

A novel algorithm for sensorless motion control of flexible structures

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Abstract

This article demonstrates the validity of using an actuator as a single platform for measurements during a motion control assignment of flexible systems kept free from any kind of measurement. System acceleration level dynamics, parameters and interaction forces with the environment are coupled in an incident reaction torque that naturally rises when a flexible system is subjected to an action imposed by an attached actuator to the system. This work attempts to decouple each of the system parameters out of the incident coupled reaction torque at the interface point of the actuator with the flexible system by way of measurements taken from the actuator, not from the system. The identified parameters, along with the estimated states, are used to achieve sensorless motion control for flexible systems.

Key Words: *Reaction torque observer, disturbance observer, motion control, flexible systems*

1. Introduction

Observers play an important role in the motion control of dynamical systems, to provide estimates of variables that are hard to physically measure. It turns out that sensorless motion control can be partially achieved if proper observers are designed and embedded in the control system. A state observer models the real observable system in order to provide an estimate of its internal states, providing that measurement of the input and output are available. The work reported in this paper attempts not to measure system's output or any measurement from the plant. The entire dynamical system is split into two portions, namely the actuator side and the flexible plant side. Measurements are allowed to be taken from the actuator side while the flexible plant is kept free from any attached sensors. Therefore, the flexible plant is kept sensorless. Strictly speaking, the word "sensorless" is inaccurate, since one must measure some variable to obtain some information as basis of the unknown variables. Therefore, the word sensorless refers to the plant, not to the entire dynamical system.

O'Connor demonstrated that dynamical systems provide a natural feedback that can be used in order to achieve precise point-to-point motion and vibration control by using the actuator to launch and absorb mechanical waves to and from the dynamical system with the right amount, at the right time [1]-[2]. However, this wave-based control technique utilizes one measurement from the dynamical system, with the assumption

that the system does not interact with its environment. [3]. Nevertheless, the wave based control technique inspires the natural feedback concept that can be either a reflected mechanical wave or an instantaneous reaction. Both natural feedbacks include information about flexible system parameters, dynamics and interaction forces with the environment. However, the physical behavior of each affects the control system design. In other words, if the natural feedback was a reflected mechanical wave, the system naturally becomes a delayed system and the controller has to be designed accordingly, which is not the case if the natural feedback was an instantaneous reaction that rises due to an imposed action by an actuator.

In this work, system's natural feedback is considered as a reaction torque from the dynamical system on the interface point of the system with the actuator.

Much effort has been expended by many researchers to achieve robust motion control. Ohnishi considered model uncertainties and system's torque load along with the interaction forces or torques with the environment as a disturbance signal which can be estimated using actuator variables [4]. Consequently, robust motion control can be achieved by using the estimated disturbance signal to generate an additional control input that, in turn, compensates for disturbances [5]. In addition, the motion control system is turned into an acceleration control system if the nominal inertia and torque coefficients are selected as unity. It turns out that, from the previous definition of the disturbance signal, that the system's reaction torque is coupled with other terms such as varied self-inertia torque and actuator's torque ripple along with many other terms which will be explained in the next section. A system's reaction torque is the signal of our interest as it naturally includes coupled information about system parameters and acceleration level dynamics. Therefore, this reaction torque is considered as a natural feedback from the dynamical system and has to be decoupled out of the total disturbance signal.

This paper attempts to accomplish motion control of flexible dynamical systems with measurements taken from the actuator not from the system itself. However, system's natural feedback is considered as the single measurement from the system. Nevertheless, it can be estimated from the actuator side using actuator variables; it is therefore an estimate not a measurement.

Remainder of this paper is organized as follows. Section 2 includes a derivation of the reaction torque estimate through measurements taken from the actuator. Flexible system parameters are identified by performing an off-line experiment using the same measurements in Section 3. Flexible system states are estimated in Section 4 through recursive observers that depend on the dynamical system model, then the estimated states are used to perform plant-sensorless motion control. Experiments conducted on a flexible system with three degrees-of-freedom are included in Section 5. The same work can be extended to systems with infinite modes such as flexible manipulators and beams that can also be modeled with lumped masses. Conclusions and final remarks are found in Section 6.

2. Reaction torque estimation

For a flexible inertial system with n degrees-of-freedom attached to an actuator with rotor inertia J_m and angular position θ_m , through an elastic element with spring constant k as shown in Figure 1-a, the equations of motion can be written in the form

$$[\mathbf{J}][\ddot{\theta}] + [\mathbf{B}][\dot{\theta}] + [\mathbf{k}][\theta] = \tau, \quad (1)$$

where \mathbf{J} , \mathbf{B} , \mathbf{K} are the inertia, damping and stiffness matrices, and θ and τ are vectors of system's generalized co-ordinates and input torques. Assuming that the damping coefficients and the joints stiffness are uniform

along the flexible dynamical system, solving for $B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1)$, we obtain the equality

$$\tau_{\text{reac}}(t) \triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) = J_1\ddot{\theta}_1 + J_2\ddot{\theta}_2 + J_3\ddot{\theta}_3 + \dots + J_n\ddot{\theta}_n, \quad (2)$$

where, $\tau_{\text{reac}}(t)$ is the incident reaction torque from the dynamical system on the point of interface between the actuator and the flexible dynamical system. $\tau_{\text{reac}}(t)$ instantaneously rises whenever the actuator imposes any action on the dynamical system. Equation (2) indicates that the reaction torque is indeed a natural feedback signal as it includes coupled information about system parameters and acceleration level dynamics. Since the purpose is to keep the plant side free from any measurement, we write the actuator's mechanical equation of motion as

$$J_{\text{mo}} \frac{d^2\theta_m}{dt^2} = k_{\text{to}} i_a - \underbrace{B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) - f_{\text{cm}} + \Delta k_t i_a - \Delta J_m \frac{d^2\theta_m}{dt^2}}_{d(t)}, \quad (3)$$

where, J_{mo} , k_{to} , i_a and f_{cm} are the nominal actuator inertia, torque constants, actuator's current and coulomb friction. ΔJ_m and Δk_t are the deviations between actuator's nominal and actual values [5]. The disturbance signal can be written as

$$d(t) \triangleq \Delta k_t i_a - \tau_{\text{reac}}(t) - f_{\text{cm}} - \Delta J_m \frac{d^2\theta_m}{dt^2}. \quad (4)$$

The following low pass filter can be used to estimate disturbance $d(t)$:

$$\widehat{d}(t) = \frac{g_{\text{dist}}}{s + g_{\text{dist}}} \left[g_{\text{dist}} J_{\text{mo}} \frac{d\theta(t)}{dt} + i_a(t) k_{\text{to}} \right] - g_{\text{dist}} J_{\text{mo}} \frac{d\theta(t)}{dt}, \quad (5)$$

where g_{dist} is the filter's corner frequency or the observer's positive gain. Consequently, the estimation error $\widetilde{d}(t)$ can be written as

$$\widetilde{d} = \widehat{d}(t) - d(t) = \frac{g_{\text{dist}}}{s + g_{\text{dist}}} \left[g_{\text{dist}} J_{\text{mo}} \frac{d\theta(t)}{dt} + i_a(t) k_{\text{to}} \right] - g_{\text{dist}} J_{\text{mo}} \frac{d\theta(t)}{dt} - J_{\text{mo}} \frac{d^2\theta(t)}{dt^2} + k_{\text{to}} i_a(t). \quad (6)$$

Therefore, the dynamics of the disturbance error is governed by the differential equation

$$\frac{d}{dt} \widetilde{d}(t) + g_{\text{dist}} \widetilde{d}(t) = \Omega(t); \quad (7)$$

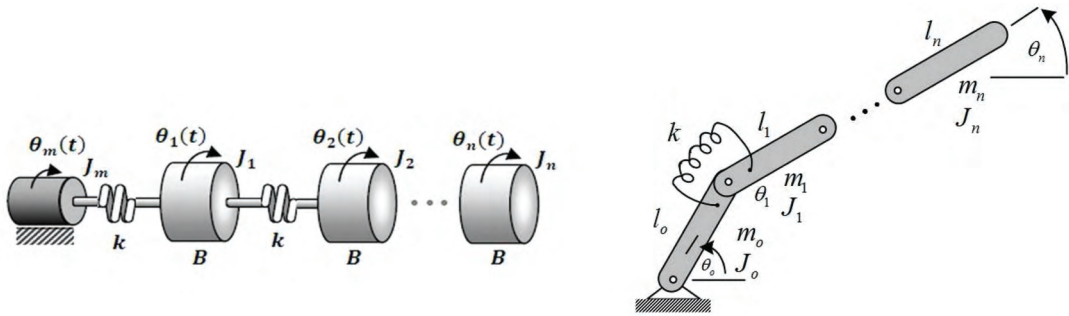


Figure 1. Diagrams of lumped dynamical systems modeled.

$$\Omega(t) \triangleq g_{\text{dist}}^2 J_{\text{mo}} \frac{d\theta(t)}{dt} + g_{\text{dist}} i_a(t) k_{\text{to}} + (s + g_{\text{dist}}) [k_t i_a(t) - J_m \frac{d^2\theta(t)}{dt^2} - g_{\text{dist}} J_{\text{mo}} \frac{d\theta(t)}{dt}].$$

Solving equation (7) for $\tilde{d}(t)$, we obtain

$$\tilde{d}(t) = c_1 e^{-g_{\text{dist}} t} + e^{-g_{\text{dist}} t} \int_0^T e^{-g_{\text{dist}} t} \Omega(t) dt, \tag{8}$$

which guarantees the exponential convergence of the estimated disturbance to the actual one by proper selection of the observer gain g_{dist} . In other words, as $t \rightarrow \infty \Rightarrow \tilde{d}(t) \rightarrow 0 \Rightarrow \hat{d}(t) \rightarrow d(t)$. Implementation of disturbance observer equation (5) is illustrated in Figure 2a, where actuator’s current along with actuator’s velocity are used to estimate the disturbance $d(t)$. Assuming that the coulomb friction is too small, the reaction torque can be decoupled out of the disturbance by subtracting the varied self-inertia torque $\Delta J_m \ddot{\theta}_m(t)$ and actuator’s torque ripple $\Delta k_t i_m(t)$ from the total disturbance $\hat{d}(t)$,

$$\widehat{\tau}_{\text{reac}}(t) = \hat{d}(t) - \Delta k_t i_m(t) + \Delta J_m \ddot{\theta}_m(t), \tag{9}$$

that in turn requires determination of deviation between actuator’s actual and nominal values $\Delta k_t, \Delta J_m$. Fortunately, the varied self-inertia torque and actuator’s torque ripple are inherent properties for the actuator, and they depend on the actuator’s parameter deviations and actuator variables. Therefore, they can be determined through an off-line experiment for the actuator when its running free from any attached load. Consequently, equation (4) can be written as

$$\widehat{d}_{\text{par}}(t) = \underbrace{\tau_{\text{reac}}(t)}_0 + \Delta k_t i_m(t) - \Delta J_m \ddot{\theta}_m(t) - D \dot{\theta}_m(t), \tag{10}$$

where $d(t)$ becomes $\widehat{d}_{\text{par}}(t)$ as the disturbance became dependent only on the parameters uncertainties as the actuator became free from any attached load $\tau_{\text{reac}}(t) = 0$. D is the viscous damping coefficient of the actuator. Putting equation (10) into the over-determined matrix form

$$\begin{bmatrix} \Delta k_t & -D & -\Delta J_m \end{bmatrix}_{1 \times 3} \begin{bmatrix} \dot{i}_m \\ \dot{\theta}_m \\ \ddot{\theta}_m \end{bmatrix}_{3 \times r} = \begin{bmatrix} \widehat{d}_{\text{par}} \end{bmatrix}_{1 \times r} ; \quad \mathbf{H} \triangleq \begin{bmatrix} \dot{i}_m \\ \dot{\theta}_m \\ \ddot{\theta}_m \end{bmatrix}, \tag{11}$$

where $\dot{i}_m(t)$, $\dot{\theta}_m(t)$ and $\ddot{\theta}_m(t)$ are vectors of actuator’s current, velocity and acceleration with r data points, the optimum Δk_t and ΔJ_m can be determined as follows through equation (11):

$$\begin{bmatrix} \widehat{\Delta k_t} & -\widehat{D} & -\widehat{\Delta J_m} \end{bmatrix} = \begin{bmatrix} \mathbf{H}^T \mathbf{H} \end{bmatrix}^{-1} \mathbf{H}^T \begin{bmatrix} \widehat{d}_{\text{par}} \end{bmatrix} = \mathbf{H}^\dagger \begin{bmatrix} \widehat{d}_{\text{par}} \end{bmatrix}, \tag{12}$$

where \mathbf{H}^\dagger is the pseudo inverse of \mathbf{H} . Using equation (12) along with equation (9), estimate of the incident reaction torque can be determined as

$$\widehat{\tau}_{\text{reac}}(t) = \widehat{d}(t) - \widehat{\Delta k_t} i_m(t) + \widehat{\Delta J_m} \ddot{\theta}_m(t) \tag{13}$$

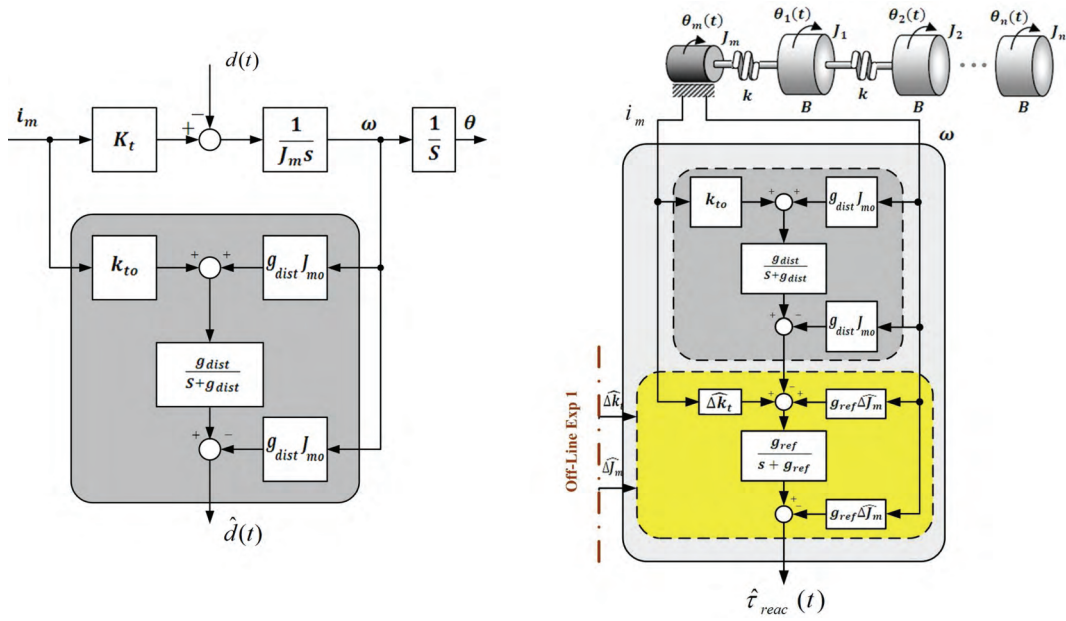


Figure 2. Block diagrams showing observers implementation. (a) Disturbance observer, (b) Reaction torque observer.

Block diagram implementation of the reaction torque observer is illustrated in Figure 2b, where disturbance is estimated then reaction torque is decoupled by subtracting the varied self-inertia and torque ripple torque. The previous torque observer provides two outputs: namely, the estimated disturbance and estimated reaction torque. The first can be used to accomplish robust motion control while the other estimate is used as a natural feedback from the dynamical system on the actuator side and will be used for further analysis and estimations. Similar procedures can be performed on a flexible manipulator illustrated in Figure 1b that can be modeled using lumped masses. However, it differs from the inertial spring-mass system due to the coupling between rotational and translational dynamics.

3. Parameters identification

In order to determine system's uniform viscous damping coefficient B and stiffness k from the reaction torque estimate, we rewrite equation (2) by replacing the actual reaction torque with the estimated torque:

$$\widehat{\tau}_{reac}(t) \triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1). \quad (14)$$

Simply, equation (14) can be used to estimate system parameters if the first angular position $\theta_1(t)$ is measured. However, this work attempts to demonstrate that flexible systems can be kept free from any measurement. Therefore, $\theta_1(t)$ is not allowed to be measured so as not to violate the natural feedback concept. In other words, system has to be kept free from any measurement in order to claim that the reaction torque can be used as a natural feedback from the dynamical system. The following analysis however will reveal that one does not have to pick a single measurement from the dynamical system. The governing equations for a single input structure with one rigid mode and n flexible modes is of the form

$$\begin{bmatrix} \dot{\theta}_o \\ \ddot{\theta}_o \\ \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \vdots \\ \ddot{\theta}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega_1^2 & -2\zeta_1\omega_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \vdots & -\omega_n^2 & -2\zeta_n\omega_n \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_o \\ \theta_1 \\ \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u, \tag{15}$$

where $\theta_o(t)$ is the rigid mode, while $\theta_1(t) \dots \theta_n(t)$ are the flexible modes. $\omega_1 \dots \omega_n$ are the corresponding natural frequencies, and $\zeta_1 \dots \zeta_n$ are the corresponding damping ratios [7]. Without any loss of generality, we can consider a flexible system with 3 degrees-of-freedom with the system matrix

$$A = \begin{bmatrix} J_1s^2 + Bs + k & -Bs - k & 0 \\ -Bs - k & J_2s^2 + 2B + 2k & -Bs - k \\ 0 & -Bs - k & J_3s^2 + Bs + k \end{bmatrix}.$$

Solving the determinant of A and setting $\det(A)=0$, assuming equal inertial masses, we obtain the characteristic equation

$$J^3s^6 + 4J^2Bs^5 + (4J^2k + 3JB^2)s^4 + 6JBks^3 + 3Jk^2s^2 = 0.$$

Solving for the roots of the previous characteristic equation assuming zero damping coefficient for computation simplicity, we obtain

$$s_{1,2} = 0 \quad , \quad s_{3,4} = \pm j\sqrt{\frac{k}{J}} \quad , \quad s_{5,6} = \pm j\sqrt{\frac{3k}{J}} \tag{16}$$

Therefore, the corresponding modal matrix is

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}. \tag{17}$$

The first eigenvector represents the so-called rigid-body mode since the three masses are moving together, while the second and third eigenvectors represent the so-called flexible modes where the three masses are moving relative to one another. Therefore, the spring mass system depicted in Figure 3a can be represented as three decoupled systems. Figure 3b illustrates the equivalent rigid-body mode when any of the flexible modes is not excited. In other words, the depicted system consists of an actuator attached to a 3 degrees-of-freedom flexible system. If the actuator didn't excite any of the 3 degrees-of-freedom system's flexible modes, the equivalent system depicted in Figure 3b can be used to describe motion of the system. Consequently, motion of the equivalent system can be estimated by integrating the estimated reaction torque:

$$\widehat{\Theta}(t) = \frac{1}{\sum_{i=1}^n J_i} \int_o^t \int_o^t \widehat{\tau}_{\text{reac}}(\tau) \, d\tau d\tau + c_2t + c_3, \tag{18}$$

where $\widehat{\Theta}(t)$ is the position estimate of the rigid system which can be obtained through actuator's current and velocity. Rewriting equation (14) by replacing the first mass position with the estimated system's rigid motion $\widehat{\Theta}(t)$, we obtain

$$\widehat{\tau}_{\text{reac}}(t) = B(\dot{\theta}_m(t) - \dot{\widehat{\Theta}}(t)) + k(\theta_m(t) - \widehat{\Theta}(t)), \tag{19}$$

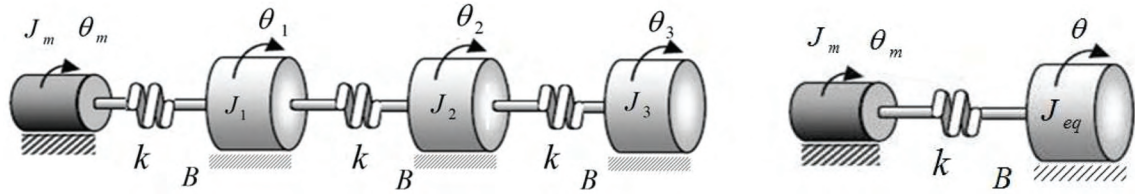


Figure 3. Diagram illustrating flexible lumped system attached to an actuator. (a) Mass spring system, (b) Equivalent rigid system.

we define $\underline{\xi} \triangleq (\theta_m - \hat{\Theta})$, $\underline{\eta} \triangleq (\dot{\theta}_m - \hat{\dot{\Theta}})$ and $\mathbf{G} \triangleq \begin{bmatrix} \underline{\xi} \\ \underline{\eta} \end{bmatrix}$. Therefore the estimated system uniform damping coefficient and stiffness can be computed

$$\begin{bmatrix} \hat{k} \\ \hat{B} \end{bmatrix} = \begin{bmatrix} \mathbf{G}^T \mathbf{G} \end{bmatrix}^{-1} \mathbf{G}^T \begin{bmatrix} \widehat{\underline{\tau}}_{\text{reac}} \end{bmatrix} = \mathbf{G}^\dagger \begin{bmatrix} \widehat{\underline{\tau}}_{\text{reac}} \end{bmatrix}. \quad (20)$$

Here, \mathbf{G}^\dagger is the pseudo inverse of \mathbf{G} .

First, an arbitrary rigid motion maneuver has to be performed so as to estimate $\hat{\Theta}(t)$, which can be performed by either using a low pass filter or by Fourier synthesizing the control input in order to guarantee that it doesn't contain any energy at the system's resonant frequencies [8], [9]. Both of these techniques guarantee that any of the system flexible modes will not be excited. However, in this paper the control input is pre-filtered with a low pass filter in order to take away any energy at the resonant frequencies of the system such that flexible modes will not be excited. Figure 4 illustrates the physical interpretation of the modal matrix \mathbf{M} . The rigid mode is illustrated in Figure 4a where no relative motion exists between system's lumped masses. Indeed, such motion can be obtained by either pre-filtering the control input or any other existing technique that can be found with more details in [11]. In this work a low pass filter is used to obtain the rigid motion depicted in Figure 4a, consequently the number of generalized coordinates required to describe the motion of the system are minimized to two, namely the actuator's angular position and the equivalent system's position $\hat{\Theta}(t)$. This allows determination of system uniform parameters through equation (19) or equation (20). On the other hand, if any of the system modes was excited Figure 4b,c, the number of unknowns will be increased which, in turn, adds more complicity to the problem. Therefore, system uniform parameters are identified at the system's low frequency range.

Equation (20) can be implemented by measuring actuator's angular position along with estimating system's rigid motion. The two measurements along with their derivatives are used to formulate vectors $\underline{\xi}$ and $\underline{\eta}$ which in turn are used to formulate the data matrix \mathbf{G} . System parameter identification block diagram implementation is illustrated in Figure 5, where any arbitrary rigid motion has to be performed. Therefore, the control signal is passed through a low pass filter to remove any energy at the system resonant frequencies, disturbance is estimated then system's reaction torque is decoupled. The estimated reaction torque is then double integrated over time and divided by the system's equivalent inertia to obtain system rigid motion estimate that in turn is used to estimate system uniform parameters through equation (20).

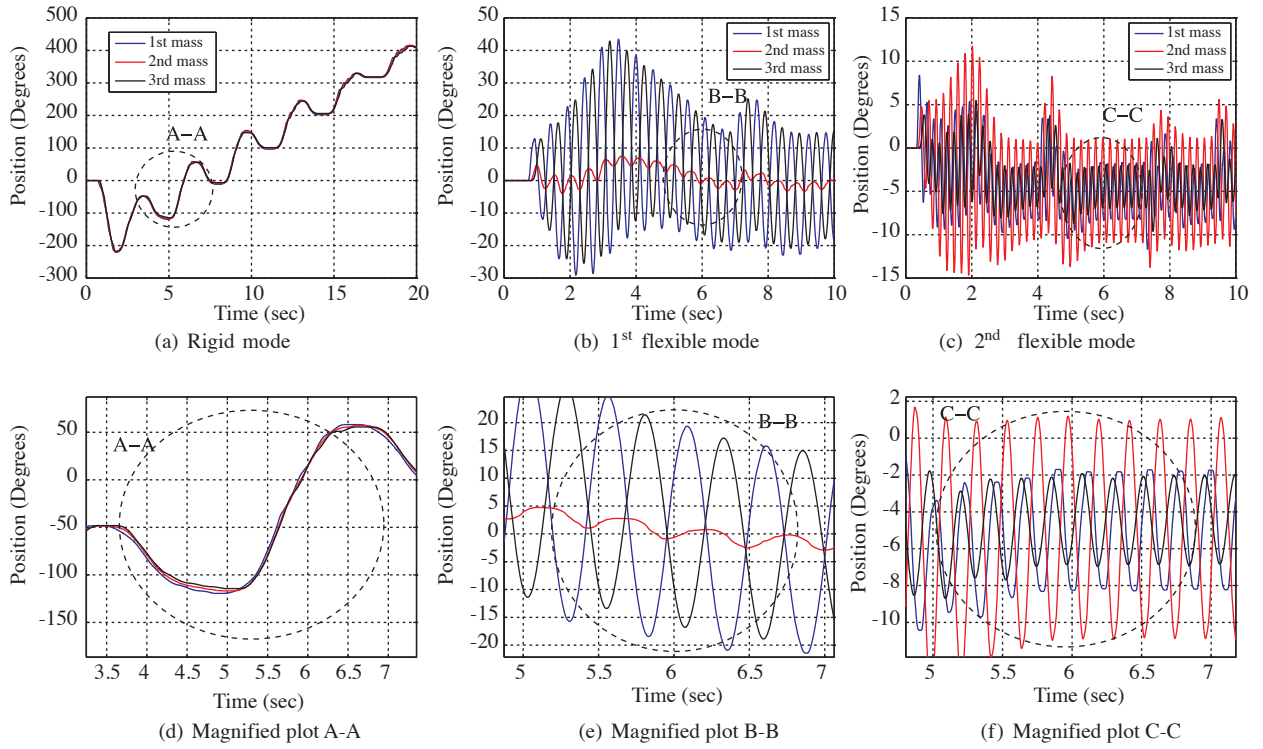


Figure 4. Modal analysis of a 3 DOF flexible system.

4. States estimation

Using the estimated viscous damping and stiffness in equation (14), then solving for $\theta_1(t)$, we obtain the following first mass position estimate:

$$\hat{\theta}_1(t) = c_4 e^{-\frac{\hat{B}}{k}t} + \int_0^T \beta(\tau) e^{\frac{\hat{B}}{k}(t-\tau)} d\tau. \quad (21)$$

$$\beta(t) \triangleq \dot{\theta}_m(t) + \frac{\hat{K}}{\hat{B}} \theta_m(t) - \frac{1}{\hat{B}} \widehat{\tau}_{\text{reac}}(t)$$

As the model of the flexible dynamical system is known, estimate of the reaction torque along with viscous damping coefficient and stiffness can be used to obtain the following recursive observers for position of each lumped mass of the flexible system [10]:

$$\hat{\theta}_2(t) = c_5 e^{-\frac{\hat{B}}{k}t} + \int_0^T \xi(\tau) e^{\frac{\hat{B}}{k}(t-\tau)} d\tau, \quad (22)$$

$$\xi(t) \triangleq \frac{J_1}{\hat{B}} \hat{\theta}_1 - (\dot{\theta}_o - \hat{\theta}_1) - \frac{\hat{k}}{\hat{B}} (\theta_o - \theta_1) + \hat{\theta}_1 + \frac{\hat{k}}{\hat{B}} \hat{\theta}_1,$$

$$\hat{\theta}_3(t) = c_6 e^{-\frac{\hat{B}}{k}t} + \int_0^T \varepsilon(\tau) e^{\frac{\hat{B}}{k}(t-\tau)} d\tau, \quad (23)$$

$$\varepsilon(t) \triangleq \frac{J_2}{\widehat{B}} \widehat{\theta}_2 - (\widehat{\theta}_1 - \widehat{\theta}_2) - \frac{\widehat{k}}{\widehat{B}} (\widehat{\theta}_1 - \widehat{\theta}_2) + \widehat{\theta}_2 + \frac{\widehat{k}}{\widehat{B}} \widehat{\theta}_2,$$

$$\widehat{\theta}_i(t) = c_i e^{-\frac{\widehat{B}}{k} t} + \int_0^T \Omega(\tau) e^{\frac{\widehat{B}}{k} (t-\tau)} d\tau, \quad (24)$$

$$\Omega(t) \triangleq \frac{1}{\widehat{B}} g(J_{i-1}, \widehat{\theta}_{i-1}, \dot{\widehat{\theta}}_{i-1}, \ddot{\widehat{\theta}}_{i-1}, \widehat{k}, \widehat{B}),$$

where $\widehat{\theta}_i(t)$ is position estimate of the i^{th} lumped mass, and $c_4, c_5, c_6 \dots, c_i$ are the integration constants. The recursive equations (equation (24)) can be used to estimate the position of the i^{th} mass regardless to the frequency content of the control input, unlike equation (18) which is used just to observe the rigid motion of the flexible system in order to estimate B and k . In addition, only two measurements from the actuator are required in order to estimate any lumped mass's position without taking any measurement from the flexible plant side. Figure 6 illustrates implementation of recursive equation (24) for estimating position. Two measurements are taken from the actuator, and the reaction torque signal is decoupled out of the disturbance. The estimated reaction torque is then used to estimate both system parameters and lumped masses positions.

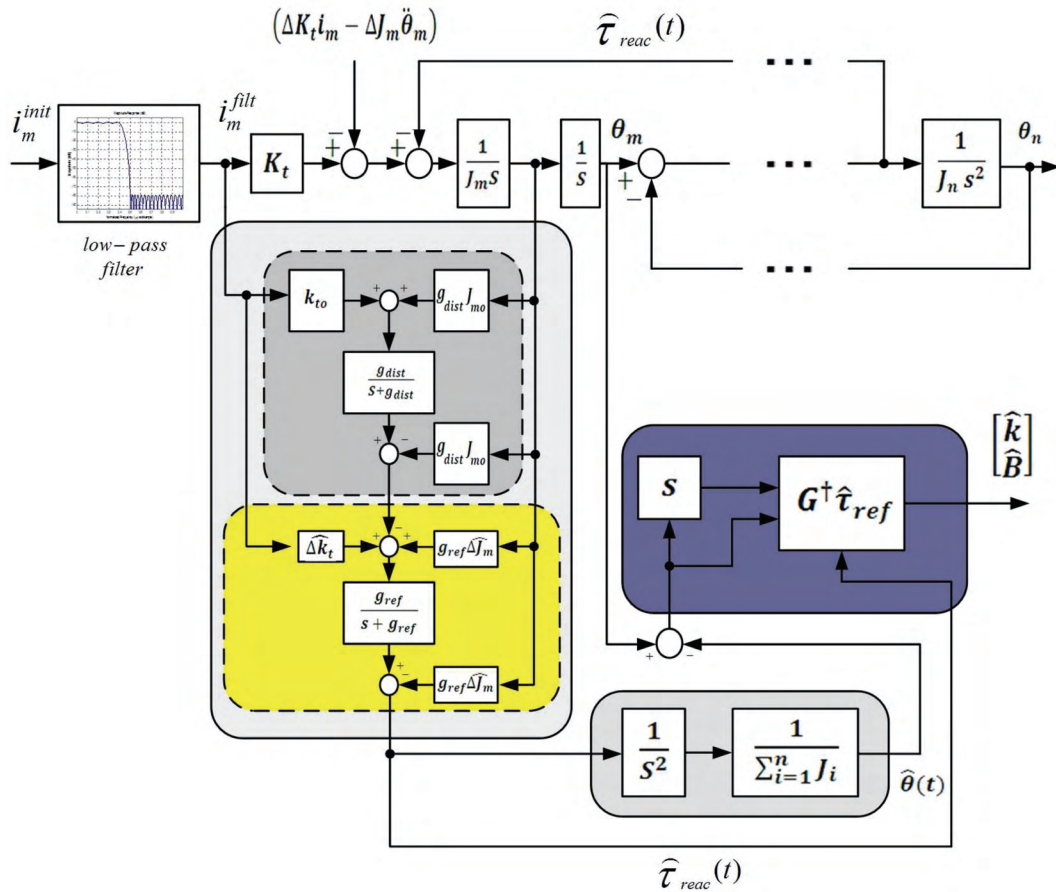


Figure 5. Experimental diagram showing off-line identification of parameters.

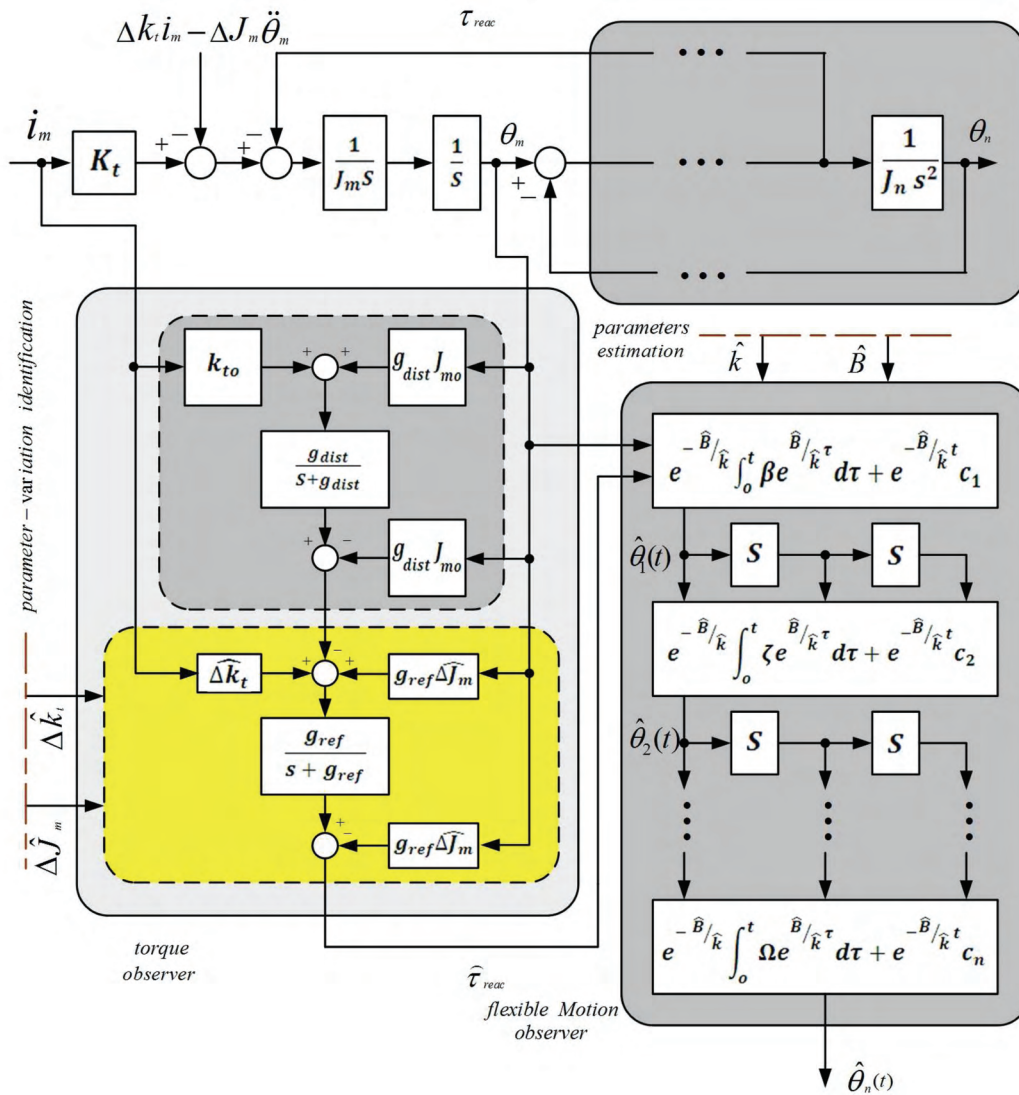


Figure 6. States estimation.

4.1. Sensorless motion control

Since system parameters are estimated through equation (20) by performing a rigid motion maneuver, and where the position of each lumped mass is observed through the recursion equations (24), a motion control assignment can be performed without attaching any sensor to the plant. Only actuator’s current and velocity need to be measured. However, the incident reaction torque that naturally rises from the flexible system is considered as an alternative to any measurement. The recursive formula equation (24) can be used to control i^{th} point of interest of the lumped system. Moreover, the rest of the system can be monitored as position estimate of each mass is available. A sensorless PID control law with a disturbance compensation can be expressed as

$$u(t) = k_p(\theta_{ref} - \hat{\theta}_i) + k_d \frac{d}{dt}(\theta_{ref} - \hat{\theta}_i) + k_p \int_0^t (\theta_{ref} - \hat{\theta}_i) d\tau + u(t)^{comp}, \quad (25)$$

$$u(t)^{comp} = \frac{1}{k_{to}} \hat{d}(t). \quad (26)$$

Figure 7 illustrates the block diagram implementation of the sensorless-plant control law (equation (25)). An actuator's current and velocity are measured while the flexible plant is kept free from any measurement, except the incident natural feedback that can be determined using the same actuator measurements.

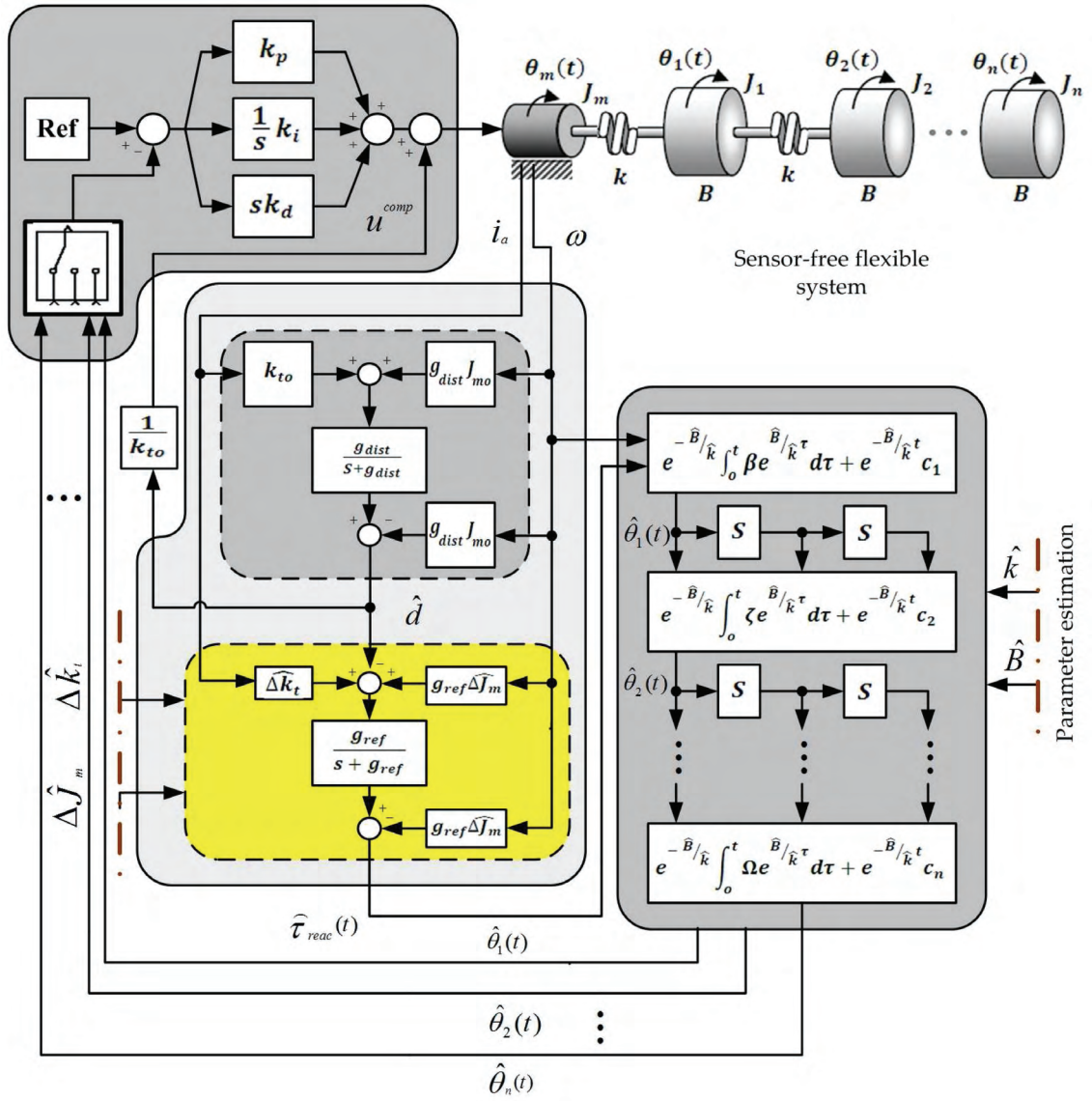


Figure 7. Sensorless-Plant motion control.

Control law equation (25) and equation (26) show an interesting feature of the proposed control technique where disturbance is used to achieve robustness along with keeping the plant sensorless. In other words, disturbance signal is used in equation (26) to generate an additional control law to compensate for disturbances. Meanwhile, disturbance is used to estimate reaction torque, system parameters and states that are used as a replacement to any attached sensor to the flexible plant.

5. Experimental results

In order to verify the validity of the proposed motion control technique, experiments are performed on a Multi-degrees-of freedom flexible system as shown in Figure 8. The experimental setup consists of three inertial masses (1-6-7) connected with springs (4) with theoretical spring constant $k_{th} = 1.62$ kN/m. In addition, each mass is connected to an encoder (2-3-5) in order to compare the observed results obtained through equation (24) with the actual measurements. The flexible plant is attached to a Maxon Ec motor (8). Table 1 includes the experimental parameters. Figure 9 illustrates the experimental procedure that has to be followed in order to control motion of i^{th} mass of the lumped flexible system without taking any measurement from the plant side. First, the control input is filtered in order to remove any energy at the system's resonant frequencies, the previous step guarantees that any of the system's flexible modes will not be excited. Consequently, rigid motion of the system can be observed through equation (20) and used to estimate system uniform damping coefficient and stiffness. The previous step is nothing but an off-line experiment to identify system parameters. Therefore, the control input doesn't have to be filtered as soon as the parameters are identified. Then, the estimated parameters along with the observed reaction torque are used as inputs for chain of recursive position observers (equation (24)) which provide a position estimate for each lumped mass of the system as shown in Figure 9. Consequently, position estimates are compared with the actual measurements taken by attached encoders to

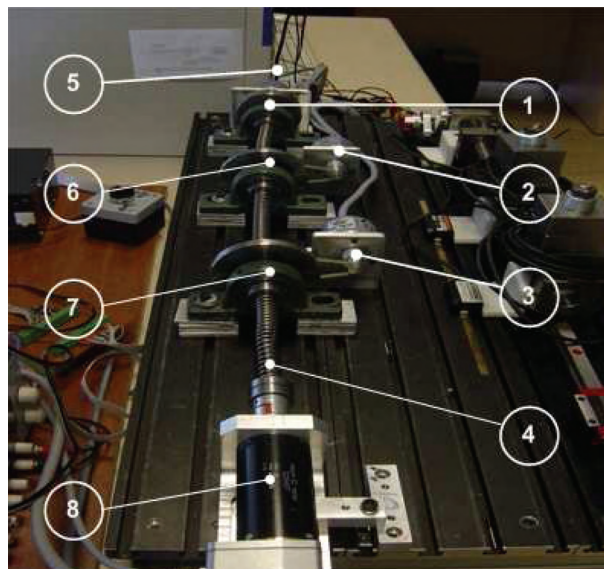


Figure 8. Experimental setup.

each mass as depicted in Figure 8. Estimates are then used as replacements to measurements taken by any attached sensors. The following steps explains how to turn the control system into plant-sensorless control system:

1. Filtering or Fourier synthesis of control input;
2. Rigid motion estimation through equation (18);
3. Parameters identification through equation (20);
4. States estimation through recursive formula (24);
5. Sensorless motion control through equation (25).

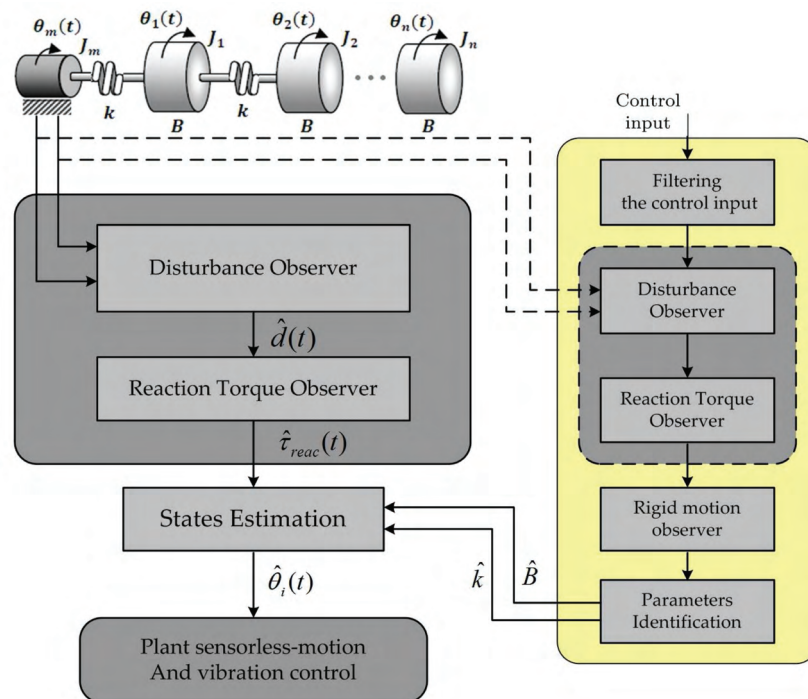


Figure 9. Experimental procedure.

5.1. Rigid motion estimation experimental results

In order to estimate system uniform-viscous damping and stiffness, the system has to perform a rigid motion maneuver. That can be performed by passing the control input through a low pass filter with a corner frequency $1 \text{ rad}\cdot\text{s}^{-1}$ that removes any energy at system resonant frequencies. The experimental setup is illustrated in Figure 8, where an encoder is attached to each lumped mass. Measurements from the encoders can be used to verify that no relative motion exists between the lumped masses, as shown in Figure 4a. Any of the attached encoder's reading can be used to verify the validity of equation (18). Figure 10 illustrates the difference between the estimated rigid motion obtained through equation (18) and the actual position.

Table 1. Experimental parameters.

Parameter	Value	Parameter	Value
J_1	5152.99 gcm ²	g_{dist}	100 rad/sec
J_2	5152.99 gcm ²	g_{lpf}	1 rad/sec
J_3	6192.707 gcm ²	f_{init}	1 rad/sec
J_m	209 gcm ²	k_{th}	1.627 KN/m
k_b	235 rpm/v	k_t	40.6 mNm/A
8	Maxon-Ec-Motor 229427	1,6,7	Inertial loads
4	Spring- $k_{th} = 1.62kN/m$	2,3,5	Optical encoders

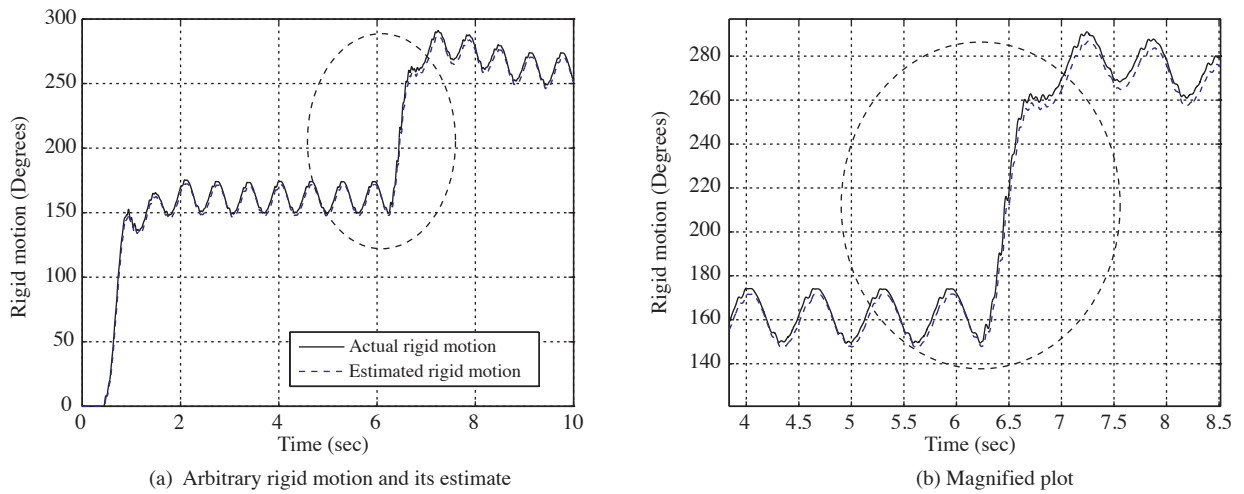


Figure 10. Rigid motion estimation experimental result.

5.2. Parameters identification experimental results

Figure 10 indicates the validity of equation (18) where the actual and estimated positions are identical. Then the data points vector of the rigid motion signal and actuator angular position along with the reaction torque are used to formulate $\underline{\xi}$, $\underline{\eta}$ and G . Consequently, the system's uniform viscous damping and stiffness can be determined through equation (20). Experimentally, the average value of the viscous damping coefficient and joints stiffness turn out to be 0.084 N·s/m and 1.546 kN/m, respectively. The difference between the actual known-before-hand parameters and those identified through equation (20) turns out to be less than 5 percent. Equation (19) represents an over-determined system, since we consider $\underline{\theta}_m(t)$, $\widehat{\underline{\Theta}}(t)$ and $\widehat{\underline{\tau}_{reac}}(t)$ as vector of data points. Therefore, the number of equations is larger than number of unknowns. Consequently, equation (20) computes the optimum parameters that minimizes the norm squares-of-error. Figure 11 illustrates the actual reaction torque signal versus the reconstructed one using the estimated parameters \widehat{B} and \widehat{k} .

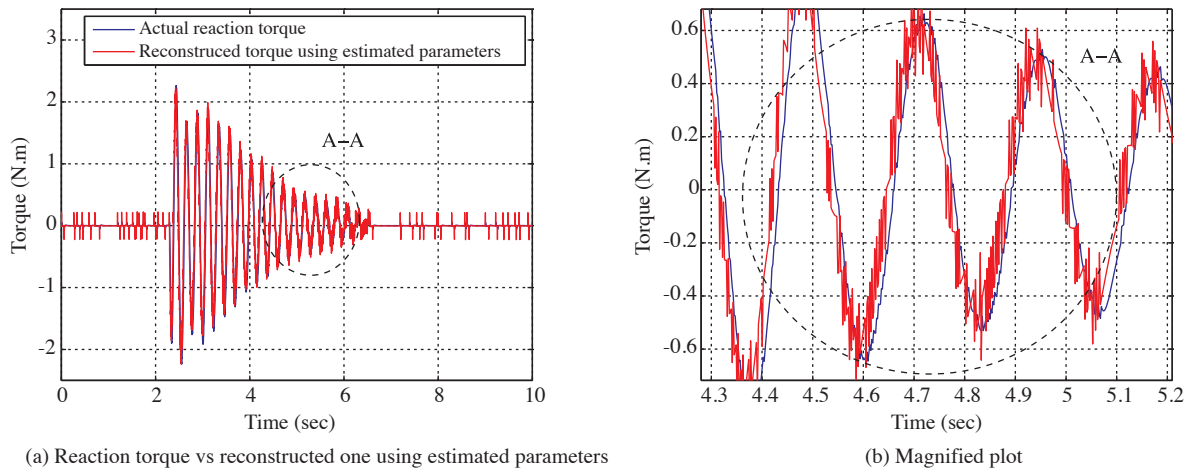


Figure 11. Torque as a function of time, as the experimental result from parameter identification.

5.3. States estimation experimental results

The recursive formula equation (24) can be used to observe position of any lumped mass along a flexible system by using the estimated reaction torque and the identified system parameters. In order to experimentally examine the validity of equation (24), system flexible modes have to be excited. Unlike the filtered control input that is used to perform a rigid motion maneuver for the parameter identification experiment, the control input in this experiment has to contain some energy at the system resonant frequencies to excite system's flexible modes so as to verify the validity of equation (24), as shown in Figure 12, where relative motion exists between system's lumped masses. Figure 13 illustrates the difference between the actual measurements taken with attached sensors with the observed positions obtained through equation (24).

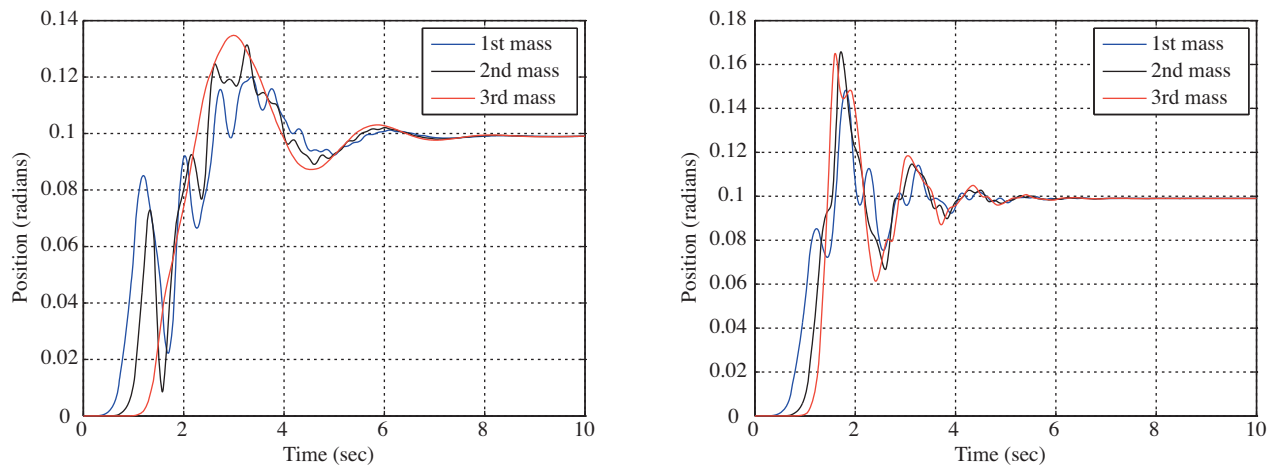


Figure 12. Relative phase position as a function time showing the flexible oscillation of the examined 3-DOF dynamical system.

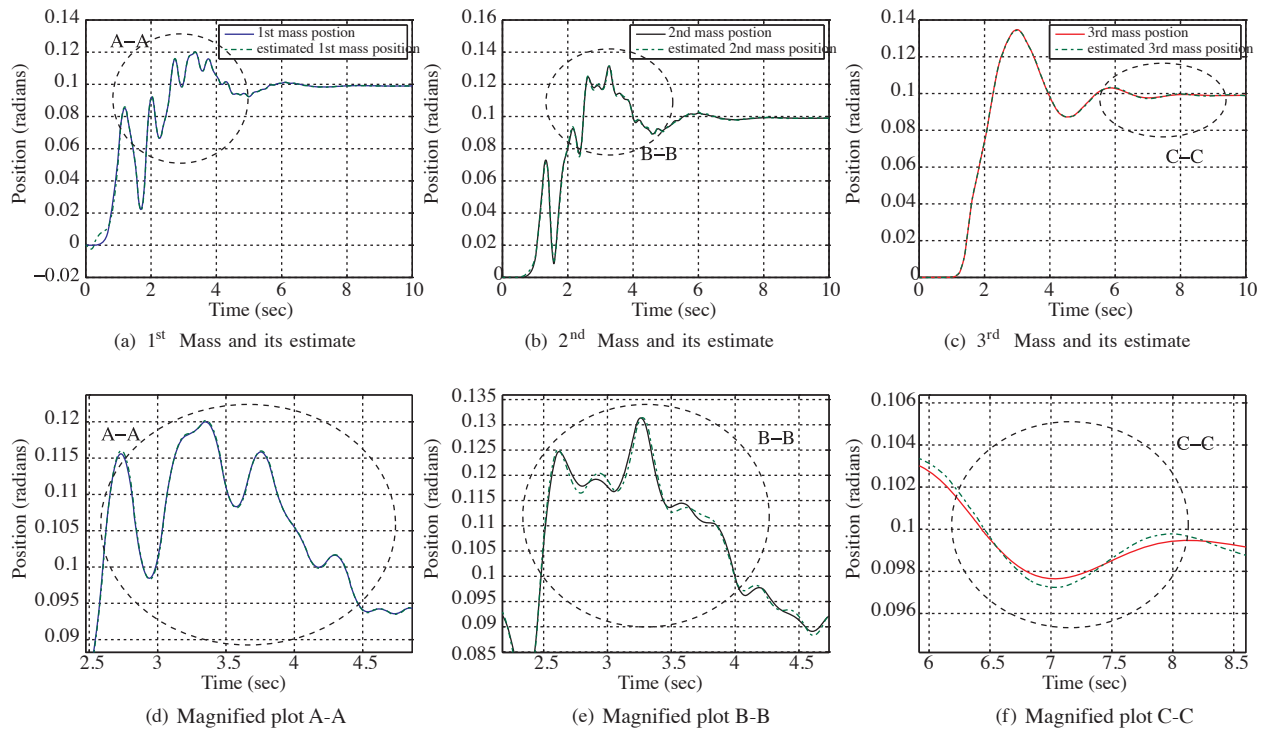


Figure 13. Flexible body motion estimation experimental results.

5.4. Sensorless motion control

One can obtain motion control of any lumped mass along the flexible system due to availability of the position estimates. The sensorless control law equation (25) can be used to position any point of interest along the flexible dynamical system to the required reference position. In this experiment, point of interest can be any one of the three lumped masses. Position estimate of each is fed back to the controller equation (25) while the other estimates are used to monitor the rest of the system's behavior. Indeed, merging the virtual or the actual sensor from the first mass to the last mass affects the performance of the controller, as can be noticed from the following experimental results. First, the non-collocated first mass is controlled by feeding back its position estimate to the controller. Figure 14a illustrates the experimental result of controlling the first non-collocated mass; rest of the system response is shown in Figure 14b,c. In order to control the second mass, its position estimate has to be fed back to the controller, which is nothing but a switching process between the on-hand position estimates. Figure 15b illustrates the experimental result of controlling the second mass. Similarly, Figure 16c shows the experimental result of controlling the last non-collocated mass that has oscillatory behavior due to the non-collocated nature of the control system. The previous results are obtained using the sensorless PID control law equation (25). Indeed, any other control law can be used since all the states are available.

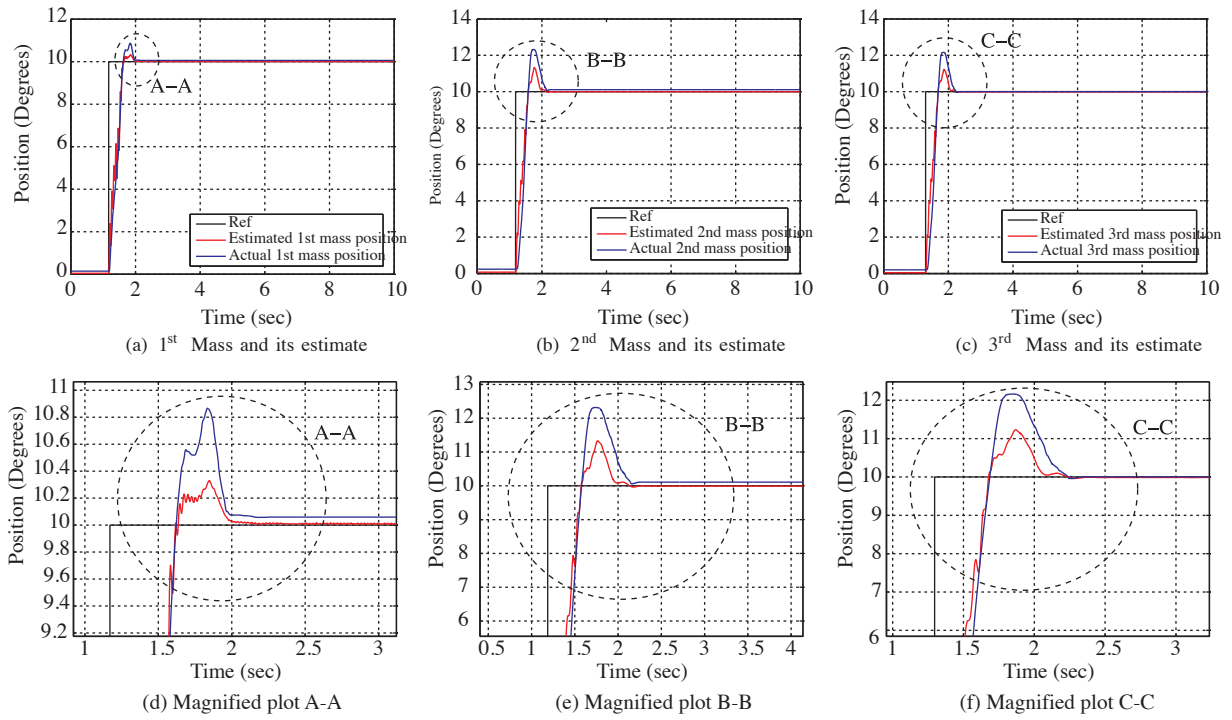


Figure 14. Plots of angular position as a function of time showing sensorless control of the first lumped mass.

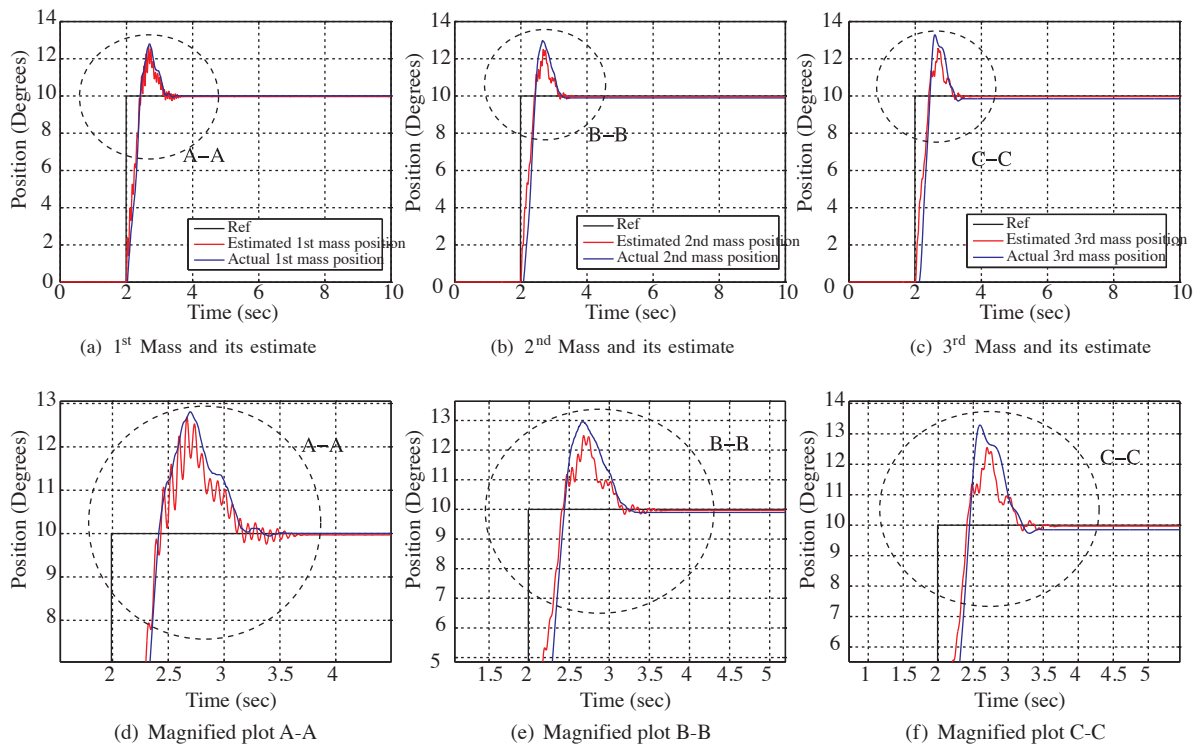


Figure 15. Plots of angular position as a function of time showing sensorless control of the second lumped mass.

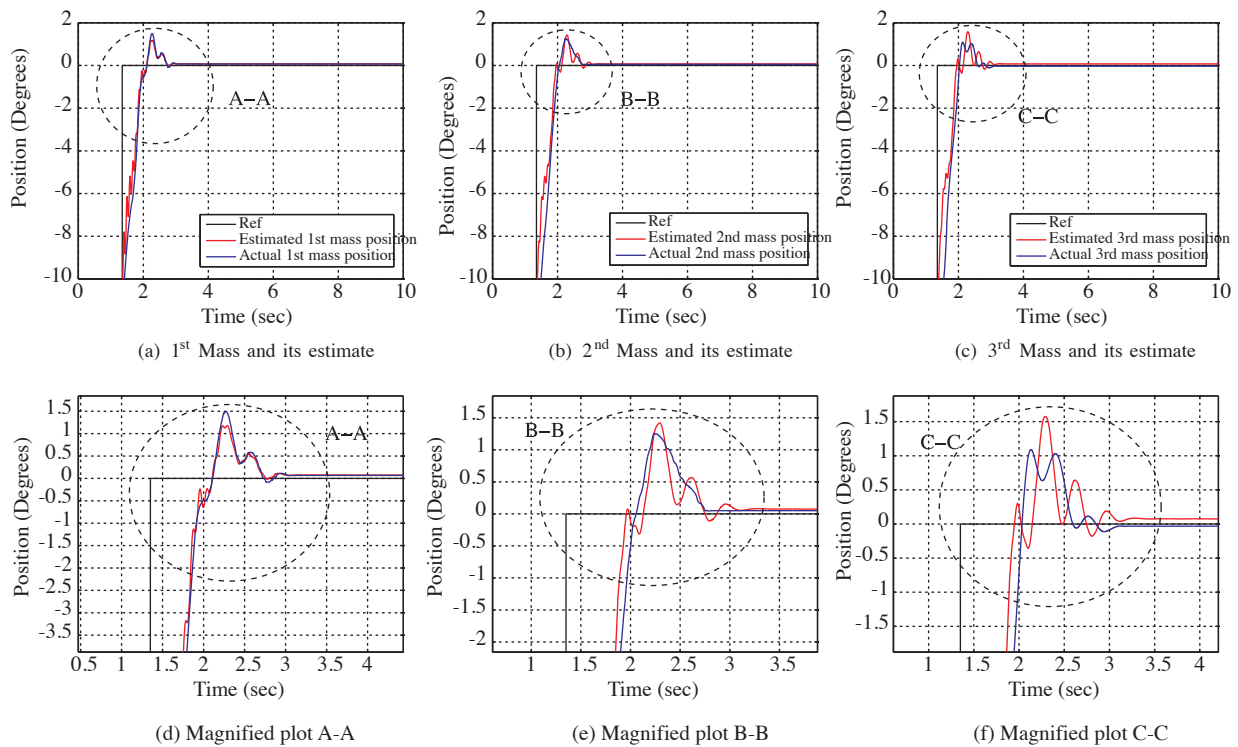


Figure 16. Plots of angular position as a function of time showing sensorless control of the third lumped mass.

6. Conclusion

The problem of keeping flexible systems free from any measurement while considering the actuator as a single platform for measurement is addressed in this work. Disturbance, flexibility and the Newtonian Action-Reaction principle are combined to formulate a framework which allows identifying system parameters and observing system states through measurements taken from the actuator side. The flexible system's reaction due to an action imposed by the actuator is investigated. Moreover, a model based mathematical representation of the reaction signal is derived for a simple system with finite flexible modes. It turns out that reaction signal carries sufficient coupled information about the flexible system, such as system parameter, dynamics and externally applied torques or forces. Furthermore, the entire coupled signal denoted as the incident reaction torque or force is determined or estimated from the interface point of the actuator with the flexible plant using actuator's current and velocity. System parameters and dynamics are decoupled out of the reaction torque. The experimental results demonstrate the validity of the proposed technique where the difference between the identified parameters and the actual known-before-hand parameters is less than five percent. In addition, on-line comparison of the observed positions with the actual measurements demonstrates the possibility of keeping these flexible systems free from any attached sensors while performing motion and vibration control assignment.

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References

- [1] W. J. O'Connor, *Wave-based analysis and control of lump-modeled flexible robots*, IEEE Transaction on Robotics, vol. 23, pp. 1552-3098, 2007.
- [2] W. J. O'Connor, *Wave-Based Analysis and Control of Lump- Modeled Flexible Robots*, in Proc Int. Conf. IEEE Industrial Electronics Society (IECON95), vol. 2, pp. 1260-1265, 1995.
- [3] W. J. O'Connor, *A gantry crane problem solved*, in ASME Journal Dynamic Systems, Measurement, and Control, vol. 4, pp. 569-576. 2004.
- [4] K. Ohnishi, M. Shibata, T. Murakami, "Motion Control for Advanced Mechatronics", In IEEE/ASME Transactions On Mechatronics, vol. 1, no. 1, pp. 56-67, March 1996.
- [5] M. Murakami, T. Murakami and K. Ohnishi, *Vibration suppression and disturbance rejection control of flexible link arm*, in Proc Int. Conf. IEEE Industrial Electronics Society (IECON95), vol. 2, pp 1260-1265, November 1995.
- [6] T. Murakami and K. Ohnishi, *Observer-Based Motion Control-Application to Robust Control and parameter Identification*, in Proc Int. Conf. IEEE Industrial Electronics Society, vol. 1, pp. 1-6, November 1993.
- [7] S. P. Bhat and D. K. Miu, *Precise Point-to-Point Positioning Control of Flexible Structures*, in ASME Journal Dynamic Systems, Measurement, and Control, vol. 112, pp. 667-673, December 1990.
- [8] S. P. Bhat and D. K. Miu, *Experiments on Point-to-Point Position Control of Flexible Beam Using Laplace Transform Technique*, in ASME Journal Dynamic Systems, Measurement, and Control, vol. 113-N13, pp.438-443, 1991.
- [9] S. P. Bhat and D. K. Miu, *Solutions to Point-to-Point Control Problems using Laplace Transform Technique*, in ASME Journal Dynamic Systems, Measurement, and Control, vol. 113-N13, pp.425-431, 1991.
- [10] Islam. S. M. Khalil, E. D. Kunt and Asif Sabanovic, "Estimation Based Control of Flexible System-Sensorless Wave Based Technique," in *ETFA "IEEE International Conference on Emerging Technologies and Factory Automation"* pp. 1-6, September 22-26, 2009.
- [11] D. K .Miu, *Mechatronics - Electromechanics and Contromechanics*, 3rd ed. New York, USA: Springer-Verlag, 1993.