

## Thermal noise engines

Laszlo B. Kish <sup>(a)</sup>

Texas A&M University, Department of Electrical Engineering, College Station, TX 77843-3128,  
USA; email: [Laszlo.Kish@ece.tamu.edu](mailto:Laszlo.Kish@ece.tamu.edu)

**Abstract.** Electrical heat engines driven by the Johnson-Nyquist noise of resistors are introduced. They utilize Coulomb's law and the fluctuation-dissipation theorem of statistical physics that is the reverse phenomenon of heat dissipation in a resistor. No steams, gases, liquids, photons, combustion, phase transition, or exhaust/pollution are present here. In these engines, instead of heat reservoirs, cylinders, pistons and valves, resistors, capacitors and switches are the building elements. For the best performance, a large number of parallel engines must be integrated to run in a synchronized fashion and the characteristic size of the elementary engine must be at the 10 nanometers scale. At room temperature, in the most idealistic case, a two-dimensional ensemble of engines of 25 nanometer characteristic size integrated on a 2.5x2.5 cm silicon wafer with 12 Celsius temperature difference between the warm-source and the cold-sink would produce a specific power of about 0.4 Watt. Regular and coherent (correlated-cylinder states) versions are shown and both of them can work in either four-stroke or two-stroke modes. The coherent engines have properties that correspond to coherent quantum heat engines without the presence of quantum coherence. In the idealistic case, all these engines have Carnot efficiency, which is the highest possible efficiency of any heat engine, without violating the second law of thermodynamics.

**Keywords:** Johnson-Nyquist noise, fluctuation-dissipation, energy-harvesting, quantum heat engines.

### 1. Introduction: recent applications of thermal noise

The thermal noise voltage of resistors [1,2] (Johnson-Nyquist noise) was discovered/understood several years later than the foundations of quantum physics [3]. Similarly, recent applications of the thermal noise for various tasks of unconventional informatics (features [4-6]; papers [7-10]), which are potential competitors of quantum communication and quantum computing, have emerged many years after the introduction of the corresponding quantum schemes. The situation is somewhat similar with the present paper. Some of the thermal noise engines (namely the coherent versions) we

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<sup>a)</sup> Until 1999: L.B. Kiss

introduce here have very similar properties to those of the coherent quantum heat engines that have been introduced and extensively featured during the last decade [11-15]. However, though this resemblance is interesting, the main goal of this paper is different: it is to show that the unavoidable nuisance, the thermal noise, can be put to work and it can drive a "clean", purely electrical heat engine when thermal gradient is present.

## 2. General considerations and assumptions

### 2.1 The source of the work in a thermal noise engine

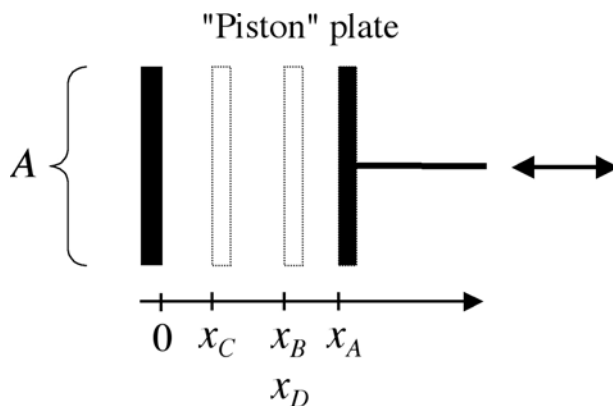
The key element is the Johnson-Nyquist noise, which is a stochastic voltage fluctuation, that appears between the terminals of an open-ended resistor. It is a manifestation of the thermal motion of electrons (and/or holes) in the resistor and it is mathematically described by the fluctuation-dissipation theorem [2,16]. When a resistor is shunted by a capacitor, the capacitor will be charged (thermalized) within the relaxation time constant  $RC$  to a steady-state, randomly fluctuating voltage level. In accordance with Boltzmann's energy equipartition theorem, this will result in  $kT/2$  mean energy  $E_C$  in the capacitor because the capacitor is an energy storage element with (one thermodynamical degree of freedom):

$$E_C = \frac{1}{2}kT = \frac{1}{2}C\langle U^2(t) \rangle = \frac{1}{2C}\langle Q^2(t) \rangle \quad , \quad (1)$$

where  $k=1.38*10^{-23}$  (J/K) is the Boltzmann constant;  $R$  is the resistance,  $C$  is the capacitance;  $U(t)$  is the instantaneous voltage amplitude on the capacitor;  $Q(t)$  is the corresponding instantaneous charge on the capacitor plates; and the symbol  $\langle \dots \rangle$  represents either time average in a single system, or ensemble average (at a given time moment) in a large number of identical independent systems. The capacitor plates are charged to oppositely with equal absolute instantaneous charge value, which, in accordance with Equation 1, is given as:

$$\langle Q^2(t) \rangle = kTC \quad (2)$$

In accordance with the Coulomb law, opposite charges will attract each other thus allowing one of the capacitor plates to move closer toward the other one will result in a positive mechanical work executed by the moving plate. Equation 2 indicates that the force is proportional to the absolute temperature therefore we can utilize this fact to construct electrical heat engines directly utilizing the fluctuation-dissipation theorem. Having a purely electrical heat engine has certain advantages. For example, no steams, gases, liquids, photons, phase transition, are used, no exhaust/pollution are present, and we can use *voltage*-controlled switches (*free from any mechanical motion*) instead of valves or levers to control the engine.



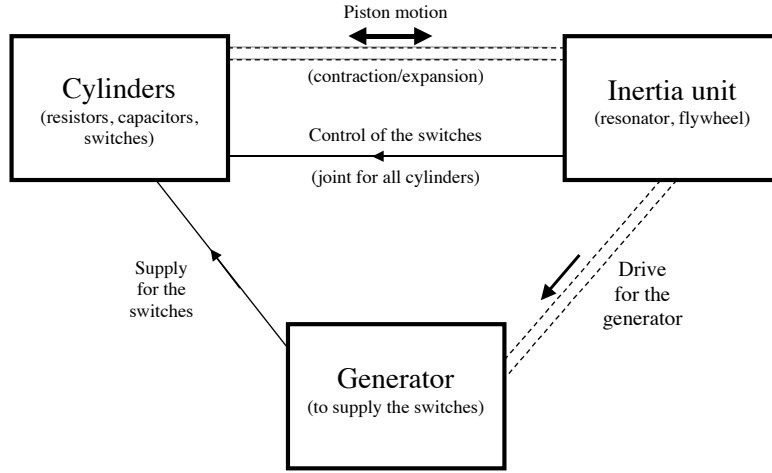
**Figure 1.** A parallel-plate capacitor's right plate (with plate size  $A$ ) is moving along the axis  $x$  and functioning as a "piston" driven by the resultant of the Coulomb force and external force. The left plate is fixed with its surface positioned at coordinate  $x=0$ . The other labels on the  $x$  coordinate and the pistons with dashed lines indicate the positions of the piston surface at the boundaries of different strokes in a 4-stroke cycle.

In the engines to be introduced below, the moving capacitor plate has the role of the piston and the unidirectional motion of the piston during a step within a full cycle will be called stroke, see Figure 1. We will show 4-stroke and 2-stroke cycles. Positive work is provided during strokes when the capacitor is contracting and negative work during expansion; situations opposite to the ones in usual heat engines.

## 2.2. Block diagram of thermal noise engines and the energy price of switch control

During the analysis of the engines below, similarly to published works on quantum heat engines, we focus on the physical principle and the fundamental properties of the idealized scheme, thus we generally do not discuss its engineering design or its mechanical engineering feasibility with today's nanomechanics technology. Still, we have to make one exception and analyze the energy dissipation of driving the ensemble of synchronized switches because of the large number of switches and the small work a single cylinder offers during a single cycle.

The block diagram of the generic thermal noise engine is shown in Figure 2. It is supposed that a large number  $N$  of parallel cylinders work in the engine to provide a large ensemble for a good statistics, deterministic operation, and negligible switching losses. The moving plates in the  $N$  independent capacitors are mechanically coupled to a single inertia unit, a *mechanical resonator* (not shown), which we call *flywheel* as in classical heat engines. It has sufficiently large inertia to keep a nearly constant angular frequency of the sinusoidal oscillation of the piston during the strokes with positive or negative work. During contraction, the cylinders contribute energy to the flywheel's kinetic energy and during expansion, the flywheel covers the necessary work on the capacitor plate against the Coulomb attraction. The electrical generator is a necessity because some electricity is needed to drive the electronic switches. Note, in a practical design, the generated electricity maybe the main energy output of the engine due to the high frequency of engine motion and piezoelectric generators are one of the candidates due to their high (>95%) efficiency [17].



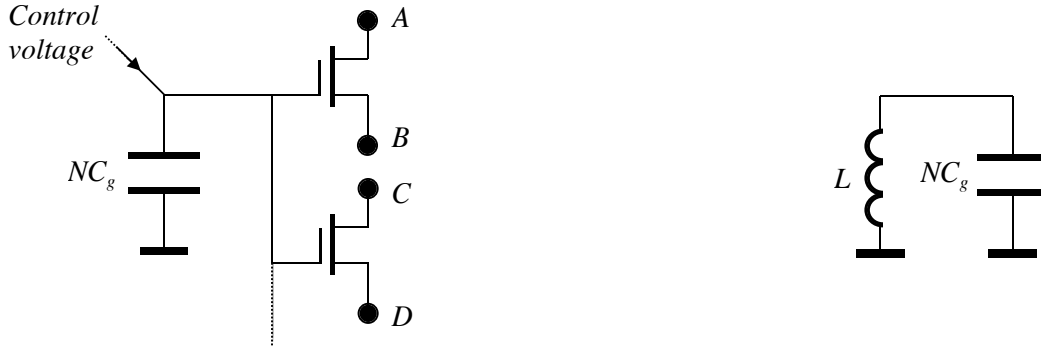
**Figure 2.** Block diagram of the generic thermal noise engine.  $N \gg 1$  cylinders are working parallel in a synchronous way and driving the inertia unit, which is a fly wheel in usual heat engines, however, in thermal noise engines, it would probably be a (nano-) mechanical resonator integrated on the chip. To see the way how to utilize the "space-time" periodicity of the system to reduce the energy need of driving the switches, see the text.

These engines are similar to the Maxwell demon [18] (and its newer versions such as the engines of Szilard [19], and Gea-Banacloche and Leff [20]) in one aspect: in a single "cylinder" of the engine, during a single cycle, the available work is in the order of  $kT$ . However, in all the other aspects, the thermal noise engines are very different from these demons because neither measurements nor related decisions and separate actions about the different cylinders are needed during the work cycle. In this way, the Carnot efficiency limit can be approached because, due to the synchronous operation of the cylinders and their large number, the energy requirement of driving the switches can be made a negligible, see below.

Due to the "space-time periodicity" of this system, that is, the parallel, synchronous and periodic operation of cylinders, and their large number  $N$ , the energy dissipation of switches is a negligible loss for the following reasons. First we show that the energy requirement for switching operations with controlled error probability is negligible. For the synchronized switches of the  $N$  cylinders, field effect transistors (MOSFETs, single-electron transistors, etc) can be used with parallel-connected gates where each drain/source electrode pair acts as a corresponding synchronized switch in one of the cylinders. The joint gate represents a single capacitor (see Figure 3) with  $kT/2$  total thermal energy, independently of the number  $N$  of synchronized switches and parallel gates. During the on/off operation of the switch, the control voltage on the capacitor must be alternated between two different levels, which implies an energy difference  $E_s$  necessary to drive the switch. To provide such an operation with error probability  $\varepsilon$ , the minimal energy is  $E_s = -kT \ln\left(\frac{\sqrt{3}}{2}\varepsilon\right)$  [21], which, for  $\varepsilon = 0.1, 0.01, \text{ and } 0.001$  yields  $E_s \approx 2.5kT, 4.8kT$  and  $7.1kT$ , respectively. Because this energy requirement is independent of  $N$ , in the case of sufficiently large number of cylinders, it is a

negligible loss compared to the total work offered by the  $N$  cylinders. On the other hand, increasing  $N$  at fixed energy in the resultant capacitor implies a decreasing control voltage on the capacitor. That means, beyond a certain  $N$  value, the voltage difference will not be enough to control the switches (to open or close the transistor). The voltage cannot be allowed to decrease beyond this limit and this requirement implies that, for greater  $N$  values, the energy loss will grow proportionally with  $N$ . However, the periodic operation in time can be utilized to decrease this energy loss: the joint gate capacitor can be connected parallel with an inductor to form an  $LC$  resonator with the same resonance frequency  $1/\sqrt{2\pi LC}$  as that of the cyclic motion of the engine. If the quality factor of the  $LC$  circuit is  $Q$  then the energy dissipation during a cycle will decrease by  $Q$ -fold. Supposing  $Q=100$ , for  $\varepsilon=0.1, 0.01$ , and  $0.001$  and using the energy limits given above, we get  $E_s \approx 0.025kT, 0.048kT$  and  $0.071kT$ , respectively, as the lower limits of switching energy loss.

In conclusion, in the large  $N$  and  $Q$  limit, the energy loss due to the operation of switches is negligible.



**Figure 3.** Running the  $N$  synchronized switches built of  $N$  separate MOSFET transistors with parallel gates by charging and discharging a single capacitor (resultant of  $N$  parallel gate capacitors). The switches are turned on/off in a synchronous fashion by controlling the voltage on the resultant of the gate capacitors. In the figure on the left, the MOSFET transistors (two shown) act as synchronized switches connecting  $A$  with  $B$  and  $C$  with  $D$  while the error-related energy requirement to control these switches remains the same as for a single switch. The time-periodicity of the switching operation allows more energy saving by utilizing a resonator with high quality factor, see figure on the right. In the case of 2 switches/cylinder only two inductors are needed in the whole engine of  $N$  cylinders.

### 2.3 Practical assumptions and their implications

These assumption hold for all the different engines and cycles to be shown below. The duration of a single stroke  $\tau_w$  and characteristic on/off transition time  $\tau_s$  of switches must satisfy:

$$\tau_s \ll RC_m < RC_M \ll \tau_w \quad (3)$$

where  $R$  is the driving resistance, and  $C_m$  and  $C_M$  are the minimum and maximum values of the capacitance during the cycle, respectively. The right inequality guarantees that the energy in the

capacitor is in thermal equilibrium (isothermal operation) during connected resistor and the left inequality guarantees that the switching is so abrupt that it does not interfere with the functioning of the engine. For the sake of simplicity (but without limiting generality) we suppose to have parallel-plate capacitors (Figure 1) with square-shaped plates and suppose that the distance  $x$  between the plates is always much less than the size  $A$  of the plates:

$$x \ll A \quad (4)$$

In this case, the capacitance and the attractive force  $F$  between the capacitor plates are:

$$C = \epsilon_0 A^2 / x \quad , \quad (5)$$

$$F = Q^2 / 2\epsilon_0 A^2 \quad (6)$$

where  $\epsilon_0$  is the dielectric constant of vacuum.

#### 2.4 On the size of the cylinders

Since the work output during a cycle does not depend on the size of cylinders, but on their numbers (see the details in Section 3), the cylinder size should be as small as possible. The small size makes large  $N$  and high cycle frequency, therefore, high power output possible. A lower limit of size is implied by the mean free path of charge carriers, which is in the order of 10 nanometers.

### 3. The standard thermal noise engine (non-coherent cylinder states)

#### 3.1 The cylinders

The *standard thermal noise engine* is the simplest version of these engines. The schematic of the "cylinders" (two of the  $N$  cylinders) is shown in Figure 4. Each cylinder has 4 elements: a "hot" resistor thermally contacted to the warm-source; a "cold" resistor thermally contacted to the cold-sink; the capacitor with the piston plate; and a 3-stage switch that is synchronized with the switches of the other cylinders and the flywheel (or mechanical resonator). The 3 stages of the switch are:

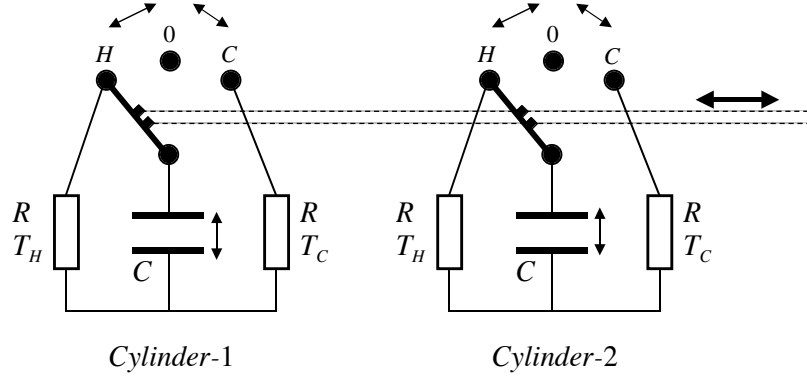
*H (Hot)*: The capacitor is connected to the "hot" resistor with temperature  $T_H$  for an isothermal stroke at temperature  $T_H$ .

0: The capacitor is open-ended for adiabatic stroke.

*C (Cold)*: The capacitor is connected to the "cold" resistor with temperature  $T_C$  for an isothermal stroke at temperature  $T_C$ .

The mean force provided by the pistons in the  $N$  parallel and synchronized cylinders is:

$$\langle F \rangle = N \frac{\langle Q^2 \rangle}{2\epsilon_0 A^2} = N \frac{C^2 \langle U^2 \rangle}{2\epsilon_0 A^2} = N \frac{kT}{2} \frac{C}{\epsilon_0 A^2} = N \frac{kT}{2} \frac{1}{x}. \quad (7)$$



**Figure 4.** Cylinders of the standard thermal noise driven engine. Only two cylinders are shown of the  $N$  identical and parallel units. The double dashed lines symbolize synchronous switch operation via a mechanical connection to the flywheel section (not shown). Similar synchronous mechanics connects the flywheel with the capacitor plates in the cylinders. The expansion and contraction of the capacitors is done in synchronized and continuous fashion while the switching steps are done in a synchronized and abrupt way at the beginning of each stroke.

### 3.2 The 4-stroke operation (Carnot-cycle)

The four-stroke operation is a Carnot-cycle [22], see its entropy-temperature graph in Figure 5. The cycle of the cylinders begins at point A, with switch in the position *Hot*, the temperature is  $T_H$ , and the piston is at position  $x_A$ . The 4 strokes are as follows:

1. Stroke  $A \Rightarrow B$ : Isothermal contraction, with switch in the position *Hot*, to  $x_B = x_A / \alpha$  (where  $\alpha > 1$ ). The engine will execute a positive work:

$$W_{AB} = T_H (S_B - S_A) = - \int_{x_A}^{x_B} N \frac{kT_H}{2x} dx = N \frac{kT_H}{2} \ln \left( \frac{x_A}{x_B} \right) = N \frac{kT_H}{2} \ln \alpha \quad (8)$$

This process is isothermal thus the required heat  $Q_{AB}$  from the warm-source is equal to the work produced:

$$Q_{AB} = W_{AB} = \frac{Nk}{2} T_H \ln \alpha \quad (9)$$

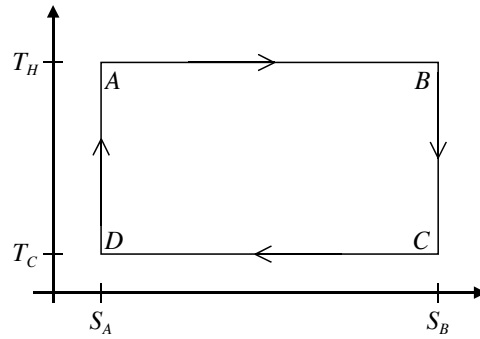
2. Stroke  $B \Rightarrow C$ : Adiabatic, reversible ( $S_C = S_B$ ) contraction, with switch in the position 0, to  $x_C = x_B / \beta$ , where  $\beta (> 1)$  is chosen properly to decrease the equivalent temperature of the system of  $N$  capacitors from  $T_H$  to  $T_C$  and their energy from  $NkT_H/2$  to  $NkT_L/2$ . The engine will execute a positive work  $W_{BC} = Nk(T_H - T_L)/2$ .
3. Stroke  $C \Rightarrow D$ : Isothermal expansion, with switch in the position  $C$  (*Cold*, the temperature is  $T_C$ ), to  $x_D = x_C \alpha$ . The work of the engine will be negative (the engine must use some of the flywheel's kinetic energy for expansion):

$$W_{CD} = T_C(S_D - S_C) = - \int_{x_C}^{x_D} N \frac{kT_C}{2x} dx = N \frac{kT_C}{2} \ln\left(\frac{x_D}{x_C}\right) = -N \frac{kT_C}{2} \ln \alpha \quad (10)$$

This process is isothermal thus the engine converts this work into heat  $Q_{CD}$  which is dissipated in the resistors  $R_C$ , and absorbed by the cold-sink:

$$Q_{CD} = -W_{CD} = \frac{Nk}{2} T_C \ln \alpha \quad (11)$$

4. Stroke  $D \Rightarrow A$ : Adiabatic, reversible ( $S_A = S_D$ ) expansion, with switch in the position 0, to  $x_A = x_D \beta$ . This is the exactly reversed operation of the second stroke thus the equivalent temperature of the system of  $N$  capacitors will increase to  $T_C$  from  $T_H$ . The work of the engine will be negative  $W_{DA} = -W_{BC}$ . Thus the total work of the engine during the 2nd and the 4th strokes is zero because of the reversed operations.



**Figure 5.** Entropy-temperature graph of the standard engine in Carnot-cycle operation mode (four-strokes).  $AB$  stroke: isothermal contraction (executing positive work);  $BC$  stroke: adiabatic contraction (executing positive work);  $CD$  stroke: isothermal expansion (negative work);  $DA$  stroke: adiabatic expansion (negative work exactly compensating the positive work during  $BC$ ).

In conclusion, the total work produced by of the 4-stroke operation during a full cycle is:

$$W_{tot} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = \frac{Nk}{2} (T_H - T_C) \ln \alpha \quad (12)$$



The total input heat consumed from the warm-source is:

$$Q_H = \frac{Nk}{2} T_H \ln \alpha \quad , \quad (13)$$

and the waste heat, which is transmitted to the cold-sink, is:

$$Q_C = Q_H \frac{T_C}{T_H} \quad (14)$$

The energy efficiency of the engine:

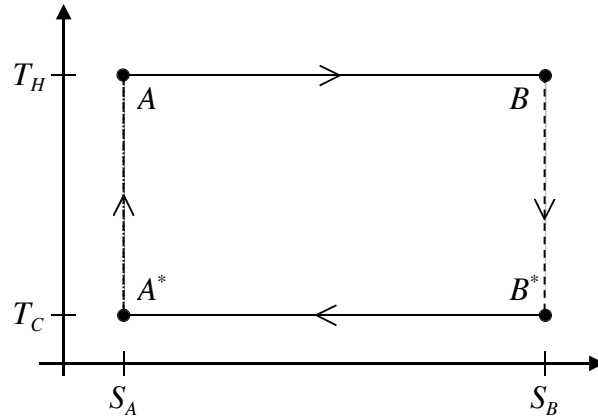
$$\eta_{clas} = \frac{W_{tot}}{Q_H} = 1 - \frac{T_C}{T_H} \quad (15)$$

which is exactly the efficiency of the idealistic Carnot-engine [22]; the highest possible efficiency of any heat engine without violating the second law of thermodynamics.

### 3.2 The 2-stroke operation

The 2nd stroke, the adiabatic contraction, and its reversed operation, the 4th stroke, can be left out from the cycle. The switch is alternating between the  $H$  and  $C$  positions and the 0 position is not used. After a completed stroke, the switch position is alternated and the capacitor system will be "thermalized" (heated or cooled, accordingly) to the new temperature with the actual  $RC$  relaxation time constant before the next stroke takes place. Relation 3 guarantees that these heating and cooling processes are much shorter than the stroke durations thus they can be done at the beginning/end of the stroke without the interruption of the mechanical motion of the engine. The entropy-temperature graph is given in Figure 6. The piston positions are:

$$x_B = x_A / \alpha \quad , \quad x_{B^*} = x_B \quad , \quad x_{A^*} = x_A \quad (16)$$



**Figure 6.** Entropy-temperature graph of the two-stroke thermal noise engine. The full cycle contains 4 steps but only 2 strokes: the isothermal constriction ( $AB$  stroke) and the isothermal expansion ( $B^*A^*$  stroke). The cooling ( $BB^*$ ) and heating ( $A^*A$ ) steps are done without piston motion and work. The solid lines represent the same strokes and piston motion as in the 4-stroke cycle. The dashed lines represent the cooling and the heating steps without motion and work.

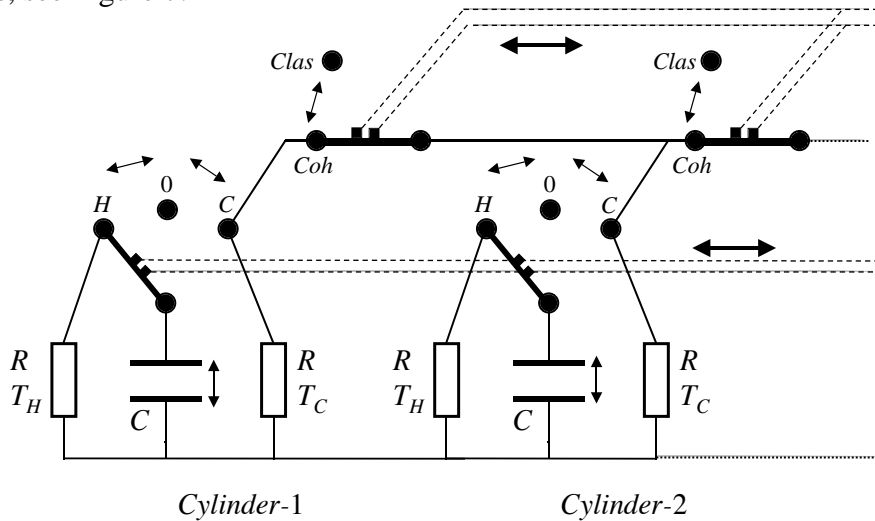
The heat transfer and the net work are zero during the omitted strokes, the total energy and work balance and the energy efficiency remain the same, as described by Equations 12-15.

#### 4. Thermal noise engines with coherent (correlated) cylinder states

The coherent thermal noise engine is inspired by coherent quantum heat engines [11-15]. During the stroke where the work of the engine is negative ( $CD$  stroke in Figure 5 and  $B^*A^*$  stroke in Figure 6), the original number  $N$  of thermodynamical degrees of freedom is reduced to one by switching all the capacitors parallel to form a single capacitor with  $N$ -times greater capacitance. Thus, in large systems, the negative work component approaches zero and the engine *seemingly* provides efficiency better than the Carnot limit and *seemingly* can work with a single temperature. For the resolution of this paradox, see the section "*The catch...*" below.

##### 4.1. Cylinders and operation: 2-heat-reservoir and single-heat-reservoir engines

The schematic of the "cylinders" (two of the  $N$  cylinders) is shown in Figure 4. Each cylinder has 5 elements: the same as the 4 elements that applied in the standard engine (see Figure 4) and an extra "coherence" switch that, in the "*Coh*" stage, connects the capacitor in parallel with the capacitors in the other cylinders. Similarly to the original switch, the coherent switches are also synchronized in all the cylinders, see Figure 7.



**Figure 7.** The coherent engine. During the isothermal expansion stroke ( $CD$  in Figure 5 and  $B^*A^*$  in Figure 6, when the engine consumes work from the flywheel/resonator unit) the upper switches connect all the capacitors parallel in their *Coh* (Coherent) position. Then the thermal noise in all the capacitors will be identical, coherent. For further clarification, see Section 4.2. During the rest of the strokes these switches are in the *Clas* (Classical) position and the operation of the

engine remains the same as in the standard engine.

Thus, in the "Coh" stage, the number of thermodynamical degrees of freedom in the system is reduced from the original number  $N$  to 1, and that yields the reduction of force and the negative work by the same factor. Therefore, in the coherent engine, the negative work, instead of Equation 11, becomes:

$$W_{CD,coh} = -Q_{CD} = -\int_{x_C}^{x_D} \frac{kT_C}{2x} dx = \frac{kT_C}{2} \ln\left(\frac{x_D}{x_C}\right) = -\frac{kT_C}{2} \ln \alpha \quad . \quad (17)$$

The total work of the coherent engine is greater than in Equation 12:

$$W_{tot,coh} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = \frac{1}{2} k(N T_H - T_C) \ln \alpha \quad , \quad (18)$$

and, because the input remains the same as described by Equation 13, the energy efficiency appears to be greater than the Carnot-efficiency and, for large  $N$ , it approaches 100% which contradicts to the Second Law of Thermodynamics (for clarification, see below):

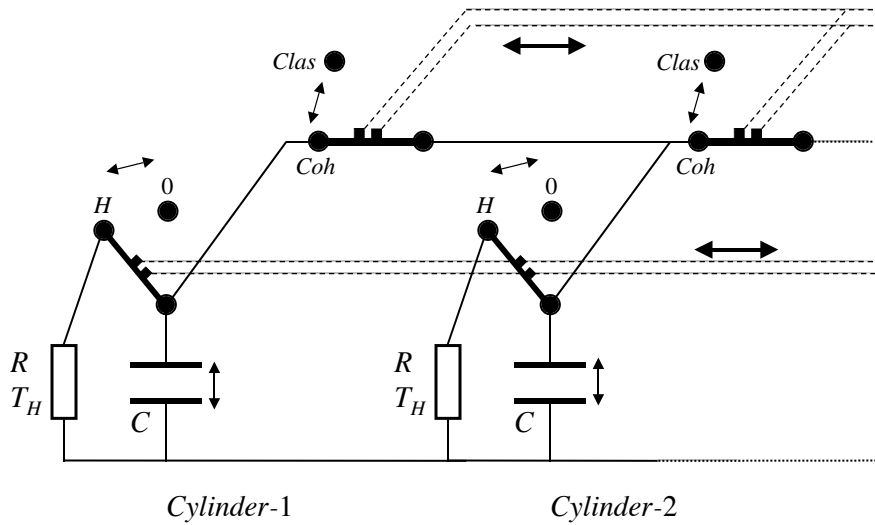
$$\eta_{coh} = \frac{W_{tot,coh}}{Q_H} = \frac{k(N T_H - T_C)}{N k T_H} = 1 - \frac{T_C}{N T_H} \quad (19)$$

Similar statements had emerged about quantum-heat-engines before the issues got clarified [11-15]. But the analogy goes further: the above equations clearly indicate that the coherent engine can produce useful work and has seemingly near 100% efficiency even when  $T_H = T_C$ , which are a direct violations of the Second Law:

$$W_{tot,coh,T_H} = \frac{1}{2} k T_H (N - 1) \ln \alpha \quad \text{and} \quad (20)$$

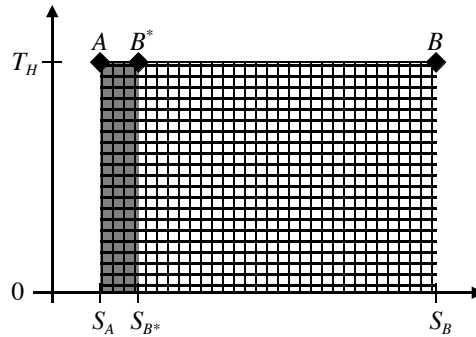
$$\eta_{coh,T_H} = \frac{W_{tot,coh}}{Q_H} = \frac{k T_H (N - 1)}{N k T_H} = 1 - \frac{1}{N} \quad (21)$$

Accordingly, a simplified coherent engine, with seemingly a single heat reservoir, can be constructed, see Figure 8, that is also described by Equations 20-21.



**Figure 8.** Coherent engine with (seemingly) a single heat reservoir. During the isothermal expansion stroke ( $CD$  in Figure 5 and  $B^*A^*$  in Figure 6, when the engine consumes work from the flywheel/resonator unit) the upper switches connect all the capacitors parallel in their  $Coh$  (Coherent) position and the lower switch is in the  $H$  position.

Figure 9 shows the temperature-entropy graph of the single-heat-reservoir-two-stroke engine with coherent cylinders during the stroke with negative work. There is a positive work during the  $AB$  stroke. Then, when the coherence is established, the system jumps to point  $B^*$  and the second stroke  $B^*A$  needs a reduced work from the inertia system and, accordingly, a reduced entropy change to restore the initial condition  $A$ .



**Figure 9.** The entropy-temperature graph of the two-strokes engine with a single heat reservoir and coherent (correlated) cylinders, cf with the non-coherent version in Figure 6. The first stroke goes from  $A$  to  $B$  and executes the work represented by the area below the  $AB$  section (grid pattern). Then, at the introduction of coherence, the entropy jumps from  $B$  to  $B^*$  and the second stroke goes from  $B^*$  to  $A$  and it requires the smaller amount of work represented by the area below the  $B^*A$  section (gray filling).

#### 4.2. The catch: where does the entropy and the heat go?

To solve the paradox of the seeming violation of the Second Law by the coherent engines, first the following question must be answered: *Where does the entropy (see Figure 6) and the energy go when the coherence is established switched on?* Due to Equation 1, at point  $B$ , the *rms* charge in a

single capacitor is:

$$\sqrt{\langle Q_i^2 \rangle_N} = \sqrt{kT_H C} \quad (22)$$

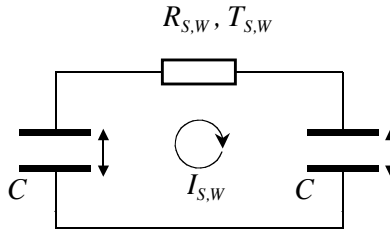
where the average is taken over the ensemble of the  $N$  capacitors. At the moment when the coherence is switched on and the system jumps from  $B$  to point  $B^*$ , due to  $\langle Q_i \rangle_N = 0$  and the charge conservation the central limit theorem applies for the effective charge at  $B^*$  in the resultant single capacitor with capacitance  $C_{B^*} = NC$ :

$$Q_{B^*} = \sqrt{NkT_H C} \quad , \quad (23)$$

and

$$E_{B^*} = \frac{Q_{B^*}^2}{2C_{B^*}} = \frac{NkT_H C}{2NC} = \frac{kT_H}{2} . \quad (24)$$

That means,  $(N - 1)kT_H / 2$  energy disappeared from the system and, because the same result holds even when no resistors are connected to the capacitors, the disappeared energy does not go back to the heat-source. The solution is well known, namely, that the energy at charge equilibration in parallel capacitors is dissipated in the stray resistance of the connecting wires (and switches) and that the energy loss is independent of the actual stray resistance values. This fact indicates that it is necessary to install a heat reservoir (with temperature  $T_{S,W}$ ) around the wires and the coherence switches otherwise they will overheat. This originally hidden heat reservoir and the stray resistances  $R_{S,W}$  will also act as thermal noise generators and cause thermal noise loop-currents through the capacitors, in accordance with the fluctuation-dissipation theorem, see Figure 10. The system of parallel capacitors will be thermalized to temperature  $T_{S,W}$  in a very short time, in the order of  $R_{S,W}C$  which is the shortest electrical time constant in the system.



**Figure 10.** The loop currents generated by the thermal noise of stray resistance of switches and wires.

Thus, equations 17-21 and the enhanced properties of the coherent engines are valid only with idealistic switches and wires that have zero thermal noise. This condition is satisfied only at absolute zero temperature,  $T_{S,W} = 0$  Kelvin. This is an indication that, when the coherence is introduced with the assumed properties, a new, hidden thermal reservoir around the wire connections is implicitly

introduced and that takes over the role of the cold sink during the coherent stroke. In this way, Carnot's work and efficiency are restored and the Second Law is satisfied. The relevance of this explanation for the case of coherent quantum-heat engines is also obvious: the thermal reservoir (or possibly a heat pump) around the coherent medium is necessary to remove the exhaust heat and this new heat reservoir will play the role of the cold sink during the coherent stroke and preserves the Second Law and the Carnot properties.

The coherent engine turns out to be not better than the standard engine. However, at certain practical designs the fact may be utilized that the connecting wires and switches can be used as cold resistors and connected to the cold sink in a coherent engine.

### **5. Estimation of work and efficiency at an idealistic example**

Suppose a 1 square inch (2.5x2.5cm) chip on which thermal noise engines of 25 nanometer effective size are integrated, thus  $N=10^{12}$  thus the Carnot efficiency can be approached if the quality factors of the LC and mechanical resonators are sufficiently large which is a reasonable assumption. At this length scale, it is practical to suppose 10 GHz mechanical resonator frequency which is the cycle frequency of the engines. The noise bandwidth of electronic element of this size is >500 GHz thus Equation 3 is satisfied. Let us suppose we want to harvest the energy of a source with human body temperature (37 °C) and the cold-sink is at room temperature, 25 C. Then the Carnot-efficiency is only 3.9%. The work by a single cylinder of contraction rate  $\alpha = 0.5$  during a single cycle is  $8.2 \cdot 10^{-23}$  J and because each of the  $10^{12}$  cylinders on the chip executes  $10^{10}$  cycles in a second, the power of the engine is 0.42 Watt. Note, the engine will extract 21.4 Watt thermal energy from the source due to the low Carnot efficiency at the given conditions.

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