Illiquidity Discount in the Czech Republic: Comparison of Stock Prices at the Prague Stock Exchange and the Off-Exchange RMS

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Introduction and definition of terms

Estimation and use of shareholder-level discounts and premiums in business valuation is a widely discussed and controversial topic not only in the Czech Republic with its' transitional economy but in highly developed market economies like in the USA as well. On one hand there is the necessity to deal with the topic originating from an evidence of cases when a discount or premium was realized in the market, on the other hand the evidence is not wide and unambiguous enough to help us estimate the magnitude of a premium or discount in specific cases. Since the observed magnitudes of discounts and premiums have a wide spread and since they are applied directly to the value of an interest, they can have a significant effect on the estimated value of the business interest. In the USA as a country with a highly developed market economy with effective and liquid capital markets the discussion on these topics has been taking place for decades and the level of knowledge in this field has been through these years continuously developing. There are also numerous empirical data and studies on discounts at hand of business valuers. In contrast to that, the Czech Republic – a transitional economy with an emerging market – suffers from a low level of efficiency and liquidity of capital markets together with a high level of information asymmetry in the market in general.

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In this paper I deal with shareholder-level discounts, specifically the lack of marketability discount which is the difference between the price of a publicly traded stock and the price of an otherwise identical stock that is not tradable.

In the Czech Republic business valuation practice we do not distinguish clearly between marketability and liquidity. These terms are usually used interchangeably. In my opinion marketability is a little wider term than liquidity because it covers not only the "possibility" of liquidating (selling) an asset but also the "right" to do it. When we are estimating liquidity of an asset we implicitly suppose the right exists. On the other hand the right to sell an asset might be unlimited thus the asset is hypothetically marketable but if there is no market, or no buyer, the asset is illiquid and has a very low level of marketability. As we can see the marketability is a condition of liquidity and at the same time the level of liquidity influences the level of marketability of an asset. That is why I also use these terms interchangeably if not stated otherwise.

Lack of marketability discount is a shareholder level discount. While company level discounts influence every single share of a company in the same way, shareholder level discounts influence only certain interests in a company – they influence the value of shares unevenly.

We should distinguish between discounts for lack of marketability for minority and controlling interests (Pratt, 2001) because these have a different theoretical reasoning and the magnitude thus differs too. The controlling interests are usually traded at M&A market whereas there are minority interests traded at capital markets. Controlling and minority interests have a different constituency of potential buyers and the owners of minority interests have usually different possibilities of exiting the business. Moreover, the process of selling a controlling interest takes a significantly longer time period during which the owner is forced to face additional risk and bear high transaction costs relative to sale at a capital market. Yet, the success of the process, the price, the terms of payment and the length of the time period are not certain. These risks and additional costs create the base of the marketability discount for controlling interests. There are no empirical studies or data sets that would be usable to estimate the magnitude of controlling interests lack of marketability discounts either in the Czech Republic, or the USA. That is why I concentrate in the paper on lack of marketability discounts of minority interests. When dealing with discounts it is necessary to pay attention to the

base to which the discount is applied. In this paper, though, I would like to concentrate only on the empirical studies of marketability and their practical aspects.

When trying to estimate the magnitude of the lack of marketability discounts, the valuers have a choice to either use some of the theoretical models or to rely on the historical data from the empirical studies. In the following chapters I briefly introduce the methods and on the example of the Czech Republic I discuss whether and how they are used and usable in less effective market conditions.

Theoretical models of determining the lack of marketability discount

The theoretical models try to infer the discount from other input variables, such as for example the value of the risk aroused by the illiquidity, the cost of eliminating the risk, required rate of return, transactional costs, or equity risk premium.

Up to now I have managed to find these theoretical quantitative models in foreign literature:

- European put-option model of B. H. Chaffee III. (1993) which estimates the maximum discount equal to the price of a European put-option, as the cost of eliminating the risk of price change of the stock during the period of illiquidity,
- Francis A. Longstaff's (1995) model which quantifies the financial difference between the situation of an investor who can sell a security in an optimal point in the future and reinvest the revenue at risk-free rate to a situation of an investor to an illiquid security.
- Christopher Z. Mercer's (1997) model of required rate of return during the illiquidity period, expected length of the holding period and expected growth.
- Jay B. Abrams' (1994) Economic Components Model, which specifies 4 basic components of the discount as the costs of delay to sale, monopsony power of the buyers and buyers' and sellers' transaction costs.
- David I. Tabak's (2002) model based on the CAPM and equity risk premium.

Unfortunately, these models are not known and used by the valuers in the Czech Republic which was confirmed at the last annual Business Valuation Conference held by the University of Economics in Prague in October 2005. Only an imponderable percentage of business valuers stated in my enquiry that they had heard about some of these theoretical models before and nobody stated that he had used any of these models for the quantification of the discount in practice.

Besides technical problems such as the language barrier and relatively high cost of foreign literature I believe the reasons are:

1. undeveloped financial market which excludes using especially the option methods,
2. replacing the problem of estimating the discount with the problem of estimating other variables, such as for example the length of holding period (in the Czech Republic there are almost no IPOs), expected growth, transaction costs,
3. higher mathematical intensiveness of the methods.

Despite the above stated problems I believe even in the market conditions of the Czech Republic I believe these theoretical models can be valuable for us because they reveal the factors of origin and magnitude of the discount which can help us in a qualitative analysis of the discount.

Empirical evidence on marketability

In the USA as a country with long market economy history and functional and most liquid financial markets in the world there have been published several empirical studies on the theme. There are basically three types of empirical studies on marketability – restricted stock studies, pre-initial public offering studies and acquisition prices studies.

Restricted stock studies in the USA have been published since 1960's. They compare the prices of stocks (accomplished in private transactions) that are not registered for public trading or are restricted on public trading, with the prices of identical stocks that are publicly traded.

Pre-IPO studies have been undertaken since 1975. They compare prices of stocks of private companies in private transactions. These companies eventually went public, so this price of private transaction is compared to the public offering price.

The acquisition prices studies compare the prices of private and publicly traded comparable companies (for example Koeplin, 2000).

The use of these studies for estimation of the magnitude of a marketability discount for a particular company is connected with a high number of application problems even in the USA. I assume there are additional problems we have to face if we want to apply results of these studies in the conditions like the ones of the Czech Republic.

- Firstly, the Czech capital markets are not so efficient, liquid either the business is transparent. I believe this should result in lower stock prices due to higher implied marketability discount.
- The stocks at US capital market are believed to be sold at their control value (Nath, 1997). Therefore the discount observed should include not only lack of marketability component, but also a lack of control component. In the Czech Republic with its' low effectiveness market and high information asymmetry we cannot suppose the stocks are traded at their control values, the application of these empirical studies would thus lead to overestimation of the magnitude of the discount.
- There are different possibilities of exit from the business (for example almost no prospect for an IPO)
- There is a different legal environment resulting in different restrictions on stocks sales.

Despite all these cons that make us believe the results of US empirical studies are not usable for estimating the magnitude of the discount in the conditions of the Czech Republic, there is one major pro, which is the existence itself of this evidence. Since we do not have other choice, we have to rely on these data and especially on what statistical analysis of these data can bring us. Most of the newer studies contain also some statistical analysis and try to specify the determinants of the discount, which is the information we can use even in the Czech Republic. Although it is certainly difficult to quantify the influence of each separate factor on the discount, we at least get a lead what to assess when estimating the discount.

In the Czech Republic we do not have any evidence on historical discounts achieved since the prices of private transactions are not usually disclosed and the amount of shares registered at the capital market is very low.

That is why we decided to make the first step and prepare an analysis of the Czech two capital markets in search for local evidence on marketability, which is contained in the following chapters.

Input data of the Czech capital markets study

The two capital markets in the Czech Republic are the Prague Stock Exchange (hereinafter PSE) and the RM-SYSTEM (hereinafter RMS).The PSE is Czech Republics main securities market organizer. It organizes the supply of and the demand for securities registered at the PSE in accordance with the terms of a license issued by the Securities Commission of the Czech Republic. The PSE is a full member of the Federation of European Stock Exchanges (FESE). The RMS is an organizer of off-exchange trading and has been active at the securities market since 1993.

The input data consists of daily prices, total trade values in CZK and total trade pieces of stocks exchanged in the PSE and in RMS starting 1.1.2000 to the present. For the purpose of the study I picked 3 groups of stocks.

First group is created by the stocks of the most liquid companies (Prague's blue chips) that are included in the base of the Prague Stock Exchange index PX-D (index of the main market) and at the same time are traded in RMS and are included in the base of RMS index PK-30. The total number of such companies at the present time is five.

Tab. 1: Sample groups

Group 1 – SPAD and PK30	Group 2 – PX-50 and PK 30	Group 3 – PK 30
ČEZ (CEZ)	Česká pojišťovna (CPO)	OKD (OKD)
Komerční banka (KOB)	Severočeské Doly (SDO)	Pražská Energetika (PRE)
Český Telecom (CTE)	STČ Energetická (STE)	ZČ Energetika (ZCE)
Philip Morris ČR (PMO)	SČ Energetika (SCE)	Metalimex (MTL)
Unipetrol (UNI)	Sokolovská uhelná (SOU)	(starting 6.7.2001)
	SSŽ (SSZ)	United Energy (UEN)
	Paramo (PAR)	Toma (TOM)

Second group consists of stocks that create the base of Prague Stock Exchange index PX-50 (secondary market of the stock exchange) and at the same time are included in the base of RMS index PK-30 excluding those, that are included in group one. At the present time there are seven such companies. The last group consists of 6 companies that are included in the base of RMS index PK-30 and are traded in Prague Stock Exchange, excluding those included already in group one and two.

In order to have comparable data from all the markets we calculated the daily rate (price) of the shares as weighted average price of trades accomplished on the market, where the volume of the trade was used as the weight. We also needed to exclude all data of days when there was no trade accomplished on the market resulting in no price change at the market, which would cause a bias in the results.

Statistical Description of the data

Using these data we first provide a description of the groups based on average trade volumes, average percentage of equity the traded stocks represent and number of days as an approximation of the number of trades with the stocks for both capital markets.

Tab. 2: Trade days and trade volumes group 1: SPAD and PK30

Group 1: SPAD and PK30	Trade days PSE (max. 1330) (days)	Trade days RMS (max. 1330) (days)
ČEZ (CEZ)	1 327	1 330
Komerční banka (KOB)	1 327	1 329
Český Telecom (CTE)	1 327	1 328
Philip Morris ČR (PMO)	1 299	1 280
Unipetrol (UNI)	1 324	1 327
Total	**1 321**	**1 319**

Group 1: SPAD and PK30	Average trade volume PSE (shares)	Average trade volume RMS (shares)	Average trade volume PSE (CZK)	Average trade volume RMS (CZK)	Average trade volume RMS/ PSE (%shares)
CEZ	1 537 699	11 693	243 082 992	1 550 840	0,76%
KOB	180 448	1 348	349 235 777	1 628 546	0,75%
CTE	951 852	3 890	367 007 309	1 634 941	0,41%
PMO	4 920	49	63 336 979	474 588	0,99%
UNI	633 941	12 754	45 454 038	787 857	2,01%
Total	661 772	5 947	213 623 419	1 215 354	0,98%

As we can see in the table above all the stocks in group 1 have in common a high number of days at which a trade was accomplished. In most cases the trade days are very close to the maximum of 1330 days in both markets the PSE as well as the RMS. The average trade volume at the PSE was substantially higher than the average trade volume at the RMS (on average 111 times more shares traded at PSE than at RMS). We cannot use the average trade volumes in CZK to make any conclusions since they are biased due to the stock price development through the testing period of almost 4.5 years. We use provide them only for the sake of completeness. Except for Unipetrol the average daily trade volume in shares at RMS reaches only up to 1% of the trades accomplished at the PSE.

Tab. 3: Trade days and trade volumes group 2: PX50 and PK30

Group 1: SPAD and PK30	Trade days PSE (max. 1330) (days)	Trade days RMS (max. 1330) (days)
Česká pojišťovna (CPO)	510	982
Severočeské doly (SDO)	383	1 114
STČ Energetická (STE)	235	1 096
SČ Energetika (SCE)	162	947
Sokolovská uhelná (SOU)	499	1 277

Group 1: SPAD and PK30	Trade days PSE (max. 1330) (days)	Trade days RMS (max. 1330) (days)
SSŽ (SSZ)	179	782
Paramo (PAR)	218	1 167
Total	**312**	**1 052**

Group 1: SPAD and PK30	Average trade volume PSE (shares)	Average trade volume RMS (shares)	Average trade volume PSE (CZK)	Average trade volume RMS (CZK)	Average trade volume RMS/ PSE (%shares)
CPO	273	27	1 854 601	115 715	9,84%
SDO	335	126	309 595	84 400	37,56%
STE	42	431	76 915	866 375	1028,36%
SCE	27	14	52 003	23 745	52,36%
SOU	883	637	447 669	273 029	72,17%
SSZ	534	496	464 352	384 754	93,04%
PAR	74	933	71 849	169 408	1265,12%
Total	**309**	**381**	**468 141**	**273 918**	**365,49%**

Stocks at group 2 have substantially higher number of days when a trade was accomplished at RMS than at PSE. This is in contrast to group 1 where the trade days where approximately equal at both markets. Even at RMS the number of trade days is 20% lower than in case of group number 1. At the PSE the trade days group average is 75% lower than in case of group 1. Concerning average trade volumes in shares, there is not such an abysmal difference in PSE and RMS as in case of group one. Moreover we can see that the average trade volume in shares for the group was higher at RMS. However, when we look at the data closer we find out, that only two stocks out of seven had higher average trade volume at RMS. Also the trade volume index is biased by the results of the two companies with extreme results. However, the trade volume index reaches significantly higher percentage than in case of group one. If we exclude the two extreme stocks (Paramo and STČ Energetická) the average group index for group 2 will reach 53% in contrast to 0,98% of group 1.

Tab. 4: Trade days and trade volumes group 3: PK30

Group 1: SPAD and PK30	Trade days PSE (max. 1330) (days)	Trade days RMS (max. 1330) (days)
OKD (OKD)	276	1 267
Pražská energetika (PRE)	114	982
ZČ Energetika (ZCE)	117	916
Metalimex (MTL)	37	404
United Energy (UNE)	35	523
Toma (TOM)	122	1 134
Total	**117**	**871**

Group 1: SPAD and PK30	Average trade volume PSE (shares)	Average trade volume RMS (shares)	Average trade volume PSE (CZK)	Average trade volume RMS (CZK)	Average trade volume RMS/ PSE (%shares)
OKD	270	839	137 710	183 615	311,19%
PRE	10	20	18 968	40 679	196,92%
ZCE	7	49	20 879	43 539	716,33%
MTL	27	21	63 896	43 539	79,88%
UNE	25	38	17 658	34 339	151,57%
TOM	112	169	29 749	21 039	151,34%
Total	**75**	**189**	**48 143**	**61 125**	**267,87%**

The stocks of Metalimex included in group 3 entered the PSE later than the testing period started so the data are calculated for a shorter period of time, but this should not influence the results of the study. In group 3 the number of trade days is lower at the PSE as well as the RMS when compared to group 2. The difference between the number of trade days at RMS and the PSE is even larger. The average trade volumes are very low at both markets, 5 out of six companies had higher daily average trade volume at the RMS than at the PSE, which is confirmed by the magnitude of the trade volume index – the group average is 270%.

Tab. 5: Percentage of equity traded

	Average percentage of equity PSE	Average percentage of equity RMS	Total percentage equity traded since 1.1.2000	Total percentage equity traded since 1.1.2000
SPAD a PK30				
CEZ	0,2596828%	0,0019748%	344,60%	2,63%
KOB	0,4914340%	0,0039154%	652,13%	5,20%
CTE	0,2955236%	0,0012078%	392,16%	1,60%
PMO	0,1792123%	0,0017814%	232,80%	2,28%
UNI	0,3493335%	0,0070281%	462,87%	9,33%
PX-50 a PK 30				
CPO	0,0080571%	0,0007921%	4,109%	0,778%
SDO	0,0037182%	0,0013992%	1,424%	1,559%
STE	0,0013068%	0,0134347%	0,307%	14,724%
SCE	0,0008193%	0,0004290%	0,133%	0,406%
SOU	0,0130165%	0,0093936%	6,495%	11,996%
SSŽ	0,0384940%	0,0358150%	6,890%	28,007%
PAR	0,0055474%	0,0701810%	1,209%	81,901%
PK 30				
OKD	0,0011097%	0,0034534%	0,31%	4,38%
PRE	0,0002591%	0,0005102%	0,03%	0,50%
ZCE	0,0004275%	0,0030620%	0,05%	2,80%
MTL	0,0028767%	0,0022979%	0,11%	0,93%
UNE	0,0013144%	0,0019922%	0,05%	1,04%
TOM	0,0075733%	0,0114617%	0,92%	13,00%

Table 5 confirms, that daily average percentages of equity traded through the test period (1.1.2000 to 18.4.20005) are very low for all sample groups. In the first group of Prague blue chips we can see that it was through the PSE that the most equity was traded. In both groups 2 and 3 the more important market concerning percentage of equity traded was

the RMS. In group 3 the total percentage of equity traded at the PSE through approximately 4.5 years is in all cases less than 1 %.

Based on these data we can conclude, that:

- the group 1 – companies of SPAD (PXD) and PK30 are the most liquid stocks at both markets the PSE and the RMS. They are traded daily at both markets, the number of trades is the highest of all groups. Also the daily trade volumes are higher than for the other groups. This group has the highest average daily percentage of equity traded at the PSE. The daily average percentage of equity traded at the RMS basically does not differ from the other groups.
- the group 2 – companies of PX-50 and PK30 are less liquid measured by number of trade days that those of group one, but more liquid than those in group 3. The total traded percentage of equity is roughly even at both markets so we cannot say that the PSE would be the more liquid market for these stocks.
- the group 3 – companies of PK30 is the least liquid group concerning all the statistics. Stocks in this group are traded irregularly in both markets, the more liquid market for these stocks is the RMS. The trade volumes and total percentage of equity traded are very low, but higher at the RMS than the PSE.
- both markets suffer from a very low average daily trade volumes (percentages of equity traded).

We cannot say in general that one of the markets would be more liquid than the other; it more depends on the type of stock.

Calculation of price indices between the RMS and the PSE

Now we can move to calculation of the difference between the weighted average price for which a stock was sold at the PSE and the RMS on the same day. Table 6 shows the mean price discount counted as one minus the index of daily weighted average prices of the stock at the RMS to the weighted average price of the stock at the PSE. Only days when there was a trade closed at both markets were included in the calculation to avoid a bias. The stocks are ranked according to the magnitude of the discount.

Tab. 6: Price indices (discounts)

SPAD and PK30	rate (price) RMS/ PSE % average	average discount 100% – (RMS/ PSE) %	Average trade volume PSE (shares)	Average trade volume RMS (shares)	Average trade volume RMS/ PSE (shares) %	Group
SCE	97,41%	2,59%	27	14	52,36%	PX-50 and PK 30
MTL	97,41%	2,59%	27	21	79,88%	PK 30
SDO	99,03%	0,97%	335	126	37,56%	PX-50 and PK 30
STE	99,22%	0,78%	42	431	1028,36%	PX-50 and PK 30
SSZ	99,38%	0,62%	534	496	93,04%	PX-50 and PK 30
KOB	99,65%	0,35%	180 448	1 348	0,75%	SPAD and PK30
CPO	99,96%	0,04%	273	27	9,84%	PX-50 and PK 30
CEZ	99,97%	0,03%	1 537 699	11 693	0,76%	SPAD and PK30
UNI	100,00%	0,00%	633 941	12 754	2,01%	SPAD and PK30
CTE	100,05%	–0,05%	951 852	3 890	0,41%	SPAD and PK30
PRE	100,22%	–0,22%	10	20	196,92%	PK 30
PMO	100,38%	–0,38%	4 920	49	0,99%	SPAD and PK30
SOU	100,64%	–0,64%	883	637	72,17%	PX-50 and PK 30
ZCE	101,37%	–1,37%	7	49	716,33%	PK 30
OKD	101,50%	–1,50%	270	839	311,19%	PK 30
PAR	101,79%	–1,79%	74	933	1265,12%	PX-50 and PK 30
UNE	105,09%	–5,09%	25	38	151,57%	PK 30
TOM	107,28%	–7,28%	112	169	151,34%	PK 30

A discount more than 1 means that shares were on average sold for a higher price at the PSE than at the RMS, whereas a discount less than 1 means that the stock was on average sold for a higher price at the RMS. As we can see it were mostly stocks of group 3 that were sold with negative discount and stocks of group 2 that were sold with a positive discount. Stocks of SPAD and PK30 were sold approximately for the same price at both markets. Stocks with a negative discount have average trade volume at the RMS higher than at the PSE, stocks with a positive discount have higher trade volume at the PSE than the RMS. These results would confirm our assumption, that we should judge liquidity of a stock according to the trade volumes and number of trades at each market, we cannot say, that the PSE would be in general more liquid than the RMS. The magnitude of the discount we calculated is not very significant, but we have not expected to find out a significant magnitude, since it is always the same stocks that are traded at both markets.

Analysis of variance

We have also analyzed the variance[1] of the price RMS/PSE index between and within the three sample groups.

Tab. 7: Group price indices (summary statistics)

	Count	Average
index_pk30_px50	1745	1.00053
index_sd_pk30	6554	1.00007
pk30	768	1.01395
Total	9067	1.00133

	Variance	Standard deviation
index_pk30_px50	0.00368884	0.0607358
index_sd_pk30	0.000226671	0.0150556
pk30	0.00876155	0.0936037
Total	0.00162948	0.0403668

[1] We used Statgraphics software.

Tab. 8: Decomposition of variance price indices (ANOVA table)

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Between groups	0.133992	2	0.0669962	41.48	0.0000
Within groups	14.6389	9064	0.00161506		
Total (Corr.)	14.7729	9066			

The F-ratio is a ratio of the between-group estimate to the within-group estimate. In our case it equals to 41.4822. Since the P-value of the F-test is less than 0.05, there is a statistically significant difference between the means of the three variables at the 95% confidence level. To determine which means are statistically different from the others, we used a multiple range test.

Tab. 9: Multiple range test

	Count	Mean	Homogeneous Groups
index_sd_pk30	1745	1.00007	X
index_pk30_px50	6554	1.00053	X
pk30	768	1.01395	X

Contrast	Difference	+/– Limits
index_pk30_px50 – index_sd_pk30	0.000467077	0.0021218
index_pk30_px50 – pk30	*–0.013422	0.00341084
index_sd_pk30 – pk 30	*–0.013889	0.00300417

* denotes a statistically significant difference.

Method: 95,0 percent LSD

The bottom half of the output shows the estimated difference between each pair of means. The two pairs of means with an asterisk are the pairs showing a statistically significant difference at the 95% confidence level. It is between group3 (PK30 stocks) and group 2 (PX50 – PK30 stocks) and between group 3 (PK3 stocks) and group 1 (SPAD –PX50). Between the group 1 and group 2 there is not a statistically significant difference.

At the upper part of Table 9, two homogenous groups are identified using columns of X's. Group 1 and group 2 were identified as homogenous so the difference between their means is not statistically significant. The method used to discriminate among the means is Fisher's least significant difference (LSD) procedure. With this method, there is a 5% risk of calling each pair of means significantly different when the actual difference equals 0.

Since the skewness and kurtosis of the data were out of rage, which signifies the distribution might not be normal, we also used the Kruskal-Wallis Test to compare medians instead of means. The Kruskal-Wallis Test tests the null hypothesis that the medians within each of the three groups are the same. The data in the groups were first combined and ranked from smallest to largest. The average rank was then computed for the data in each column.

Tab. 10: Kruskal-Wallis Test

	Sample Size	Average Bank
index_pk30_px50	1745	4318.41
index_sd_pk30	6554	4593.98
pk30	768	4511.95

Test statistic = 15,3333 P-Value= 0,000468181

Since the P-value is less than 0.05, there is a statistically significant difference amongst the medians at the 95% confidence level. The Kruskal-Wallis Test has confirmed the results of the multiple range test.

Correlation analysis of number of trades and the price

At last we prepared a correlation analysis between prices of stocks at the PSE (or the RMS) and the number of trades. As an approximation of number of trades we used the number of trade days for each stock per calendar month. For each company stock we calculated the Pearson product moment correlations and Spearman rank correlations.

The Pearson product moment correlation coefficients range between –1 and +1 and measure the strength of the linear relationship between the variables (number of trade days per month and price). The Spearman rank

correlation coefficients range between –1 and +1 and measure the strength of the association between the variables. In contrast to the more common Pearson correlations, the Spearman coefficients are computed from the ranks of the data values rather than from the values themselves. Consequently, they are less sensitive to outliers than the Pearson coefficients. We also show the number of pairs of data values used to compute each coefficient as well as the P-value which tests the statistical significance of the estimated correlations (Appendix 1, Appendix 2). P-values below 0.05 indicate statistically significant non-zero correlations at the 95% confidence level.

As we do not suppose linear dependency, we consider the Spearman correlation more concise for our purpose since the values themselves are not as important as the rank. Interestingly, the results of testing the null hypothesis of zero correlation are equal for both markets just for the group 1. In all five cases the hypothesis was not rejected.

This could have two explanations:

- The stocks are traded so often, that it is impossible to empirically find and measure the price corresponding with lower values of trade days.
- The stocks are so liquid, that the number of trade days cannot influence the price.

When we make a closer look at the data we have to come to conclusion that the first point is probable. The approximation of trade numbers by number of trade days is for the first group too rough since there was at least one trade accomplished almost every day.

For the stocks of group 2 and 3 the results are puzzling. But as for groups 2 and 3 we find out that the null hypothesis was rejected in 40 – 60% of cases. For the group 2 we can say, that at the PSE if the hypothesis was not rejected then the correlation coefficient is positive. (For STCEnerg the correlation coefficient is so low that we can also assume independence). Positive correlation coefficient would confirm that the more trades are closed with the stock, the higher price of the stock (the lower implied liquidity discount). However, at the RMS the results are exactly opposite. The percentage of hypothesis that was rejected is about the same (for group 2 and 3), but with particular companies in 10 cases out of 13 there are opposite conclusions for the RMS. Moreover, if the

hypothesis was not rejected, the correlation coefficients are negative. Firstly, we tried to explain the opposite rejection of the hypothesis at the markets. In order to test correlation of a variable, it is necessary that the variable (number of days) would have a sufficient variance. We presume, that the price of the stock depends, besides number of trades, on other factors as well. If the price is variable due to other factors and the tested variable stays about the same, we have to come to conclusion about zero correlation. That is why we added an analysis of variance. Since we could not use variation coefficients (if the mean is 2 days, 50% variability means 1 day; if the mean is 10 days, the 50% variability means 5 days), we opted for an absolute measure of variance 18 and added data with the variation coefficient more than 0,5. With the help of analysis of variance we found out, that in RMS the zero correlation was not rejected in cases when the factor "trade days" did not have sufficient variance. In cases where the variance was sufficient at the PSE (companies of group2: Cpoj, SCDoly, SokolU, Paramo; companies of group 3: OKD, Metalimex) the correlation coefficients are positive and confirming our expectations about lower implied liquidity discount. However, companies of group 3 in general have very low average trade days so the results might be biased. At the RMS after the variance analysis only 4 companies remain with significant results of which 3 are in group 3 (PrazskaE, ZCEnerg, UnitedE) and 1 in group 2 (SSZ). The correlation coefficients of these are negative which we for now leave unresolved for further analysis.

As the next step we were planning to make a correlation analysis of the price and the percentage of equity traded but we came to conclusion that the percentages traded are in all cases so low that such an analysis would have no practical use.

Conclusions

Both capital markets, the PSE and the RMS suffer from a rather low liquidity, low registered number of companies and low trade volumes. The conclusion of the study is that the stocks reach higher prices at the market where more trades with a higher volume are realized (that is more liquid market) whether it is Prague Stock Exchange or the off-exchange RMS. However, the price difference magnitude is not significant. For business valuation purposes we suggest that there should not be a higher marketability discount for stocks traded only in RMS as the off-exchange than there is for stocks traded in Prague Stock Exchange. Analysis of the PSE

data we confirmed mainly for companies of second group that number of trades and price have positive correlation, which means that a lower liquidity discount is implied. The correlation analysis of group 1 does not provide good evidence. We believe it is since the approximation of number of trades by the number of trade days was too rough. As to minority discounts, we have to conclude that the interests traded at the Czech capital markets are so microscopical and the number of companies traded is so low, that by an analysis of data from the capital markets we cannot get results usable either for estimation or for identification of factors influencing minority discounts.

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Appendix 1: Correlation analysis: the PSE

Company	CEZ	KOB	CTE	PMO	UNI	CPO
Pearson correlation coefficient	–0,2354	–0,1012	–0,0695	0,151	0,0129	0,5399
Number of pairs	64	64	64	62	60	62
P-value	0,0611	0,4263	0,5851	0,2414	0,9219	0
Hypothesis at 95% confidence level	not rejected	not rejected	not rejected	not rejected	not rejected	rejected
Spearman rank correlation coefficient	–0,0353	0,0028	–0,015	0,163	–0,03	0,4634
Number of pairs	64	64	64	62	60	62
P-value	0,7793	0,9821	0,9052	0,2029	0,818	0,00003
Hypothesis at 95% confidence level	not rejected	not rejected	not rejected	not rejected	not rejected	rejected

Company	SDO	STE	SCE	SOU	SSZ	PAR
Pearson correlation coefficient	0,5309	–0,2234	–0,1106	0,6362	0,2245	0,4736
Number of pairs	59	55	44	61	48	52
P-value	0	0,101	0,4749	0	0,125	0,0004
Hypothesis at 95% confidence level	rejected	not rejected	not rejected	rejected	not rejected	rejected
Spearman rank correlation coefficient	0,4962	–0,243	–0,0905	0,5658	0,0149	0,436
Number of pairs	59	55	44	61	48	52
P-value	0,0002	0,0742	0,5531	0	0,9189	0,0018
Hypothesis at 95% confidence level	rejected	not rejected	not rejected	rejected	not rejected	rejected

Company	OKD	PRE	ZCE	MTL	UNE	TOM
Pearson correlation coefficient	0,5528	–0,256	–0,3133	0,6965	0,3515	0,1083
Number of pairs	49	38	31	31	19	49
P-value	0	0,1209	0,0861	0	0,14	0,4591
Hypothesis at 95% confidence level	rejected	not rejected	not rejected	rejected	not rejected	not rejected
Spearman rank correlation coefficient	0,5053	–0,3268	–0,2816	0,8679	0,2006	0,1462
Number of pairs	49	38	31	31	19	49
P-value	0,0005	0,0468	0,123	0	0,3946	0,311
Hypothesis at 95% confidence level	rejected	rejected	not rejected	rejected	not rejected	not rejected

Appendix 2: Correlation analysis: the RMS

Company	CEZ	KOB	CTE	PMO	UNI	CPO
Pearson correlation coefficient	–0,2484	–0,1044	–0,0265	–0,2052	–0,2662	–0,3976
Number of pairs	64	64	64	64	64	64
P-value	0,0478	0,4118	0,8355	0,1039	0,0335	0,0011
Hypothesis at 95% confidence level	rejected	not rejected	not rejected	not rejected	rejected	rejected
Spearman rank correlation coefficient	–0,0331	0,0043	–0,085	–0,187	–0,021	–0,548
Number of pairs	64	64	64	64	64	64
P-value	0,7927	0,9729	0,5	0,1377	0,8675	0
Hypothesis at 95% confidence level	not rejected	not rejected	not rejected	not rejected	not rejected	rejected

Company	SDO	STE	SCE	SOU	SSZ	PAR
Pearson correlation coefficient	–0,1439	–0,6093	–0,5836	–0,15	–0,446	0,0158
Number of pairs	64	64	64	64	64	64
P-value	0,2566	0	0	0,2369	0,0002	0,9016
Hypothesis at 95% confidence level	not rejected	rejected	rejected	not rejected	rejected	not rejected
Spearman rank correlation coefficient	–0,0952	–0,6005	–0,5722	–0,0841	–0,5636	–0,027
Number of pairs	64	64	64	64	64	64
P-value	0,45	0	0	0,5045	0	0,8303
Hypothesis at 95% confidence level	not rejected	rejected	rejected	not rejected	rejected	not rejected

Company	OKD	PRE	ZCE	MTL	UNE	TOM
Pearson correlation coefficient	–0,1453	–0,6623	–0,5108	–0,141	–0,5708	0,0754
Number of pairs	64	64	64	45	64	64
P-value	0,252	0	0	0,3557	0	0,5537
Hypothesis at 95% confidence level	not rejected	rejected	rejected	not rejected	rejected	not rejected
Spearman rank correlation coefficient	0,0533	–0,6937	–0,4658	–0,1226	–0,5965	0,0085
Number of pairs	64	64	64	45	64	64
P-value	0,6724	0	0,0002	0,4162	0	0,9464
Hypothesis at 95% confidence level	not rejected	rejected	rejected	not rejected	rejected	not rejected

Illiquidity Discount in the Czech Republic: Comparison of Stock Prices at the Prague Stock Exchange and the Off-Exchange RMS

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ABSTRACT

The Czech Republic – a transitional economy with an emerging market – suffers from a low level of efficiency and liquidity of capital markets together with a high level of information asymmetry in the market in general. The questions I am raising and trying to answer are following. 1. What knowledge from the USA on shareholder-level discounts and premiums can also be accepted in the specific conditions of the Czech emerging market? 2. Can we without hesitation use the US data sets and empirical studies originating in developed markets for quantifying the marketability discounts and apply them to the basis of value originating in our emerging market? 3. Are there any possibilities of quantifying or at least identifying the factors of discounts and premiums with local data? In the final part of the paper I introduce the datasets, methods and the results of the Czech capital markets study focused on marketability (liquidity). The conclusion of the study is that the stocks reach higher prices at the market where more trades with a higher volume are realized (that is more liquid market) whether it is Prague Stock Exchange or the off-exchange RMS. However, the price difference magnitude is not significant. For business valuation purposes we suggest that there should not be a higher marketability discount for stocks traded only in RMS as the off-exchange than there is for stocks traded in Prague Stock Exchange. As to minority discounts, we have to conclude that the interests traded at the Czech capital markets are so microscopic and the number of companies traded, is so low, that by an analysis of data from the capital markets we cannot get results usable either for estimation or for identification of factors influencing minority discounts.

Key words: Lack of marketability discount; Share-holder level discount; Liquidity.

JEL classification: G34.