Universal Fluctuations of the FTSE100

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We compute the analytic expression of the probability distributions $F_{FTSE100,+}$ and $F_{FTSE100,-}$ of the normalized positive and negative FTSE100 (UK) index daily returns $r(t)$. Furthermore, we define the α re-scaled FTSE100 daily index positive returns $r(t)^\alpha$ and negative returns $(-r(t))^\alpha$ that we call, after normalization, the α positive fluctuations and α negative fluctuations. We use the Kolmogorov-Smirnov statistical test, as a method, to find the values of α that optimize the data collapse of the histogram of the α fluctuations with the Bramwell-Holdsworth-Pinton (BHP) probability density function. The optimal parameters that we found are $\alpha^+ = 0.55$ and $\alpha^- = 0.55$. Since the BHP probability density function appears in several other dissimilar phenomena, our results reveal an universal feature of the stock exchange markets.

I. INTRODUCTION

The modeling of the time series of stock prices is a main issue in economics and finance and it is of a vital importance in the management of large portfolios of stocks [10, 19, 20]. Here, we analyze the well known FTSE100 Index also called FTSE100, FTSE, or, informally, the "footsie" that corresponds to a share index of the 100 most highly capitalised UK companies listed on the London Stock Exchange. It is the most widely used of the FTSE Group's indices and is frequently reported as a measure of business prosperity. The FTSE100 companies represent about 81 % of the market capitalisation of the whole London Stock Exchange. The time series to investigate in our analysis is the *FTSE100 index* from April of 1984 to September of 2009. Let $Y(t)$ be the FTSE100 index adjusted close value at day t. We define the *FTSE100 index daily return* on day t by

$$r(t) = \frac{Y(t) - Y(t-1)}{Y(t-1)}.$$

We define the α *re-scaled FTSE100 daily index positive returns* $r(t)^\alpha$, for $r(t) > 0$, that we call, after normalization, the α *positive fluctuations*. We define the α *re-scaled FTSE100 daily index negative returns* $(-r(t))^\alpha$, for $r(t) < 0$, that we call, after normalization, the α *negative fluctuations*. We analyze, separately, the α positive and α negative daily fluctuations that can have different statistical and economic natures due, for instance, to the leverage effects (see, for example, [1, 2, 21, 22]). Our aim is to find the values of α that optimize the data collapse of the histogram of the α positive and α negative fluctuations to the universal, non-parametric, Bramwell-Holdsofworth-Pinton (BHP) probability density function. To do it, we apply the Kolmogorov-Smirnov statistic test to the null hypothesis claiming that the probability distribution of the α fluctuations is equal to the (BHP) distribution. We observe that the P values of the Kolmogorov-Smirnov test vary continuously with α. The highest P values $P^+ = 0.19...$ and $P^- = 0.14...$ of the Kolmogorov-Smirnov test are attained for the values $\alpha^+ = 0.55...$ and $\alpha^- = 0.55...$, respectively, for the positive and negative fluctuations. Hence, the null hypothesis is not rejected for values of α in small neighborhoods of $\alpha^+ = 0.55...$ and $\alpha^- = 0.55...$. Then, we show the data collapse of the histograms of the α^+ positive fluctuations and α^- negative fluctuations to the BHP pdf. Using this data collapse, we do a change of variable that allow us to compute the analytic expressions of the probability density functions $f_{FTSE100,+}$ and $f_{FTSE100,-}$ of the normalized positive and negative FTSE100 index daily returns

$$f_{FTSE100,+}(x) = 8.73x^{-0.45}f_{BHP}(30.87x^{0.55} - 1.95)$$
$$f_{FTSE100,-}(x) = 8.74x^{-0.45}f_{BHP}(28.88x^{0.55} - 1.82)$$

in terms of the BHP pdf f_{BHP}. We exhibit the data collapse of the histogram of the positive and negative returns to our proposed theoretical pdfs $f_{FTSE100,+}$ and $f_{FTSE100,-}$. Similar results are observed for some other stock indexes, prices of stocks, exchange rates and commodity prices (see [13, 14]). Since the BHP probability density function appears in several other dissimilar phenomena (see, for instance, [4, 7, 8, 11, 15–17, 21]), our result reveals an universal feature of the stock exchange markets.

II. POSITIVE FTSE100 INDEX DAILY RETURNS

Let T^+ be the set of all days t with positive returns, i.e.

$$T^+ = \{t : r(t) > 0\}.$$

Let $n^+ = 3367$ be the cardinal of the set T^+. The α *re-scaled FTSE100 daily index positive returns* are the returns $r(t)^\alpha$ with $t \in T^+$. Since the total number of observed days is $n = 6442$, we obtain that $n^+/n = 0.52$. The *mean* $\mu_\alpha^+ = 0.063...$ of the α re-scaled FTSE100 daily index positive returns is given by

$$\mu_\alpha^+ = \frac{1}{n^+} \sum_{t \in T^+} r(t)^\alpha \tag{1}$$

The *standard deviation* $\sigma_\alpha^+ = 0.032...$ of the α re-scaled FTSE100 daily index positive returns is given by

$$\sigma_\alpha^+ = \sqrt{\frac{1}{n^+} \sum_{t \in T^+} r(t)^{2\alpha} - (\mu_\alpha^+)^2} \tag{2}$$

We define the α *positive fluctuations* by

$$r_\alpha^+(t) = \frac{r(t)^\alpha - \mu_\alpha^+}{\sigma_\alpha^+} \tag{3}$$

for every $t \in T^+$. Hence, the α *positive fluctuations* are the normalized α re-scaled $FTSE100$ daily index positive returns. Let $L_\alpha^+ = -1.88...$ be the *smallest* α positive fluctuation, i.e.

$$L_\alpha^+ = \min_{t \in T^+} \{r_\alpha^+(t)\}.$$

Let $R_\alpha^+ = 6.68...$ be the *largest* α positive fluctuation, i.e.

$$R_\alpha^+ = \max_{t \in T^+} \{r_\alpha^+(t)\}.$$

We denote by $F_{\alpha,+}$ the *probability distribution of the α positive fluctuations*. Let the *truncated BHP probability distribution* $F_{BHP,\alpha,+}$ be given by

$$F_{BHP,\alpha,+}(x) = \frac{F_{BHP}(x)}{F_{BHP}(R_\alpha^+) - F_{BHP}(L_\alpha^+)}$$

where F_{BHP} is the BHP probability distribution. We apply the Kolmogorov-Smirnov statistic test to the null hypothesis claiming that the probability distributions $F_{\alpha,+}$ and $F_{BHP,\alpha,+}$ are equal. The Kolmogorov-Smirnov P *value* $P_{\alpha,+}$ is plotted in Figure 1. Hence, we observe that $\alpha^+ = 0.55...$ is the point where the P value $P_{\alpha,+} = 0.19...$ attains its maximum.

It is well-known that the Kolmogorov-Smirnov P value $P_{\alpha,+}$ decreases with the distance $\|F_{\alpha,+} - F_{BHP,\alpha,+}\|$ between $F_{\alpha,+}$ and $F_{BHP,\alpha,+}$. In Figure 2, we plot $D_{\alpha^+,+}(x) = \left|F_{\alpha^+,+}(x) - F_{BHP,\alpha^+,+}(x)\right|$ and we observe that $D_{\alpha^+,+}(x)$ attains its highest values for the α^+ positive fluctuations above or close to the mean of the probability distribution.

In Figures 3 and 4, we show the data collapse of the histogram $f_{\alpha^+,+}$ of the α^+ positive fluctuations to the truncated BHP pdf $f_{BHP,\alpha^+,+}$.
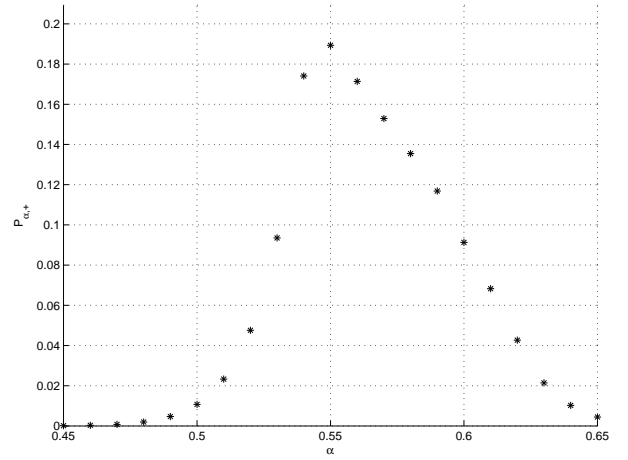

FIG. 1. The Kolmogorov-Smirnov P value $P_{\alpha,+}$ for values of α in the range $[0.45, 0.65]$.
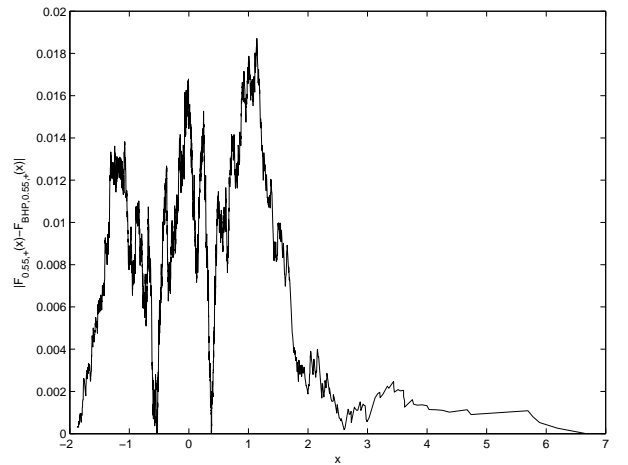

FIG. 2. The map $D_{0.55,+}(x) = |F_{0.55,+}(x) - F_{BHP,0.55,+}(x)|$.

Assume that the probability distribution of the α^+ positive fluctuations $r_{\alpha^+}^+(t)$ is given by $F_{BHP,\alpha^+,+}$ (see [11]). The pdf $f_{FTSE100,+}$ of the FTSE100 daily index positive returns $r(t)$ is given by

$$f_{FTSE100,+}(x) = \frac{\alpha^+ x^{\alpha^+-1} f_{BHP}\left(\left(x^{\alpha^+} - \mu_{\alpha^+}^+\right)/\sigma_{\alpha^+}^+\right)}{\sigma_{\alpha^+}^+ \left(F_{BHP}\left(R_{\alpha^+}^+\right) - F_{BHP}\left(L_{\alpha^+}^+\right)\right)}.$$

Hence, taking $\alpha^+ = 0.55...$, we get

$$f_{FTSE100,+}(x) = 8.73...x^{-0.45...} f_{BHP}(30.87...x^{0.55...} - 1.95...).$$

In Figures 5 and 6, we show the data collapse of the histogram $f_{1,+}$ of the positive returns to our proposed theoretical pdf $f_{FTSE100,+}$.
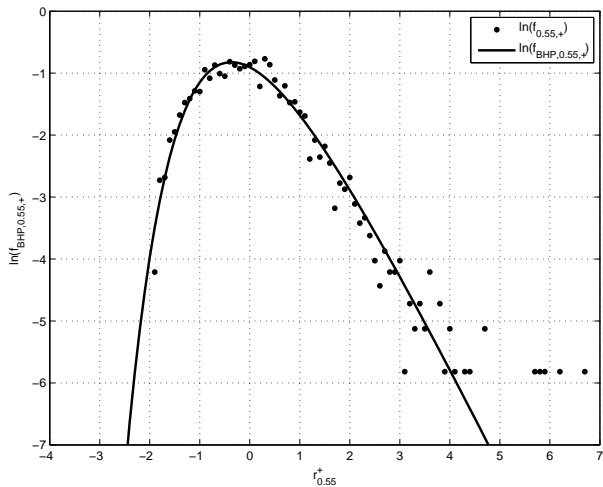
FIG. 3. The histogram of the α^+ positive fluctuations with the truncated BHP pdf $f_{BHP,0.55,+}$ on top, in the semi-log scale.
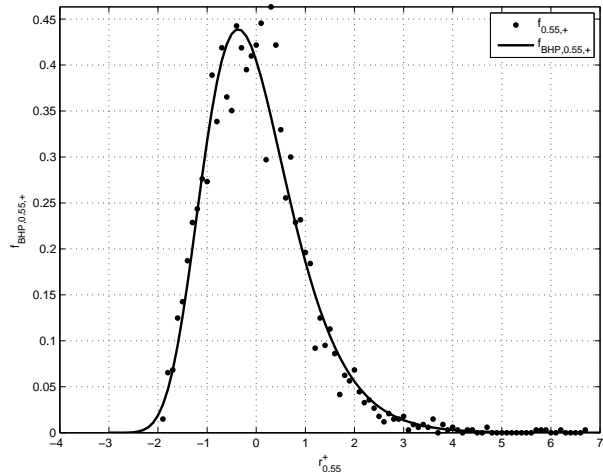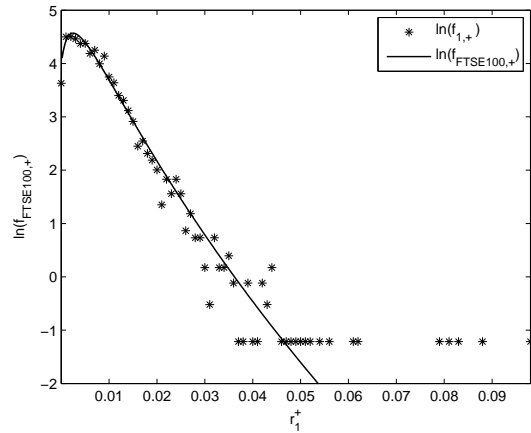

FIG. 5. The histogram of the fluctuations of the positive returns with the pdf $f_{FTSE100,+}$ on top, in the semi-log scale.

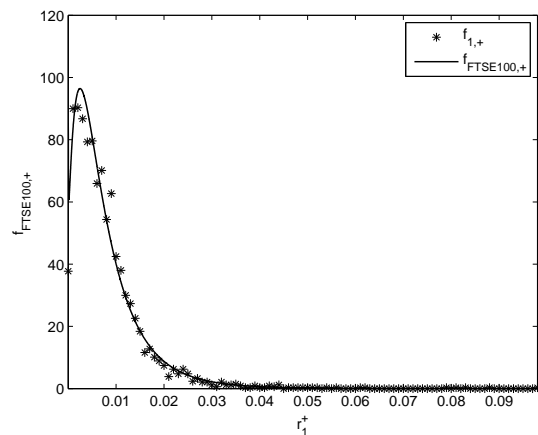FIG. 4. The histogram of the α^+ positive fluctuations with the truncated BHP pdf $f_{BHP,0.55,+}$ on top.

FIG. 6. The histogram of the fluctuations of the positive returns with the pdf $f_{FTSE100,+}$ on top.

III. NEGATIVE FTSE100 INDEX DAILY RETURNS

Let T^- be the set of all days t with negative returns, i.e.

$$T^- = \{t : r(t) < 0\}.$$

Let $n^- = 3074$ be the cardinal of the set T^-. Since the total number of observed days is $n = 6442$, we obtain that $n^-/n = 0.48$. The α re-scaled FTSE100 daily index negative returns are the returns $(-r(t))^\alpha$ with $t \in T^-$. We note that $-r(t)$ is positive. The mean $\mu_\alpha^- = 0.063...$ of the α re-scaled FTSE100 daily index negative returns

is given by

$$\mu_\alpha^- = \frac{1}{n^-} \sum_{t \in T^-} (-r(t))^\alpha \tag{4}$$

The *standard deviation* $\sigma_\alpha^- = 0.035...$ of the α re-scaled FTSE100 daily index negative returns is given by

$$\sigma_\alpha^- = \sqrt{\frac{1}{n^-} \sum_{t \in T^-} (-r(t))^{2\alpha} - (\mu_\alpha^-)^2} \tag{5}$$

We define the α *negative fluctuations* by

$$r_\alpha^-(t) = \frac{(-r(t))^\alpha - \mu_\alpha^-}{\sigma_\alpha^-} \tag{6}$$

for every $t \in T^-$. Hence, the α *negative fluctuations* are the normalized α re-scaled $FTSE100$ daily index nega-

tive returns. Let $L_\alpha^- = -1.74...$ be the *smallest* α negative fluctuation, i.e.

$$L_\alpha^- = \min_{t \in T^-} \{r_\alpha^-(t)\}.$$

Let $R_\alpha^- = 7.27...$ be the *largest* α negative fluctuation, i.e.

$$R_\alpha^- = \max_{t \in T^-} \{r_\alpha^-(t)\}.$$

We denote by $F_{\alpha,-}$ the *probability distribution of the α negative fluctuations*. Let the *truncated BHP probability distribution* $F_{BHP,\alpha,-}$ be given by

$$F_{BHP,\alpha,-}(x) = \frac{F_{BHP}(x)}{F_{BHP}(R_\alpha^-) - F_{BHP}(L_\alpha^-)}$$

where F_{BHP} is the BHP probability distribution. We apply the Kolmogorov-Smirnov statistic test to the null hypothesis claiming that the probability distributions $F_{\alpha,-}$ and $F_{BHP,\alpha,-}$ are equal. The Kolmogorov-Smirnov P value $P_{\alpha,-}$ is plotted in Figure 7. Hence, we observe that $\alpha^- = 0.55...$ is the point where the P value $P_{\alpha^-} = 0.68...$ attains its maximum. The Kolmogorov-Smirnov P value
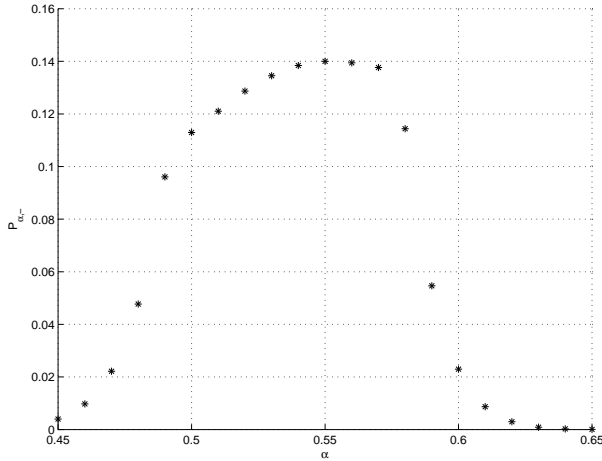
FIG. 7. The Kolmogorov-Smirnov P value $P_{\alpha,-}$ for values of α in the range $[0.45, 0.65]$.

$P_{\alpha,-}$ decreases with the distance $\|F_{\alpha,-} - F_{BHP,\alpha,-}\|$ between $F_{\alpha,-}$ and $F_{BHP,\alpha,-}$. In Figure 8, we plot $D_{\alpha^-,-}(x) = |F_{\alpha^-,-}(x) - F_{BHP,\alpha^-,-}(x)|$ and we observe that $D_{\alpha^-,-}(x)$ attains its highest values for the α^- negative fluctuations below the mean of the probability distribution.

In Figures 9 and 10, we show the data collapse of the histogram $f_{\alpha^-,-}$ of the α^- negative fluctuations to the truncated BHP pdf $f_{BHP,\alpha^-,-}$.

Assume that the probability distribution of the α^- negative fluctuations $r_{\alpha^-}^-(t)$ is given by $F_{BHP,\alpha^-,-}$, (see [11]).
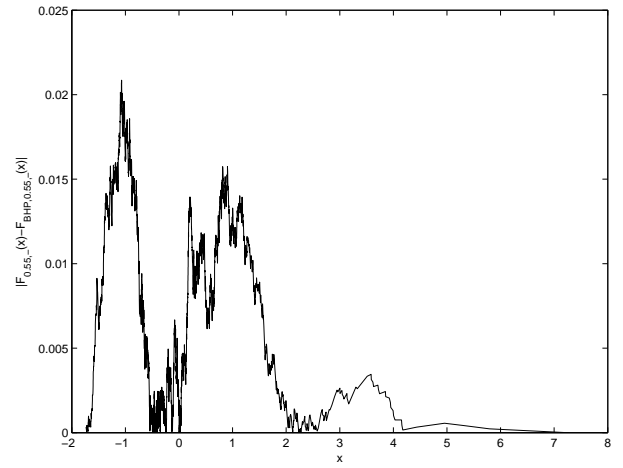
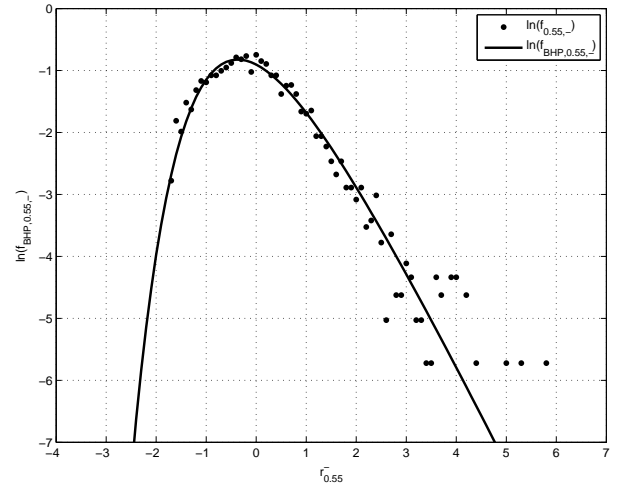FIG. 8. The map $D_{0.55,-}(x) = |F_{0.55,-}(x) - F_{BHP,0.55,-}(x)|$.

FIG. 9. The histogram of the α^- negative fluctuations with the truncated BHP pdf $f_{BHP,0.55,-}$ on top, in the semi-log scale.

The pdf $f_{FTSE100,-}$ of the FTSE100 daily index (symmetric) negative returns $-r(t)$, with $T \in T^-$, is given by

$$f_{FTSE100,-}(x) = \frac{\alpha^- x^{\alpha^- 1} f_{BHP}\left(\left(x^{\alpha^-} - \mu_{\alpha^-}^-\right) / \sigma_{\alpha^-}^-\right)}{\sigma_{\alpha^-}^-\left(F_{BHP}\left(R_{\alpha^-}^-\right) - F_{BHP}\left(L_{\alpha^-}^-\right)\right)}.$$

Hence, taking $\alpha^- = 0.55...$, we get

$$f_{FTSE100,-}(x) = 8.74...x^{-0.45...} f_{BHP}(28.88...x^{0.55...} - 1.82...)$$

In Figures 11 and 12, we show the data collapse of the histogram $f_{1,-}$ of the negative returns to our proposed theoretical pdf $f_{FTSE100,-}$.
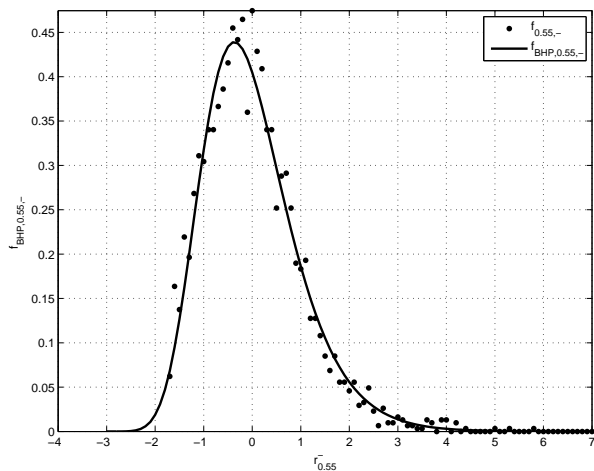
FIG. 10. The histogram of the α^- negative fluctuations with the truncated BHP pdf $f_{BHP,0.55,-}$ on top.
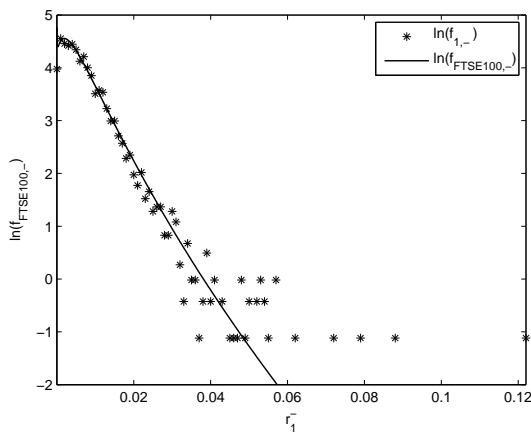

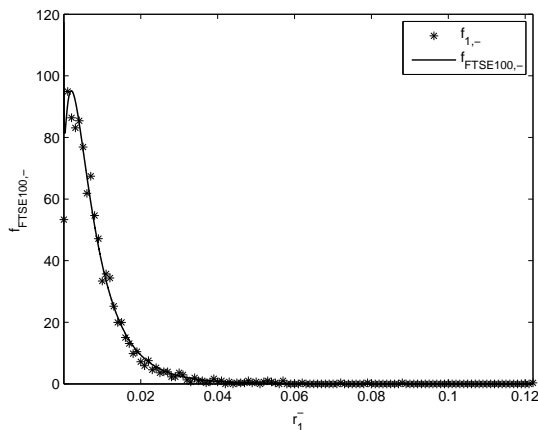FIG. 11. The histogram of the negative returns with the pdf $f_{FTSE100,-}$ on top, in the semi-log scale.

FIG. 12. The histogram of the negative returns with the pdf $f_{FTSE100,-}$ on top, in the semi-log scale.

IV. CONCLUSIONS

We used the Kolmogorov-Smirnov statistical test to compare the histogram of the α positive fluctuations and α negative fluctuations with the universal, non-parametric, Bramwell-Holdsworth-Pinton (BHP) probability distribution. We found that the parameters $\alpha^+ = 0.55...$ and $\alpha^- = 0.55...$ for the positive and negative fluctuations, respectively, optimize the P value of the Kolmogorov-Smirnov test. We obtained that the respective P values of the Kolmogorov-Smirnov statistical test are $P^+ = 0.19...$ and $P^- = 0.14....$ Hence, the null hypothesis was not rejected. The fact that α^+ is different from α^- can be do to leverage effects. We presented the data collapse of the corresponding fluctuations histograms to the BHP pdf. Furthermore, we computed the analytic expression of the probability distributions $F_{FTSE100,+}$ and $F_{FTSE100,-}$ of the normalized FTSE100 index daily positive and negative returns in terms of the BHP pdf. We showed the data collapse of the histogram of the positive and negative returns to our proposed theoretical pdfs $f_{FTSE100,+}$ and $f_{FTSE100,-}$. The results obtained in daily returns also apply to other periodicities, such as weekly and monthly returns as well as intraday values.

In [9, 13], it is found the data collapses of the histograms of some other stock indexes, prices of stocks, exchange rates, commodity prices and energy sources [12] to the BHP pdf.

Bramwell, Holdsworth and Pinton [3] found the probability distribution of the fluctuations of the total magnetization, in the strong coupling (low temperature) regime, for a two-dimensional spin model (2dXY) using the spin wave approximation. From a statistical physics point of view, one can think that the stock prices form a non-equilibrium system [6, 18, 19, 23]. Hence, the results presented here lead to a construction of a new qualitative and quantitative econophysics model for the stock market based in the two-dimensional spin model (2dXY) at criticality (see [14]).

Appendix: Definition of the Bramwell-Holdsworth-Pinton probability distribution

The universal nonparametric BHP pdf was discovered by Bramwell, Holdsworth and Pinton [3]. The *BHP probability density function (pdf)* is given by

$$f_{BHP}(\mu) = \int_{-\infty}^{\infty} \frac{dx}{2\pi} \sqrt{\frac{1}{2N^2} \sum_{k=1}^{N-1} \frac{1}{\lambda_k^2}} e^{ix\mu \sqrt{\frac{1}{2N^2} \sum_{k=1}^{N-1} \frac{1}{\lambda_k^2}}}$$

$$.e^{-\sum_{k=1}^{N-1} \left[\frac{ix}{2N} \frac{1}{\lambda_k} - \frac{i}{2} \arctan\left(\frac{x}{N\lambda_k}\right) \right]} .e^{-\sum_{k=1}^{N-1} \left[\frac{1}{4} \ln\left(1 + \frac{x^2}{N^2 \lambda_k^2}\right) \right]}$$

(A.1)

where the $\{\lambda_k\}_{k=1}^{L}$ are the eigenvalues, as determined in [5], of the adjacency matrix. It follows, from the formula of the BHP pdf, that the asymptotic values for large deviations, below and above the mean, are exponential and double exponential, respectively (in this article, we use the approximation of the BHP pdf obtained by taking $L = 10$ and $N = L^2$ in equation (A.1)). As we can see, the BHP distribution does not have any parameter (except the mean that is normalize to 0 and the standard deviation that is normalized to 1) and it is universal, in the sense that appears in several physical phenomena. For instance, the universal nonparametric BHP distribution is a good model to explain the fluctuations of order parameters in theoretical examples such as, models of self-organized criticality, equilibrium critical behavior, percolation phenomena (see [3]), the Sneppen model (see [3] and [7]), and auto-ignition fire models (see [24]). The universal nonparametric BHP distribution is, also, an explanatory model for fluctuations of several phenomenon such as, width power in steady state systems (see [3]), fluctuations in river heights and flow (see [5, 8, 11, 15, 16]), for the plasma density fluctuations and electrostatic turbulent fluxes measured at the scrape-off layer of the Alcator C-mod Tokamaks (see [25]) and for Wolf's sunspot numbers fluctuations (see [17]).

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