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Viscous and Joule Heating Effects over an Isothermal Cone in Saturated Porous Media

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Abstract

In this paper, the magnetohydrodynamic mixed convection flow about an isothermal cone embedded in a saturated porous medium is considered. The Darcian model including the viscous dissipation effects in the energy equation is used. The governing equations are transformed by using a set of nonsimilarity variables and solved by using Keller box method. The entire mixed convection regime is covered by nonsimilarity parameter χ, where χ=0 corresponds to pure free convection and χ=1 corresponds to pure forced convection. Numerical results for the dimensionless velocity and temperature profiles and the local Nusselt number for various values of the mixed convection parameter χ, the cone angle parameter m, the magnetic field strength parameter H, and the modify Gebhart number Ge^* are drawn. It is found that increasing the magnetic strength decreased the heat transfer rates, while increasing the cone angle increased the heat transfer rates.

Keywords: viscous heating; Joule heating; mixed convection;

Nomenclature

Alphabetic symbols

B_y	: Magnetic field flux density
Cf_z	: Local skin friction factor
c_p	: Specific heat capacity
f	: Dimensionless stream function
g	: Gravitational acceleration
Ha	: Hartman number, $\sqrt{\sigma B_y^2 K / \rho \upsilon}$
K	: Permeability coefficient of the porous medium
m	: Cone angle parameter
q_w	: Dimensionless local surface heat flux
r	: Local radius of the cone
Ra_x	: Modified local Rayleigh number
Ge^*	: Modified Gebhart number.
k	:Thermal Conductivity
Nu_z	: Local Nusselt number, *hx/k*
$\overline{Nu}$	: Average Nusselt numbers, $\bar{h}L/k$
p	: Pressure
Pe_z	: Local Peclet number, $U_\infty x/\alpha$
Pr	: Prandtl number, υ/α
T	: Temperature
T_∞	: Free stream temperature
T_w	: Wall temperature
u,V	: Velocity components in *x*-and *y*-directions
U_∞	: Free stream velocity

Greek symbols

α	: Thermal diffusivity
β	: Coefficient of thermal expansion
η	: Pseudo-similarity variable
θ	: Dimensionless temperature
μ	: Dynamic viscosity
υ	: Kinematic viscosity
ρ	: Fluid density
σ	: Electrical conductivity
τ_w	: Local wall shear stress
Ψ	: Dimensional stream function
ϕ	: Half angle of the cone
ε	: Porosity
χ	: Mixed convection parameter

1. Introduction

Magnetohydrodynamic mixed convection flow of an electrically conducting fluid about different geometries is of important considerations in the thermal design of a

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variety of industrial equipment and also in nuclear reactors, geophysical fluid dynamics. Many articles have been published [1-9] dealing with the problem of MHD-convection heat transfer from different surfaces.

The problem of convection flow along a vertical cone has been investigated by many researchers. Merk and Prins[10] have studied the problem of laminar free convection flow along vertical cone. They found the similarity solutions for the case of an isothermal cone. Hering and Grosh [11] have obtained a number of similarity solutions for cones with prescribed wall temperatures being a power function of the distance from the apex along the generator. Further results were obtained by Hering [12] and by Roy [13], respectively, Pop and Takhar[14] have studied the compressibility effects in laminar free convection from a vertical cone, while Hossain and Paul [15] and Watanabe [16] have considered the effect of suction and injection when the cone surface is permeable. Wang [17] and Wang et al. [18] have presented results for the free convection boundary-layer flow due to a rotating cone with constant or variable surface temperature. Yih [19] studied the radiation effect on mixed convection flow over an isothermal cone embedded in a saturated porous media; the transformed nonlinear system of equations is solved using an implicit finite difference method. Increasing the cone angle increases the heat transfer rates, and the local Nusselt number is significantly increased for the large values of the radiation conduction parameters. Recently, Duwairi and Al-Kablawi [20] have studied the MHD conjugate mixed convection heat transfer from vertical hollow cylinder embedded in a saturated porous media.

In the present work, the effects of magnetic field strength are investigated on mixed convection heat transfer rates over an isothermal cone in a saturated porous medium with viscous dissipation effects included in the energy equation. The governing equations, which describe the problem, wastransformed and solved numerically using the Keller box scheme.

2. Problem Formulation

Consider a vertical isothermal cone with half angle Φ and local radius r, embedded in a saturated porous medium in presence of transverse uniform magnetic field B_y. Figure.1. shows the physical model and the coordinate system. The flow over the cone is assumed to be two-dimensional, laminar, steady and incompressible; the wall temperature of the cone T_w is higher than the ambient temperature T_∞. Assuming low velocity and porosity, the Darcy model is employed in the analysis. The properties of the fluid are assumed to be constant except the density variation in the buoyancy force term. The viscous dissipation effect is included. And the boundary layer approximations is applicable $(v << u, \partial/\partial x << \partial/\partial y)$.

The normal buoyancy force to the surface of the cone is neglected;this approximation is valid for a wide range of cone angle except near $\Phi=90^o$ and in the region $\Phi \to 0^o$. With the above approximations, the governing equations based on Darcy law and the boundary conditions can be written as follows:

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} = 0 \qquad (1)$$

$$H\frac{\partial u}{\partial y} = \frac{g\cos\phi\,\beta\,K}{\upsilon}\frac{\partial T}{\partial y} \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_y^2}{\rho c_p}u^2 + \frac{\upsilon}{Kc_p}u^2 \qquad (3)$$

$$y = 0 \;:\; v = 0,\; T = T_w \qquad (4)$$

$$y \to \infty \;:\; u = U_\infty,\; T = T_\infty \qquad (5)$$

Where u, v are the velocity components along the x and y axes, g is the acceleration due to gravity, α is the thermal diffusivity, β, the coefficient of thermal expansion, K, the permeability of the porous medium, ν, the kinematic viscosity, T, the temperature, B_y is the magnetic field density in y direction, H the magnetic field parameter $(H=1+Ha^2/\varepsilon)$, Ha, is the Hartman number. Where for the cone problem, $U_\infty=Bx^m$ is the velocity of the potential flow outside the boundary layer. Here , B is prescribed constant and m is the cone angle parameter. The tabulated values: Φ and m are given by Hess and Faulkner [21]. The cone angles of 15, 30, 45, 60 and 75 are discussed in this paper; therefore, m is 0.0316314, 0.1156458, 0.2450773, 0.4241237, and 0.66672777, respectively.

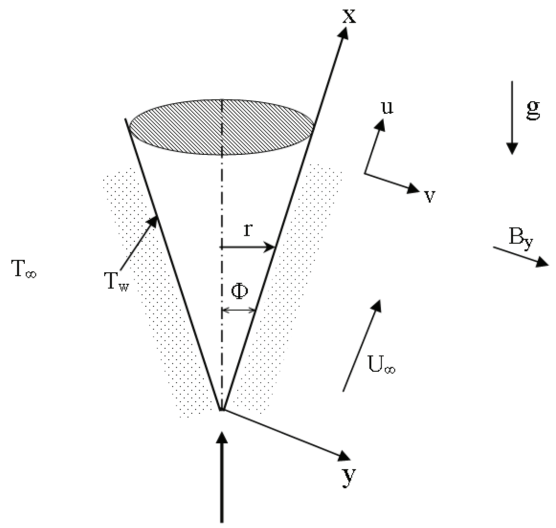

Figure. 1 physical model and coordinate system

3. Nonsimilarity Transformation

In the above system of equations and in order to satisfy the continuity equation defining the stream function as $ru = \partial\psi/\partial y$, and $rv = -\partial\psi/\partial x$ .The following dimensionless variables are introduced in the transformation:

$$\chi = \left[1 + \left(\frac{Ra_x}{Pe_x}\right)^{1/2}\right]^{-1} \qquad (6)$$

$$\eta = (y/x) Pe_X^{1/2} \ \chi^{-1} \tag{7}$$

$$\psi = \alpha\, r\, Pe_X^{1/2} f(\chi,\eta)\, \chi^{-1}$$
$$\theta(\chi,\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

And substituting equations (6), (7) and (8) into equations (1)-(5), we obtain the following transformed governing equations:

$$H \ f'' = (1-\chi)^2 \theta' \tag{9}$$

$$\theta'' + \frac{1}{2}(3 + m\chi) f\theta' + \overset{*}{Ge}_x [1 + \varepsilon(H-1)](f')^2 = \frac{m\chi}{2}(1-\chi)\left[f' \frac{\partial\theta}{\partial\chi} - \theta' \frac{\partial f}{\partial\chi} \right] \tag{10}$$

The boundary conditions are:

$$\eta = 0 \quad : \quad f = 0 \ , \ \theta = 1$$
$$\eta \to \infty : \ f' = \chi^2 \ , \ \theta = 0 \tag{11}$$

In the above equations, the primes denote the differentiation with respect to η. H is the magnetic field parameter. χ the mixed convection parameter, when χ=0 corresponds to pure free convection and χ =1 corresponds to the other limit of pure forced convection. Between the two limits correspond to the mixed convection regime. Ge^* is the modified Gebhart number defined as:

$$\overset{*}{Ge}_x = g\cos\phi\beta\, x / c_p (1-\chi)^2$$
$$Ra_x = g\cos\phi\, \beta (T_w - T_\infty) K\, x/\nu\alpha$$
$$Pe_x = U_\infty x/\alpha$$

These are the modified local Rayleigh number and the local Peclet number. The viscous heating effects are excluded from the energy equation by setting $Ge^* = 0$, where the Joule heating effects can be excluded by setting *H=1* which represent the case of absent magnetohydrodynamic forces from the presented cone. Also, it is clear that the Joule heating effects are incorporated in the problem when viscous heating effects are important.

Some of the physical quantities of practical interest include the wall shear stress $\tau_w = \mu(\partial u/\partial y)_{Y=0}$, and the surface heat flux, $h(T_w - T_\infty) = q_w$. They are given by:

$$Nu_x = \frac{hx}{k} = \frac{q_w x}{k(T_w - T_\infty)} \tag{12}$$

$$\frac{Nu_X}{[Pe_x^{1/2} + Ra_x^{1/2}]} = -\theta(\chi,0) \tag{13}$$

$$\frac{Cf_X \Pr^{-1} Pe_X^2}{[Pe_x^{1/2} + Ra_x^{1/2}]^3} = 2 f''(\chi,0) \tag{14}$$

4. Methods of Solution

The nonlinear partial differential equations (9-10) under the corresponding boundary conditions (11) are solved numerically using an implicit tri-diagonal finite-difference method (Cebeci and Bradshaw, 1984; Keller, 1988). In this method, any quantity g at point (χ_n, η_j) is written as g_j^n. Quantities at the midpoints of grid segments are approximated to second order as:

$$g_j^{n-1/2} = \frac{1}{2}\left(g_j^n + g_j^{n-1}\right)$$
$$g_{j-1/2}^n = \frac{1}{2}\left(g_j^n + g_{j-1}^n\right) \tag{15}$$

And the derivatives are approximated to second order as

$$\left(\frac{\partial g}{\partial \chi}\right)_j^{n-1/2} = k_n^{-1}\left(g_j^n - g_j^{n-1}\right)$$
$$(g')_{j-1/2}^n = h_j^{-1}\left(g_j^n - g_{j-1}^n\right) \tag{16}$$

Where g is any dependent variable and n and j are the node locations along χ and η directions, respectively. First the second order partial differential equation is converted in a first order by substitutions f'=s, the difference equations that are to approximate the previous equations are obtained by averaging about the midpoint (χ_n, $\eta_{j-1/2}$), and those to approximate the resulting equations by averaging about ($\chi_{n-1/2}$, $\eta_{j-1/2}$). At each line of constant χ, a system of algebraic equations is obtained. With the nonlinear terms evaluated at the previous iteration, the algebraic equations are solved iteratively. The same process is repeated for the next value of χ, and the problem is solved line by line until the desired χ value is reached. A convergence criterion based on the relative difference between the current and the previous iterations is employed. When this difference reaches 10^{-5}, the solution is assumed to have converged and the iterative process is terminated

5. Results and Discussion

The viscous heating effects, Joule heating effects and cone half angle are studied on the mixed convection heat transfer over a vertical cone embedded in a saturated porous medium.

Figure 2. Shows the dimensionless velocity and temperature profiles for various values of m. It is found that increasing the cone angle parameter m was decreased and the velocity increased heat transfer rates inside the boundary layer. This is due to change of the flow toward stagnation point flow heat transfer problem.

Figure 3. shows the dimensionless velocity and temperature profiles for various values of magnetic field parameter H. It is found that the increasing of the magnetic field strength decreased the velocity and the heat transfer rates. The magneto effect on this problem is found to retard the motion of the fluid and to heat it at the same time. Figure 4. shows the effect of the viscous dissipation on the dimensionless velocity and temperature profiles, where increasing the effect of the viscous dissipation parameter increased the velocity and decreased the heat transfer rates inside the boundary layerdue to increasing the effect of the buoyancy force and excessive heating of flow.

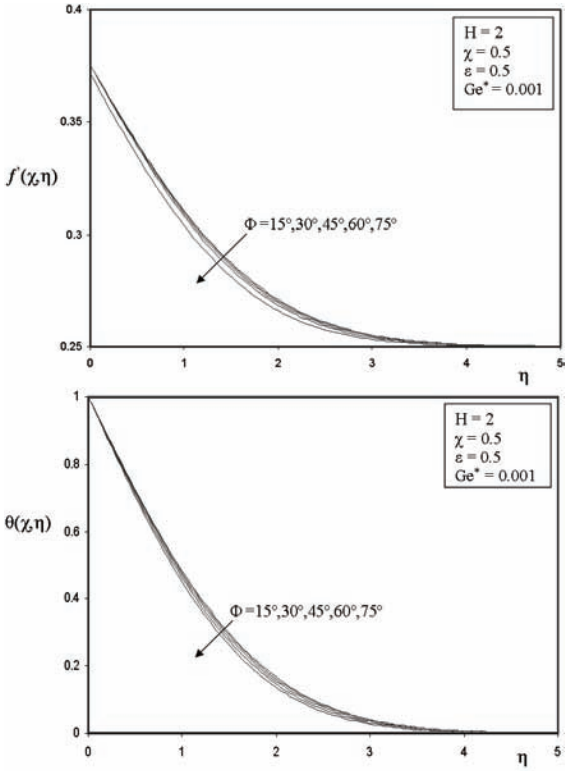

Fig. 2 Dimensionless velocity and temperature profiles at selected values of Φ

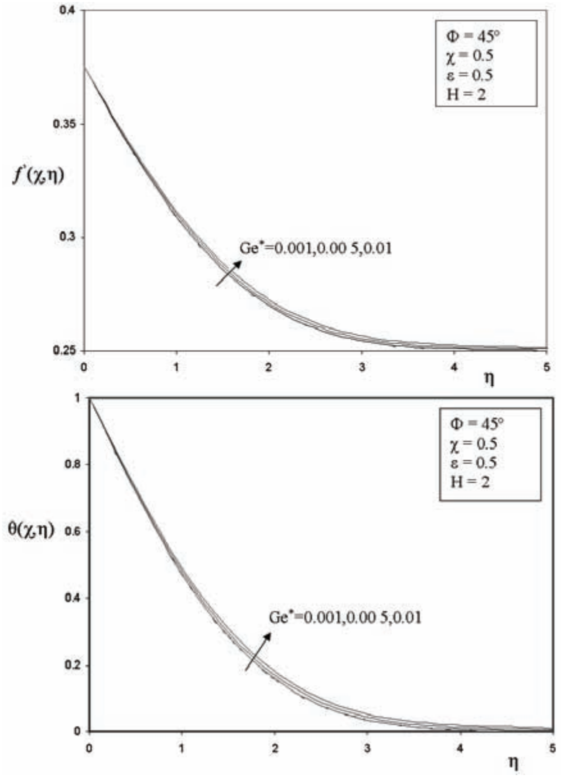

Fig. 4 Dimensionless velocity and temperature profiles at selected values of Ge*

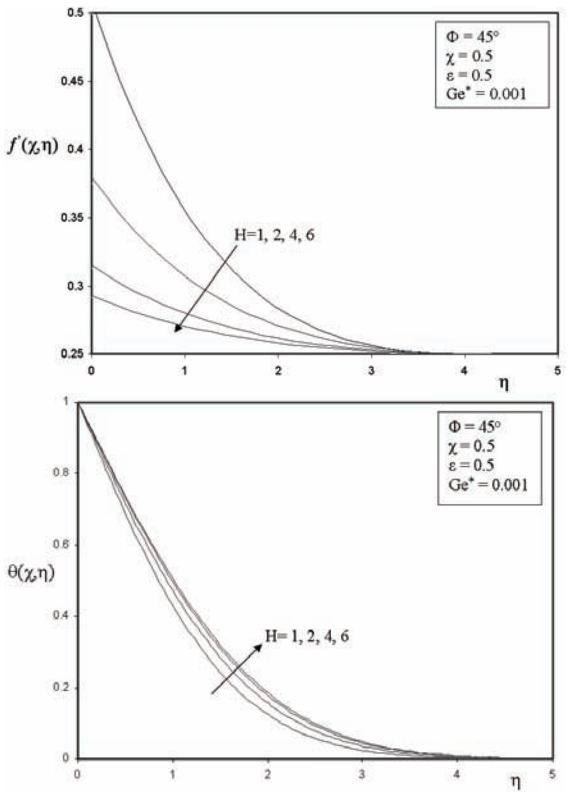

Fig. 3 Dimensionless velocity and temperature profiles at selected values of H

Figure 5. shows that increasing the cone angle had increased the local Nusselt numbers. Fig. 6 shows the effect of Joule heating on the local Nusselt numbers. This behavior is due to heating the fluid and increasing the velocity inside the boundary layer. In addition, figure .7 shows the effect of modified Gebhart number of? viscous dissipation effects on the local Nusselt numbers. As shown, the increasing of the viscous parameter affects the forced flow more than free flow, and decreased the local Nusselt number and local coefficient of friction.

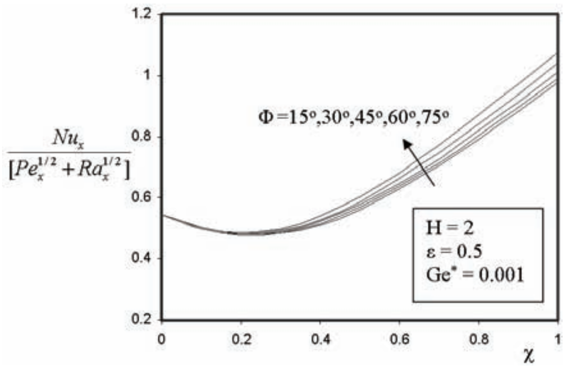

Fig. 5 Local Nusselt number at selected values of Φ

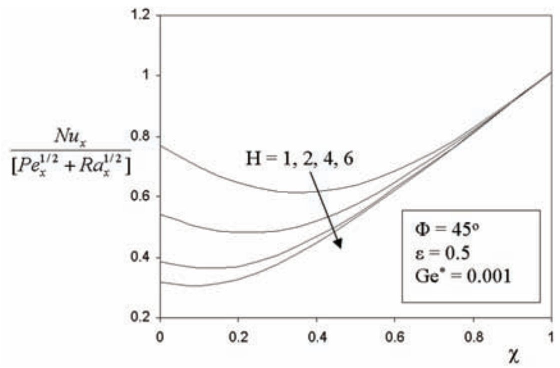

Fig. 6 Local Nusselt number at selected values of H

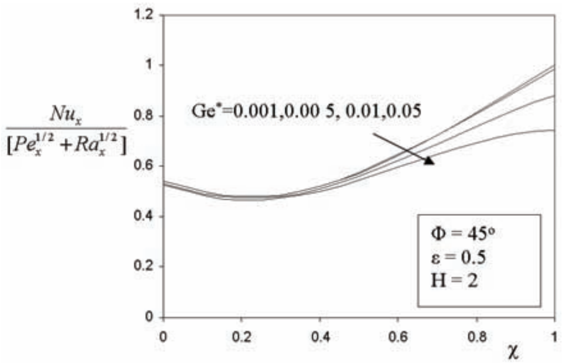

Fig. 7 Local Nusselt number at selected values of Ge*

Table 1. shows the comparison between the $Nu_x/[Pe_x^{1/2}+Ra_x^{1/2}]$ calculated by the present method and that of the Yih (2001). For various values χ, with m=0.4241237, H=1. Iit is seen that the present results are in agreement with those obtained by Yih (2001).

Table 1. Comparison between the $Nu_x/[Pe_x^{1/2}+Ra_x^{1/2}]$ calculated by the present method and that of Yih (2001) for the case of, m=0.4241237, H=1

χ	Present method	Yih (2001)
0	0.7683	0.7686
0.1	0.6990	0.6997
0.2	0.6488	0.6496
0.3	0.6222	0.6228
0.4	0.6218	0.6222
0.5	0.6478	0.6480
0.6	0.6974	0.6975
0.7	0.7659	0.7661
0.8	0.8487	0.8491
0.9	0.9417	0.9427
1	1.0423	1.0440

6. Conclusions

The MHD-mixed convection heat transfer problem is analyzed with viscous dissipation and Joule heating effects included in the energy equation. It is found that increasing the magnetic field strength or the joule heating effect had decreased the heat transfer rates from the surface of the vertical cone. The increasing of the cone angle parameter m enhances the heat transfer rates. Including the viscous dissipation effect in the energy equation was found to decrease the heat transfer rates.

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