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# Efficiency of Atkinson Engine at Maximum Power Density using Temperature Dependent Specific Heats

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**Abstract**

Thermodynamic analysis of an ideal air-standard Atkinson cycle with temperature dependant specific heat is presented in this paper. The paper outlines the effect of maximizing power density on the performance of the cycle efficiency. The power density is defined as the ratio of the power output to the maximum cycle specific volume. It showed significant effect on the performance of the cycle over the constant specific heat model. The results obtained from this work can be helpful in the thermodynamic modeling and in the evaluation of real Atkinson engines over other engines.



*Keywords:* Atkinson engine; power density; temperature dependant specific heat;

## 1. Introduction

The Atkinson cycle (the complete expansion cycle) is named after its inventor James Atkinson in 1882 [1]. As shown in Fig. 1, the cycle involves isentropic compression followed by isochoric heat addition. The expansion occurs isentropically, and finally the cycle has isobaric heat rejection.

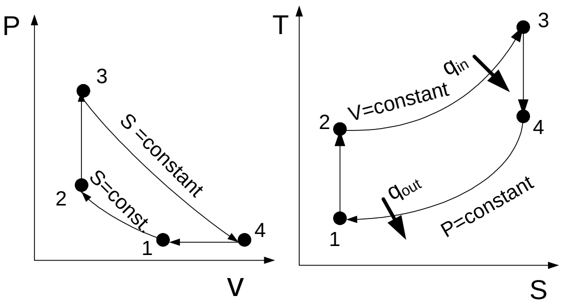


Figure 1: Sketch of *P-V* and *T-S* diagrams of Atkinson cycle

Optimization and performance analysis can be applied using finite time thermodynamic techniques. They are used to study performance of various air-standard power cycles [2-8]. For example, Chen *et al*. [6] and Sahin *et al*. [7] examined Atkinson cycle and Joule-Brayton cycle, respectively, at maximum power density. Both studies found that the efficiency at the maximum power density is greater than that at the maximum power output. Constant specific heats of air (the working fluid) were used. The maximization of the power density is defined as the ratio of the maximum power to the maximum specific volume in the cycle. It takes into consideration the engine size instead of just maximizing its power output. The inclusion of the engine size in the calculation of its performance is a very important factor from an economical point of view. Many researchers have performed finite-time thermodynamic analysis on Atkinson engines. Most of them assumed constant specific heats for the working fluid in their studies. Performance analysis of an Atkinson cycle with heat transfer, friction and variable specific-heats of the working fluid was studied by Ge et al. [9]. Their results showed that the effects of variable specific-heats of working fluid and friction-like term losses on the irreversible cycle performance are significant.

Zhao and Chen [10] performed analysis and parametric optimum criteria of an irreversible Atkinson heat-engine using finite-time thermodynamics processes. The optimum criteria of some important parameters, such as the power output, efficiency and pressure ratio were given in their study. Constant specific heat in their model was assumed in their model. Wang and Hou [11] studied the performance analysis and comparison of an Atkinson cycle coupled to variable temperature heat reservoirs under maximum power and maximum power density conditions, assuming a constant specific heat, too. Their results showed an engine design based on maximum power density is better than that based on maximum power conditions, from the view points of engine size and thermal efficiency. However, due to the higher compression ratio and maximum temperature in the cycle,

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an engine design based on maximum power density conditions requires tougher materials for engine construction than one based on maximum power conditions. Performance analyses under maximum power and power density have been also performed on Brayton cycle and an endo-reversible Braysson cycle, respectively [12,13]. Both studies assumed constant specific heat of the working fluid. Recent studies were involved in performing thermodynamic analysis using temperature dependent specific heats on various conditions. It was found that temperature dependent specific heat gives better approximation to actual cycles than using constant temperature specific heat [14-19].

This paper examines the significance of using the temperature dependant specific heat on the performance of an Atkinson engine under maximum power density. The results are compared to those obtained from recent study by Chen et al. using constant specific heat [6]. Different parameters affecting cycle performance and net work output at maximum power density will be considered. The obtained results will be presented as performance characteristic curves for the Atkinson cycle using numerical examples

## 2. Thermodynamic Analysis

Figure 1 presents pressure-volume (*P-V*) and temperature-entropy (*T-S*) diagrams for the thermodynamic processes performed by an ideal air-standard Atkinson cycle. All four processes are reversible. Process 1-2 is an adiabatic (isentropic) compression; process 2-3 is a heat addition at a constant volume; process 3-4 adiabatic (isentropic) expansion; process 4-1 is heat rejection at a constant pressure. The employed variable specific heat model assumes variation of specific heat with temperature in a linear fashion. The best straight line fit was determined. It is plotted in Fig. 2.

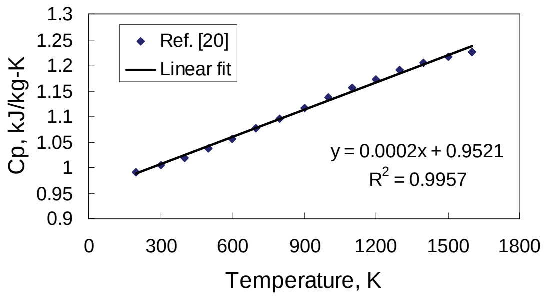


Figure 2: The best linear fit for temperature dependant specific heat of air in the temperature range of 300 to 1500 K.

The variations of specific heats with temperatures are presented in equations (1) and (2), as

$$C_p = a_1 + a_2 T \tag{1}$$

$$C_p - C_v = R \tag{2}$$

where $a_1$, $a_2$ are constants [9] and $R$ is the gas constant.

Equation (1) is taken as the best straight line that fit the variable $C_p$ as in Çengel [20];

The heat input of Atkinson cycle is given as

$$Q_{in} = m\int_{T_2}^{T_3} C_v dT \tag{3}$$

and the heat rejected from the cycle during the process 4–1 is

$$Q_{out} = m\int_{T_1}^{T_4} C_p dT \tag{4}$$

Then by substituting the variable specific heat equation and performing the integration the heat addition and rejection will be computed as in equations (5) and (6), respectively.

$$Q_{in} = m\left\{(a_1 - R)(T_3 - T_2) + \frac{a_2}{2}(T_3^2 - T_2^2)\right\} \tag{5}$$

$$Q_{out} = m\left\{a_1(T_4 - T_1) + \frac{a_2}{2}(T_4^2 - T_1^2)\right\} \tag{6}$$

The power output, *W*, of this cycle can be written as

$$W = m\left\{\begin{matrix}(a_1 - R)(T_3 - T_2) + \frac{a_2}{2}(T_3^2 - T_2^2) \\ -a_1(T_4 - T_1) - \frac{a_2}{2}(T_4^2 - T_1^2)\end{matrix}\right\} \tag{7}$$

The power density, *P*, is defined as the power per maximum specific volume in the cycle

$$P = \frac{W}{v_4} \tag{8}$$

where, $v_4$, is the maximum specific volume

$$P = \frac{m}{v_4}\left\{\begin{matrix}(a_1 - R)(T_3 - T_2) + \frac{a_2}{2}(T_3^2 - T_2^2) \\ -a_1(T_4 - T_1) - \frac{a_2}{2}(T_4^2 - T_1^2)\end{matrix}\right\} \tag{9}$$

for simplicity and since most of the temperature changes occur during the isochoric ( 2 to 3) and isobaric process (4 to 1) a constant specific heat can be assumed during the isentropic processes (i.e. from 1 to 2 and from 3 to 4). The total entropy change in the cycle is equal to zero, therefore,

$$\frac{T_4}{T_1} = \left(\frac{T_3}{T_2}\right)^{1/k} \tag{10}$$

where,

$$k = \frac{C_p}{C_v} \tag{11}$$

Let

$$\theta = \frac{T_2}{T_1} \qquad (12)$$

and

$$\tau = \frac{T_3}{T_1} \qquad (13)$$

Then the power density (equation (6)) in terms of $\theta$ and $\tau$, becomes

$$P = \frac{mT_1}{v_4}\left( \begin{array}{l} (a_1 - R)(\tau - \theta) + \frac{a_2}{2}T_1(\tau^2 - \theta^2) \\ -a_1\left[ (\frac{\tau}{\theta})^{1/k} - 1 \right] - \frac{a_2 T_1}{2}\left[ (\frac{\tau}{\theta})^{2/k} - 1 \right] \end{array} \right) \qquad (14)$$

where $T_1$ is the temperature of the working fluid at sate1 (atmospheric condition) in the cycle.

## 3. Power density maximization

For a given $\tau$, the power density will be differentiated with respect to $\theta$ and the result will be equating to zero ($dP/d\theta = 0$) this gives

$$R - a_1 - a_2 T_1 \theta + \frac{a_1}{k}\tau^{1/k}\theta^{(-1-k)/k} + \frac{a_2}{k}T_1\tau^{(2/k)}\theta^{(-2-k)/k} = 0 \qquad (15)$$

the root of equation (15) is ($\theta_p$) is the ($\theta$) at which the power density is maximum. Substituting ($\theta_p$) in equation (14) gives the maximum power density $P_{max}$.

The cycle efficiency ($\eta_p$) at maximum power density point ($\theta_p$) is

$$\eta_p = 1 - \left[ \frac{a_1\left((\tau/\theta_p)^{1/k} - 1\right) + \frac{a_2 T_1}{2}\left((\tau/\theta_p)^{2/k} - 1\right)}{(a_1 - R)(\tau - \theta_p) + \frac{a_2 T_1}{2}(\tau^2 - \theta_p{}^2)} \right] \qquad (16)$$

for the numerical calculation in the present study the following values will be used

$a_1 = 0.9521$ kJ/(kg-K) , $a_2 = 0.0002$ kJ/kg, and $R = 0.287$ kJ/(kg-K)

## 4. Performance Comparison

The derived formula above is used and plotted in order to compare Atkinson engine with variable specific heat with those results assuming a constant specific heat as shown in Figures 3 through 9. The following constants and range of parameters are selected: $k$ = 1.4, $\tau$ = 1 to 6 and $T_1 = 298$ K. By varying isentropic temperature ratio ($\theta$) or thermal efficiency ($\eta$) and for a given value of cycle temperature ratio ($\tau$) the normalized power density ($P/P_{max}$) is plotted. The normalized power density is the ratio of power density to the maximum power density at ($\theta = \theta_p$). The results are presented in figure 3 through 9.

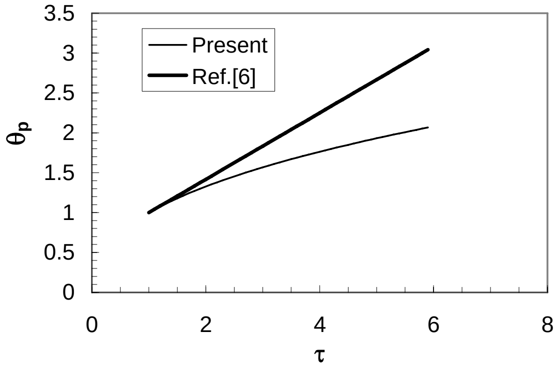


Figure 3: Variations of the isentropic temperature ratios for the variable and constant specific heats with cycle temperature ratio

Figure 3 shows the variations of the isentropic temperature ratios at maximum power density ($\theta_p$), for variable and constant specific heats with temperature ratio ($\tau$). The deviation between ($\theta_p$) using the variable and constant specific heat increases with increasing the cycle temperature ratio ($\tau$). The value of ($\theta_p$) using constant specific is larger than that using variable specific heat. Figure 4 shows the variation of the thermal efficiency with cycle temperature ratio ($\tau$). The thermal efficiency at maximum power density of Atkinson cycle using the constant specific heat is over predicted as compared to the variable specific heat.

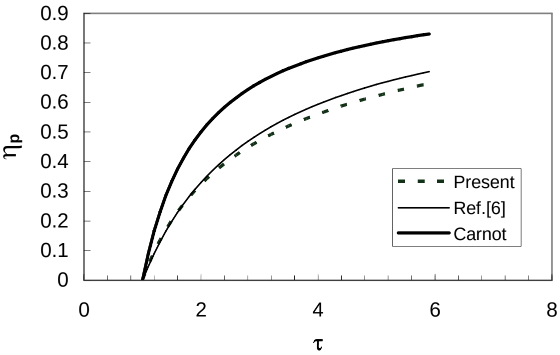


Figure 4: variation of the cycle thermal efficiency at maximum power density point compared to Carnot cycle.

Figure 5 shows the variation of the percentage difference in ($\eta_p$) between constant and variable specific heats with cycle temperature ratio ($\tau$). The percentage difference is defined as

$$\Delta\eta_p = \frac{\left(\eta_{p-\text{variable}-\text{cp}} - \eta_{p-\text{constant}-\text{cp}}\right)}{\eta_{p-\text{variable}-\text{cp}}} \times 100\% \qquad (17)$$

The maximum percentage difference occurs at low values of $\tau$. The percentage difference remains constant at around 6% beyond $\tau$ = 4. The minimum difference occurs at $\tau$ = 1.7. The behavior of this curve suggests that the efficiency at maximum power density is underestimated at low values of $\tau$ and it is over estimated for high values of $\tau$ (above 1.7).

  

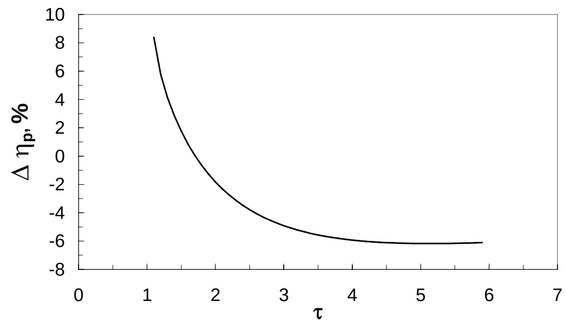


Figure 5: % error in thermal efficiency at max. power density point-variable theta; Variation of the percentage difference in $\eta_p$ between constant and variable specific heat

Normalized power density variation with isentropic temperature ratio at $\tau$ = 2 is shown in Figure 6. The normalized power density is different at same temperature ratio $\theta$. Both constant and variable specific heat curves of normalized power density have parabolic trend. Similar trend for $\tau$ = 4 is shown in Figure 7 but greater difference appears at higher values of $\tau$ .

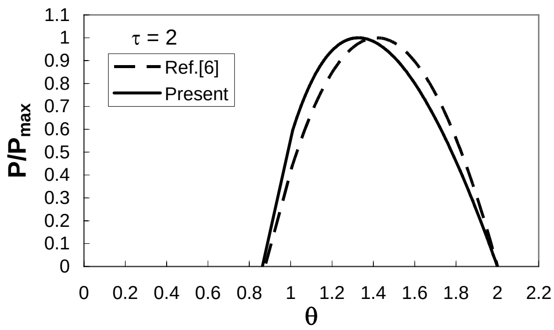


Figure 6: Normalized power density variation with isentropic temperature ratio at τ = 2

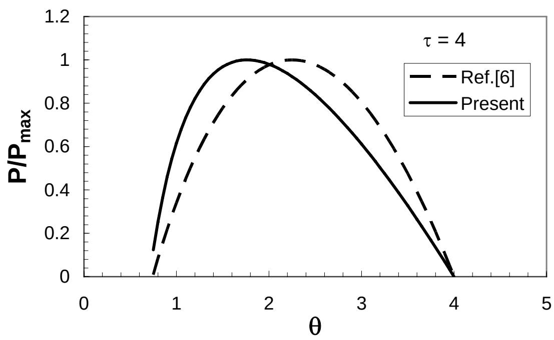


Figure 7: Normalized power density variation with isentropic temperature ratio at τ = 4

Figure 8 presents the variation of the percentage difference in the normalized power density between constant and variable specific heat for three cases, $\tau$ =1.4, 2 and 4. All curves start with high percentage difference at low values of $\theta$ (i.e. P/Pmax is underestimated) and the then the percentage difference decreases until it reaches a minimum point and then increases again with increasing $\theta$ (i.e. P/Pmax is overestimated). According to this figure, there is a point where both variable and constant specific heat will give the approximately the same result.

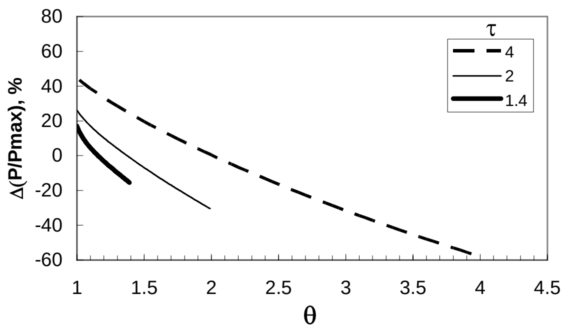


Figure 8: variation of the percentage difference in the normalized power density between constant and variable specific heat

Figure 9 shows the variation of normalized power density with thermal efficiency at τ = 4. The maximum point of the normalized power density is not at the same point of maximum efficiency. Both carves are not identical. At a small range of cycle thermal efficiency approximately from $\eta$ = 0.58 to 0.66 where both constant and variable specific heat will give approximately the same result but most of the range of the curve will give different value.

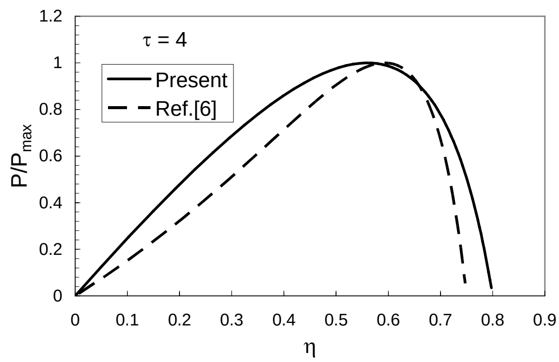


Figure 9: Variation of normalized power density with thermal efficiency at τ = 4

Figure 10 shows the variation of percentage difference of the normalized power density using the constant and variable specific with thermal efficiency at τ = 4. The show small difference at low value of thermal efficiency and the difference then increases and the decreases then increases in harmonic like behavior. This figure clearly indicates the needs for using variable specific heat in calculations of cycle performance.

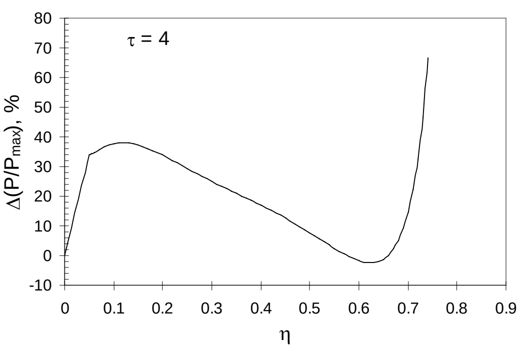


Figure 10: Variation of percentage difference of the normalized power density with thermal efficiency at τ = 4.

## 5. Conclusion

Using constant or temperature dependant specific heats will affect the Atkinson maximum power density calculation. The differences in the results using the two methods are significant. Therefore, temperature dependant specific heat must be used in modeling the performance of Atkinson engine at maximum power density. The efficiency at maximum power density point when using the constant specific heat is over predicted. The maximum power density occurs at higher isentropic temperature ratios for the case of constant specific heat which also leads to incorrect optimum power density. It is recommended that other parameters be considered for future work.

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