description applies in the context of laboratory analogues of Hawking radiation [14, 15] and it was noted that the power output from the analogue Hawking process coincides with the optimal energy current through a single quantum channel [15]. The concept of a 1D quantum channel was first considered in the modeling of electrical transport in mesoscopic circuits by Landauer and others [16, 17] and subsequently extended to describe thermal transport [18]. Quantum mechanics places upper limits on the 1D energy and entropy currents. These upper limits are attained in the absence of backscattering for bosonic channels[13, 19], and are independent of the material nature of the channel due to the mutual cancellation of the group velocity and density of states factors entering the current formulae in 1D. Furthermore, these upper limits can be independent of whether the particles are bosons or fermions, and thus are deemed "universal" [20].

Motivated by these connections, in this letter we consider a Landauer-transport model for black hole entropy flow and production rates, describing the Hawking effect in terms of currents flowing in 1D quantum channels connecting thermal reservoirs at each end. We in particular emphasize the conditions under which the 1D currents are independent of particle statistics. In essence, the Landauer approach extends the application of thermodynamic principles to black holes [21], enabling the description of certain non-equilibrium, steady state flow properties independent of the microscopic physics.

Near-horizon conformal symmetry and the Hawking flux.— For an observer near the horizon of a spherically symmetric Schwarzschild black hole of mass M, the original 4D metric ($G = c = 1$),

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega^2, \qquad (1)$$

can be reduced to that of a (1+1)-dimensional spacetime through the coordinate transformation $r = 2M + x^2/8M$, where near $x = 0$, $1-2M/r \approx x^2/16M^2$. Thus, the near-horizon form of the metric is given by

$$ds^2 = -(\kappa x)^2 dt^2 + dx^2, \qquad (2)$$

where $\kappa = 1/4M$ is the surface gravity and the t-x portion of the metric defines the flat (1+1)-dimensional

Rindler spacetime. Excitations and dimensional quantities transverse to the t-x plane are redshifted with respect to those in the Rindler spacetime and can be ignored [22]. Equation (2) can be brought into conformal form by defining the coordinate $x = \kappa^{-1}\exp(\kappa\xi)$ and forming null coordinates, $u = t - \xi$ and $v = t + \xi$, under which the metric takes the form

$$ds^2 = -C(u,v)dudv = -e^{-\kappa(v-u)}dudv, \tag{3}$$

where $C(u,v)$ is the conformal factor. Here we ignore the effects of the radial potential as it is blue-shifted away by the conformal symmetry [6]. The regularized expectation values for the stress-energy tensor can be immediately evaluated from the conformal structure of Eq. (3) [23], $\langle T_{ii}\rangle = -(1/12)C^{1/2}\partial_i^2 C^{-1/2}$ for $i = u, v$. For a Schwarzschild black hole, these expectation values with respect to the Unruh vacuum are given as [5],

$$\langle T_{uu}\rangle_U = \frac{\pi}{12}T_{\mathrm{H}}^2\left[1-\frac{2M}{r}\right]^2\left[1+\frac{4M}{r}+\frac{12M^2}{r^2}\right] \tag{4}$$

$$\langle T_{vv}\rangle_U = \frac{\pi}{12}T_{\mathrm{H}}^2\left[\frac{48M^4}{r^4}-\frac{32M^3}{r^3}\right], \tag{5}$$

where $T_{\mathrm{H}} = \kappa/2\pi$. The power emitted through Hawking radiation as seen by an inertial observer at $r = \infty$ is obtained from Eq. (4) as

$$\langle T_{uu}\rangle_U = \frac{\pi k_{\mathrm{B}}^2}{12\hbar}T_{\mathrm{H}}^2, \tag{6}$$

where, reintroducing dimensional constants for later convenience, we have $T_{\mathrm{H}} = \hbar c^3/8\pi k_{\mathrm{B}}GM$. Additionally, Eq. (5) represents the corresponding influx of negative energy across the horizon, $\langle T_{vv}\rangle_U|_{r=2M} = -(\pi k_{\mathrm{B}}^2/12\hbar)T_{\mathrm{H}}^2$, responsible for the evaporation of the black hole.

One-dimensional quantum channels.— As a model for a single 1D quantum channel we will consider two thermal reservoirs characterized by the temperatures T_{L} and T_{R} and with chemical potentials μ_{R} and μ_{L}. The reservoirs are coupled adiabatically through an effectively 1D connection supporting the bidirectional propagation of particles. The subscripts L and R denote the left and right thermal reservoirs respectively. Here we will assume $T_{\mathrm{L}} > T_{\mathrm{R}}$ and that the transport through the 1D-connection is ballistic.

Although our focus is on fundamental fields/particles, for complete generality we will assume interpolating fractional statistics where the distribution function is [24] $f_g(E) = \{w[(E-\mu)/k_{\mathrm{B}}T] + g\}^{-1}$, with $w(x)^g[1+w(x)]^{1-g} = e^x$. Here, $g = 0$ and $g = 1$ describe bosons and fermions respectively. The individual single-channel energy and entropy currents flowing from the left (L) and right (R) reservoirs may be written as [19]

$$\dot{E}_{\mathrm{L(R)}} = \frac{\left(k_{\mathrm{B}}T_{\mathrm{L(R)}}\right)^2}{2\pi\hbar}\int_{x^0_{\mathrm{L(R)}}}^{\infty}dx\left(x+\frac{\mu_{\mathrm{L(R)}}}{k_{\mathrm{B}}T_{\mathrm{L(R)}}}\right)f_g(x) \tag{7}$$

and

$$\begin{aligned}\dot{S}_{\mathrm{L(R)}} =& -\frac{k_{\mathrm{B}}^2T_{\mathrm{L(R)}}}{2\pi\hbar}\int_{x^0_{\mathrm{L(R)}}}^{\infty}dx\,\{f_g\ln f_g + (1-gf_g)\ln(1-gf_g) \\ &-[1+(1-g)f_g]\ln[1+(1-g)f_g]\},\end{aligned} \tag{8}$$

where $x^0_{\mathrm{L(R)}} = -\mu_{\mathrm{L(R)}}/k_{\mathrm{B}}T_{\mathrm{L(R)}}$. We define the zero of energy with respect to the longitudinal component of the kinetic energy. For the case of photons (or phonons) with $\mu_{\mathrm{L}} = \mu_{\mathrm{R}} = 0$, the net power and entropy flow through the quantum channel, $\dot{E}_{1D} = \dot{E}_{\mathrm{L}} - \dot{E}_{\mathrm{R}}$ and $\dot{S}_{1D} = \dot{S}_{\mathrm{L}} - \dot{S}_{\mathrm{R}}$ respectively, become

$$\dot{E}_{\mathrm{1D}} = \frac{\pi k_{\mathrm{B}}^2}{12\hbar}\left(T_{\mathrm{L}}^2 - T_{\mathrm{R}}^2\right) \tag{9}$$

and

$$\dot{S}_{\mathrm{1D}} = \frac{\pi k_{\mathrm{B}}^2}{6\hbar}\left(T_{\mathrm{L}} - T_{\mathrm{R}}\right). \tag{10}$$

It can be seen immediately that Eq. (9) is identical to the outgoing Hawking flux Eq. (6) for $T_{\mathrm{L}} = T_{\mathrm{H}}$ and $T_{\mathrm{R}} = T_{\mathrm{E}} = 0$, where T_{E} is defined to be the temperature of the thermal environment surrounding the black hole. Thus, the (1+1)-dimensional conformal symmetry of the metric near the horizon allows for a 1D-quantum channel description of the Hawking process. Here, the currents are generated by temperature differences rather than chemical potential differences typically considered in Landauer transport. The emitted power Eq. (9) holds for all bosonic quantum channels since the group velocity and density of states mutually cancel in 1D. This insensitivity to the channel's longitudinal dispersion relation should make Eq. (9) valid not just in flat but in arbitrary curved spacetimes [25], however the conformal symmetry of the near-horizon region suggests that the production of Hawking radiation is itself essentially a flat-space process.

The power $\pi k_{\mathrm{B}}^2T_{\mathrm{L}}^2/(12\hbar)$ and the entropy current

$$\dot{S}_{\mathrm{1D}} = \frac{\pi k_{\mathrm{B}}^2}{6\hbar}T_{\mathrm{L}}, \tag{11}$$

are the maximum possible rates for single-channel bosonic flow. The unidirectional entropy current (11) is in fact the maximum possible rate for single-channel fermionic flow as well, i.e., it is independent of the particle statistics [20, 26]. To see this, we make a change of integration variables in Eq. (8), $x = (E-\mu)/k_{\mathrm{B}}T \to w$, upon which the entropy current can be simplified to [20]

$$\dot{S}_{\mathrm{L}} = \frac{k_{\mathrm{B}}^2T_{\mathrm{L}}}{2\pi\hbar}\int_{w_g\left(\frac{-\mu_{\mathrm{L}}}{k_{\mathrm{B}}T_{\mathrm{L}}}\right)}^{\infty}dw\left[\frac{\ln(1+w)}{w} - \frac{\ln w}{1+w}\right]. \tag{12}$$

We can see that the statistics of the particles shows up only in the lower integration bound of Eq. (12). The maximum current (11) is obtained in the degenerate limit where the statistics-dependence vanishes, since $-\mu_{\rm L}/k_{\rm B}T_{\rm L} \to 0^+$, $w_{g=0}(0) = 0$ for bosons, and $-\mu_{\rm L}/k_{\rm B}T_{\rm L} \to -\infty$, $w_{g=1}(-\infty) = 0$ for fermions. This same statistics independence in the degenerate limit does not hold for the unidirectional power Eq. (7), however. If instead one considers bidirectional current flow for fermions with $\mu_{\rm R} = \mu_{\rm L}$ and $T_{\rm R} = 0$, then in the degenerate limit one recovers the same maximum rate as for bosons: $\pi k_{\rm B}^2 T_{\rm L}^2/(12\hbar)$ [19]. If the maximum energy and entropy current expressions are combined by eliminating $T_{\rm L}$, then one obtains equality for the bound $\dot{S}_{\rm 1D}^2 \le \left(\pi k_{\rm B}^2/3\hbar\right) \dot{E}_{\rm 1D}$. This bound holds for 1D quantum channels with arbitrary reservoir temperatures and chemical potentials and particle statistics [13, 19]. We note in passing that this bound is similar in form to the conjectured Bekenstein holographic bound [27].

Assuming that Eq. (12) holds for black holes, what can be said about the net entropy outflow rate for black holes radiating into the vacuum (i.e., $T_{\rm E} = \mu_{\rm E} = 0$)? The electrochemical potential of the black hole reservoir is $\mu_{\rm BH} = q\Phi$, where q is the electric charge of the field under consideration and Φ is the electrostatic potential corresponding to the charge of the black hole [3]. For a Schwarzschild black hole with $\mu_{\rm BH} = 0$, bosons such as photons and gravitons have a maximum rate given by Eq. (11) with $T_{\rm L} = T_{\rm H}$. For fermions such as neutrinos and electrons (i.e. leptons), setting $\mu_{\rm BH} = 0$ gives a lower integration limit of $w_{g=1}(0) = 1$ in Eq. (12), resulting in entropy and energy rates that are reduced by a factor of 1/2 from the maximum values (11) and (6). This result for the energy rate was established in earlier calculations for massless fermions [28]. Subsequently, it was pointed out [29] that in a (1+1)-dimensional curved spacetime, the fermionic field describing a massless particle plus its antiparticle is equivalent to a single massless bosonic field. From the Landauer viewpoint, the combined fermionic particle/antiparticle single channel currents can therefore be thought of as a single effective bosonic channel that satisfies the maximum rates, Eqs. (6) and (11), when $\mu_{\rm BH} = 0$. Although leptons are massive particles, the conformal symmetry removes the length scale set by the particle mass [6]; the particles are effectively massless. In the case of ballistic transport, multiple channels can be treated independently. Thus, the net Schwarzschild black hole energy and entropy outflow rates are bounded by $N\left(T_{\rm H}\right)$ times the single channel rates given by Eq. (6) and Eq. (11), respectively. Here, $N\left(T_{\rm H}\right)$ is the total number of effective bosonic channels spontaneously produced by a black hole at temperature $T_{\rm H}$. This temperature dependence arises due to the requirement that $k_{\rm B}T_{\rm H} \gtrsim 2mc^2$ for pair production of particles with mass m. The full Landauer approach, relating transport to scattering processes, can incorporate interchannel scattering due to particle interactions and back scatter from the radial potential barrier not considered here.

For a black hole with nonzero electrochemical potential, charged particle/antiparticle rates differ so as to cause the black hole net charge to decrease over time. The maximum entropy rate for a single charged fermionic channel coincides with the maximum rate for a single bosonic channel as shown above, giving Eq. (11). The extent to which these maximum rates can be achieved depends on how close to degenerate is the thermal Hawking reservoir of the black hole for charged particles. Similar reasoning applies to a black hole with angular momentum which can be characterized by an effective chemical potential [3]. A special case is provided by extremal charged black holes [3] satisfying $Q^2/M^2 \approx 1$, where Q is the non-dimensional black hole charge. In this limt $T_{\rm H} \to 0$ giving $-\mu_{\rm BH}/k_{\rm B}T_{\rm H} \to -\infty$, the degenerate limit for fermions. Charged fermions then satisfy Eq. (11). It may be possible to reach the degenerate limit for other choices of black hole parameters.

Net entropy production in (1+1)-dimensions.— Following Ref. [1], the entropy production ratio for radiation emitted by a Schwarzschild black hole is given by

$$R = \frac{dS}{dS_{\rm BH}} = T_{\rm H}\frac{dS}{dE} = T_{\rm H}\frac{\dot{S}}{\dot{E}} \qquad (13)$$

where we have used the first law of thermodynamics $dE_{\rm BH} = T_{\rm H}dS_{\rm BH}$ and assumed energy conservation, $dE = dE_{\rm BH}$. For a 3D black hole radiating into a thermal environment with temperature $T_{\rm E}$ the power and entropy currents are [1]

$$\dot{E}_{\rm 3D} \sim a\left(T_{\rm H}^4 - T_{\rm E}^4\right) \qquad (14)$$

$$\dot{S}_{\rm 3D} \sim \frac{4a}{3}\left(T_{\rm H}^3 - T_{\rm E}^3\right), \qquad (15)$$

where a is a constant. Upon substitution into Eq. (13) this yields the 3D black hole entropy production ratio

$$R_{\rm 3D} = \frac{4}{3}\frac{1 - \left(T_{\rm E}/T_{\rm H}\right)^3}{1 - \left(T_{\rm E}/T_{\rm H}\right)^4}, \qquad (16)$$

which reduces to the earlier stated $R_{\rm 3D} = 4/3$ as $T_{\rm E}/T_{\rm H} \to 0$. However, as we have shown above, the emission of Hawking radiation is more appropriately characterized as a 1D radiative process. For comparison we set $\mu_{\rm E} = \mu_{\rm BH} = 0$, and the net energy and entropy currents are given by Eqs. (9) and (10) respectively. The factors of 1/2 in the fermion rates will drop out when evaluating the ratio Eq. (13). The 1D entropy production ratio is then

$$R_{\rm 1D} = 2\frac{1 - \left(T_{\rm E}/T_{\rm H}\right)}{1 - \left(T_{\rm E}/T_{\rm H}\right)^2}, \qquad (17)$$

which yields a larger value of $R_{1D} = 2$ when radiating into vacuum. The 3D and 1D entropy rates, Eqs. (16) and (17) respectively, for various ratios of $T_{\rm E}/T_{\rm H}$ are presented in Fig. 1. In the case where $T_{\rm H} \approx T_{\rm E}$, both Eqs. (16) and (17) give approximately $R \approx 1 + \delta/T_{\rm H}$ to

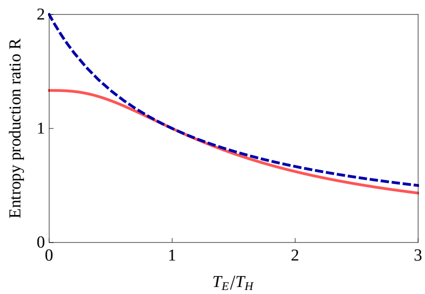

FIG. 1. (Color online) Entropy production ratio for a black hole characterized as 1D quantum channel $R_{1\mathrm{D}}$ (dashed-blue) compared to the standard 3D answer $R_{3\mathrm{D}}$ (red). Both results agree near thermal equilibrium $T_{\mathrm{H}} \approx T_{\mathrm{E}}$.

first order in $\delta = (T_{\mathrm{H}} - T_{\mathrm{E}})/2$. As to be expected, in equilibrium ($\delta = 0$), there is no net entropy production ($R = 1$).

Near thermal equilibrium we can make use of linear response for small temperature differences, $(T_{\mathrm{H}} - T_{\mathrm{E}}) \ll \bar{T}$ where $\bar{T} = (T_{\mathrm{H}} + T_{\mathrm{E}})/2$, to relate the energy and entropy flows by $\dot{S}_{1\mathrm{D}} = \dot{E}_{1\mathrm{D}}/\bar{T}$. In this regime the unidirectional entropy rate Eq. (11) allows us to recover the quantum of thermal conductance for a single effective bosonic channel [20]:

$$G_Q = \frac{\dot{E}_{1\mathrm{D}}}{T_{\mathrm{H}} - T_{\mathrm{E}}} = \frac{\left(\dot{S}_{\mathrm{H}} - \dot{S}_{\mathrm{E}}\right)\bar{T}}{T_{\mathrm{H}} - T_{\mathrm{E}}} = \frac{\pi k_{\mathrm{B}}^2}{6\hbar}\bar{T}. \tag{18}$$

From the statistics independence of Eq. (11), it follows that Eq. (18) provides a general upper bound on the thermal conductance that is independent of the particle statistics [30, 31].

Conclusion.— The conformal symmetry arising in the near-horizon region of a Schwarzschild black hole in vacuum generates a Hawking radiation energy flux that is identical to the power flowing in a single 1D quantum channel connected to a thermal bath with the Hawking temperature at one end and zero temperature at the other. A Schwarzschild black hole in vacuum radiates power and entropy as a collection of effective bosonic channels. The unidirectional entropy current leads to a statistics independent heat flow near thermal equilibrium characterized by the quantum of thermal conductance. The energy and entropy currents in 1D give a Hawking radiation entropy production ratio that is twice the corresponding value lost by the black hole when radiating into vacuum. These results are a direct consequence of the reduced dimensionality in the near-horizon region and its conformal symmetry. Given the intimate connection between entropy and information, the present findings may have implications for the information loss problem [32, 33].

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