Stabilization of Discrete-time 2-D T-S Fuzzy Systems Based on New Relaxed Conditions

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Abstract This paper is concerned with the problem of stabilization of the Roesser type discrete-time nonlinear 2-D system which plays an important role in many practical applications. Firstly, a discrete-time 2-D T-S fuzzy model is proposed to represent the underlying nonlinear 2-D system. Secondly, new quadratic stabilization conditions are proposed by applying relaxed quadratic stabilization technique for 2-D case. Thirdly, for sake of further reducing conservatism, new non-quadratic stabilization conditions are also proposed by applying a new parameter-dependent Lyapunov function, matrix transformation technique and relaxed technique for the underlying discrete-time 2-D T-S fuzzy system. Finally, a numerical example is provided to illustrate the effectiveness of the proposed results.

Key words Roesser model, 2-D discrete systems, Takagi-Sugeno (T-S) fuzzy model, relaxed stabilization conditions

In the past three decades, the two-dimensional (2-D) systems[1−2] have been investigated by many researchers since it could represent a wide range of practical systems, such as those in image data processing and transmission, thermal process, signal filtering, etc. Recently, The 2-D system theory is also frequently used as an analysis tool to some problems, e.g., iterative learning control[3] and repetitive process control[4]. The PI control of discrete linear repetitive processes has been investigated in [5]. In [6], the problem of H_∞ control for 2-D discrete state delay systems described by the second FM state-space model has been studied. Due to the application in modeling hybrid systems, H_∞ filtering for 2-D Markovian jump systems has also been investigated in [7]. Moreover, stability analysis of 2-D discrete systems described by the Fornasini-Marchesini (FM) second model with state saturation is studied in [8]. However, the aforementioned results are only for linear 2-D systems. As well known, most of the actual 2-D systems are nonlinear and the above results don't work in this case. To the best of the author's knowledge, the corresponding problems on nonlinear 2-D systems have not been fully investigated yet, research in this area should be very important and useful for researchers and designers in this field, which motivates us to carry out the present work.[9−14]

On the other hand, the stability analysis and systematic design of nonlinear systems, with a design model given by the T-S fuzzy model [9], have been studied by many researchers. The authors in [10] have proved that the T-S fuzzy systems can be approximate to any continuous functions in a compact set of $\mathbb{R}^n$ at any preciseness. This allows the designers to take advantage of conventional linear systems to analyze and design the nonlinear systems. Therefore T-S fuzzy control has become one of the most popular and promising research platform in the model-based fuzzy control and the theoretic researches on the issue have been conducted actively by many fuzzy control theorists. Among these exiting stabilization conditions for T-S fuzzy systems, most of the works proposed the use of a common quadratic Lyapunov function (CQLF)[11−12]. Other works can be found in dealing with piecewise quadratic Lyapunov functions[13]. Recently, some works[14−17] are also dealing with nonquadratic stability and stabilization with the purpose of further releasing the conservatism. As stated in [14], there is still some conservatism to be lifted if we change "something" either the control law, or the Lyapunov function or the form of introducing additional variables.

In this paper, the problem of stabilization for Roesser type discrete-time nonlinear 2-D systems will be investigated. A discrete-time 2-D T-S fuzzy model is proposed to represent the underlying discrete-time nonlinear 2-D systems. Based on the attained fuzzy model, new stabilization conditions via CQLF and parallel distributed compensation (PDC) scheme are derived by using relaxed quadratic stabilization techniques. With the sake of further releasing the conservatism, less conservative stabilizations are also obtained by using the non-PDC scheme and new relaxed non-quadratic techniques. Unlike to the usual 1-D T-S fuzzy systems, the systems information is propagated along two independent directions and this fact makes the controller synthesis more complicated, especially for non-quadratic stabilization. Fortunately, this obstacle is overcome by designing appropriate controller gain matrices' structure. Furthermore, the fact that the relaxed technique provided in this paper is prior to those ones provided in the existing literature is also proved.

The rest of this paper is organized as follows: following the introduction, the discrete-time 2-D T-S fuzzy system is proposed to represent the underlying nonlinear 2-D systems and some important definitions and lemmas are also given in Section 1. In Section 2, new quadratic stabilization conditions are proposed by using the PDC scheme and CQLF. With the purpose of further reducing the conservatism, non-quadratic stabilization conditions are also investigated by using non-PDC scheme and parameter-dependent Lyapunov function (PDLF) in Section 3. In Section 4, an example is given to demonstrate the effectiveness of the results proposed in Section 2 and 3 respectively. Finally, some conclusions are drawn in Section 5.

For simplicity, the notations used are fair standard. For example, $X > 0$(or $X \geq 0$) means the matrix X is symmetric and positive definite(or symmetric and positive semidefinite). X^{T} denotes the transpose of X. The symbol I represents the identity matrix with appropriate dimension. a star $*$ in a symmetric matrix denotes the transposed element in the symmetric position. For a matrix P, $\min(P)$ (respectively, $\max(P)$) means the smallest (respectively, largest) eigenvalue of P.

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1 Problem statement

1.1 Discrete-time 2-D T-S fuzzy model

Consider a class of Roesser type discrete-time nonlinear 2-D systems described as follows:

$$\boldsymbol{x}^+(k,l) = z(\boldsymbol{x}(k,l)) + s(\boldsymbol{x}(k,l))\boldsymbol{u}(k,l) \quad (1)$$

$$\boldsymbol{x}^h(0,l) = f(l), \boldsymbol{x}^v(k,0) = g(k) \quad (2)$$

with

$$\boldsymbol{x}(k,l) = \begin{bmatrix} \boldsymbol{x}^h(k,l) \\ \boldsymbol{x}^v(k,l) \end{bmatrix}, \ \boldsymbol{x}^+(k,l) = \begin{bmatrix} \boldsymbol{x}^h(k+1,l) \\ \boldsymbol{x}^v(k,l+1) \end{bmatrix},$$

where $\boldsymbol{x}^h(\cdot)$ is the horizonal state in $\mathbf{R}^{n_1}$, $\boldsymbol{x}^v(\cdot)$ is the vertical state in $\mathbf{R}^{n_2}$, $\boldsymbol{u}(\cdot)$ is the control input in $\mathbf{R}^m$. $z(\cdot)$ and $s(\cdot)$ are general nonlinear functions satisfying $z, s \in C^1$. $f(l)$ and $g(k)$ are corresponding boundary conditions along two independent indirections.

Extending the usual 1-D T-S fuzzy modeling method to the 2-D case, a discrete-time 2-D T-S fuzzy model described by the following rules is proposed to represent discrete-time nonlinear 2-D systems (1):

$$\begin{aligned} &IF\ z_1(k,l)\ is\ M_{i1},\ and...,\ and\ z_L(k,l)\ is\ M_{iL},\ Then, \\ &\boldsymbol{x}^+(k,l) = A_i\boldsymbol{x}(k,l) + B_i\boldsymbol{u}(k,l),\ i = 1,\cdots,r \quad (3) \\ &\boldsymbol{x}^h(0,l) = f(l), \boldsymbol{x}^v(k,0) = g(k) \end{aligned}$$

with

$$A_i = \begin{bmatrix} A_i^{11} & A_i^{12} \\ A_i^{21} & A_i^{22} \end{bmatrix}, \ B_i = \begin{bmatrix} B_i^1 \\ B_i^2 \end{bmatrix}.$$

where $z_p(k,l)$, for $p = 1,\cdots,L$ are the premise variables, M_{ip} is the fuzzy set, r is the number of IF-THEN rules. k, l are two integers in $\mathbf{Z}_+$, and $A_i^{11} \in \mathbf{R}^{n_1\times n_1}, A_i^{12} \in \mathbf{R}^{n_1\times n_2}, A_i^{21} \in \mathbf{R}^{n_2\times n_1}, A_i^{22} \in \mathbf{R}^{n_2\times n_2}, B_i^1 \in \mathbf{R}^{n_1\times m}, B_i^2 \in \mathbf{R}^{n_2\times m}$, respectively.

By using product of inference, singleton fuzzifier, and center-average defuzzifer, the overall discrete-time 2-D T-S fuzzy systems can be expressed as follows:

$$\boldsymbol{x}^+(k,l) = \sum_{i=1}^{r} h_i(z(k,l))\{A_i\boldsymbol{x}(k,l) + B_i\boldsymbol{u}(k,l)\} \quad (4)$$

$$\boldsymbol{x}^h(0,l) = f(l), \boldsymbol{x}^v(k,0) = g(k)$$

where $h_i(z(k,l)) = \frac{\beta_i(z(k,l))}{\sum_{i=1}^{r}\beta_i(z(k,l))}$, $\beta_j(z(k,l)) = \Pi_{k=1}^{L} M_{kj}(z(k,l))$.

In this paper, for a matrix X, the following notations will be adopted for simplicity:

$$h_i = h_i(z(k,l)),\ X_z = \sum_{i=1}^{r} h_iX_i, X_z^{-1} = (\sum_{i=1}^{r} h_iX_i)^{-1}. \quad (5)$$

Remark 1. Based on the discrete-time 2-D T-S fuzzy model (4), the problem of controller synthesis for systems (1) could be implemented under the framework for linear 2-D systems. However, it is worth noting that membership functions play an important part in systems (4), hence how to make good use of the information of them in the process of controller synthesis seems meaningful and interesting[18]. Furthermore, the underlying 2-D T-S system's information is propagated along two independent directions and this fact makes the problem of stabilization more complicated, especially for the case of non-quadratic stabilization.

1.2 Definition and lemma

Denote $X_r = \sup\{\|\boldsymbol{x}(k,l)\| : r = k+l\}$, and we firstly give out the definition of asymptotically stability for system (4).

Definition 1. The discrete-time 2-D T-S fuzzy systems (4) is asymptotically stable if $\lim_{r\to\infty} X_r = 0$ with the initial and boundary conditions (2).

We end this section with an useful lemma which will play an important part in the derivation of one of our results.

Lemma 1[14]. For a symmetric matrix $P > 0$, the inequality $A^{\mathrm{T}}PA - P < 0$ holds, if there exist a matrix G such that $\begin{bmatrix} P & (*) \\ GA & G+G^{\mathrm{T}}-P \end{bmatrix} > 0$.

2 Stabilization conditions via PDC scheme and CQLF

In this section, new quadratic stabilization conditions for systems (4) via PDC scheme and CQLF will be proposed by using some relaxed quadratic stabilization techniques.

With the idea of extending the so-called PDC scheme for usual 1-D T-S fuzzy systems to the 2-D case, we use the controller structure incorporating a set of fuzzy rules expressed as follows:

$$\begin{gathered} IF\ \ z_1(k,l)\ is\ M_{i1},\ and\ ...,\ and\ z_L(k,l)\ is\ M_{iL}, THEN \\ \boldsymbol{u}(k,l) = K_i\boldsymbol{x}(k,l),\ i = 1,...,r. \end{gathered}$$

So the overall state feedback 2-D fuzzy control law is represented by :

$$\boldsymbol{u}(k,l) = \sum_{i=1}^{r} h_iK_i\boldsymbol{x}(k,l) = K_z\boldsymbol{x}(k,l), \quad (6)$$

Then the closed-loop system of (4) and (6) is shown as follows:

$$\begin{aligned} \boldsymbol{x}^+(k,l) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j(A_i + B_iK_j)\boldsymbol{x}(k,l) \\ &= (A_z + B_zK_z)\boldsymbol{x}(k,l). \quad (7) \end{aligned}$$

Hence, the problem which we are dealing with now is how to design the gain matrix of K_z that stabilizes the 2-D closed-loop systems (7) with less conservative quadratic stabilization conditions.

Theorem1. The discrete-time 2-D T-S fuzzy systems (4) is asymptotically stable via the controller (6) if there exists appropriately dimensional matrices $X_1 > 0, X_2 > 0, F_i, Q_{ii}$, and $Q_{ij} = Q_{ji}^{\mathrm{T}}$, with

$$F_i = \begin{bmatrix} F_i^1 & F_i^2 \end{bmatrix},$$

$$Q_{ii} = \begin{bmatrix} Q_{ii}^{11} & Q_{ii}^{12} & Q_{ii}^{13} & Q_{ii}^{14} \\ * & Q_{ii}^{22} & Q_{ii}^{23} & Q_{ii}^{24} \\ * & * & Q_{ii}^{33} & Q_{ii}^{34} \\ * & * & * & Q_{ii}^{44} \end{bmatrix},$$

$$Q_{ij} = \begin{bmatrix} Q_{ij}^{11} & Q_{ij}^{12} & Q_{ij}^{13} & Q_{ij}^{14} \\ Q_{ij}^{21} & Q_{ij}^{22} & Q_{ij}^{23} & Q_{ij}^{24} \\ Q_{ij}^{31} & Q_{ij}^{32} & Q_{ij}^{33} & Q_{ij}^{34} \\ Q_{ij}^{41} & Q_{ij}^{42} & Q_{ij}^{43} & Q_{ij}^{44} \end{bmatrix},$$

such that the following LMIs hold,

$$\Theta_{ii} \leqslant Q_{ii}, i = 1, \cdots, r, \quad (8)$$
$$\Theta_{ij} + \Theta_{ji} \leqslant Q_{ij} + Q_{ji}, i \neq j, i, j = 1, \cdots, r, \quad (9)$$
$$\begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1r} \\ Q_{21} & Q_{22} & \cdots & Q_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{r1} & Q_{r2} & \cdots & Q_{rr} \end{bmatrix} < 0, \quad (10)$$

where, for $i, j = 1, \cdots, r$, we have
$$\Theta_{ij} = \begin{bmatrix} -X_1 & 0 & \Theta_{ij}^{13} & \Theta_{ij}^{14} \\ * & -X_2 & \Theta_{ij}^{23} & \Theta_{ij}^{24} \\ * & * & -X_1 & 0 \\ * & * & * & -X_2 \end{bmatrix},$$
$\Theta_{ij}^{13} = (A_i^{11} X_1 + B_i^1 F_j^1)^{\mathrm{T}}, \Theta_{ij}^{14} = (A_i^{21} X_1 + B_i^2 F_j^1)^{\mathrm{T}}$,
$\Theta_{ij}^{23} = (A_i^{12} X_2 + B_i^1 F_j^2)^{\mathrm{T}}, \Theta_{ij}^{24} = (A_i^{22} X_2 + B_i^2 F_j^2)^{\mathrm{T}}$.

Moreover, the controller gain matrices could be given by $K_i = \begin{bmatrix} F_i^1 X_1^{-1} & F_i^2 X_2^{-1} \end{bmatrix}$.

Proof. Consider a CQLF given as follows:

$$V(\boldsymbol{x}(k,l)) = \boldsymbol{x}^{\mathrm{T}}(k,l) P \boldsymbol{x}(k,l) \quad (11)$$

where P is a positive definite matrix of the following form:

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

here $P_1 \in \mathbf{R}^{n_1 \times n_1}$ and $P_2 \in \mathbf{R}^{n_2 \times n_2}$.

The variation of (11) is given by

$$\begin{aligned} \triangle V(\boldsymbol{x}(k,l)) &= \boldsymbol{x}^{+^{\mathrm{T}}}(k,l) P \boldsymbol{x}^{+}(k,l) - \boldsymbol{x}^{\mathrm{T}}(k,l) P \boldsymbol{x}(k,l) \\ &= \boldsymbol{x}^{\mathrm{T}}(k,l)[(A_z + B_z K_z)^{\mathrm{T}} P (A_z + B_z K_z) - P]\boldsymbol{x}(k,l). \end{aligned} \quad (12)$$

Then, it is easy to see that systems (4) is asymptotically stable if we have

$$(A_z + B_z K_z)^{\mathrm{T}} P (A_z + B_z K_z) - P < 0. \quad (13)$$

Using the Schur complement lemma, (13) is equivalent to the following inequality:

$$\begin{bmatrix} -P & (A_z + B_z K_z)^{\mathrm{T}} P \\ * & -P \end{bmatrix} < 0. \quad (14)$$

Pre- and post- multiplying both sides of (14) with $\mathtt{diag}\{P^{-1}, P^{-1}\}$ and applying the change of variables $X = P^{-1}, F_i = K_i X$ $(i = 1, \cdots, r)$, leads to

$$\sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \Theta_{ij} = \begin{bmatrix} -X & (A_z X + B_z F_z)^{\mathrm{T}} \\ * & -X \end{bmatrix} < 0 \quad (15)$$

where $X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}$ and Θ_{ij} have been defined in (8)-(9).

Reordering the expression of $\sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \Theta_{ij}$ and using (8) and (9), we can obtain

$$\begin{aligned} \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \Theta_{ij} &= \sum_{i=1}^{r} h_i^2 \Theta_{ii} + \sum_{i=1}^{r-1} \sum_{j>i} h_i h_j (\Theta_{ij} + \Theta_{ji}) \\ &\leq \sum_{i=1}^{r} h_i^2 Q_{ii} + \sum_{i=1}^{r-1} \sum_{j>i} h_i h_j (Q_{ij} + Q_{ji}) \\ &= \begin{bmatrix} h_1 I \\ h_2 I \\ \vdots \\ h_r I \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1r} \\ Q_{21} & Q_{22} & \cdots & Q_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{r1} & Q_{r2} & \cdots & Q_{rr} \end{bmatrix} \begin{bmatrix} h_1 I \\ h_2 I \\ \vdots \\ h_r I \end{bmatrix} \end{aligned} \quad (16)$$

Thus, if (10) holds, $\sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \Theta_{ij} < 0$ evidently holds. In other words, the discrete-time 2-D system (4) is asymptotically stable via the fuzzy controller (6). □

Remark 2. By extending the relaxed quadratic stabilization technique for 1-D T-S fuzzy systems[12] to the 2-D case and modifying somewhat in view of adapting to the 2-D setting, new quadratic stabilization conditions proposed in Theorem 1 are less conservative than those ones only using common quadratic stabilization methods. However, in the process of the derivation, the nonlinear functions (membership functions) used to blend the linear models are not involved in the LMI conditions. Thus, these conditions remain conservative and how to further reduce the conservatism seems meaningful and interesting.

3 Stabilization conditions via non-PDC scheme and PDLF

As well known, membership functions (MFs) play important parts in the T-S fuzzy systems. It has a chance to further reduce the conservatism if we consider information of MFs in the process of controller design. With the purpose of further releasing the conservatism, new stabilization conditions for system (4) will be proposed by using non-PDC scheme, PDLF and new relaxed techniques in this section. Here, the non-quadratic control law is designed as:

$$\begin{aligned} \boldsymbol{u}(k,l) &= \left(\sum_{i=1}^{r} h_i F_i \right) \left(\sum_{i=1}^{r} h_i G_i \right)^{-1} \boldsymbol{x}(k,l) \\ &= F_z G_z^{-1} \boldsymbol{x}(k,l), \end{aligned} \quad (17)$$

where F_i, G_i are appropriately dimensional matrices to be determined and have the following matrix structures:

$$F_i = \begin{bmatrix} F_i^1 & F_i^2 \end{bmatrix}, G_i = \begin{bmatrix} G_i^1 & 0 \\ 0 & G_i^2 \end{bmatrix}. \quad (18)$$

Theorem 2. The discrete-time 2-D T-S fuzzy system (4) with the non-quadratic controller (17) is asymptotically stable if there exists appropriately dimensional matrices $P_i > 0, F_i, G_i, R_{ii}^{mn}, R_{ij}^{mn} = (R_{ji}^{mn})^{\mathrm{T}}$, with

$$P_i = \begin{bmatrix} P_i^1 & 0 \\ 0 & P_i^2 \end{bmatrix}, P_i^1 \in \mathbf{R}^{n_1 \times n_1}, P_i^2 \in \mathbf{R}^{n_2 \times n_2},$$

$$R_{ii}^{mn} = \begin{bmatrix} R_{ii}^{mn}(11) & \cdots & R_{ii}^{mn}(14) \\ * & \ddots & R_{ii}^{mn}(24) \\ * & * & R_{ii}^{mn}(44) \end{bmatrix},$$

$$R_{ij}^{mn} = \begin{bmatrix} R_{ij}^{mn}(11) & R_{ij}^{mn}(12) & R_{ij}^{mn}(13) & R_{ij}^{mn}(14) \\ R_{ij}^{mn}(21) & R_{ij}^{mn}(22) & R_{ij}^{mn}(23) & R_{ij}^{mn}(24) \\ R_{ij}^{mn}(31) & R_{ij}^{mn}(32) & R_{ij}^{mn}(33) & R_{ij}^{mn}(34) \\ R_{ij}^{mn}(41) & R_{ij}^{mn}(42) & R_{ij}^{mn}(43) & R_{ij}^{mn}(44) \end{bmatrix},$$

such that the following LMIs hold,

$$\Upsilon_{ii}^{mn} > R_{ii}^{mn}, \quad i, m, n = 1, \cdots, r, \tag{19}$$

$$\Upsilon_{ij}^{mn} + \Upsilon_{ji}^{mn} > R_{ij}^{mn} + R_{ji}^{mn}, \tag{20}$$
$$i \neq j, \ i, j, m, n = 1, \cdots, r,$$

$$R^{mn} = \begin{bmatrix} R_{11}^{mn} & R_{12}^{mn} & \cdots & R_{1r}^{mn} \\ R_{21}^{mn} & R_{22}^{mn} & \cdots & R_{2r}^{mn} \\ \vdots & \vdots & \ddots & \vdots \\ R_{r1}^{mn} & R_{r2}^{mn} & \cdots & R_{rr}^{mn} \end{bmatrix} > 0, \tag{21}$$
$$m, n = 1, \cdots, r,$$

where, for $i, j, m, n = 1, \cdots, r$, we have

$$\Upsilon_{ij}^{mn} = \begin{bmatrix} P_i^1 & 0 & \Upsilon_{ij}^{mn}(1,3) & \Upsilon_{ij}^{mn}(1,4) \\ * & P_i^2 & \Upsilon_{ij}^{mn}(2,3) & \Upsilon_{ij}^{mn}(2,4) \\ * & * & \Upsilon_{ij}^{mn}(3,3) & 0 \\ * & * & * & \Upsilon_{ij}^{mn}(4,4) \end{bmatrix}, \text{ and}$$

$$\Upsilon_{ij}^{mn}(1,3) = (A_i^{11} G_j^1 + B_i^1 F_j^1)^{\mathrm{T}},$$
$$\Upsilon_{ij}^{mn}(1,4) = (A_i^{21} G_j^1 + B_i^2 F_j^1)^{\mathrm{T}},$$
$$\Upsilon_{ij}^{mn}(2,3) = (A_i^{12} G_j^2 + B_i^1 F_j^2)^{\mathrm{T}},$$
$$\Upsilon_{ij}^{mn}(2,4) = (A_i^{22} G_j^2 + B_i^2 F_j^2)^{\mathrm{T}},$$
$$\Upsilon_{ij}^{mn}(3,3) = G_m^1 + (G_m^1)^{\mathrm{T}} - P_m^1,$$
$$\Upsilon_{ij}^{mn}(4,4) = G_n^2 + (G_n^2)^{\mathrm{T}} - P_n^2.$$

Proof. Consider a new non-quadratic Lyapunov function for discrete-time 2-D T-S systems as follows:

$$V(\boldsymbol{x}(k,l)) = \boldsymbol{x}^{\mathrm{T}}(k,l) G_z^{-\mathrm{T}} P_z G_z^{-1} \boldsymbol{x}(k,l). \tag{22}$$

Firstly, let us check the existence of G_z^{-1}. Noting that if these conditions of Theorem 2 hold true, we have with inequalities (19): $G_m^1 + (G_m^1)^{\mathrm{T}} - P_m^1 > 0$ $(m = 1, \cdots, r)$ and $G_n^2 + (G_n^2)^{\mathrm{T}} - P_n^2 > 0$ $(n = 1, \cdots, r)$. Therefore, $\sum_i h_i(G_i + G_i^{\mathrm{T}} - P_i) > 0$ $(i = 1, \cdots, r)$, which ensures that G_z^{-1} exists.

Secondly, we check the non-quadratic Lyapunov function (22)'s validity. We can write

$$\boldsymbol{x}^{\mathrm{T}}(k,l) \underline{\lambda} G_z^{-\mathrm{T}} G_z^{-1} \boldsymbol{x}(k,l) \leqslant V \leqslant \boldsymbol{x}^{\mathrm{T}}(k,l) \overline{\lambda} G_z^{-\mathrm{T}} G_z^{-1} \boldsymbol{x}(k,l) \tag{23}$$

where $\underline{\lambda} = \min_z(P_z)$ and $\overline{\lambda} = \max_z(P_z)$.

As $(G_z^{-\mathrm{T}} G_z^{-1})^{-1} = G_z G_z^{\mathrm{T}}$ and with $\underline{\mu} = \min_z(G_z G_z^T)$ and $\overline{\mu} = \max_z(G_z G_z^T)$, (23) becomes $\underline{\lambda}\overline{\mu}^{-1}\|\boldsymbol{x}(k,l)\|^2 \leqslant V \leqslant \overline{\lambda}\underline{\mu}^{-1}\|\boldsymbol{x}(k,l)\|^2$ that ensures $V(\boldsymbol{x}(k,l))$ to be a candidate Lyapunov function.

Then, its variation is written as

$$\triangle V(\boldsymbol{x}(k,l)) = \boldsymbol{x}^{\mathrm{T}}(k,l)[(A_z + B_z F_z G_z^{-1})^{\mathrm{T}} G_{z+}^{-\mathrm{T}} P_{z+} G_{z+}^{-1} (A_z + B_z F_z G_z^{-1}) - G_z^{-\mathrm{T}} P_z G_z^{-1}] \boldsymbol{x}(k,l). \tag{24}$$

where

$$G_{z+} = \begin{bmatrix} \sum_{i=1}^{r} h_i(z(k+1,l)) G_i^1 & 0 \\ 0 & \sum_{j=1}^{r} h_j(z(k,l+1)) G_j^2 \end{bmatrix},$$

$$P_{z+} = \begin{bmatrix} \sum_{i=1}^{r} h_i(z(k+1,l)) P_i^1 & 0 \\ 0 & \sum_{j=1}^{r} h_j(z(k,l+1)) P_j^2 \end{bmatrix}.$$

Here, $h_i(z(k+1,l))$ and $h_j(z(k,l+1))$ are two different one-step ahead membership functions produced by the fact that the 2-D systems's information is propagated along two independent directions. Therefore, in the derivation of the relaxed non-quadratic stabilization conditions, we should consider this difference via solving more LMIs as a tradeoff.

Multiplying left by G_z^{T} and right by G_z to (24), it is easy to verify that the systems (4) with the non-quadratic controller (17) is asymptotically stable if we have the following inequality:

$$(G_z^{\mathrm{T}} A_z^{\mathrm{T}} + F_z^{\mathrm{T}} B_z^{\mathrm{T}}) G_{z+}^{-\mathrm{T}} P_{z+} G_{z+}^{-1} (A_z G_z + B_z F_z) - P_z < 0 \tag{25}$$

and using lemma 1 with $A = G_{z+}^{-1}(A_z G_z - B_z F_z)$, leads to

$$\begin{bmatrix} P_z & * \\ A_z G_z + B_z F_z & G_{z+} + G_{z+}^T - P_{z+} \end{bmatrix} = \sum_{m=1}^{r} \sum_{n=1}^{r} h_m(z(k+1,l)) h_n(z(k,l+1)) (\sum_{i=1}^{r} h_i^2 \Upsilon_{ii}^{mn} + \sum_{i=1}^{r-1} \sum_{j>i} h_i h_j (\Upsilon_{ij}^{mn} + \Upsilon_{ji}^{mn})) > 0 \tag{26}$$

where $\Upsilon_{ii}^{mn}, \Upsilon_{ij}^{mn}$ and Υ_{ji}^{mn} have been defined in (19) and (20).

On the other hand, noting (19)-(20) and applying the Schur complement lemma, we can obtain:

$$\begin{bmatrix} P_z & * \\ A_z G_z + B_z F_z & G_{z+} + G_{z+}^{\mathrm{T}} - P_{z+} \end{bmatrix} \geqslant \sum_{m=1}^{r} \sum_{n=1}^{r} h_m(z(k+1,l)) h_n(z(k,l+1)) (\sum_{i=1}^{r} h_i^2 R_{ii}^{mn} + \sum_{i=1}^{r-1} \sum_{j>i} h_i h_j (R_{ij}^{mn} + R_{ji}^{mn})) = \sum_{m=1}^{r} \sum_{n=1}^{r} h_m(z(k+1,l)) h_n(z(k,l+1)) \eta^{\mathrm{T}} R^{mn} \eta \tag{27}$$

where $\eta^{\mathrm{T}} = \begin{bmatrix} h_1 I & h_2 I & \cdots & h_r I \end{bmatrix}$ and R^{mn} has been defined in (21).

Thus, if (21) holds, (25) evidently holds. In other words, the discrete-time 2-D system (4) is asymptotically stable via the fuzzy controller (17). □

Remark 3. Unlike to the usual 1-D T-S fuzzy systems, the key feature of a 2-D T-S fuzzy systems is that the system information is propagated along two independent directions. For the underlying Roesser type 2-D T-S fuzzy systems (4) which could model a wide range of practical systems, two different one-step ahead membership functions($h_i(z(k+1,l))$ and $h_j(z(k,l+1))$ for horizonal and vertical directions, respectively) are produced for conceiving relaxed non-quadratic stabilization conditions. Furthermore, due to the structure of system matrices A_i, B_i in (3), those matrices Υ_{ij}^{mn} in Theorem 2 also represent the information along two independent directions. These two facts lead to more LMIs and complicated matrices structures than the usual 1-D case. Moreover, less conservative non-quadratic stabilization conditions are proposed by applying a new PDLF, non-PDC scheme and form of introducing additional variables for discrete-time 2-D T-S fuzzy systems (4). This fact will also be illustrated in Section 4.

In [15], the authors proposed a novel form of introducing additional variables for usual 1-D T-S fuzzy systems. The following corollary will be attained if we extend their technique to the 2-D setting:

Corollary 1. The discrete-time 2-D T-S fuzzy system (4) with the non-quadratic controller (17) is asymptotically stable if there exists appropriately dimensional matrices $P_i > 0, F_i, G_i, Q_{ij}^{mn}(i \neq j)$ and symmetric matrices Q_{ii}^{mn}, with

$$P_i = \begin{bmatrix} P_i^1 & 0 \\ 0 & P_i^2 \end{bmatrix}, P_i^1 \in \mathbf{R}^{n_1 \times n_1}, P_i^2 \in \mathbf{R}^{n_2 \times n_2},$$

$$Q_{ii}^{mn} = \begin{bmatrix} Q_{ii}^{mn}(11) & \cdots & Q_{ii}^{mn}(14) \\ * & \ddots & Q_{ii}^{mn}(24) \\ * & * & Q_{ii}^{mn}(44) \end{bmatrix},$$

$$Q_{ij}^{mn} = \begin{bmatrix} Q_{ij}^{mn}(11) & Q_{ij}^{mn}(12) & Q_{ij}^{mn}(13) & Q_{ij}^{mn}(14) \\ Q_{ij}^{mn}(21) & Q_{ij}^{mn}(22) & Q_{ij}^{mn}(23) & Q_{ij}^{mn}(24) \\ Q_{ij}^{mn}(31) & Q_{ij}^{mn}(32) & Q_{ij}^{mn}(33) & Q_{ij}^{mn}(34) \\ Q_{ij}^{mn}(41) & Q_{ij}^{mn}(42) & Q_{ij}^{mn}(43) & Q_{ij}^{mn}(44) \end{bmatrix},$$

such that the following LMIs hold,

$$\Upsilon_{ii}^{mn} > Q_{ii}^{mn}, \quad i, m, n = 1, \cdots, r, \tag{28}$$

$$\Upsilon_{ij}^{mn} + (\Upsilon_{ij}^{mn})^{\mathrm{T}} > Q_{ij}^{mn} + (Q_{ij}^{mn})^{\mathrm{T}}, \tag{29}$$

$$i \neq j, \ i, j, m, n = 1, \cdots, r,$$

$$\begin{bmatrix} 2Q_{11}^{mn} & \cdots & Q_{1r}^{mn} + (Q_{r1}^{mn})^{\mathrm{T}} \\ \vdots & \ddots & \vdots \\ * & \cdots & 2Q_{rr}^{mn} \end{bmatrix} > 0, \tag{30}$$

$$m, n = 1, \cdots, r,$$

where those terms $\Upsilon_{ij}^{mn}(i, j, m, n = 1, \cdots, r)$ have the same definitions as in Theorem 2.

Proof. From the proof of Theorem 2, it is easy to see that the discrete-time 2-D T-S fuzzy system (4) with the non-quadratic controller (17) is asymptotically stable if the inequality (26) holds.

Reordering the expression of the term $\begin{bmatrix} P_z & * \\ A_z G_z + B_z F_z & G_{z+} + G_{z+}^T - P_{z+} \end{bmatrix}$, we can obtain:

$$\begin{bmatrix} P_z & * \\ A_z G_z + B_z F_z & G_{z+} + G_{z+}^T - P_{z+} \end{bmatrix} = \sum_{m=1}^{r}\sum_{n=1}^{r} h_m(z(k+1,l))h_n(z(k,l+1))(\sum_{i=1}^{r} h_i^2 \Upsilon_{ii}^{mn} + \sum_{i=1}^{r-1}\sum_{j\neq i} h_i h_j \Upsilon_{ij}^{mn}) \tag{31}$$

with (28-29) and the fact $W + W^{\mathrm{T}} > 0$ is equivalent to $W > 0$ for a real matrix, we have

$$\sum_{i=1}^{r} h_i^2 \Upsilon_{ii}^{mn} + \sum_{i=1}^{r-1}\sum_{j\neq i} h_i h_j \Upsilon_{ij}^{mn} > \sum_{i=1}^{r} h_i^2 Q_{ii}^{mn} + \sum_{i=1}^{r-1}\sum_{j\neq i} h_i h_j Q_{ij}^{mn}. \tag{32}$$

Similar to the proof of Theorem 2, (30) guarantees $\begin{bmatrix} P_z & * \\ A_z G_z + B_z F_z & G_{z+} + G_{z+}^T - P_{z+} \end{bmatrix} > 0$. Hence, the discrete-time 2-D T-S fuzzy system (4) with the non-quadratic controller (17) is asymptotically stable. □

Remark 4. The number of additional variables Q_{ij}^{kl} (Corollary 1) is r^4 while the number of additional variables R_{ij}^{kl} $(i \leq j)$ (Theorem 2) is $\frac{r^3(r+1)}{2}$. It is easy to see that $\frac{r^3(r+1)}{2} < r^4$ for all $r \geq 2$, i.e, Theorem 2 requires less computational load. Moreover, the fact that stabilization conditions derived by Theorem 2 are less conservative will be proved in the following proposition.

Proposition 1. Stabilization conditions (19)-(21) hold, if stabilization conditions (28)-(30) hold.

Proof. Recalling the stabilization conditions (28)-(29) proposed in Corollary 1, we have

$$\Upsilon_{ii}^{kl} > Q_{ii}^{kl}, \quad i, k, l \in \{1, \cdots, r\}, \tag{33}$$

$$\Upsilon_{ij}^{kl} + (\Upsilon_{ij}^{kl})^{\mathrm{T}} + \Upsilon_{ji}^{kl} + (\Upsilon_{ji}^{kl})^{\mathrm{T}} > Q_{ij}^{kl} + (Q_{ij}^{kl})^{\mathrm{T}} + Q_{ji}^{kl} + (Q_{ji}^{kl})^{\mathrm{T}}, \tag{34}$$

$$i \neq j, \ i, j, k, l \in \{1, \cdots, r\},$$

where $\Upsilon_{ii}^{kl}, \Upsilon_{ij}^{kl}$ are the same definitions as in Theorem 2.

It is worth noting that $\Upsilon_{ij}^{kl} + \Upsilon_{ji}^{kl}$ are symmetric matrices. Then, from (34), it is easy to see that

$$\Upsilon_{ij}^{kl} + \Upsilon_{ji}^{kl} > \frac{Q_{ij}^{kl} + (Q_{ij}^{kl})^{\mathrm{T}} + Q_{ji}^{kl} + (Q_{ji}^{kl})^{\mathrm{T}}}{2} = \frac{Q_{ij}^{kl} + (Q_{ji}^{kl})^{\mathrm{T}}}{2} + \frac{(Q_{ij}^{kl})^{\mathrm{T}} + Q_{ji}^{kl}}{2}. \tag{35}$$

Let $R_{ii}^{kl} = Q_{ii}^{kl}, R_{ij}^{kl} = \frac{Q_{ij}^{kl} + (Q_{ji}^{kl})^{\mathrm{T}}}{2}, R_{ji}^{kl} = \frac{(Q_{ij}^{kl})^{\mathrm{T}} + Q_{ji}^{kl}}{2}$ and substitute them in (30), (33), (35) respectively, all the stabilization conditions (19)-(21) for Theorem 2 are satisfied, i.e., conditions (28)-(30) provided in Corollary 1 are sufficient conditions for those ones provided in Theorem 2. □

4 Numerical Examples

Example 4.1: Consider the following nonlinear differential equation:

$$\frac{\partial^2 q(x,t)}{\partial x \partial t} = a_1 \frac{\partial q(x,t)}{\partial t} + a_2 \frac{\partial q(x,t)}{\partial x} + a_0 \sin^2(q(x,t)) + b f(x,t)$$

where the initial and boundary conditions $q(x,0) = q_1(x)$ and $q(0,t) = q_2(t)$, $q(x,t)$ is the variable function, a_0, a_1, a_2, b are real coefficients, $f(x,t)$ is the input function.

Next, we will establish the state space model for the above nonlinear differential equation. Let us define

$$x_c^h(x,t) = \frac{\partial q(x,t)}{\partial t} - a_2 q(x,t),$$

$$x_c^v(x,t) = q(x,t)$$

Table 1 Feasible parameter intervals of a_2

Methods	usual PDC	Theorem 1	Theorem 2	Corollary 1
feasible intervals	[-1.990, -0.512]	[-2.012, -0.494]	[-2.492, -0.014]	[-2.292, -0.212]

Then, the following 2-D state space model can be easily obtained

$$\begin{bmatrix} \frac{\partial x_c^h(x,t)}{\partial x} \\ \frac{\partial x_c^v(x,t)}{\partial t} \end{bmatrix} = \begin{bmatrix} a_1 & a_1a_2 + a_0\sin^2(x_c^v(x,t)) \\ 1 & a_2 \end{bmatrix} \begin{bmatrix} x_c^h(x,t) \\ x_c^v(x,t) \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u_c(x,t)$$

with boundary conditions: $x_c^h(0,t) = \dot{q}_2(t) - a_2q_2(t)$, $x_c^v(x,t) = q_1(x)$.

To obtain a 2-D T-S fuzzy representation for this 2-D nonlinear systems, consider the two following rules obtained for $\sin^2(x_c^v(x,t))$:

IF $\sin^2(x_c^v(x,t))$ is about 0, THEN

$$\begin{bmatrix} \frac{\partial x_c^h(x,t)}{\partial t} \\ \frac{\partial x_c^v(x,t)}{\partial t} \end{bmatrix} = A_1^c \begin{bmatrix} x_c^h(x,t) \\ x_v^h(x,t) \end{bmatrix} + B_1^c u_c(x,t),$$

IF $\sin^2(x_c^v(x,t))$ is about ∓ 1, THEN

$$\begin{bmatrix} \frac{\partial x_c^h(x,t)}{\partial t} \\ \frac{\partial x_c^v(x,t)}{\partial t} \end{bmatrix} = A_2^c \begin{bmatrix} x_c^h(x,t) \\ x_v^h(x,t) \end{bmatrix} + B_2^c u_c(x,t),$$

here, $A_1^c = \begin{bmatrix} a_1 & a_1a_2 \\ 1 & a_2 \end{bmatrix}, B_1^c = \begin{bmatrix} b \\ 0 \end{bmatrix}$, $A_2^c = \begin{bmatrix} a_1 & a_1a_2 + a_0 \\ 1 & a_2 \end{bmatrix}, B_2^c = B_1^c$.

Without loss of generality, we choose the membership functions as: $h_1(x,t) = 1 - \sin^2(x_c^v(x,t)), h_2(x,t) = \sin^2(x_c^v(x,t))$.

The above 2-D T-S fuzzy systems are discretized with sampling times T_1 and T_2 corresponding to variables x and t respectively. The obtained discrete-time 2-D fuzzy systems are given by:

IF $\mathtt{sin}^2(x^v(k,l))$ is about 0, THEN

$$\begin{bmatrix} x^h(k+1,l) \\ x^v(k,l+1) \end{bmatrix} = A_1 \begin{bmatrix} x^h(k,l) \\ x^v(k,l) \end{bmatrix} + B_1u(k,l),$$

IF $\mathtt{sin}^2(x^v(k,l))$ is about ∓ 1, THEN

$$\begin{bmatrix} x^h(k+1,l) \\ x^v(k,l+1) \end{bmatrix} = A_2 \begin{bmatrix} x^h(k,l) \\ x^v(k,l) \end{bmatrix} + B_2u(k,l),$$

here, $A_1 = \begin{bmatrix} 1 + a_1T_1 & a_1a_2T_1 \\ T_2 & 1 + a_2T_2 \end{bmatrix}, B_1 = \begin{bmatrix} bT_1 \\ 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 + a_1T_1 & (a_1a_2 + a_0)T_1 \\ T_2 & 1 + a_2T_2 \end{bmatrix}, B_2 = B_1$.

Then, the membership functions of the attained discrete-time 2-D T-S fuzzy system become: $h_1(k,l) = 1 - \sin^2(x^v(k,l)), h_2(k,l) = \sin^2(x^v(k,l))$. Consider the following parameter values: $a_0 = -2, a_1 = -3, b = -1, T_1 = 0.5, T_2 = 0.8$. We can calculate the feasible parameter intervals by evaluating the feasibility of the associated LMI problems with Theorem 1-2 and Corollary 1 for varying values of a_2 .

Table 1 shows the parameter feasible intervals of a_2 in which the fuzzy state feedback stabilizing controllers of the above system can be found by using those results provided in theorem 1-2 and Corollary 1, respectively. Moreover, feasible interval via PDC scheme without applying relaxed technique is also given in Table 1. From Table 1, it can be seen that Theorem 2 provides the most relaxed results.

Next, choosing $a_2 = -2.3$ which is feasible for Theorem 2 but unfeasible for Theorem 1 and Corollary 1, and solving (19)-(21) by the Matlab LMI solver, the corresponding controller gain matrices are attained as follows:

$$F_1 = \begin{bmatrix} -14.28 & 238.23 \end{bmatrix}, F_2 = \begin{bmatrix} -7.72 & 155.87 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 32.85 & 0 \\ 0 & 79.41 \end{bmatrix}, G_2 = \begin{bmatrix} 17.76 & 0 \\ 0 & 74.97 \end{bmatrix}.$$

Then, under the controller of (17), Fig 1-2 show the evolution of two state $x^h(k,l)$ and $x^v(k,l)$ respectively with the initial and boundary conditions to be

$$\begin{aligned} &x^h(0,l) = 0.2,\ 0 \leqslant l \leqslant 30, \\ &x^v(k,0) = 0.3,\ 0 \leqslant k \leqslant 30, \\ &x^h(0,l) = 0.05,\ x^v(k,0) = 0.1,\ i,j > 30. \end{aligned}$$

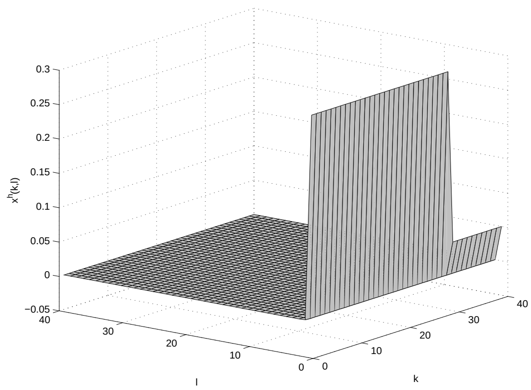

Fig. 1 Trajectory of the state $x^h(k,l)$

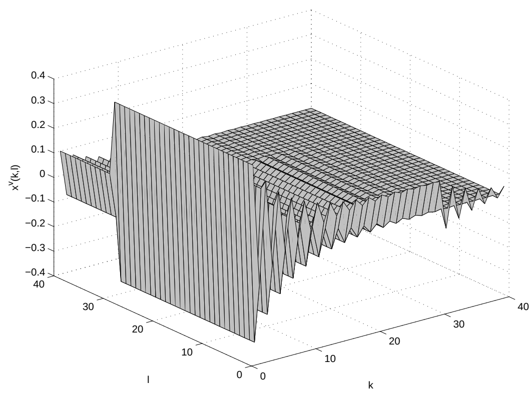

Fig. 2 Trajectory of the state $x^v(k,l)$

From Fig 1 and 2, it is easy to see that the closed-loop 2-D T-S fuzzy system is asymptotically stable via the attained non-quadratic controller.

5 Conclusion

This paper has presented a solution to the problem of stabilizing the Roesser type discrete-time nonlinear 2-D system. The underlying nonlinear system is represented by a discrete-time 2-D T-S fuzzy model, and then two kinds of stabilization conditions are derived by using new relaxed techniques respectively. Numerical example shows the effectiveness of the proposed results.

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