

Nuclear Collective Excitation by a Short Strong Laser Pulse

Hans A. Weidenmüller¹

¹Max-Planck-Institut für Kernphysik, D-69029 Heidelberg, Germany

We derive the conditions on the average laser energy and the mean photon number such that a strong short laser pulse causes collective nuclear excitation. We use the nuclear Giant Dipole Resonance as a representative example, and a random-matrix description of the fine-structure states and perturbation theory as tools.

PACS numbers: 42.50Ct, 24.30Cz, 24.60Dr

Purpose. Qualitative Considerations. With the start of the construction of ELI (the “extreme light infrastructure”) [1], nuclear spectroscopy using intense beams of high-energy gamma rays or even intense laser light has become a realistic possibility. Indeed, laser light of several MeV energy is expected to be produced by coherent Thomson backscattering [2, 3]. These developments call for a theoretical analysis of the absorption mechanism of laser light in nuclei. Naively one might expect that for a laser pulse containing $N \gg 1$ photons, the dominant process is multiple excitation of individual nucleons moving independently in the nuclear mean field since the time between successive photon absorption processes becomes too short for the nuclear system to relax collectively. We show that that naive expectation may fail: Even a strong short laser pulse may lead to single or multiple collective nuclear excitation.

We focus attention on dipole absorption, the dominant photon absorption process in nuclei. The dipole mode $|10\rangle$ is the normalized product of the dipole operator and the wave function $|0\rangle$ of the nuclear ground state. That is not an eigenstate of the nuclear Hamiltonian H_{nuc} , and the dipole mode is spread over the eigenstates $|\mu\rangle$ of H_{nuc} with eigenvalues E_μ , $\mu = 1, \dots$. Gross features versus excitation energy E of that spreading are measured by the strength function $S(E) = \sum_\mu \overline{|\langle 10|\mu\rangle|^2 \delta(E - E_\mu)}$. The average (overbar) is taken over an energy interval large compared to the average nuclear level spacing d . In the simplest model adopted here, $S(E)$ has Lorentzian shape and is characterized by two experimentally determined parameters [4]: The peak energy $E_{\text{dip}} \approx 80 A^{-1/3}$ MeV (where A is the nuclear mass), and the width $\Gamma^\downarrow \approx 5$ MeV (the “spreading width”). The resulting broad peak of $S(E)$ is referred to as the Giant Dipole Resonance (GDR). By the uncertainty relation, the time for the dipole mode to spread over the eigenstates of H_{nuc} (the “equilibration time”) is $\tau_{\text{eq}} = \hbar/\Gamma^\downarrow$. If photons within the same laser pulse are successively absorbed at time intervals larger than τ_{eq} , the nucleus relaxes after every step. Absorption of the first photon then excites the dipole mode, and successive absorption of several photons leads to multiple excitation of that mode. As a result, nuclei may absorb laser light in a manner characteristic of a strongly interacting many-body system. In the present

paper we show that that process is indeed expected to occur, and we derive the conditions for it to happen.

The width Γ_{dip} for gamma decay of the GDR to the nuclear ground state is estimated below and has a typical value of 10 keV. With N photons in the laser pulse, the characteristic time scale for photon absorption is $\tau_{\text{dip}} = \hbar/(N\Gamma_{\text{dip}})$. Naively one would expect that excitation of the GDR (as opposed to independent excitation of individual nucleons) dominates whenever $\tau_{\text{dip}} > \tau_{\text{eq}}$, i.e., whenever $N\Gamma_{\text{dip}} < \Gamma^\downarrow$. That simple estimate is modified by two factors, however. (i) For a short laser pulse with average energy E_L and energy spread σ (where we take $\sigma \approx 10$ keV corresponding to a time length of the laser pulse of $\approx 10^{-19}$ s, a realistic estimate [5]), the Lorentzian shape of the GDR produces for $E_L < E_{\text{dip}}$ an additional factor $[\Gamma^\downarrow/(E_L - E_{\text{dip}})]^2$. (ii) The characteristic cubic dependence of Γ_{dip} on energy yields an additional factor $(E_L/E_{\text{dip}})^3$. In total the criterion for collective excitation of the GDR at energy E_L reads $N < (E_L/E_{\text{dip}})^3 [(E_L - E_{\text{dip}})^2/(\Gamma_{\text{dip}}\Gamma^\downarrow)]$. With $\Gamma_{\text{dip}} = 10$ keV, $\Gamma^\downarrow = 5$ MeV, $E_{\text{dip}} = 14$ MeV, $E_L = 7$ MeV that yields $N < 5 \times 10^3$. That bound on N is significantly larger than the bound $N < \Gamma^\downarrow/\Gamma_{\text{dip}} \approx 700$ obtained from the naive estimate and shows that even for an intense laser pulse, excitation of the collective GDR is a realistic alternative in nuclei to excitation of individual nucleons provided only that E_L is sufficiently far below E_{dip} . And even for E_L near E_{dip} a laser pulse of sufficiently low intensity (i.e., sufficiently small N) would excite the GDR. Thus, varying both N and E_L provides the exciting possibility to investigate the dynamical interplay between collective and single-particle dipole excitation in nuclei.

Our argument is not restricted to single excitation of the GDR but applies likewise to double photon absorption. We demonstrate that fact by calculating the probabilities P_1 for single-quantum dipole excitation and P_2 for double-quantum dipole excitation as functions of N , of E_L , and of σ . To calculate P_2 we use the Brink-Axel hypothesis [6, 7]. It states that every nuclear state (and not only the nuclear ground state) possesses a GDR. The hypothesis applies, in particular, to the configurations that mix with the dipole mode. As a consequence, single excitation of the dipole mode may be followed either by double excitation of that mode (i.e., formation of the

second harmonic) or by dipole excitation of the configurations mixed with the single dipole mode. We account for both possibilities and show that for $\sigma \ll \Gamma^\downarrow$ the contribution from the Brink–Axel mechanism dominates and yields $P_2 = (1/2)P_1^2$. For values of N and of E_L such that $P_1 \ll 1$ that relation implies that single photon absorption is the dominant process even if $N \gg 1$. (Observable consequences of single–photon absorption were recently investigated theoretically in Ref. [5]). Our result suggests that the probability for nuclear excitation by n –fold dipole absorption may be approximately given by $P_n \approx 2^{-n}P_1^n$. That would imply that in the regime where our approximations apply ($P_1 < 1/2$ or so) multiple collective nuclear excitation is unlikely.

Our calculations are based upon a random–matrix model for the complex configurations that mix with the single or double dipole modes. Every random–matrix model is based upon the implicit assumption that the equilibration time (here τ_{eq}) is short compared to the time scale of the physical process of interest (here τ_{dip}). Our use of random–matrix theory is justified if the above–mentioned conditions for collective excitation of the GDR are met. We also use perturbation theory to calculate P_1 and P_2 . That is justified if P_1 and P_2 are sufficiently small compared to unity. The resulting constraint is the same as for the use of random–matrix theory.

Hamiltonian. We write the total time–dependent Hamiltonian as

$$\mathcal{H}(t) = H_{\text{nuc}} + H(t) \quad (1)$$

where $H(t)$ stands for the time–dependent interaction with the laser light. In constructing H_{nuc} we are guided by the following qualitative picture [8]. In a closed–shell nucleus, the dipole mode $|10\rangle$ is a superposition of one–particle one–hole (1p 1h) excitations. That mode is embedded in a sea of 2p 2h excitations $|0k\rangle$ where $k = 1, \dots, K$ and $K \gg 1$. (Here and in what follows the first label of the state vector counts the number of absorbed dipole quanta and the second enumerates the states). The mixing of both kinds of excitations causes the dipole mode to be distributed over the eigenstates of H_{nuc} . The absorption of a second dipole quantum may either lead from the dipole mode $|10\rangle$ to the double dipole mode $|20\rangle$ (a 2p 2h state), or it may lead from one of the 2p 2h states $|0k\rangle$ to the dipole mode $|1k'\rangle$ of that same state (a 3p 3h state). The double dipole mode $|20\rangle$ is similarly embedded in a sea of 3p 3h states $|0\alpha\rangle$ with $\alpha = 1, \dots, L$. All of the states $|1k'\rangle$ are embedded in a sea of 4p 4h states $|0\rho\rangle$ where $\rho = 1, \dots, M$ and $M \gg K$. The residual interaction of the nuclear shell model mixes these configurations, and both the double dipole mode and the states $|1k'\rangle$ are spread out over the eigenstates of H_{nuc} . In modeling this qualitative picture we disregard the fact that single or double dipole excitation may populate states with different spin and isospin values. H_{nuc}

is accordingly schematically written in matrix form as follows.

$$H_{\text{nuc}} = \begin{pmatrix} E_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_1 & V_{1l} & 0 & 0 & 0 & 0 \\ 0 & V_{k1} & \tilde{H}_{kl}^{(1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_2 & V_{2\beta} & 0 & 0 \\ 0 & 0 & 0 & V_{\alpha 2} & \tilde{H}_{\alpha\beta}^{(2)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{\mathcal{H}}_{k'l'} & W_{k'\sigma} \\ 0 & 0 & 0 & 0 & 0 & W_{\rho l'} & \tilde{h}_{\rho\sigma} \end{pmatrix}. \quad (2)$$

Here E_0 is the energy of the nuclear ground state, while E_1 and E_2 are the mean excitation energies of the single and of the double dipole modes. For simplicity we use a harmonic–oscillator picture so that $E_2 - E_1 = E_1 - E_0 = E_{\text{dip}}$. Moreover we put $E_0 = 0$. The real matrix elements V_{1l} mix the dipole mode with the 2p 2h states $|0l\rangle$. These are governed by the K –dimensional Hamiltonian matrix $\tilde{H}_{kl}^{(1)}$. Similarly, the matrix elements $V_{2\beta}$ mix the double dipole mode with the 3p 3h states $|0\beta\rangle$. These are governed by the L –dimensional Hamiltonian matrix $\tilde{H}_{\alpha\beta}^{(2)}$. We write $\tilde{H}_{kl}^{(1)} = E_1\delta_{kl} + H_{kl}^{(1)}$ and $\tilde{H}_{\alpha\beta}^{(2)} = E_2\delta_{\alpha\beta} + H_{\alpha\beta}^{(2)}$ and assume that both $H_{kl}^{(1)}$ and $H_{\alpha\beta}^{(2)}$ are random matrices, members of the Gaussian Orthogonal Ensemble (GOE), with no correlations between the elements of $H_{kl}^{(1)}$ and of $H_{\alpha\beta}^{(2)}$. The spectra of $E_1\delta_{kl} + H_{kl}^{(1)}$ and of $E_2\delta_{\alpha\beta} + H_{\alpha\beta}^{(2)}$ both have the shape of a semicircle centered at E_1 and E_2 , respectively. The last diagonal block in Eq. (2) describes similarly the mixing of the states $|1k'\rangle$ with the 4p 4h states $|0\rho\rangle$. We write $\tilde{\mathcal{H}}_{k'l'} = E_2\delta_{k'l'} + \mathcal{H}_{k'l'}$ and $\tilde{h}_{\alpha\beta} = E_2\delta_{\alpha\beta} + h_{\alpha\beta}$. We implement the Brink–Axel hypothesis by putting $\mathcal{H} = H^{(1)}$. Again, the M –dimensional matrix $h_{\rho\sigma}$ is assumed to be a member of the GOE. We calculate the excitation probabilities P_1 and P_2 as ensemble averages for $K, L, M \rightarrow \infty$. In that limit, the spreading widths of the single and double dipole mode and of each of the states $|1k'\rangle$ are given by the generic expression [9] $\Gamma^\downarrow = 2\pi v^2 \rho$ where v^2 stands for the mean square of the relevant mixing matrix elements and ρ for the mean level density in the center of the semicircle. To avoid unnecessary complexity we assume that all spreading widths have the same value Γ^\downarrow . That schematic picture can be refined if the need arises. We disregard the fact that the states excited by gamma absorption may decay by particle or by gamma emission. That is justified because the time scales associated with such decay are orders of magnitude larger than both τ_{eq} and τ_{dip} .

For the time–dependent interaction Hamiltonian $H(t)$ we use a semiclassical description (justified for $N \gg 1$) and write

$$H(t) = \sqrt{N}g(t)H_{\text{dip}}. \quad (3)$$

Here H_{dip} is the time–independent electromagnetic interaction operator for a single–photon dipole transition.

The factor \sqrt{N} accounts for the presence of N photons and the ensuing factor N in the transition rate. (For $N \gg 1$ single and double transitions have the same amplification factor). The dimensionless function $g(t)$ describes the time dependence of the short laser pulse. We use the ansatz

$$g(t) = \exp[-\sigma^2 t^2 / (2\hbar^2) - i\omega_L t]. \quad (4)$$

Fourier transformation of $g(t)$ shows that the mean energy of the laser pulse is $E_L = \hbar\omega_L$, the spread in energy has width σ . Actually the interaction H_{dip} depends on energy, too, via the wave number k . For $\sigma \approx 10$ keV we may put $k \approx k_L$ where $k_L = E_L c / \hbar$.

In the scheme of Eq. (2) the non-zero matrix elements of the dipole operator are $\langle 10 | H_{\text{dip}} | 0 \rangle$, $\langle 20 | H_{\text{dip}} | 10 \rangle$, and $\langle 1k' | H_{\text{dip}} | 0k \rangle$. We use the Brink-Axel hypothesis to write $\langle 1k' | H_{\text{dip}} | 0k \rangle = \delta_{kk'} \langle 1k | H_{\text{dip}} | 0k \rangle$. We assume that all non-zero matrix elements of the dipole operator have the same value written as $\langle H_{\text{dip}} \rangle$. That corresponds to a harmonic-oscillator approximation. To estimate $\langle H_{\text{dip}} \rangle$, we write the Hamiltonian H_{int} describing the interaction with the electromagnetic field in Coulomb gauge as $H_{\text{int}} = -(1/c) \vec{j} \vec{A}$. Here \vec{j} is the current and \vec{A} the vector potential. In our time-dependent approach the latter has the form of a wave packet,

$$\vec{A}(\vec{x}, \Omega, t) = \alpha \int d\omega \exp[-i\omega t] \tilde{g}(\omega) \exp[i\vec{k}\vec{r}] \vec{\chi}. \quad (5)$$

The unit vector $\vec{\chi}$ describes the polarization, Ω indicates the direction of the vector \vec{k} , and $k = \sqrt{k^2}$ and ω are related by $k = \omega/c$. The function \tilde{g} is the Fourier transform of $g(t)$ in Eq. (4). We determine the normalization constant α from the requirement that the energy carried by \vec{A} be equal to E_L . We use the dipole approximation. That yields $\alpha^2 = (\sigma E_L) / (\pi^{1/2} \hbar c)$. Quantization of the electromagnetic field for individual quanta that have the form of the wave packet (5) yields for the energy density the expression $n(E) = 1 / (4\pi^{3/2} \sigma)$. From Fermi's golden rule, the total width for dipole decay is $\Gamma_{\text{dip}} = 2\pi n(E_L) |\langle H_{\text{dip}} \rangle|^2$. Thus,

$$|\langle H_{\text{dip}} \rangle| = \sqrt{2\pi^{1/2} \Gamma_{\text{dip}} \sigma}. \quad (6)$$

For the dipole width we use the Weisskopf estimate, $\Gamma_{\text{dip}} = \frac{3}{4} \frac{e^2}{\hbar c} (kR)^2 E_L$. With $R = 3 \times 10^{-13}$ cm and $E_L = 15$ MeV that gives $\Gamma_{\text{dip}} \approx 10$ keV, so that $|\langle H_{\text{dip}} \rangle| \approx 10$ keV, too. A somewhat larger value for Γ_{dip} results when the Thomas-Reiche-Kuhn sum rule is taken into account. Here we are interested in order-of-magnitude estimates only, however.

Perturbation Expansion. We solve the time-dependent Schrödinger equation in the interaction representation where the perturbation has the form

$$\tilde{H}(t) = \exp[iH_{\text{nuc}} t / \hbar] H(t) \exp[-iH_{\text{nuc}} t / \hbar]. \quad (7)$$

We assume that at time $t = -\infty$ the nucleus is in the ground state $|0\rangle$. We determine perturbatively the probabilities P_1 and P_2 that at time $t = +\infty$ one or two dipole quanta have been absorbed.

At $t = +\infty$, the probability amplitudes for occupation of the states $|10\rangle$ and $|0k\rangle$ reached after single-dipole absorption are

$$\begin{aligned} b_1 &= \frac{1}{i\hbar} \langle 10 | \int_{-\infty}^{+\infty} dt \tilde{H}(t) | 0 \rangle \\ &= \frac{\sqrt{N}}{i\hbar} \langle H_{\text{dip}} \rangle \int_{-\infty}^{+\infty} dt g(t) \langle 10 | \exp[iH_{\text{nuc}} t / \hbar] | 10 \rangle, \\ b_{0k} &= \frac{1}{i\hbar} \langle 0k | \int_{-\infty}^{+\infty} dt \tilde{H}(t) | 0 \rangle \\ &= \frac{\sqrt{N}}{i\hbar} \langle H_{\text{dip}} \rangle \int_{-\infty}^{+\infty} dt g(t) \langle 0k | \exp[iH_{\text{nuc}} t / \hbar] | 10 \rangle. \end{aligned} \quad (8)$$

The corresponding amplitudes for occupation of the states $|20\rangle$, $|0\alpha\rangle$ and $|1k'\rangle$, $|0\rho\rangle$ reached after double-dipole absorption are denoted by b_2 , b_α , $b_{1k'}$ and $b_{0\rho}$. For brevity we give here only the expression for $b_{0\rho}$ as an example.

$$\begin{aligned} b_{0\rho} &= \left(\frac{1}{i\hbar} \right)^2 \langle 0\rho | \int_{-\infty}^{+\infty} dt_1 \tilde{H}(t_1) \int_{-\infty}^{t_1} dt_2 \tilde{H}(t_2) | 0 \rangle \\ &= \left(\frac{\sqrt{N}}{i\hbar} \right)^2 \langle H_{\text{dip}} \rangle^2 \int_{-\infty}^{+\infty} dt_1 g(t_1) \int_{-\infty}^{t_1} dt_2 g(t_2) \\ &\quad \times \sum_{l'l'} \langle 0\rho | \exp[-iH_{\text{nuc}} t_1 / \hbar] | 1l' \rangle \delta_{l'l'} \\ &\quad \times \langle 0l | \exp[i\{H_{\text{nuc}}(t_1 - t_2)\} / \hbar] | 10 \rangle. \end{aligned} \quad (9)$$

The average probabilities for single and double dipole absorption are, thus, given by

$$\begin{aligned} P_1 &= \left\langle |b_1|^2 + \sum_k |b_{0k}|^2 \right\rangle, \\ P_2 &= \left\langle |b_2|^2 + \sum_\alpha |b_{0\alpha}|^2 \right. \\ &\quad \left. + \sum_{k'} |b_{1k'}|^2 + \sum_\rho |b_{0\rho}|^2 \right\rangle. \end{aligned} \quad (10)$$

The big angular brackets indicate the ensemble average. The first (last) two terms that contribute to P_2 are due to double excitation of the dipole mode and to the Brink-Axel hypothesis, respectively.

Averages. By way of example we perform the ensemble average for P_1 and focus attention on the sum of the squares of the time-dependent matrix elements in Eqs. (8). Using completeness and a simple identity we obtain for these

$$\left\langle \langle 10 | \exp[iH_{\text{nuc}}(t_1 - t_2) / \hbar] | 10 \rangle \right\rangle =$$

$$\int_{-\infty}^{+\infty} d\varepsilon \exp[i\varepsilon(t_1 - t_2)/\hbar] \left(\frac{1}{2i\pi} \left\langle \langle 10 | \frac{1}{\varepsilon^- - H_{\text{nuc}}} | 10 \rangle - \langle 10 | \frac{1}{\varepsilon^+ - H_{\text{nuc}}} | 10 \rangle \right\rangle \right). \quad (11)$$

We use Eq. (2) to write

$$\begin{aligned} & \left\langle \langle 10 | \frac{1}{\varepsilon^\pm - H_{\text{nuc}}} | 10 \rangle \right\rangle \\ &= \left\langle \langle 10 | \frac{1}{\varepsilon^\pm - E_{\text{dip}} - V_1(\varepsilon^\pm - H^{(1)})^{-1}V_1^\dagger} | 10 \rangle \right\rangle \\ &= \frac{1}{\varepsilon - E_{\text{dip}} \pm (i/2)\Gamma^\downarrow}. \end{aligned} \quad (12)$$

Using Eq. (4) for $g(t)$ and carrying out the time integrals (see Eqs. (8)), we find that ε is confined to an interval of size σ around E_L . Since $\sigma \ll \Gamma^\downarrow$, the argument of the expression in Eq. (12) can be taken at $\varepsilon = E_L$. The remaining integration can be done. With the help of Eq. (6) that yields

$$P_1 = \frac{2\pi N \Gamma_{\text{dip}} \Gamma^\downarrow}{(E_L - E_{\text{dip}})^2 + (1/4)(\Gamma^\downarrow)^2}. \quad (13)$$

The result (13) is intuitively appealing and clearly displays the suppression factors $\Gamma^{\downarrow 2}/(E_L - E_{\text{dip}})^2$ and $(E_L/E_{\text{dip}})^3$ mentioned above that come into play for $E_L < E_{\text{dip}}$.

The calculation of P_2 proceeds similarly but is more involved. We use operator identities such as

$$\begin{aligned} & \langle 10 | \frac{1}{\varepsilon_2^- - H_{\text{nuc}}} | 0k \rangle \\ &= \langle 10 | \frac{1}{\varepsilon_2^- - E_{\text{dip}} - V_1(\varepsilon_2^- - E_{\text{dip}} - H^{(1)})^{-1}V_1^\dagger} | 10 \rangle \\ & \quad \times \langle 10 | V_1 \frac{1}{\varepsilon_2^- - E_{\text{dip}} - H^{(1)}} | 0k \rangle. \end{aligned} \quad (14)$$

That leads to products of terms each containing $H^{(1)}$ in the denominator. We neglect the correlations between eigenvalues of $H^{(1)}$ in different factors because such correlations extend over an energy range measured in units of the mean level spacing d while the range of the terms in Eq. (14) is given by $\Gamma^\downarrow \gg d$. For the last two terms in the second of Eqs. (10) we obtain

$$\left\langle \sum_{k'} |b_{1k'}|^2 + \sum_{\rho} |b_{0\rho}|^2 \right\rangle = \frac{1}{2} P_1^2 \quad (15)$$

with P_1 given by Eq. (13). The calculation of the first two terms yields a contribution that in comparison to Eq. (15) is small of order σ/Γ^\downarrow . Thus for all values of E_L the contribution to P_2 from double excitation of the dipole mode is negligibly small in comparison with that from the Brink–Axel mechanism in Eq. (15). As a result we find

$$P_2 = \frac{1}{2} P_1^2. \quad (16)$$

The factor 1/2 in Eq. (16) is due to the time ordering in Eqs. (9). Thus, we expect that for arbitrary positive integer n we have $P_n = 2^{-n} P_1^n$.

Conclusions. We have used a random–matrix model to calculate the probabilities P_1 and P_2 for single and double nuclear dipole absorption from a strong laser pulse containing N photons. The assumptions and approximations we have used require both P_1 and P_2 to be small compared to unity. Eq. (13) shows that in the tails of the GDR that condition is easily met even for an intense laser pulse. That is due to the suppression factors mentioned in the introduction. Ways of detecting such collective nuclear excitation experimentally are discussed in Ref. [5]. Double photon absorption is dominantly due to the Brink–Axel mechanism (as opposed to double excitation of the dipole mode). The remarkable result (16) shows that if our approximations hold for the calculation of P_1 , i.e., if P_1 is small compared to unity, they are valid *a fortiori* for the calculation of P_2 . We speculate that for integer $n > 2$ we have $P_n = 2^{-n} P_1^n$.

With increasing N , the time for dipole absorption τ_{dip} will eventually become small compared to the nuclear equilibration time τ_{eq} , and the collective mechanism studied above will not apply. Then photons are absorbed by nucleons moving independently in the mean field of the nuclear shell model. Eq. (13) shows that in the center of the GDR, i.e., for $E_L = E_{\text{dip}}$, that will happen already for fairly small values of $N \approx 10$ or so. As N is increased, the process spreads to the tails of the GDR. It would be of considerable interest to investigate the transition between the two regimes.

Acknowledgments. I thank D. Habs and P. Thirolf for stimulating discussions and A. Richter and B. Dietz for helpful suggestions.

-
- [1] Scientific Advisory Committee of ELI, the Extreme Light Infrastructure: Report on the ELI Science, <http://www.extreme-light-infrastructure.eu>
 - [2] H.-C. Wu, J. Meyer-ter-Vehn, J. Fernandez, and B. M. Hegelich, Phys. Rev. Lett. **104**, 234801 (2010).
 - [3] D. Kiefer *et al.*, Eur. Phys. J. D **55**, 427 (2009).
 - [4] M. N. Harakeh and A. van der Woude, *Giant Resonances*, Oxford University Press, Oxford, 2001.
 - [5] B. Dietz and H. A. Weidenmüller, Phys. Lett. B **693**, 316 (2010).
 - [6] D. Brink, PhD thesis, Oxford University (1955) unpublished.
 - [7] P. Axel, Phys. Rev. **126**, 671 (1962).
 - [8] J. Z. Gu and H. A. Weidenmüller, Nucl. Phys. A **690**, 382 (2001).
 - [9] H. A. Weidenmüller and G. E. Mitchell, Rev. Mod. Phys. **81**, 539 (2009).