Delay-Dependent Stability for Systems with Fast-Varying Neutral-Type Delay via A PTVD Compensation

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Abstract The stability for a class of linear neutral systems with time-varying delays is studied in this paper, where delay in neutral-type term includes a fast-varying case (i.e., the derivative of delay is more than 1), which is never considered in current literature. The less conservative delay-dependent stability criteria for this systems are proposed by applying a new Lyapunov-Krasovskii functional and a novel polynomials with time-varying delay (PTVD) compensation technique. The aim dealt with systems with fast-varying neutral-type delay can be achieved by using the new functional. And the benefit brought by applying the PTVD compensation technique is that some useful elements can be included in criteria, which are generally ignored when estimating the upper bound of derivative of Lyapunov-Krasovskii functional. A numerical example is provided to verify the effectiveness of the proposed results.

Key words Linear neutral systems, stability, delay-dependent, fast-varying neutral-type delay, PTVD compensation technique

There are two kinds of discrete time delays in systems, retarded-type delay and neutral-type delay. Retarded-type delay means that the delay is in the states of systems, whereas neutral-type delay means that the delay is in the derivatives of states of systems. In recent years, the neutral systems with delay (i.e., systems with retarded-type delay and neutral-type delay) have received much attention, due to it can be found in many fields, such as population ecology[1], distributed networks containing lossless transmission lines[2], propagation and diffusion models[3], partial element equivalent circuits in VLSI systems[4], and etc. Thus, the stability of linear neutral systems with delay has developed a hot topic both in theory and in practice[5]. At present, the stability results for linear neutral systems with delay can be generally classified into two types: delay-independent case which can be applied at delay with arbitrary size, and delay-dependent case which makes use of the size of delay. Generally speaking, the delay-dependent case is less conservative than the delay-independent one. Therefore, researches on delay-dependent stability for linear neutral systems with delay had been extensively carried out. For example, literature [6] and [7] proposed a descriptor system approach to deal with linear neutral systems with delay. In [8] and [9], the Lyapunov-Krasovskii functional with term $\boldsymbol{x}(t) - C\boldsymbol{x}(t-\tau)$ was employed, where $\boldsymbol{x}(t)$ was the state of systems, C was constant matrix with $\|C\| < 1$, and τ was delay. Then, a free-weighting matrix approach[10][11] was proposed to deal with linear delay systems.

Recently, some new techniques had been used in stability analysis of systems with delay. Those results were included in [12]−[23]. In [12] and [13], the methods based on characteristic function (or transfer function) were used to deal with linear neutral systems with constant delay. In [14] and [15], the authors considered the term $-\int_{t-d_M}^{t-d(t)} \dot{\boldsymbol{x}}^{\mathrm{T}}(t)Z\dot{\boldsymbol{x}}(t)\mathrm{d}s$ in Lyapunov functional, which was usually neglected in previous literature, where $d(t)$ denoted time-varying delay and d_M denoted the upper bound of $d(t)$, i.e., $d(t) \in [0,\ d_M]$. The augmented Lyapunov-Krasovskii functional was employed to reduce the conservativeness of stability results in [14], [16]–[18]. In [19] and [20], the robust stability for the neutral systems with delay and nonlinear perturbations were studied. And then, neutral systems with distributed delay and interval delay were found in [21] and [22]. In [23], the absolute stability of neutral systems was studied. To the best of our knowledge, there is no stability criterion which can deal with systems with fast-varying neutral-type delay, i.e., the derivative of neutral-type delay is more than 1. How to obtain the stability results dealt with fast-varying neutral-type delay and reduce their conservativeness, which motivate the present study.

In this paper, the stability of linear neutral system with time-varying retarded-type delay and time-varying neutral-type delay (including the fast-varying neutral-type delay) is studied. By employing a new Lyapunov-Krasovskii functional and a novel polynomials with time-varying delay (PTVD) compensation technique, the less conservative stability criteria are obtained. Compared with previous results, it is the first time to consider the fast-varying neutral-type delay in neutral system with delay, which is achieved by the new functional. And since the PTVD compensation technique is used, some useful terms can be introduced by using some polynomials with time-varying delays in system, which are usually ignored at the process of estimating the upper bound of derivative of Lyapunov-Krasovskii functional. Obviously, the criteria are less conservative by applying this novel technique. A numerical example shows that our results are effective and less conservative than the other reports in previous literature.

In the following, $D = [d_{ij}]_{n\times n}$ denotes an $n \times n$ real matrix. D^{T} and $\|D\|$ represent the transpose and norm of matrix, where $\|\cdot\|$ is Euclidean norm. $D > 0$ ($D < 0$) denotes that D is a positive (negative) definite matrix. $D \geqslant 0$ ($D \leqslant 0$) denotes that D is a positive (negative) semidefinite matrix. I denotes the identity matrix with appropriate dimensions.

1 Systems Description and Preliminaries

Considering the following linear neutral system with time-varying delays:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + B\boldsymbol{x}(t-d(t)) + C\dot{\boldsymbol{x}}(t-\tau(t)), \quad t \geqslant 0 \\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t) \qquad \forall t \in [-\max{(d_M, \tau_M)}, 0], \end{aligned} \tag{1}$$

where $\boldsymbol{x}(\cdot) = [x_1(\cdot)\ \ x_2(\cdot)\ \ \cdots\ \ x_n(\cdot)]^T$ is the state vector of system, A, B, and C are constant matrices, and $\|C\| < 1$. The initial condition $\boldsymbol{\phi}(t)$ is a continuous and differentiable vector-valued function of $t \in [-\max{(d_M, \tau_M)},\ 0]$. The time-delays $d(t)$ and $\tau(t)$ are two irrelevant differentiable functions that satisfy

$$0 \leqslant d(t) \leqslant d_M, \quad \dot{d}(t) \leqslant \mu, \tag{2}$$

and

$$0 \leqslant \tau(t) \leqslant \tau_M, \quad \eta_1 \leqslant \dot{\tau}(t) \leqslant \eta_2 \tag{3}$$

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where $d_M > 0$ and $\tau_M > 0$. For parameters η_1 and η_2 in (3), only two cases are considered in this paper. There are 1) $\eta_1 \leqslant \eta_2 < 1$ (slowly-varying delay, i.e., $\dot{\tau}(t) < 1$) and 2) $1 < \eta_1 \leqslant \eta_2$ (fast-varying delay, i.e., $\dot{\tau}(t) > 1$). Especially for case 2), it is the first time to be discussed in the systems with neutral-type time-varying delay.

The following lemmas will be used to prove the results of this paper.

Lemma 1 (Jensen's Inequality[24]). For any constant matrix $\Omega > 0$, vector function $\boldsymbol{\chi}(t)$ with appropriate dimensions, and function $\sigma(t) \in \mathbf{R}$ satisfies $0 < \sigma(t) \leqslant \delta$, we have

$$\begin{aligned}&\left[\int_{t-\sigma(t)}^{t} \boldsymbol{\chi}(s)\mathrm{d}s\right]^{\mathrm{T}} \Omega \left[\int_{t-\sigma(t)}^{t} \boldsymbol{\chi}(s)\mathrm{d}s\right]\\ \leqslant&\sigma(t)\int_{t-\sigma(t)}^{t} \boldsymbol{\chi}^{\mathrm{T}}(s)\Omega\boldsymbol{\chi}(s)\mathrm{d}s.\end{aligned}$$

Lemma 2. The following inequalities

$$\begin{cases}\Delta + \beta X_1 < 0,\\ \Delta + \beta X_2 < 0,\end{cases} \tag{4}$$

are equivalent to the following condition

$$\Delta + zX_1 + (\beta - z)X_2 < 0, \tag{5}$$

where X_1, X_2, Δ are constant matrices with appropriate dimensions, variable $z \in [0,\ \beta] \in \mathbf{R}$, and $\beta > 0$.

Proof: See Appendix A. □

Remark 1. Lemma 2 is proposed based on the idea of convex combination[25]. Since the proof isn't given in [25], the detailed proof is provided in this paper. Some similar results have been employed in literature [26]–[28].

2 Main Results

In this section, the new stability criteria can be proposed to deal with linear neutral systems with time-varying delay. An augmented Lyapunov-Krasovskii functional and PTVD compensation technique will be used in proposed criteria. Firstly, the case of slow-varying neutral-type delay will be considered, i.e., $\eta_1 \leqslant \eta_2 < 1$.

Theorem 1. The system (1) with time-varying delays $d(t)$ and $\tau(t)$ satisfying (2) and (3) is asymptotically stable, for the given scalar parameters d_M, μ, τ_M, and $\eta_1 \leqslant \eta_2 < 1$, if there exist some matrices

$$P = P^{\mathrm{T}} = \begin{bmatrix} P_{11} & P_{12} & P_{13}\\ P_{12}^{\mathrm{T}} & P_{22} & P_{23}\\ P_{13}^{\mathrm{T}} & P_{23}^{\mathrm{T}} & P_{33}\end{bmatrix} > 0,$$

$$Q_1 = Q_1^{\mathrm{T}} > 0, Q_2 = Q_2^{\mathrm{T}} > 0, R_1 = R_1^{\mathrm{T}} > 0, R_2 = R_2^{\mathrm{T}} > 0,$$

$$Y_1 = Y_1^{\mathrm{T}} > 0, Y_2 = Y_2^{\mathrm{T}} > 0, Z_1 = Z_1^{\mathrm{T}} > 0, Z_2 = Z_2^{\mathrm{T}} > 0,$$

$$S_1 = S_1^{\mathrm{T}} > 0, S_2 = S_2^{\mathrm{T}} > 0, S_3 = S_3^{\mathrm{T}} > 0$$

such that the following matrix inequalities hold:

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_1^{\mathrm{T}} Z_1 e_1 - e_3^{\mathrm{T}} Z_2 e_3 < 0 \tag{6}$$

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_1^{\mathrm{T}} Z_1 e_1 - e_4^{\mathrm{T}} Z_2 e_4 < 0 \tag{7}$$

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_2^{\mathrm{T}} Z_1 e_2 - e_3^{\mathrm{T}} Z_2 e_3 < 0 \tag{8}$$

$$\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_2^{\mathrm{T}} Z_1 e_2 - e_4^{\mathrm{T}} Z_2 e_4 < 0 \tag{9}$$

where Φ is shown at the bottom of this page,

$$\begin{aligned}
\Phi_1 =& P_{11}A + A^{\mathrm{T}}P_{11} + Q_1 + Q_2 + R_1 - Z_1 - Z_2 + \eta_2 S_1\\
\Phi_2 =& -(1-\mu)Q_1 - 2Z_1\\
\Phi_3 =& -Q_2 - Z_1\\
\Phi_4 =& -(1-\eta_2)R_1 + (1-\eta_1)R_2 - 2Z_2 + \eta_2 S_2\\
\Phi_5 =& -R_2 - Z_2 + \eta_2 S_3\\
\Phi_6 =& -(1-\eta_2)Y_1 + (1-\eta_1)Y_2 + \eta_2 P_{12}^{\mathrm{T}} S_1^{-1} P_{12}\\
&+ \eta_2 P_{22} S_2^{-1} P_{22} + \eta_2 P_{23} S_3^{-1} P_{23}^{\mathrm{T}}\\
\bar{A} =& \begin{bmatrix} A & B & 0 & 0 & 0 & C & 0\end{bmatrix},\\
e_1 =& \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0\end{bmatrix},\\
e_2 =& \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0\end{bmatrix},\\
e_3 =& \begin{bmatrix} I & 0 & 0 & -I & 0 & 0 & 0\end{bmatrix},\\
e_4 =& \begin{bmatrix} 0 & 0 & 0 & I & -I & 0 & 0\end{bmatrix},
\end{aligned}$$

and $\star$ denotes the symmetric terms in a symmetric matrix.

Proof. Construct the following Lyapunov-Krasovskii functional:

$$V(\boldsymbol{x}(t)) = V_1(\boldsymbol{x}(t)) + V_2(\boldsymbol{x}(t)) + V_3(\boldsymbol{x}(t)) + V_4(\boldsymbol{x}(t)), \tag{10}$$

where

$$\begin{aligned}
V_1(\boldsymbol{x}(t)) =& \boldsymbol{\delta}^{\mathrm{T}}(t)P\boldsymbol{\delta}(t),\\
V_2(\boldsymbol{x}(t)) =& \int_{t-d(t)}^{t} \boldsymbol{x}^{\mathrm{T}}(s)Q_1\boldsymbol{x}(s)\mathrm{d}s + \int_{t-d_M}^{t} \boldsymbol{x}^{\mathrm{T}}(s)Q_2\boldsymbol{x}(s)\mathrm{d}s,\\
V_3(\boldsymbol{x}(t)) =& \int_{t-\tau(t)}^{t} \left(\boldsymbol{x}^{\mathrm{T}}(s)R_1\boldsymbol{x}(s) + \dot{\boldsymbol{x}}^{\mathrm{T}}(s)Y_1\dot{\boldsymbol{x}}(s)\right)\mathrm{d}s\\
&+ \int_{t-\tau_M}^{t-\tau(t)} \left(\boldsymbol{x}^{\mathrm{T}}(s)R_2\boldsymbol{x}(s) + \dot{\boldsymbol{x}}^{\mathrm{T}}(s)Y_2\dot{\boldsymbol{x}}(s)\right)\mathrm{d}s,\\
V_4(\boldsymbol{x}(t)) =& d_M\int_{-d_M}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta\\
&+ \tau_M\int_{-\tau_M}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta,
\end{aligned}$$

where $\boldsymbol{\delta}^{\mathrm{T}}(t) = \begin{bmatrix}\boldsymbol{x}^{\mathrm{T}}(t) & \boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) & \boldsymbol{x}^{\mathrm{T}}(t-\tau_M)\end{bmatrix}$, $P = P^{\mathrm{T}} = \begin{bmatrix} P_{11} & P_{12} & P_{13}\\ P_{12}^{\mathrm{T}} & P_{22} & P_{23}\\ P_{13}^{\mathrm{T}} & P_{23}^{\mathrm{T}} & P_{33}\end{bmatrix} > 0$, $Q_k = Q_k^{\mathrm{T}} > 0$, $R_k = R_k^{\mathrm{T}} > 0$, $Y_k = Y_k^{\mathrm{T}} > 0$, $Z_k = Z_k^{\mathrm{T}} > 0$, and $k = 1, 2$.

$$\Phi = \begin{bmatrix}
\Phi_1 & P_{11}B + Z_1 & 0 & A^{\mathrm{T}}P_{12} + Z_2 & A^{\mathrm{T}}P_{13} & P_{11}C + P_{12} & P_{13}\\
\star & \Phi_2 & Z_1 & B^{\mathrm{T}}P_{12} & B^{\mathrm{T}}P_{13} & 0 & 0\\
\star & \star & \Phi_3 & 0 & 0 & 0 & 0\\
\star & \star & \star & \Phi_4 & Z_2 & P_{12}^{\mathrm{T}}C + P_{22} & P_{23}\\
\star & \star & \star & \star & \Phi_5 & P_{13}^{\mathrm{T}}C + P_{23}^{\mathrm{T}} & P_{33}\\
\star & \star & \star & \star & \star & \Phi_6 & 0\\
\star & \star & \star & \star & \star & \star & -Y_2
\end{bmatrix}$$

Calculating the time derivatives of $V_i(\boldsymbol{x}(t))$ $(i = 1, 2, 3, 4)$ along the trajectories of system (1) yields

$$\begin{aligned}\dot{V}_1(\boldsymbol{x}(t)) =&2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-\tau(t))\\ \boldsymbol{x}(t-\tau_M)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{12}^{\mathrm{T}} & P_{22} & P_{23}\\ P_{13}^{\mathrm{T}} & P_{23}^{\mathrm{T}} & P_{33}\end{bmatrix}\\ &\times\begin{bmatrix}A\boldsymbol{x}(t)+B\boldsymbol{x}(t-d(t))+C\dot{\boldsymbol{x}}(t-\tau(t))\\ (1-\dot{\tau}(t))\dot{\boldsymbol{x}}(t-\tau(t))\\ \dot{\boldsymbol{x}}(t-\tau_M)\end{bmatrix}\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_2(\boldsymbol{x}(t)) \leqslant&\boldsymbol{x}^{\mathrm{T}}(t)(Q_1+Q_2)\boldsymbol{x}(t)\\ &-(1-\mu)\boldsymbol{x}^{\mathrm{T}}(t-d(t))Q_1\boldsymbol{x}(t-d(t))\\ &-\boldsymbol{x}^{\mathrm{T}}(t-d_M)Q_2\boldsymbol{x}(t-d_M)\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{V}_3(\boldsymbol{x}(t)) \leqslant&\boldsymbol{x}^{\mathrm{T}}(t)R_1\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-\tau_M)R_2\boldsymbol{x}(t-\tau_M)\\ &-(1-\eta_2)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))R_1\boldsymbol{x}(t-\tau(t))\\ &+(1-\eta_1)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))R_2\boldsymbol{x}(t-\tau(t))\\ &+\dot{\boldsymbol{x}}^{\mathrm{T}}(t)Y_1\dot{\boldsymbol{x}}(t)-\dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_M)Y_2\dot{\boldsymbol{x}}(t-\tau_M)\\ &-(1-\eta_2)\dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))Y_1\dot{\boldsymbol{x}}(t-\tau(t))\\ &+(1-\eta_1)\dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))Y_2\dot{\boldsymbol{x}}(t-\tau(t))\end{aligned} \tag{13}$$

$$\begin{aligned}\dot{V}_4(\boldsymbol{x}(t)) =&d_M^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)Z_1\dot{\boldsymbol{x}}(t)-d_M\int_{t-d_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &+\tau_M^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)Z_2\dot{\boldsymbol{x}}(t)-\tau_M\int_{t-\tau_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\end{aligned} \tag{14}$$

For the terms with $\dot{\tau}(t)$ in (11), by using some matrices $S_1 = S_1^{\mathrm{T}} > 0$, $S_2 = S_2^{\mathrm{T}} > 0$, and $S_3 = S_3^{\mathrm{T}} > 0$, there is the following inequality,

$$\begin{aligned}&-2\dot{\tau}(t)\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-\tau(t))\\ \boldsymbol{x}(t-\tau_M)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}P_{12}\\ P_{22}\\ P_{23}^{\mathrm{T}}\end{bmatrix}\dot{\boldsymbol{x}}(t-\tau(t))\\ \leqslant&\eta_2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-\tau(t))\\ \boldsymbol{x}(t-\tau_M)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}S_1 & 0 & 0\\ 0 & S_2 & 0\\ 0 & 0 & S_3\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-\tau(t))\\ \boldsymbol{x}(t-\tau_M)\end{bmatrix}\\ &+\eta_2\dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\Big(P_{12}^{\mathrm{T}}S_1^{-1}P_{12}+P_{22}S_2^{-1}P_{22}\\ &+P_{23}S_3^{-1}P_{23}^{\mathrm{T}}\Big)\dot{\boldsymbol{x}}(t-\tau(t)).\end{aligned} \tag{15}$$

To utilize the information that is ignored in previous results, we apply two following polynomials,

$$\rho_1 = d(t)\int_{t-d_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s-\tau(t)\int_{t-\tau_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s, \tag{16}$$

$$\rho_2 = \tau(t)\int_{t-\tau_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s-d(t)\int_{t-d_M}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s, \tag{17}$$

where the polynomials ρ_1 and ρ_2 are named as PTVD compensation terms. It is clear that $\rho_1 + \rho_2 = 0$. Then, using PTVD compensation terms ρ_1 and ρ_2, Jensen's inequality[24], and Leibniz-Newton formula, $\dot{V}_4(\boldsymbol{x}(t))$ can be rewritten as follows

$$\begin{aligned}\dot{V}_4(\boldsymbol{x}(t)) =&\dot{V}_4(\boldsymbol{x}(t))+\rho_1+\rho_2\\ =&d_M^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)Z_1\dot{\boldsymbol{x}}(t)+\tau_M^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)Z_2\dot{\boldsymbol{x}}(t)\\ &-d(t)\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &-(d_M-d(t))\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &-(d_M-d(t))\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &-d(t)\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &-\tau(t)\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &-(\tau_M-\tau(t))\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &-(\tau_M-\tau(t))\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ &-\tau(t)\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)Z_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\\ \leqslant&d_M^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)Z_1\dot{\boldsymbol{x}}(t)+\tau_M^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)Z_2\dot{\boldsymbol{x}}(t)\\ &-\left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}}Z_1\left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]\\ &-\left[\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}}Z_1\left[\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]\\ &-\left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}}Z_2\left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]\\ &-\left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}}Z_2\left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]\\ &-\frac{d_M-d(t)}{d(t)}\left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}}Z_1\\ &\times\left[\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]-\frac{d(t)}{d_M-d(t)}\\ &\times\left[\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}}Z_1\left[\int_{t-d_M}^{t-d(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]\\ &-\frac{\tau_M-\tau(t)}{\tau(t)}\left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}}Z_2\\ &\times\left[\int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]-\frac{\tau(t)}{\tau_M-\tau(t)}\\ &\times\left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]^{\mathrm{T}}Z_2\left[\int_{t-\tau_M}^{t-\tau(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\right]\\ \leqslant&d_M^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)Z_1\dot{\boldsymbol{x}}(t)+\tau_M^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t)Z_2\dot{\boldsymbol{x}}(t)-\boldsymbol{\zeta}^{\mathrm{T}}(t)Z_0\boldsymbol{\zeta}(t)\\ &-\frac{d_M-d(t)}{d_M}\boldsymbol{\zeta}^{\mathrm{T}}(t)e_1^{\mathrm{T}}Z_1e_1\boldsymbol{\zeta}(t)\\ &-\frac{d(t)}{d_M}\boldsymbol{\zeta}^{\mathrm{T}}(t)e_2^{\mathrm{T}}Z_1e_2\boldsymbol{\zeta}(t)\\ &-\frac{\tau_M-\tau(t)}{\tau_M}\boldsymbol{\zeta}^{\mathrm{T}}(t)e_3^{\mathrm{T}}Z_2e_3\boldsymbol{\zeta}(t)\\ &-\frac{\tau(t)}{\tau_M}\boldsymbol{\zeta}^{\mathrm{T}}(t)e_4^{\mathrm{T}}Z_2e_4\boldsymbol{\zeta}(t)\end{aligned} \tag{18}$$

where $\boldsymbol{\zeta}^{\mathrm{T}}(t) = [\boldsymbol{x}^{\mathrm{T}}(t) \;\; \boldsymbol{x}^{\mathrm{T}}(t-d(t)) \;\; \boldsymbol{x}^{\mathrm{T}}(t-d_M) \;\; \boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) \;\; \boldsymbol{x}^{\mathrm{T}}(t-\tau_M) \;\; \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t)) \;\; \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau_M)]$ and

$$Z_0 = \begin{bmatrix} Z_1+Z_2 & -Z_1 & 0 & -Z_2 & 0 & 0 & 0 \\ -Z_1 & 2Z_1 & -Z_1 & 0 & 0 & 0 & 0 \\ 0 & -Z_1 & Z_1 & 0 & 0 & 0 & 0 \\ -Z_2 & 0 & 0 & 2Z_2 & -Z_2 & 0 & 0 \\ 0 & 0 & 0 & -Z_2 & Z_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus, according to (11)–(13), (15), and (18), $\dot{V}(\boldsymbol{x}(t))$ can be rewritten as follows

$$\begin{aligned} \dot{V}(\boldsymbol{x}(t)) \leqslant \boldsymbol{\zeta}^{\mathrm{T}}(t) \Big[& \Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} \\ & - \frac{d_M - d(t)}{d_M} e_1^{\mathrm{T}} Z_1 e_1 - \frac{d(t)}{d_M} e_2^{\mathrm{T}} Z_1 e_2 \\ & - \frac{\tau_M - \tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4 \Big] \boldsymbol{\zeta}(t) \end{aligned} \tag{19}$$

Obviously, if matrix inequality $\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - \frac{d_M - d(t)}{d_M} e_1^{\mathrm{T}} Z_1 e_1 - \frac{d(t)}{d_M} e_2^{\mathrm{T}} Z_1 e_2 - \frac{\tau_M - \tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4 < 0$, it means that $\dot{V}(\boldsymbol{x}(t)) < 0$. Based on Lemma 2, it is equivalent to the following matrix inequalities

$$\begin{aligned} & \Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_1^{\mathrm{T}} Z_1 e_1 \\ & - \frac{\tau_M - \tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4 < 0, \end{aligned} \tag{20}$$

and

$$\begin{aligned} & \Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - e_2^{\mathrm{T}} Z_1 e_2 \\ & - \frac{\tau_M - \tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4 < 0, \end{aligned} \tag{21}$$

when $d(t) = 0$ and $d(t) = d_M$, respectively. Then, applying Lemma 2 again, (20) and (21) are equivalent to (6)−(9), i.e., $\Phi + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1 + \tau_M^2 Z_2)\bar{A} - \frac{d_M - d(t)}{d_M} e_1^{\mathrm{T}} Z_1 e_1 - \frac{d(t)}{d_M} e_2^{\mathrm{T}} Z_1 e_2 - \frac{\tau_M - \tau(t)}{\tau_M} e_3^{\mathrm{T}} Z_2 e_3 - \frac{\tau(t)}{\tau_M} e_4^{\mathrm{T}} Z_2 e_4 < 0$ is equivalent to (6)–(9). Thus, if the (6)−(9) are satisfied, then $\dot{V}(\boldsymbol{x}(t)) < 0$, i.e., system (1) is asymptotically stable. □

Remark 2. In Theorem 1, there are two points which are different from the previous results for linear neutral systems with time-varying delay.

1. An augmented Lyapunov-Krasovskii functional V_1 is used to deal with stability problem of linear neutral systems with time-varying delays. Meanwhile, the functionals used in previous literature are listed as follows,

$$\boldsymbol{x}^{\mathrm{T}}(t) P \boldsymbol{x}(t) = \boldsymbol{\delta}^{\mathrm{T}}(t) \begin{bmatrix} P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{\delta}(t), \tag{22}$$

$$\begin{aligned} & (\boldsymbol{x}^{\mathrm{T}}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau_M) C^{\mathrm{T}}) P (\boldsymbol{x}(t) - C\boldsymbol{x}(t-\tau_M)) \\ = & \boldsymbol{\delta}^{\mathrm{T}}(t) \begin{bmatrix} P & 0 & -PC \\ 0 & 0 & 0 \\ -C^{\mathrm{T}} P & 0 & C^{\mathrm{T}} P C \end{bmatrix} \boldsymbol{\delta}(t). \end{aligned} \tag{23}$$

Obviously, (22) and (23) are just the special cases of functional V_1. That is to say, compared with results employed (22) and (23), the criterion used functional V_1 has larger solution set.

2. Theorem 1 does not only depend on delay $d(t)$, but also depends on neural-type delay $\tau(t)$. Since the delay-dependent criteria are less conservative than delay-independent ones, it also means that Theorem 1 is less conservative than the criteria independent of neural-type delay. The numerical example in Section 3 can also verify this point.

Remark 3. When Jensen's inequality is used to deal with $-d_M \int_{t-d_M}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) Z_1 \dot{\boldsymbol{x}}(s) \mathrm{d}s$ and $-\tau_M \int_{t-\tau_M}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) Z_2 \dot{\boldsymbol{x}}(s) \mathrm{d}s$ in (14), it can be dealt with as follows,

$$\begin{aligned} & - d_M \int_{t-d_M}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) Z_1 \dot{\boldsymbol{x}}(s) \mathrm{d}s - \tau_M \int_{t-\tau_M}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) Z_2 \dot{\boldsymbol{x}}(s) \mathrm{d}s \\ \leqslant & - \left[\int_{t-d(t)}^{t} \dot{\boldsymbol{x}}(s) \mathrm{d}s \right]^{\mathrm{T}} Z_1 \left[\int_{t-d(t)}^{t} \dot{\boldsymbol{x}}(s) \mathrm{d}s \right] \\ & - \left[\int_{t-d_M}^{t-\tau(t)} \dot{\boldsymbol{x}}(s) \mathrm{d}s \right]^{\mathrm{T}} Z_1 \left[\int_{t-d_M}^{t-\tau(t)} \dot{\boldsymbol{x}}(s) \mathrm{d}s \right] \\ & - \left[\int_{t-\tau(t)}^{t} \dot{\boldsymbol{x}}(s) \mathrm{d}s \right]^{\mathrm{T}} Z_2 \left[\int_{t-\tau(t)}^{t} \dot{\boldsymbol{x}}(s) \mathrm{d}s \right] \\ & - \left[\int_{t-\tau_M}^{t-\tau(t)} \dot{\boldsymbol{x}}(s) \mathrm{d}s \right]^{\mathrm{T}} Z_2 \left[\int_{t-\tau_M}^{t-\tau(t)} \dot{\boldsymbol{x}}(s) \mathrm{d}s \right]. \end{aligned} \tag{24}$$

According to (18), it means that the terms

$$\begin{aligned} & - d(t) \int_{t-d_M}^{t-d(t)} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) Z_1 \dot{\boldsymbol{x}}(s) \mathrm{d}s, \\ & - (d_M - d(t)) \int_{t-d(t)}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) Z_1 \dot{\boldsymbol{x}}(s) \mathrm{d}s, \\ & - \tau(t) \int_{t-\tau_M}^{t-\tau(t)} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) Z_2 \dot{\boldsymbol{x}}(s) \mathrm{d}s, \\ & - (\tau_M - \tau(t)) \int_{t-\tau(t)}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) Z_2 \dot{\boldsymbol{x}}(s) \mathrm{d}s \end{aligned}$$

can be ignored at the process of estimating the upper bound of $\dot{V}(\boldsymbol{x}(t))$ in the previous literature. Obviously, it leads to the increase of conservativeness in stability results. In this paper, we apply the novel PTVD compensation technique shown as (18) to compensate these ignored terms. Thus, a new stability criterion applied this method can be obtained. Furthermore, since there are two irrelevant time-varying delays $d(t)$ and $\tau(t)$, PTVD compensation technique can be used only for function $d(t)$ or $\tau(t)$. Thus, on the premise of reducing the load of calculation, the satisfactory stability results can be derived.

In Theorem 1, the stability result does not only depend on retarded-type delay but also depend on neutral-type delay. Then, the only dependent on retarded-type delay stability criterion will be proposed. Especially, the case of fast-varying neutral-type delay will be first considered in following theorem.

Theorem 2. The system (1) with time-varying delays $d(t)$ and $\tau(t)$ satisfying (2) and (3) is asymptotically stable, for the given scalar parameters d_M, μ, and $\eta_1 \leqslant \eta_2 < 1$ or $1 < \eta_1 \leqslant \eta_2$, if there exist some matrices

$$P = P^{\mathrm{T}} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12}^{\mathrm{T}} & P_{22} & P_{23} \\ P_{13}^{\mathrm{T}} & P_{23}^{\mathrm{T}} & P_{33} \end{bmatrix} > 0,$$

Table 1 Maximum allowable upper bound d_M for different μ and $\tau_M = 0.1$

	Method	$\mu = 0.7$	$\mu = 0.8$	$\mu = 0.9$	Unknown μ
$\eta_1 = \eta_2 = 0(\tau(t) = \tau_M)$	[8]	$d_M = 0.3890$	$d_M = 0.2547$	$d_M = 0.1253$	–
	[19]	$d_M = 1.0071$	$d_M = 0.9201$	$d_M = 0.8347$	$d_M = 0.7603$
	[22]	$d_M = 1.0425$	$d_M = 0.9515$	$d_M = 0.8596$	$d_M = 0.7652$
	[20]	$d_M = 1.0628$	$d_M = 0.9642$	$d_M = 0.8642$	$d_M = 0.7652$
	Theorem 2	$d_M = 1.1281$	$d_M = 1.0941$	$d_M = 1.0882$	$d_M = 1.0882$
	Theorem 1	$d_M = 1.1642$	$d_M = 1.1294$	$d_M = 1.1210$	$d_M = 1.1208$
$\eta_1 = 0,\ \eta_2 = 0.5$	[22]	$d_M = 0.9938$	$d_M = 0.9083$	$d_M = 0.8224$	$d_M = 0.7345$
	Theorem 2	$d_M = 1.0448$	$d_M = 1.0160$	$d_M = 1.0093$	$d_M = 1.0093$
	Theorem 1	$d_M = 1.1071$	$d_M = 1.0755$	$d_M = 1.0688$	$d_M = 1.0688$
$\eta_1 = 1.1,\ \eta_2 = 2.0$	Theorem 2	$d_M = 1.2170$	$d_M = 1.1824$	$d_M = 1.1774$	$d_M = 1.1774$

$$Q_1 = Q_1^{\mathrm{T}} > 0, Q_2 = Q_2^{\mathrm{T}} > 0, R_1 = R_1^{\mathrm{T}} > 0, R_2 = R_2^{\mathrm{T}} > 0,$$
$$Y_1 = Y_1^{\mathrm{T}} > 0, Y_2 = Y_2^{\mathrm{T}} > 0, Z_1 = Z_1^{\mathrm{T}} > 0,$$
$$S_1 = S_1^{\mathrm{T}} > 0, S_2 = S_2^{\mathrm{T}} > 0, S_3 = S_3^{\mathrm{T}} > 0$$

such that the following matrix inequalities hold:

$$\bar{\Phi} + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1)\bar{A} - e_1^{\mathrm{T}} Z_1 e_1 < 0 \tag{25}$$
$$\bar{\Phi} + \bar{A}^{\mathrm{T}}(Y_1 + d_M^2 Z_1)\bar{A} - e_2^{\mathrm{T}} Z_1 e_2 < 0 \tag{26}$$

where $\bar{\Phi}$ is shown at the bottom of this page,

$$\bar{\Phi}_1 = P_{11}A + A^{\mathrm{T}}P_{11} + Q_1 + Q_2 + R_1 - Z_1 + \eta_2 S_1$$
$$\bar{\Phi}_4 = -(1-\eta_2)R_1 + (1-\eta_1)R_2 + \eta_2 S_2$$

and the other parameters are the same as those defined in Theorem 1.

Proof. Construct the following Lyapunov-Krasovskii functional:

$$\bar{V}(\boldsymbol{x}(t)) = V_1(\boldsymbol{x}(t)) + V_2(\boldsymbol{x}(t)) + V_3(\boldsymbol{x}(t)) + \bar{V}_4(\boldsymbol{x}(t)), \tag{27}$$

where $V_1(\boldsymbol{x}(t))$, $V_2(\boldsymbol{x}(t))$, and $V_3(\boldsymbol{x}(t))$ are the same as the definition in Theorem 1, and

$$\bar{V}_4(\boldsymbol{x}(t)) = d_M \int_{-d_M}^{0} \int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) Z_1 \dot{\boldsymbol{x}}(s) \mathrm{d}s \mathrm{d}\theta.$$

The process of proof is similar with Theorem 1. □

Remark 4. The systems with fast-varying neutral-type delay (i.e., $\dot{\tau}(t) > 1$) can be first considered in Theorem 2, which is achieved by using condition (3) and functional V_3. So far, there is no literature referred to this case.

Remark 5. For Theorems 1 and 2, only by setting $Q_1 = 0$, the criteria independent of derivative of delay function $d(t)$ can be derived.

3 Numerical Example

In this section, an example will be given to verify the proposed criteria.

Consider linear neutral system (1) with the following parameters, see [9],

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},$$
$$C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix},$$

delay functions $d(t)$ and $\tau(t)$ satisfy the condition (2) and (3).

Applying LMI Toolbox of MATLAB, we can solve the maximum allowable upper bounds d_M by setting τ_M, μ, η_1, and η_2. Let $\tau_M = 0.1$, the stability results between [8], [19], [20], [22], and this paper will be obtained in Table 1 for different μ, η_1, and η_2, respectively. It is clear that the stability results by using our method are less conservative. When $\eta_1 \leqslant \eta_2 < 1$, the upper bound d_M obtained by using Theorem 1 is larger than that by using Theorem 2. Meanwhile, Theorem 2 is more effective and suitable to deal with linear systems with fast-varying neutral-type delay. And since only two matrix inequalities need to be solved, Theorem 2 is less load of calculation.

4 Conclusion

A class of linear neutral systems with time-varying retarded-type delay and time-varying neutral-type delay is investigated in this paper. Since a new Lyapunov-Krasovskii functional and a novel PTVD compensation technique are introduced, the less conservative stability criterion is proposed. The gain used new functional is that the stability of linear neutral systems with fast-varying neutral-type delay (i.e., $\dot{\tau}(t) > 1$) can be obtained, which is the first

$$\bar{\Phi} = \begin{bmatrix} \bar{\Phi}_1 & P_{11}B + Z_1 & 0 & A^{\mathrm{T}}P_{12} & A^{\mathrm{T}}P_{13} & P_{11}C + P_{12} & P_{13} \\ \star & \Phi_2 & Z_1 & B^{\mathrm{T}}P_{12} & B^{\mathrm{T}}P_{13} & 0 & 0 \\ \star & \star & \Phi_3 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \bar{\Phi}_4 & 0 & P_{12}^{\mathrm{T}}C + P_{22} & P_{23} \\ \star & \star & \star & \star & -R_2 + \eta_2 S_3 & P_{13}^{\mathrm{T}}C + P_{23}^{\mathrm{T}} & P_{33} \\ \star & \star & \star & \star & \star & \Phi_6 & 0 \\ \star & \star & \star & \star & \star & \star & -Y_2 \end{bmatrix}$$

time to be considered in stability criteria. And some useful terms can be considered by using the PTVD compensation technique, which are usually ignored at the process of estimating the upper bound of $\dot{V}(\boldsymbol{x}(t))$. The numerical example has proved that the proposed criteria are effective.

Appendix A

The Proof of Lemma 2

1). (5)⇒(4)

Since variable z satisfies the following condition at interval $[0,\ \beta]$

$$\Delta + zX_1 + (\beta - z)X_2 < 0.$$

Thus, when variable $z = \beta$ and $z = 0$, two following inequalities hold

$$\Delta + \beta X_1 < 0, \quad \Delta + \beta X_2 < 0.$$

2). (4)⇒(5)

Let matrices Δ_1 and Δ_2 satisfy the following conditions

$$\Delta_1 = \Delta + \beta X_1 < 0, \quad \Delta_2 = \Delta + \beta X_2 < 0.$$

We can get the following result

$$z\Delta_1 + (\beta - z)\Delta_2 < 0,$$

i.e.,

$$\beta(\Delta + zX_1 + (\beta - z)X_2) < 0.$$

Since $\beta > 0$, the following inequality holds

$$\Delta + zX_1 + (\beta - z)X_2 < 0.$$

References

1 Kuang Y. *Delay differential equations with applications in population dynamics*. Boston: Academic Press, 1993.

2 Brayton R K. Small-signal stability criterion for electrical networks containning lossless transmission lines. *IBM Journal of Research and Development*, 1968, **12**: 431–440.

3 Niculescu S I, Răsvan V. Delay-independent stability in lossless propagation models with applications (I): a complex-domain approach, In: Proceedings. MTNS 2000, Perpigna, France, 2000.

4 Yue D, Han Q L. A delay-dependent stability criterion of neutral systems and its application to apartial element equivalent circuit model. *IEEE Transactions on Circuits and Systems II, Express Briefs*, 2004, **51**(12): 685–689.

5 Hale J, Lunel S M V. *Introduction to functional differential equations*. New York: Springer-Verlay, 1993.

6 Fridman E, Shaked U. A descriptor system approach to H_∞ control of linear time-delay systems. *IEEE Transactions on Automatic Control*, 2002, **47**(2): 253–270.

7 Fridman E, Shaked U. Delay-dependent stability and H_∞ control: constant and time-varying delays. *International Journal of Control*, 2003, **76**(1): 48–60.

8 Han Q L. On robust stability of neutral systems with time-varying discrete delay and norm-bounded uncertainty. *Automatica*, 2005, **40**(6): 1087–1092.

9 Han Q L. On stability of linear neutral systems with mixed time delays: a discretized Lyapunov functional approach. *Automatica*, 2005, **41**(7): 1209–1218.

10 He Y, Wu M, She J, Liu G P. Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays. *System & Control Letters*, 2004, **51**(1): 57–65.

11 Wu M, He Y, She J. New delay-dependent stability criteria and stabilitzing method for neutral systems. *IEEE Transactions on Automatic Control*, 2004, **49**(12): 2266–2271.

12 Fu P, Niculescu S I, Chen J. Stability of linear neutral time-delay systems: exact conditions via matrix pencil solutions. *IEEE Transactions on Automatic Control*, 2006, **51**(6): 1063–1069.

13 Hu G D, Liu M. Stability criteria of linear neutral systems with multiple delays. *IEEE Transactions on Automatic Control*, 2007, **52**(4): 720–724.

14 He Y, Wang Q G, Xie L H, Lin C. Further inmprovment of free-weighting matrices technique for systems with time-varying delay. *IEEE Transactions on Automatic Control*, 2007, **52**(2): 293–299.

15 He Y, Liu G P, Rees D, Wu M. Stability analysis for neural networks with time-varying interval delay. *IEEE Transactions on Neural Networks*, 2007, **18**(6): 1850–1854.

16 He Y, Wang Q G, Lin C. Augmented Lyapunov functional and delay-dependent stability criteria for neutral systems. *International Journal of Robust and Nonlinear Control*, 2005, **15**(18): 923–933.

17 Parlakçi M. Extensively augmented Lyapunov functional approach for the stability of neutral time-delay systems. *IET Control Theory and Applications*, 2008, **2**(5): 431–436.

18 Li X G, Zhu X J. Stability analysis of neutral systems with distributed delays. *Automatica*, 2008, **44**(8): 2197–2201.

19 Zhao Z R, Wang W, Yang B. Delay and its time-derivative dependent robust stability of neutral control system. *Applied Mathematics and Computation*, 2007, **187**(2): 1326–1332.

20 Zhang W A, Yu L. Delay-dependent robust stability of neutral systems with mixed delays and nonlinear perturbations. *ACTA Automatica SINICA*, 2007, **33**(8): 863–866.

21 Chen W H, Zheng W X. Delay-dependent robust stabilization for uncertain neutral systems with distributed delays. *Automatica*, 2007, **43**(1): 95–104.

22 Yu K W, Lien C H. Stability criteria for uncertain neutral systems with interval time-varying delays. *Chaos Solitons & Fractals*, 2008, **38**(3): 650–657.

23 Han Q L. A new delay-dependent absolute stability criterion for a calss of nonlinear neutral systems. *Automatica*, 2008, **44**(1): 272–277.

24 Gu K, Kharitonov V, Chen J. *Stability of Time-delay Systems*. Cambridge, MA: Birkhauser, 2003.

25 Park P, Ko J. Stability and robust stability for systems with a time-varying delay. *Automatica*, 2007, **43**(10): 1855–1858.

26 Zhu X L, Yang G H. New delay-dependent stability results for neural networks with time-varying delay. *IEEE Transactions on Neural Networks*, 2008, **19**(10): 1783–1791.

27 Chen Y G, Wu Y Y. Novel delay-dependent stability criteria of neural networks with time-varying delay. *Neurocomputing*, 2009, **72**(4-6): 1065–1070.

28 Tian E G, Yue D, Zhang Y J. Delay-dependent robust H_∞ control for T-S fuzzy system with interval time-varying delay. *Fussy Sets and Systems*, to be pulished.

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