Capital allocation for credit portfolios under normal and stressed market conditions

Norbert Jobst & Dirk Tasche, Lloyds Banking Group[*]

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Abstract

If the probability of default parameters (PDs) fed as input into a credit portfolio model are estimated as through-the-cycle (TTC) PDs stressed market conditions have little impact on the results of the capital calculations conducted with the model. At first glance, this is totally different if the PDs are estimated as point-in-time (PIT) PDs. However, it can be argued that the reflection of stressed market conditions in input PDs should correspond to the use of reduced correlation parameters or even the removal of correlations in the model. Additionally, the confidence levels applied for the capital calculations might be made reflective of the changing market conditions. We investigate the interplay of PIT PDs, correlations, and confidence levels in a credit portfolio model in more detail and analyse possible designs of capital-levelling policies. Our findings may of interest to banks that want to combine their approaches to capital measurement and allocation with active portfolio management that, by its nature, needs to be reflective of current market conditions.

1   Introduction

Economic Capital (EC) is typically used within banks to

- determine the amount of capital needed in order to ensure solvency,

- allocate actual capital to business units in a systematic manner, and

- facilitate origination activities by consistently assessing different opportunities within a RAROC (Risk Adjusted Return on Capital) framework.

Key ingredients to the computation of EC and RAROC are estimates for the underlying assets' probability of default (PD), loss-given-default (LGD), and correlation parameters.

Furthermore, when setting up an EC framework, it is commonly believed that these input parameters should be so-called through-the-cycle (TTC) estimates. In practice, the estimates

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are frequently derived as "historic average" estimates based on several years of observation resulting in fairly stable capital requirements for banks. While stable capital requirements are clearly desirable for a bank's longer term planning process and actual operations, producing true TTC parameter estimates proves challenging.

In practice, banks often operate several PD and LGD models, and the extent to which these models display a true TTC behaviour varies considerably. In contrast to TTC estimates, point-in-time (PIT) estimates are based on current economic conditions and aim to provide more accurate, short term estimates of an obligor's default likelihood and losses. However, most models in practice are hybrid in nature, and, henceforth, the capital requirements as determined bottom up through usage of typical credit portfolio models (e.g. KMV, CreditMetrics) may vary accordingly and fail to reflect the true risk.

In this chapter, we investigate whether or not a stable level of economic capital can be achieved within a PIT modelling environment. There is some literature on the properties of PIT and TTC PD estimates (e.g. Heitfield, 2005) but not much has been published on a definition that can be used in practice to characterise PIT or TTC estimates. We therefore begin by considering and comparing methods for PIT and TTC PD estimates and the connection between PIT PD estimates and credit portfolio risk modelling. It turns out that full consistency of PD estimation and portfolio modelling is feasible in principle but unlikely to be achieved in practice. We then demonstrate the consequences of being consistent and failing to be consistent in portfolio risk modelling and PD estimation by a numerical example. We also demonstrate the effect of one possible work-around, the "time-varying target solvency probability" as suggested by Gordy and Howells (2006) in the context of stabilising regulatory capital requirements.

2 TTC vs. PIT PD modelling

TTC estimates are appropriate for stabilising capital requirements in the mid and long term. Use of TTC estimates, however, risks to overlook immediate and near-term developments. Particularly in a crisis situation, an institution might suffer significant losses when relying on TTC estimates only. PIT estimates are appropriate when a realistic short to mid term view of risk is required. Ideally, PIT and TTC estimates would be consistent for longer horizons in that long-horizon PDs converge to TTC PDs whereas short-term estimates are more reflective of current economic conditions.

A risk measurement policy for a financial institution therefore could include the following elements:

- Capital requirements: Based on TTC estimates.

- Short term lending: Rely on PIT estimates and PIT risk assessments for decision making.

- Mid to long term lending: Rely on TTC estimates and TTC risk assessments for decision making although near-term fluctuations should be considered as indicated above.

- Credit extensions and limit management for existing obligors: Look at PIT estimates and PIT risk assessments to identify obligors who are temporarily high risk (e.g. with PIT PD of 5% or more).

Key for the implementation of a policy with such elements is the ability to estimate PIT and TTC PDs. Aguais et al. (2006) describe an approach to PIT PD modelling which is based on a structural model and uses segment-wise average indices of the so-called distance-to-default to infer the current state in the credit cycle. This approach is very attractive as it provides a way to determine TTC PD estimates at the same time. In addition, this approach ties in very naturally with the estimation of a correlation structure for a credit portfolio model.

From a causality perspective – why are obligors more vulnerable in certain time periods? – PIT PD estimation approaches based on macroeconomic factors like GDP growth are also quite interesting. Engelmann and Porath (2003), for instance, suggest that a proportional hazard rate model can be utilised very efficiently to include macroeconomic factors and thus provide PIT PD estimates.

The further discussion in this chapter draws mainly on the estimation approach suggested by Hamerle et al. (2004). This approach is based on logit or probit regression that includes macreeconomic factors but also a latent factor to better capture correlation. We adopt here the simpler version of the methodology without latent factor because it provides an intuitive framework that allows a consistent approach to PIT and TTC PD estimation on the one hand and credit portfolio risk modelling on the other hand.

The need for such a consistent approach is intensified by increased regulatory requirements for stress testing. Clarity with respect to PIT and TTC risk assessments is required to avoid "double stressing" and to more efficiently implement specific stress scenarios.

A policy to achieve consistency of PIT and TTC risk assessments could include the following elements:

- Estimate PIT PDs with (e.g.) the Hamerle et al. (2004) probit approach.

- Transform PIT PDs into TTC PDs (e.g., as described in section 3 below).

- Use the factor structure as provided by the probit regression as the factor model for the credit risk portfolio model. Note that this is different to typical portfolio models where frequently factor models based on equity or default correlations are used and estimated using long periods of historic data resulting in predominantly TTC correlation parameters.

- To study stress scenarios, evaluate certain factors or apply the distribution truncation approach as suggested by Bonti et al. (2006).

3 Portfolio credit risk modelling and PD estimation

The purpose of this section is to describe an ideal situation such that portfolio credit risk modelling and PD estimation are perfectly consistent. While such a situation is unlikely to be

found in practice, knowing an example of how it could look like nonetheless might help to improve consistency between the portfolio model in place and the way parameter estimation is done. In addition, the simple setting of this section should help to clarify the notions of point-in-time (PIT) and through-the-cycle (TTC) PDs.

3.1 A simple credit portfolio model

For the purpose of illustration we consider a simple default-mode Gaussian copula credit portfolio model with deterministic loss severities. In this model, the portfolio-wide loss realised in one observation period can be described as follows:

$$L = \sum_{i=1}^{N} u_i \, \mathbf{I}(D_i)$$

$$D_i = \left\{ \sqrt{1 - \varrho_i^2} \, \xi_i + \varrho_i \, w_i'S \le T_i \right\} \tag{1}$$

Explanation of the notation used in (1):

- L is portfolio-wide realised loss if we assume that losses occur only as a consequence of default and loss given default is deterministic.

- N is the number of obligors in portfolio.

- u_i is the (deterministic) loss in case of default associated with obligor i. In this chapter, we consider relativ losses, i.e. we assume $\sum u_i = 1$.

- D_i denotes the event that obligor i defaults.

- $\mathbf{I}(A)$ is the indicator function of the event A, i.e. $\mathbf{I}(A) = 1$ if A occurs and $\mathbf{I}(A) = 0$ otherwise.

- $S = (S_1, \ldots, S_k)$ is a random vector of *systematic factors*, assumed to be multi-variate normal with mean vector 0. The number k of factor usually is small (e.g. not greater than 10). The factors could be GDP growth, change in unemployment rate, or something similar.

- $w_i = (w_{i1}, \ldots, w_{ik})$ is the vector of weights of the systematic factors associated with obligor i. The weights are assumed to be normalised such that $\mathrm{var}[w_i'S] = 1$ ($w_i'S = \sum_{j=1}^{k} w_{ij} \, S_j$ denotes the Euclidian inner product).

- ξ_i is the *individual risk factor* associated with obligor i, assumed to be standard normal and stochastically independent of all other random variables in the model. $X = (\xi_1, \ldots, \xi_N)$ denotes the vector of all ξ_i.

- $\varrho_i \in [-1, 1]$ denotes the *sensitivity* of obligor i to the systematic risk. $\varrho_i = 0$ means that the obligor i's wealth is not vulnerable with regard to systematic risk (obligor is acyclical). $\varrho_i = 1$ means that obligors i's wealth is completely determined by the systematic risk. $\varrho_i < 0$ would imply that obligor i is counter-cyclical.

- T_i is obligor i's *default threshold*. $T = (T_1, \ldots, T_N)$ denotes the vector of all T_i.

Usually only the individual factors ξ_i and the vector S of systematic factors are considered random elements. The default threshold T_i is determined individually for each single obligor but often not considered random although it may change year on year if the obligor's perceived creditworthiness changes. *In this chapter we allow for T_i to be a random variable as this helps to consider some issues more rigorously.*

TTC portfolio risk measurement. The distribution of the portfolio loss $P[L \leq \ell], 0 < \ell < 1$, usually is estimated from an artificially generated sample of loss realisations – by Monte-Carlo simulation of realisations of the individual risk factors ξ_i and the systematic factors S but usually not the thresholds T_i. This process is equivalent to trying to calculate the probabilities $P[L \leq \ell]$ by taking the expectation of the event $\{L \leq \ell\}$ with respect to X and S. Technically speaking, as the loss $L = L(X, S, T)$ is a function of the random vectors X, S, and T, taking the expectation results in

$$\int \mathbf{I}_{[0,\ell]}\big(L(x, s, T)\big) P_{(X,S)}(dx, ds) \;=\; P\big[L(X, S, t) \leq \ell\big]\big|_{t=T} . \tag{2}$$

The right-hand side of (2) is not the unconditional probability $P[L \leq \ell]$ but – if we assume that the risk factors X and S and the thresholds T are stochastically independent – can be interpreted as the conditional probability $P[L \leq \ell \,|\, T]$. The assumption of stochastic independence is crucial for this interpretation. In general the formula for the conditional probability $P[L \leq \ell \,|\, T]$ is given by

$$P[L \leq \ell \,|\, T] \;=\; \int \mathbf{I}_{[0,\ell]}\big(L(x, s, T)\big) P_{(X,S)\,|\,T}(dx, ds). \tag{3}$$

This differs from the left-hand side of (2) by the fact that the integration is done with respect to the conditional distribution $P_{(X,S)\,|\,T}$ of (X, S) given T which in general – unless (X, S) and T are stochastically independent – is not the same as the unconditional distribution $P_{(X,S)}$ of (X, S). But it is the unconditional distribution of (X, S) that is approximated by the Monte-Carlo simulation. *Hence Monte-Carlo simulation of (X, S) with fixed T as described above yields an economically interpretable result if and only if the risk factors (X, S) and the threshold T can be assumed to be stochastically independent.*

Note that integrating with respect to the systematic factors means that a cycle-neutral (or TTC) perspective on the risks is adopted. Clearly, TTC PDs should be used for this kind of calculation.

The random variables in the definition of the default event D_i in (1) are normalised in such a way that $\sqrt{1 - \varrho_i^2}\, \xi_i + \varrho_i\, w_i' S$ is standard normal and hence the TTC PD of obligor i – more correctly the PD of obligor i conditional on T_i which is cycle-neutral after integration with respect to S – is[1]

$$PD_i = P[D_i \,|\, T_i] = \Phi(T_i). \tag{4}$$

PIT portfolio risk measurement. In the model framework given by (1), a PIT view on the actual risk of the portfolio can be achieved in two ways:

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[1] Φ denotes the standard normal distribution function.

- By taking the expectation of the event $\{L \leq \ell\}$, $0 < \ell < 1$, with respect to X and S as described above for TTC risk measurement, but with a restricted range for S to specify specific economic conditions. Details about this approach can be found in Bonti et al. (2006).

- By fixing a realisation s of the systematic risk factor vector S and taking the expectation of the event $\{L \leq \ell\}$, $0 < \ell < 1$, with respect to X only – while keeping the values of the T_i constant at the same time. As the vector X of individual risk factors is assumed to be independent of S and T, this procedure gives estimates of the conditional distribution $\mathrm{P}\big[L \leq \ell \,|\, S, T\big]$, $0 < \ell < 1$.

In section 3.2 we will see that in both approaches for the PIT risk view TTC PDs should be used as input to the portfolio model.

3.2   The probit regression model for PD estimation

We describe in this section an approach to PD modelling and estimation that is perfectly consistent with the portfolio model (1). The approach we have chosen is essentially a simplified version of the probit estimation suggested by Hamerle et al. (2004, without latent factor).

We consider obligors in a fixed segment, defined say by industry or region. With each obligor i, we associate a vector of *characteristic factors* $F_i = (F_{i1}, \ldots, F_{im})$ (m does not depend on i), $i = 1, \ldots, N$. Typically, F_{ij} will be a financial ratio from the balance sheet or a qualitative score like management quality. Denote by F the random matrix $F = (F_1, \ldots, F_N)$.

The point-in-time PDs resulting of a probit regression of default event indicators on the risk factors F_i (different for different obligors) and the vector of systematic factors S (the same vector for each obligor) can then be characterised as

$$PD_i(F, S) \;=\; \Phi(a_0 + a'F_i + b'S), \tag{5}$$

with a_0 being a constant and a and b being constant vectors of appropriate dimension (estimated by regression).

To see the connection between the probit model (5) and the Gaussian copula credit portfolio model (1) it is convenient to rewrite (5) as a probability conditional on the factor vectors F_i and S. Denote by ξ_i a standard normal random variable as in subsection 3.1 (with the same interpretation as individual risk factor). Then we have

$$
\begin{aligned}
PD_i(F, S) &= \mathrm{P}\big[\xi_i \leq a_0 + a'F_i + b'S \,|\, F_i, S\big] \\
&= \mathrm{P}\left[\frac{\xi_i - b'S}{\sqrt{1 + \mathrm{var}[b'S]}} \leq \frac{a_0 + a'F_i}{\sqrt{1 + \mathrm{var}[b'S]}} \,\Big|\, F_i, S\right] \\
&= \mathrm{P}\big[\sqrt{1 - \varrho^2}\,\xi_i + \varrho\,w'S \leq T_i \,|\, F_i, S\big],
\end{aligned} \tag{6}
$$

where

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- the sensitivity ϱ is defined by $\varrho = \frac{\sqrt{\mathrm{var}[b'S]}}{\sqrt{1+\mathrm{var}[b'S]}}$;

- the weight vector w is defined by $w = -\frac{b}{\sqrt{\mathrm{var}[b'S]}}$;

- the default threshold T_i is defined by

$$T_i = (a_0 + a'F_i)\sqrt{1 - \varrho^2} = \frac{a_0 + a'F_i}{\sqrt{1 + \mathrm{var}[b'S]}}. \tag{7}$$

Proposition 1 *Assume that the same vector of systematic factors S is used both for portfolio risk modelling and for PIT PD estimation and that the constants in the description of the default event D_i in (1) are not allowed to differ between different obligors but only between different segments of obligors. Then there is a one-to-one relation between the constants ϱ and w and the thresholds T_i in the default events D_i in (1) on the one hand and b, $\mathrm{var}[b'S]$, and $(a_0 + a'F_i)_{i=1,\ldots,N}$ on the other hand.*

Proof. We have seen above how ϱ, w and the thresholds T_i are determined by b, $\mathrm{var}[b'S]$, and $a_0 + a'F_i$. Conversely, if ϱ, w and the thresholds T_i are given then b, $\mathrm{var}[b'S]$, and $a_0 + a'F_i$ can be determined by

$$\mathrm{var}[b'S] = \frac{\varrho^2}{1 - \varrho^2}$$
$$b = -w\,\frac{\varrho}{\sqrt{1 - \varrho^2}} \tag{8}$$
$$a_0 + a'F_i = \frac{T_i}{\sqrt{1 - \varrho^2}}.$$

This provides the other direction of the one-to-one relation. □

While we have shown how to parameterise the portfolio model (1) from probit estimators (5) and to derive a PIT probit estimator from the parameterisation of (1), to show full consistency of the model (1) and the probit PD estimator (5) we have to prove that indeed the threshold T_i from (7) is related to a TTC PD that derives in an intuitive way from the PIT PD (6). The following proposition shows that this consistency property of the portfolio model and the probit PD estimation can indeed be obtained if the risk factors F_i and the systematic factors S are independent (i.e. the risk factors are through-the-cycle).

Proposition 2 *Assume that the risk factors $F_i = (F_{i1}, \ldots, F_{im})$ and the systematic factors $S = (S_1, \ldots, S_k)$ are stochastically independent. Then the TTC PD corresponding to (5) by integration with respect to the systematic factor vector S is given by (4) with threshold T_i as defined by (7).*

Proof. The TTC PD corresponding to a PIT PD is derived from the PIT PD by integration with respect to the systematic factors over the full range they can take on. By independence of

F_i and S and the fact that $b'S$ is normally distributed we can calculate the TTC PD for (5) as follows

$$
\begin{aligned}
PD_i^{\mathrm{TTC}}(F, S) &= \int PD_i(F, x)\, P_S(dx) \\
&= \int \Phi(a_0 + a'F_i + y)\, P_{b'S}(dy) \\
&= \int \mathrm{P}\big[\xi_i \le a_0 + a'F_i + b'S \,|\, b'S = y\big]\, P_{b'S}(dy) \\
&= \mathrm{P}\big[\xi_i - b'S \le a_0 + a'F_i\big] \\
&= \Phi\left(\frac{a_0 + a'F_i}{\sqrt{1 + \mathrm{var}[b'S]}}\right).
\end{aligned}
$$

In this calculation the random variable ξ_i is assumed to be standard normal and independent of all other random variables. By definition of the default threshold T_i, we then have $\Phi\left(\frac{a_0 + a'F_i}{\sqrt{1 + \mathrm{var}[b'S]}}\right) = \Phi(T_i)$. This completes the proof. $\qquad\square$

In practice, integration of (5) with respect to the vector S of systematic factors might be approximated by averaging the values of (5) across a time series with realisations of the systematic factors in both benign and poor economic conditions.

At first glance, the result of proposition 2 seems more or less self-evident. However, in practice most of the time not all assumptions underlying proposition 2 are satisfied such that consistency of portfolio modelling and PIT PD estimation might not be obtained.

- The vector of systematic factors S might not be normally distributed in reality.

- PIT PDs are not necessarily estimated by probit estimation but by logit estimation or other methods (see, e.g. Tasche, 2009). While this could be accounted for in the portfolio model by choosing another distribution for the individual risks, nonetheless a lot of further adjustments would be necessary which would make the model much less straight-forward.

- A bank typically operates several different PD models, some of which may be expert driven and less suitable for statistical modeling.

- It is unlikely that the characteristic factors F_i and the systematic factors S are stochastically independent. As a consequence, the portfolio loss distribution $\mathrm{P}[L \le \ell \,|\, T]$ when calculated according to (2) might be upward or downward biased, depending on the position in the credit cycle when the calculation is conducted.

Remark 1 *There are two other interesting consequences of proposition 1 on the one hand and proposition 2 and (7) on the other hand.*

1) *The PIT PD estimates according to (5) should never be used as an input to a model like (1), even if the model is applied to make a PIT risk measurement unless all other model parameters can be specified consistently as outlined in footnote 2 below.*

*2) The naive transformation of the PIT PD (5) into a TTC PD by just removing the sys-
tematic factors S from the right-hand side of the equation is inadequate. The simple term
$\Phi(a_0 + a'F_i)$ would underestimate the true TTC PD which according to (7) and proposition
2 is given by $\Phi\big((a_0 + a'F_i)\sqrt{1 - \varrho^2}\big)$ (it would be an underestimation as $a_0 + a'F_i$ should
be negative most of the time for even PIT PDs in general will be less than 50%).*

In this section we have discussed how – under certain assumptions – a consistent framework
of PIT PD estimators, TTC PD estimates, and the factor model for the correlation structure
of a credit portfolio model can be constructed. In particular, by proposition 1 we have shown
that there is a one-to-one mapping between suitable probit PIT PD estimators and the factor
model for the correlation structure while proposition 2 provides a recipe of how to derive TTC
PD estimates from the PIT PD estimators such that the results can be used as input to the
portfolio model. In the following section, we demonstrate the potential issues with mixing up
PIT and TTC parameters in standard portfolio models by means of a stylized example in the
context of the computation of tail risk and EC.

4 Numerical example

We look at a stylised homogeneous portfolio with 100 assets. All assets have the same exposure,
all LGDs are assumed to be 100%. We study the two cases of all assets being of investment
grade quality (TTC PD 0.3%) and all assets being of sub-investment grade quality (TTC PD
3%). The calculations are done with a one-factor version of model (1) and a sensitivity of 50%
unless explicitly stated otherwise. As the calculation is for illustrative purposes only we need
not identify the systematic factor. We calculate tail risk as Value-at-Risk (VaR) and economic
capital (EC) as *Unexpected Loss* which is defined as *VaR minus Expected Loss*.

The upper panel of table 1 shows the results for the TTC case where the input PDs for the
calculation are just the TTC PDs assigned to the assets. VaR and EC here are calculated at
99.9% confidence level. Note that the confidence level of 99.9% implies a target TTC PD of 0.1%
for the bank holding the portfolio. The VaR and capital values resulting from the calculations
seem rather high – this is a consequence of the high granularity of the portfolio, the 100% LGD
assumption and the relatively high sensitivity of 50% to the systematic factor.

The lower panels of table 1 show the results for the PIT case where the input PDs for the
calculation are a) PIT PDs determined by having the single factor taking on the value -2.33
(which corresponds roughly to a 1 in 100 scenario) and b) the TTC PDs as in the TTC analysis.
For a), the TTC PD of 0.3% is transformed into a PIT PD of 3.4% whereas the TTC PD of 3%
is transformed into a PIT PD of 20.4%. Otherwise the calculation is done exactly in the same
way as for the TTC analysis – that is why a) is labelled "PIT input, TTC calculation". For b),
TTC PDs are used as input PDs[2] to the model but the calculation is done by integration with
respect to the individual risk factors only while the systematic risk factor has the fixed value
-2.33. This is labelled "TTC input, PIT calculation" in table 1.

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[2]Actually, it might be easier to implement this calculation by using the PIT PD derived as described in a) as
input PD and then run the portfolio model in TTC mode but with sensitivity $\varrho = 0$.

Table 1: *Value-at-Risk (VaR) and economic capital (EC) results for a homogeneous port-folio with 100 assets calculated with a one-factor model under TTC and PIT assumptions. EC is determined as 'Unexpected Loss = VaR minus Expected Loss'. Details on input parameters and calculation methods are provided in section 4.*

TTC analysis		
Asset PD (TTC)	3%	0.3%
VaR (99.9%)	37%	9%
Capital (99.9%)	34%	8.7%
PIT analysis: PIT input, TTC calculation		
Input PD	20.4%	3.4%
VaR (99.9%)	81%	39%
TTC capital (99.9%)	60.6%	35.6%
VaR (98.7%)	64%	21%
PIT capital (98.7%)	43.6%	17.6%
VaR (98%)	60%	18%
PIT capital (98%)	39.6%	14.6%
PIT analysis: TTC input, PIT calculation		
Input PD	3%	0.3%
VaR (99.9%)	34%	10%
TTC capital (99.9%)	13.6%	6.6%
VaR (98.7%)	30%	8%
PIT capital (98.7%)	9.6%	4.6%
VaR (98%)	29%	7%
PIT capital (98%)	8.6%	3.6%

The rows labelled "VaR (99.9%)" and "TTC capital (99.9%)" respectively in the lower panels of table 1 display the VaR and capital values calculated for these input PDs if the confidence level of 99.9% remains the same as for the TTC analysis. With PIT input PDs the new VaR and capital values are then clearly much higher (e.g., for EC we have 60.6% to 34% for sub-investment grade and 35.6% to 8.7% for investment grade) than the ones calculated for the TTC analysis. This picture changes dramatically when the input PDs are the same TTC PDs as before in the TTC analysis (e.g., for EC 13.6% to 34% for sub-investment grade and 6.6% to 8.7% for investment grade). Note, however, that the VaR results are remarkably similar to the VaR results from the TTC analysis (34% to 37% for sub-investment grade and 10% to 9% for investment grade). The reason for these significant changes is the fact that in the PIT analysis default events become stochastically independent because the calculations are conducted with a fixed value (-2.33) for the systematic factor. Despite the higher input PDs the model (1) therefore generates lighter tail loss distributions than in the TTC analysis. In addition, expected loss is higher because it is based on the PIT PDs.

The rows labelled "VaR (98.7%)" and "PIT capital (98.7%)", and "VaR (98%)" and "PIT capital (98%)" respectively in the lower panels of table 1 display the VaR and capital values calculated for the PIT analysis if the confidence level is adapted to reflect '1 in 100' PIT target PDs for the

bank holding the portfolio. The target PD of $1.3\% = 100\% - 98.7\%$ has been derived from the bank's TTC target PD of 0.1% in the same way as the PIT transformed PDs of the assets in the portfolio. In particular the same sensitivity of 50% to the systematic factor has been applied. The target PD of $2\% = 100\% - 98\%$ has been derived from the bank's TTC target PD of 0.1% under the assumption of a higher sensitivity of 71% ($= \sqrt{50\%}$) to the systematic factor in order to reflect the fact that the systematic risk of banks is higher than the systematic risk of most other industries.

Comparison of the most upper and the most lower panels of table 1 shows that in the PIT analysis the reduced uncertainty by the realisation of the systematic factor implies that the portfolio model effectively is evaluated with stochastically independent default events and, therefore, indicates broadly the same VaR values but less capital requirements than in the TTC analysis even in a 1 in 100 stressed environment. Adopting a PIT target PD for the bank holding the portfolio further enlarges the difference between PIT capital figures and TTC capital figures, in particular if the higher systematic risk of the bank is taken into account (3.6% and 8.7% for investment grade and 8.6% and 34% for sub-investment grade). In contrast, PIT VaR figures differ not too much from the TTC analysis VaR figures even if PIT target PDs are used to determine the confidence level for VaR.

In total, the results of table 1 appear somewhat unintuitive as most people would expect to see higher capital requirements in a 1 in 100 stressed environment. Hence one should think again about whether a "conditional independence" (conditional on the systematic factors) model like (1) is really appropriate for the calculation of risk in poor economic conditions. Weakening the conditional independence by taking recourse to a latent systematic factor as suggested by Hamerle et al. (2004) could be a work-around. In addition or alternatively inclusion of contagion effects in the model structure appears as a promising approach to more realistic modelling with regard to the issue of unintuitive results.

The observations from table 1 also confirm that "time-varying target solvency probabilities" (Gordy and Howells, 2006) indeed do not only reduce regulatory capital requirements but also economic capital – at least when they are combined with consistent approach to PIT portfolio risk modelling.

5   Conclusions

We have looked in some detail at an ideal framework of a credit portfolio model and a methodology for the estimation of PIT PDs. In this framework, the same systematic factors are included in the portfolio model and in the regression approach that is applied for PD estimation. There is actually a one-to-one relation between the key portfolio model parameters and factors and the key PIT PD estimation parameters and factors. It turns out, however, that stochastic independence of the obligor-characteristic factors in the PD estimates and the systematic factors is a crucial condition both for the portfolio model to be able to generate unbiased TTC risk measures and the portfolio model and the PIT PD estimates being consistent in the sense that the TTC PD input parameters for the portfolio model can be derived from the PIT PD estimates.

We have then demonstrated the conclusions from the theoretical considerations by a numerical example. The example clearly shows that using PIT PD estimates as input to a portfolio model that is run in TTC mode leads to unrealistically high capital requirements. The example shows also that this effect can be mitigated to some extent by replacing the TTC confidence level for the capital calculation by a PIT confidence level that is derived from the bank's own PIT PD.

In addition, the results of the numerical example indicate that PIT capital levels may come out unintuitively low when calculated in the theoretically correct way. This observation, however, might be a consequence of a choosing a too simple portfolio model and a too simple PIT estimation approach for the purpose of this chapter as defaults become stochastically independent in our modelling framework when it is used for PIT calculations. Further investigation is recommend of how to modify "conditional independence" modelling in order to create appropriate levels of correlations for PIT calculations. Promising approaches are the use of latent systematic factors and the inclusion of contagion effects in the modelling. Duffie et al. (2009) and some of the references therein provide further examples of extending approaches used for PD modelling towards portfolio risk modelling.

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