Adaptive Fuzzy Control for Unknown Nonlinear Systems with Perturbed Dead-Zone Inputs

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Abstract Adaptive fuzzy control is used to control a class of unknown nonlinear systems with perturbed dead-zone inputs in this paper. A new dead-zone actuator model which contains time-varying and perturbed actuation gain is proposed. The dead-zone nonlinearity is treated as a linear-like term, a nonlinear term and a disturbance-like term, by which the robustness of the system can be obtained by less control efforts. Backstepping technique is employed to get the adaptive fuzzy controller for the considered unknown nonlinear system with triangular structure. Nonlinearly parameterized fuzzy logic systems are used to design the control scheme which ensures the stability of the closed-loop system and satisfactory tracking of the output to the given reference signal. A numerical example is included to show the effectiveness of the approach.

Key words Adaptive control, fuzzy systems, dead-zone, time-varying gain, perturbation, backstepping technique, nonlinear systems.

Dead-zone nonlinearity is ubiquitous in many of practical systems, for example, some mechanical and electrical components like valves, DC servo motors and so on are all with dead-zone inputs. The existence of such a non-differential nonlinearity has caused much difficulty in control design since the dead-zone parameters are unknown in most cases. As it may cause severe deterioration of the system performance and serious problem in high precision control, many efforts have been made to deal with dead-zone nonlinearity for various systems.

There are three main approaches to design control systems with dead-zone inputs. The first one is to construct an inverse dead-zone nonlinearity to minimize the effects of dead-zone; the second one is based on a group of fuzzy rules which describe some rude knowledge of the dead-zone characteristics; and the third one models the dead-zone as a combination of a linear term and a disturbance-like term, then robust control technique can be used to obtain the required control performance. The first approach is intuitional for control design and will be effective if the dead-zone parameters are all known. Though successful control was obtained in [1] for linear systems and in [2] for some nonlinear systems, it is assumed that the dead-zone parameters are constants. The approach based on fuzzy rules was used to control some mechanical systems in [3] and [4]. However, it depends much on the experiences of operators or experts, when no good rules can be acquired about the dead-zone nonlinearity, this method will not be feasible. The existing results using the third method[5−9] employed the upper bound of the disturbance-like term to achieve robustness of the system. Though satisfactory performance was obtained, such design is conservative to some extent.

The above mentioned results assumed that the systems under control are well-known, but in many practical systems, the dynamics of the system are not completely known. Since Wang[10] proved that adaptive fuzzy systems are universal approximators, many control strategies have been proposed for unknown nonlinear systems based on adaptive fuzzy approximation[11−15]. These results were obtained with the restriction that the system is feedback linearizable. For systems without this restriction, the authors of [16-18] employed backstepping technique to develope adaptive fuzzy tracking control for single-input-single-output (SISO) systems[16,17] and multiple-input-multiple-output (MIMO) systems[18], respectively. Though many complicated nonlinear systems have been studied in the existing works, so far there is no result on control of non-feedback-linearizable unknown systems with perturbed dead-zone inputs to the best of our knowledge.

This paper proses a control scheme for unknown nonlinear systems with dead-zone inputs. The considered systems are general as they are not required to be feedback linearizable. more general for the matching conditions are not required for the system functions and the nonlinearities in the controlled plant are all unknown. Actually, actuators are not strictly linear even without dead-zone, but may be perturbed or vary with time. We give a dead-zone model which possesses time-varying and perturbed actuation gain. The model is treated as a perturbed linear-like input, a nonlinear function, and a bounded disturbance-like term for control design. The width of the dead-zone is unknown and estimated explicitly by an adaptive law, so the control scheme has the ability to adapt the uncertainties of the width caused by circumstance changes. The unknown functions in the design are approximated by nonlinearly parameterized adaptive fuzzy system, backstepping technique is employed to derive the controller. The proposed control scheme can guarantee the stability of the closed-loop system and satisfactory output tracking to the given reference signal.

The rest of this paper is organized as follows. Section II formulates the problem first. Section III introduces the proposed adaptive fuzzy control scheme in detail. In Section IV, a simulation example illustrates the effectiveness of the control scheme. Finally, Section V concludes the paper.

1 Problem Formulation:

Consider the following nonlinear plant

$$\begin{aligned} &\dot{x}_i = f_i(\bar{\boldsymbol{x}}_i) + g_i(\bar{\boldsymbol{x}}_i)x_{i+1} \quad 1 \le i \le n-1 \\ &\dot{x}_n = f_n(\bar{\boldsymbol{x}}_n) + g_n(\bar{\boldsymbol{x}}_n)\mathscr{D}(u) \\ &y = x_1 \end{aligned} \tag{1}$$

where $x_1,\ x_2,\ \cdots,\ x_n$ are the states of the system which are available, $\bar{\boldsymbol{x}}_i = (x_1,\ \cdots,\ x_i)^T$, and $\boldsymbol{x} = \bar{\boldsymbol{x}}_n = (x_1,\ \cdots,\ x_n) \in U \subseteq \mathbf{R}^n$ is the state vector, U is a compact set in $\mathbf{R}^n$. y is the system output, u_i is the designed control law, and $\mathscr{D}(u)$ is the output of the actuator with dead-zone characteristic to the plant. The nonlinear func-

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tions $f_i(\bar{\boldsymbol{x}}_i) \in \mathbf{R}$ and $g_i(\bar{\boldsymbol{x}}_i) \in \mathbf{R}$ with $i = 1, \cdots, n$ are unknown but smooth.

The dead-zone characteristic considered in this paper is different from the existing literature because we have taken time variation and perturbation into account. The model of the dead-zone is described as follows,

$$\mathscr{D}(u) = \begin{cases} (m(t) + \phi(\boldsymbol{x}))(u - b) & u \geq b \\ 0 & -b < u < b \\ (m(t) + \phi(\boldsymbol{x}))(u + b) & u \leq -b \end{cases} \tag{2}$$

where $m(t) + \phi(\boldsymbol{x}) > 0$ with $m(t)$ being the time-varying slope and $\phi(\boldsymbol{x})$ being the perturbed term, $b > 0$ is the unknown width of the above dead-zone model. From a practical point of view, it is reasonable to make the following assumptions:

Assumption 1. There exit constants $\underline{m}$ and $\bar{m}$ satisfy $0 < \underline{m} \leq m(t) + \phi(\boldsymbol{x}) \leq \bar{m}$.

Assumption 2. There exists a constant $\bar{b}$ such that $b \leq \bar{b}$.

Remark 1. Though $m(t) + \phi(\boldsymbol{x})$ and b are bounded by some constant values, they are not required to be known to the designer, but only used for analysis.

For the control design, we rewritten the dead-zone characteristic as

$$\mathscr{D}(u) = (m(t) + \phi(\boldsymbol{x}))u + \eta(\boldsymbol{x}, u, b) \tag{3}$$

with η (short for $\eta(\boldsymbol{x}, u, b)$) defined as

$$\eta = \begin{cases} -(m(t) + \phi(\boldsymbol{x}))b & u \geq b \\ -(m(t) + \phi(\boldsymbol{x}))u & -b < u < b \\ (m(t) + \phi(\boldsymbol{x}))b & u \leq b \end{cases} \tag{4}$$

We further treat η as the sum of a hyperbolic tangent function and a disturbance-like term which is bounded. That is

$$\eta = -(m(t) + \phi(\boldsymbol{x}))b tanh(\frac{u}{b}) + \psi(\boldsymbol{x}) \tag{5}$$

where $\psi(\boldsymbol{x})$ satisfies

$$\begin{aligned} |\psi(\boldsymbol{x})| &= |\eta + [m(t) + \phi(\boldsymbol{x})]b tanh(\tfrac{u}{b})| \\ &\leq [m(t) + \phi(\boldsymbol{x})]b[1 - tanh(1)] \end{aligned} \tag{6}$$

Then from Assumptions 1-2, it is obvious that $\psi(\boldsymbol{x})$ is bounded.

The control objective is to design a feedback control law for u to ensure that all closed-loop signals are bounded and the plant output $y(t)$ tracks a given reference signal $y_r(t)$ as closely as possible though the nonlinearities of the system are unknown and the actuator are with the time-varying perturbed dead-zone characteristic described by (2).

2 Adaptive Fuzzy Control Design

2.1 Preliminaries

In this section, a new adaptive fuzzy control for system (1) will be presented in detail. Because fuzzy logic systems with adjustable parameters are used to approximate the unknown system functions, we first show the approximation property of adaptive fuzzy system in the following lemma:

Lemma 1.[10] For any given real continuous function $F(\boldsymbol{x})$, on a compact set $\Omega \subseteq \mathbf{R}^n$, there exists a fuzzy logic system $Y(\boldsymbol{x}) = \boldsymbol{\theta}^T \boldsymbol{\xi}(\boldsymbol{x})$ such that $\forall \varepsilon > 0$,

$$\sup_{\boldsymbol{x} \in \Omega} \left| F(\boldsymbol{x}) - \boldsymbol{\theta}^T \boldsymbol{\xi}(\boldsymbol{x}) \right| \leq \varepsilon \tag{7}$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_M)^T$ is the vector of connection weights, and $\boldsymbol{\xi}(\boldsymbol{x}) = (\xi_1(x), \xi_2(x), \cdots, \xi_M(x))^T$ is the vector of fuzzy basis functions, M is the number of fuzzy rules. One can refer [11] for more details.

In most existing designs, the fuzzy basis functions are assumed to be known, this implies that all the fuzzy membership functions are certain for the described fuzzy sets. However, in many cases the fuzzy membership functions are uncertain because there is no apriori knowledge available for them. In such situation, the membership function of the fuzzy set A_{ji} for x_i in the jth rule can be defined by

$$\mu_{A_{ji}}(x_i) = e^{-[\sigma_{ji}(x_i - c_{ji})]^2}$$

with σ_{ji} and c_{ji} unknown to the designer. This is the case considered in our design. We choose the fuzzy basis function for j rule as

$$\xi_j(\bar{\boldsymbol{x}}_k, \boldsymbol{c}_j, \boldsymbol{\sigma}_j) = \prod_{i=1}^{k} \mu_{A_{ji}}(x_i) \tag{8}$$

where $\boldsymbol{c}_j = (c_{j1}, c_{j2}, \cdots, c_{jk})^T$, $\boldsymbol{\sigma}_j = (\sigma_{j1}, \sigma_{j2}, \cdots, \sigma_{jk})^T$ with $1 \leq k \leq n$. Denote $\boldsymbol{c}_j^i$ and $\boldsymbol{\sigma}_j^i$ are the corresponding vectors of $\boldsymbol{c}_j$ and $\boldsymbol{\sigma}_j$ in the ith step design, and θ_j^i is the connection weight of the jth rule in step i. Supposes there are M_i rules in the ith step design, define parameter vectors $\boldsymbol{\theta}^i = (\theta_1^i, \theta_2^i, \cdots \theta_{M_i}^i)^T$, $\boldsymbol{c}^i = ({\boldsymbol{c}_1^i}^T, {\boldsymbol{c}_2^i}^T, \cdots, {\boldsymbol{c}_{M_i}^i}^T)^T$ and $\boldsymbol{\sigma}^i = ({\boldsymbol{\sigma}_1^i}^T, {\boldsymbol{\sigma}_2^i}^T, \cdots, {\boldsymbol{\sigma}_{M_i}^i}^T)^T$, where $i = 1, 2, \cdots, n$ corresponding to n step backstepping design respectively. $\boldsymbol{\theta}^{i*}$, $\boldsymbol{c}^{i*}$ and $\boldsymbol{\sigma}^{i*}$ denote the optimal parameters which minimize the following expression.

$$\sup_{\boldsymbol{x} \in U} \left| F^i(\boldsymbol{x}) - {\boldsymbol{\theta}^i}^T \boldsymbol{\xi}(\boldsymbol{x}, \boldsymbol{c}^i, \boldsymbol{\sigma}^i) \right|$$

It is obvious that fuzzy logic systems constructed by the fuzzy basis functions in the form of (8) are not linearly parameterized, which brings challenges to the control design.

Besides, the following lemmas and assumptions are needed for the design of the proposed controller.

Lemma 2.[18] Let $P(x_1, x_2, \cdots, x_n)$ be a real-value continuous function and satisfy $0 < a_m \leq P(x_1, x_2, \cdots, x_n) \leq a_M$ with a_m and a_M being two constants. Define functions $V(t)$ as follows:

$$\begin{aligned} V(t) = \textstyle\int_0^{z(t)} \rho P(x_1, x_2, \cdots, x_{k-1}, \rho \\ + \beta(t), x_{k+1}, \cdots, x_n) \mathrm{d}\rho \end{aligned}$$

where $z(t)$ and $\beta(t)$ are real-value functions with $t \in [0, \infty)$. Then the integral function $V(t)$ has the following properties.

1)

$$\frac{1}{2} a_m z^2(t) \leq V(t) \leq \frac{1}{2} a_M z^2(t)$$

2)

$$\begin{aligned} \tfrac{d}{dt} V(t) = {} & z(t) P(x_1, x_2, \cdots, x_{k-1}, z(t) + \beta(t), x_{k+1}, \cdots, \\ & x_n) \dot{z}(t) + \dot{\beta}(t) z(t) P(x_1, x_2, \cdots, x_{k-1}, \\ & z(t) + \beta(t), x_{k+1}, \cdots, x_n) + z^2(t) \textstyle\int_0^1 [\theta \sum\limits_{i=1, i \neq k}^{n} \\ & \dot{x}_i(t) \tfrac{\partial}{\partial x_i} P(x_1, x_2, \cdots, x_{k-1}, z(t) + \beta(t), x_{k+1}, \\ & \cdots, x_n)] \mathrm{d}\theta - z(t) \dot{\beta}(t) \textstyle\int_0^1 P(x_1, x_2, \cdots, x_{k-1}, \\ & \theta z(t) + \beta(t), x_{k+1}, \cdots, x_n) \mathrm{d}\theta \end{aligned}$$

The proof of Lemma 2 can be found in [18].

Lemma 3. For any $\epsilon > 0$ and any $q \in \mathbf{R}$, the hyperbolic tangent function fulfills

$$0 \leq |q| - q tanh(\frac{q}{\epsilon}) \leq \kappa\epsilon$$

where κ is a constant that satisfies $\kappa = \mathrm{e}^{-(\kappa+1)}$ (i.e.$\kappa \approx 0.2785$).

The proof of Lemma 3 is omitted for space limitations.

Assumption 3. For system functions $g_i(\bar{\boldsymbol{x}}_i)$ $(1 \leq i \leq n)$, there exist positive constants g_l and $g_u$such that $g_{il} \leq |g_i(\bar{\boldsymbol{x}}_i)| \leq g_{iu}$.

From Assumption 3 it can be concluded that the unknown functions $g_i(\bar{\boldsymbol{x}}_i)$ are not zero. Without loss of generality, it is assumed that $g_i(\bar{\boldsymbol{x}}_i) > 0$.

Assumption 4. There exist constants $\bar{\theta}^i$, $\bar{c}^i$ and $\bar{\sigma}^i$ that $\|\boldsymbol{\theta}^i\|_\infty \leq \bar{\theta}^i$, $\|\boldsymbol{c}^i\|_\infty \leq \bar{c}^i$ and $\|\boldsymbol{\sigma}^i\|_\infty \leq \bar{\sigma}^i$ for $i = 1, 2, \cdots, n$, where $\|\cdot\|_\infty$ denotes the infinite-norm of a vector.

2.2 Control Design

1)**Step 1.** Define $z_1 = x_1 - y_r$, then

$$\dot{z}_1 = f_1(\bar{x}_1) + g_1(\bar{x}_1)x_2 - \dot{y}_r \tag{9}$$

Consider a Lyapunov function candidate as

$$V_1 = \int_0^{z_1} \rho P_1(\rho + y_r)\mathrm{d}\rho + \tfrac{1}{2}\tilde{\boldsymbol{\theta}}^{1T}\Gamma_{\theta^1}^{-1}\tilde{\boldsymbol{\theta}}^1 + \tfrac{1}{2}\tilde{\boldsymbol{c}}^{1T}\Gamma_{c^1}^{-1}\tilde{\boldsymbol{c}}^1 + \tfrac{1}{2}\tilde{\boldsymbol{\sigma}}^{1T}\Gamma_{\sigma^1}^{-1}\tilde{\boldsymbol{\sigma}}^1 + \tfrac{1}{2\gamma_1}\tilde{\delta}_1^2 \tag{10}$$

where $P_1(\rho + y_r) = g_1^{-1}(\rho + y_r)$, Γ_{θ^1}, Γ_{c^1} and Γ_{σ^1} are positive definite matrices with proper dimensions, γ_1 is a positive constant, $\tilde{\boldsymbol{\theta}}^1 = \hat{\boldsymbol{\theta}}^1 - \boldsymbol{\theta}^{1*}$, $\tilde{\boldsymbol{c}}^1 = \hat{\boldsymbol{c}}^1 - \boldsymbol{c}^{1*}$ and $\tilde{\boldsymbol{\sigma}}^1 = \hat{\boldsymbol{\sigma}}^1 - \boldsymbol{\sigma}^{1*}$ with $\hat{\boldsymbol{\theta}}^1$, $\hat{\boldsymbol{c}}^1$ and $\hat{\boldsymbol{\sigma}}^1$ are the estimates of $\boldsymbol{\theta}^{1*}$, $\boldsymbol{c}^{1*}$ and $\boldsymbol{\sigma}^{1*}$, respectively; $\tilde{\delta}_1 = \hat{\delta}_1 - \delta_1^*$ with δ_1^* defined later, $\hat{\delta}_1$ is the estimate of δ_1^*.

From Lemma 2 the derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= z_1 g_1^{-1}\dot{z}_1 + \dot{y}_r z_1 g_1^{-1} - z_1\dot{y}_r \int_0^1 P_1(\vartheta z_1 + y_r)\mathrm{d}\vartheta \\ &\quad + \tilde{\boldsymbol{\theta}}^{1T}\Gamma_{\theta^1}^{-1}\dot{\hat{\boldsymbol{\theta}}}^1 + \tilde{\boldsymbol{c}}^{1T}\Gamma_{c^1}^{-1}\dot{\hat{\boldsymbol{c}}}^1 + \tilde{\boldsymbol{\sigma}}^{1T}\Gamma_{\sigma^1}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^1 + \tfrac{1}{\gamma_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 \\ &= z_1(x_2 + \Delta f_1) + \tilde{\boldsymbol{\theta}}^{1T}\Gamma_{\theta^1}^{-1}\dot{\hat{\boldsymbol{\theta}}}^1 + \tilde{\boldsymbol{c}}^{1T}\Gamma_{c^1}^{-1}\dot{\hat{\boldsymbol{c}}}^1 \\ &\quad + \tilde{\boldsymbol{\sigma}}^{1T}\Gamma_{\sigma^1}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^1 + \tfrac{1}{\gamma_1}\tilde{\delta}_1\dot{\hat{\delta}}_1\end{aligned} \tag{11}$$

where $\Delta f_1 = g_1^{-1}(x_1)f_1(x_1) - \dot{y}_r \int_0^1 P_1(\vartheta z_1 + y_r)\mathrm{d}\vartheta$. According to Lemma 1, for a given ε_1 there exists a fuzzy logic system $\boldsymbol{\theta}^{1*T}\xi(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})$ such that

$$\begin{aligned}\Delta f_1 &= \boldsymbol{\theta}^{1*T}\boldsymbol{\xi}(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*}) + \varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*}) \\ &= \hat{\boldsymbol{\theta}}^{1T}\hat{\boldsymbol{\xi}}^1 - (\hat{\boldsymbol{\theta}}^{1T}\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\theta}^{1*T}\boldsymbol{\xi}^{1*}) + \varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})\end{aligned} \tag{12}$$

with $\varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})$ being the approximation error and $|\varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})| \leq \varepsilon_1$, $\hat{\boldsymbol{\xi}}^1 = \boldsymbol{\xi}(x_1, \hat{\boldsymbol{c}}^1, \hat{\boldsymbol{\sigma}}^1)$ and $\boldsymbol{\xi}^{1*} = \boldsymbol{\xi}(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})$.

Define that $\delta_1^* = \varepsilon_1 + \|\theta^{1*}\|_1$, $\hat{\xi}'_{c^1} = \frac{\partial \boldsymbol{\xi}(x_1, \boldsymbol{c1}, \boldsymbol{\sigma}^1)}{\partial \boldsymbol{c}^1} \mid (\boldsymbol{c}^1 = \hat{\boldsymbol{c}}^1, \boldsymbol{\sigma}^1 = \hat{\boldsymbol{\sigma}}^1)$ and $\hat{\xi}'_{\sigma^1} = \frac{\partial \boldsymbol{\xi}(x_1, \boldsymbol{c}^1, \boldsymbol{\sigma}^1)}{\partial \boldsymbol{\sigma}^1} \mid (\boldsymbol{c}^1 = \hat{\boldsymbol{c}}^1, \boldsymbol{\sigma}^1 = \hat{\boldsymbol{\sigma}}^1)$. Then by Taylor series expansion of $\boldsymbol{\xi}^{1*}$ at $(\hat{\boldsymbol{c}}^1, \hat{\boldsymbol{\sigma}}^1)$, one has

$$\begin{aligned}&\hat{\boldsymbol{\theta}}^{1T}\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\theta}^{1*T}\boldsymbol{\xi}^{1*} \\ &= \tilde{\boldsymbol{\theta}}^{1T}\hat{\boldsymbol{\xi}}^1 + \boldsymbol{\theta}^{1*T}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \boldsymbol{\theta}^{1*T}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 - \boldsymbol{\theta}^{1*T}o(x_1, \tilde{\boldsymbol{c}}^1, \tilde{\boldsymbol{\sigma}}^1) \\ &= \tilde{\boldsymbol{\theta}}^{1T}\hat{\boldsymbol{\xi}}^1 + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 - \tilde{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 - \tilde{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 \\ &\quad - \boldsymbol{\theta}^{1*T}o(\cdot) \\ &= \tilde{\boldsymbol{\theta}}^{1T}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 \\ &\quad + \tilde{\boldsymbol{\theta}}^{1T}(\hat{\xi}'_{c^1}\boldsymbol{c}^{1*} + \hat{\xi}'_{\sigma^1}\boldsymbol{\sigma}^{1*}) - \boldsymbol{\theta}^{1*T}o(\cdot) \\ &= \tilde{\boldsymbol{\theta}}^{1T}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \hat{\boldsymbol{\theta}}^{1T} \\ &\quad \hat{\xi}'_{c^1}\boldsymbol{c}^{1*} - \boldsymbol{\theta}^{1*T}\hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 + \boldsymbol{\theta}^{1*T}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{\sigma^1}\boldsymbol{\sigma}^{1*} - \boldsymbol{\theta}^{1*T} \\ &\quad \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1 + \boldsymbol{\theta}^{1*T}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \boldsymbol{\theta}^{1*T}(\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\xi}^{1*} - \hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1) \\ &= \tilde{\boldsymbol{\theta}}^{1T}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 \\ &\quad + \hat{\boldsymbol{\theta}}^{1T}(\hat{\xi}'_{c^1}\boldsymbol{c}^{1*} + \hat{\xi}'_{\sigma^1}\boldsymbol{\sigma}^{1*}) - \boldsymbol{\theta}^{1*T}(\hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) \\ &\quad + \boldsymbol{\theta}^{1*T}(\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\xi}^{1*}) \\ &\leq \tilde{\boldsymbol{\theta}}^{1T}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \|\hat{\boldsymbol{\theta}}^{1T} \\ &\quad \hat{\xi}'_{c^1}\|_1\bar{\boldsymbol{c}}^1 + \|\hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{\sigma^1}\|_1\bar{\sigma}^1 + \|\hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1\|_1\bar{\theta}^1 + \|\boldsymbol{\theta}^{1*}\|_1\end{aligned} \tag{13}$$

where $o(\cdot) = o(x_1, \tilde{\boldsymbol{c}}^1, \tilde{\boldsymbol{\sigma}}^1)$, and $\|\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\xi}^{1*}\|_\infty < 1$ is used.

According to (13), (11) can be rewritten as

$$\begin{aligned}\dot{V}_1 &= z_1[x_2 + \hat{\boldsymbol{\theta}}^{1T}\hat{\boldsymbol{\xi}}^1 + \varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*}) - \tilde{\boldsymbol{\theta}}^{1T}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 \\ &\quad - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) - \hat{\boldsymbol{\theta}}^{1T}(\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1) - \tilde{\boldsymbol{\theta}}^{1T}(\hat{\xi}'_{c^1}\boldsymbol{c}^{1*} + \hat{\xi}'_{\sigma^1} \\ &\quad \boldsymbol{\sigma}^{1*}) + \boldsymbol{\theta}^{1*T}o(x_1, \tilde{\boldsymbol{c}}^1, \tilde{\boldsymbol{\sigma}}^1)] + \tilde{\boldsymbol{\theta}}^{1T}\Gamma_{\theta^1}^{-1}\dot{\hat{\boldsymbol{\theta}}}^1 + \tilde{\boldsymbol{c}}^{1T}\Gamma_{c^1}^{-1}\dot{\hat{\boldsymbol{c}}}^1 \\ &\quad + \tilde{\boldsymbol{\sigma}}^{1T}\Gamma_{\sigma^1}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^1 + \tfrac{1}{\gamma_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 \\ &\leq z_1[x_2 + \hat{\boldsymbol{\theta}}^{1T}\hat{\boldsymbol{\xi}}^1 - \tilde{\boldsymbol{\theta}}^{1T}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) \\ &\quad - \hat{\boldsymbol{\theta}}^{1T}(\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1)] + |z_1\omega_1| + |z_1\delta_1^*| + \tilde{\boldsymbol{\theta}}^{1T}\Gamma_{\theta^1}^{-1}\dot{\hat{\boldsymbol{\theta}}}^1 \\ &\quad + \tilde{\boldsymbol{c}}^{1T}\Gamma_{c^1}^{-1}\dot{\hat{\boldsymbol{c}}}^1 + \tilde{\boldsymbol{\sigma}}^{1T}\Gamma_{\sigma^1}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^1 + \tfrac{1}{\gamma_1}\tilde{\delta}_1\dot{\hat{\delta}}_1\end{aligned} \tag{14}$$

where $\omega_1 = \|\hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{c^1}\|_1\bar{c}^1 + \|\hat{\boldsymbol{\theta}}^{1T}\hat{\xi}'_{\sigma^1}\|_1\bar{\sigma}^1 + \|\hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1\|_1\bar{\theta}^1$.

Choose the virtue control variable α_1 as

$$\alpha_1 = -q_1 z_1 - \hat{\boldsymbol{\theta}}^{1T}\hat{\boldsymbol{\xi}}^1 - \omega_1 tanh(\frac{z_1\omega_1}{\pi_1}) - \hat{\delta}_1 tanh\frac{z_1\hat{\delta}_1}{\tau_1} \tag{15}$$

where q_1, π_1 and τ_1 are positive constants.

$$\begin{aligned}\dot{\hat{\boldsymbol{\theta}}}^1 &= Proj[\Gamma_{\theta^1}z_1(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) - R_{\theta^1}\hat{\boldsymbol{\theta}}^1] \\ \dot{\hat{\boldsymbol{c}}}^1 &= Proj[\Gamma_{c^1}z_1\hat{\xi}'^{\mathrm{T}}_{c^1}\hat{\boldsymbol{\theta}}^1 - R_{c^1}\hat{\boldsymbol{c}}^1] \\ \dot{\hat{\boldsymbol{\sigma}}}^1 &= Proj[\Gamma_{\sigma^1}z_1\hat{\xi}'^{\mathrm{T}}_{\sigma^1}\hat{\boldsymbol{\theta}}^1 - R_{\sigma^1}\hat{\boldsymbol{\sigma}}^1] \\ \dot{\hat{\delta}}_1 &= \gamma_1 z_1 - r_1\hat{\delta}_1\end{aligned} \tag{16}$$

where R_{θ^1}, R_{c^1} and R_{σ^1} are positive definite matrices with proper dimensions, r_1 is a positive real constant. $Proj[\cdot]$ is the projection operator to ensure that $\|\boldsymbol{\theta}^i\|_\infty \leq \bar{\theta}^i$, $\|\boldsymbol{c}^i\|_\infty \leq \bar{c}^i$ and $\|\boldsymbol{\sigma}^i\|_\infty \leq \bar{\sigma}^i$ for $1 \leq i \leq n$. Let $z_2 = x_2 - z_1$, the following inequalities can be obtained.

$$\begin{aligned}\dot{V}_1 &\leq -q_1 z_1^2 + z_1 z_2 + |z_1\hat{\delta}_1| - z_1\hat{\delta}_1 tanh(\tfrac{z_1\hat{\delta}_1}{\tau_1}) + |z_1\omega_1| \\ &\quad - z_1\omega_1 tanh(\tfrac{z_1\omega_1}{\pi_1}) - \tfrac{1}{2}\tilde{\boldsymbol{\theta}}^{1T}\Gamma_{\theta^1}^{-1}R_{\theta^1}\tilde{\boldsymbol{\theta}}^1 + \tfrac{1}{2}\boldsymbol{\theta}^{1*T}\Gamma_{\theta^1}^{-1} \\ &\quad R_{\theta^1}\tilde{\boldsymbol{\theta}}^{1*} - \tfrac{1}{2}\tilde{\boldsymbol{c}}^{1T}\Gamma_{c^1}^{-1}R_{c^1}\tilde{\boldsymbol{c}}^1 + \tfrac{1}{2}\boldsymbol{c}^{1*T}\Gamma_{c^1}^{-1}R_{c^1}\boldsymbol{c}^{1*} - \tfrac{1}{2}\tilde{\boldsymbol{\sigma}}^{1T} \\ &\quad \Gamma_{\sigma^1}^{-1}R_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \tfrac{1}{2}\boldsymbol{\sigma}^{1*T}\Gamma_{\sigma^1}^{-1}R_{\sigma^1}\boldsymbol{\sigma}^{1*} - \tfrac{1}{2\gamma_1}\tilde{\delta}_1^2 + \tfrac{1}{2\gamma_1}\delta_1^{*2} \\ &\leq -q_1 z_1^2 - \tfrac{\lambda_{\theta^1}^{min}}{2}\tilde{\boldsymbol{\theta}}^{1T}\Gamma_{\theta^1}^{-1}\tilde{\boldsymbol{\theta}}^1 - \tfrac{\lambda_{c^1}^{min}}{2}\tilde{\boldsymbol{c}}^{1T}\Gamma_{c^1}^{-1}\tilde{\boldsymbol{c}}^1 \\ &\quad - \tfrac{\lambda_{\sigma^1}^{min}}{2}\tilde{\boldsymbol{\sigma}}^{1T}\Gamma_{\sigma^1}^{-1}\tilde{\boldsymbol{\sigma}}^1 - \tfrac{1}{2\gamma_1}\tilde{\delta}_1^2 + z_1 z_2 + \kappa(\pi_1 + \tau_1) \\ &\quad + \tfrac{1}{2}\boldsymbol{\theta}^{1*T}\Gamma_{\theta^1}^{-1}R_{\theta^1}\tilde{\boldsymbol{\theta}}^{1*} + \tfrac{1}{2}\boldsymbol{c}^{1*T}\Gamma_{c^1}^{-1}R_{c^1}\boldsymbol{c}^{1*} \\ &\quad + \tfrac{1}{2}\boldsymbol{\sigma}^{1*T}\Gamma_{\sigma^1}^{-1}R_{\sigma^1}\boldsymbol{\sigma}^{1*} + \tfrac{1}{2\gamma_1}\delta_1^{*2}\end{aligned} \tag{17}$$

where Lemma 3 has been used, $\lambda_{\theta^1}^{min}$, $\lambda_{c^1}^{min}$ and $\lambda_{\sigma^1}^{min}$ are the minimal eigenvalues of R_{θ^1}, R_{c^1} and R_{σ^1}, respectively.

2)**Step 2.**

$$\dot{z}_2 = f_2(\bar{\boldsymbol{x}}_2) + g_2(\bar{\boldsymbol{x}}_2)x_3 - \dot{\alpha}_1 \tag{18}$$

Then take a Lyapunov function candidate as

$$V_2 = V_1 + \int_0^{z_2} \rho P_2(x_1, \rho + \alpha_1)\mathrm{d}\rho + \tfrac{1}{2}\tilde{\boldsymbol{\theta}}^{2T}\Gamma_{\theta^2}^{-1}\tilde{\boldsymbol{\theta}}^2 + \tfrac{1}{2}\tilde{\boldsymbol{c}}^{2T}\Gamma_{c^2}^{-1}\tilde{\boldsymbol{c}}^2 + \tfrac{1}{2}\tilde{\boldsymbol{\sigma}}^{2T}\Gamma_{\sigma^2}^{-1}\tilde{\boldsymbol{\sigma}}^2 + \tfrac{1}{2\gamma_2}\tilde{\delta}_2^2 \tag{19}$$

with $P_2(x_1, \rho+\alpha_1) = g_2^{-1}(x_1, \rho+\alpha_1)$, Γ_{θ^2}, Γ_{c^2} and Γ_{σ^2} are positive definite matrices with proper dimensions, γ_2 is a positive constant, $\tilde{\boldsymbol{\theta}}^2 = \hat{\boldsymbol{\theta}}^2 - \boldsymbol{\theta}^{2*}$, $\tilde{\boldsymbol{c}}^2 = \hat{\boldsymbol{c}}^2 - \boldsymbol{c}^{2*}$, $\tilde{\boldsymbol{\sigma}}^2 = \hat{\boldsymbol{\sigma}}^2 - \boldsymbol{\sigma}^{2*}$ and $\tilde{\delta}_2 = \hat{\delta}_2 - \delta_2^*$ respectively.

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2 g_2^{-1}\dot{z}_2 + \dot{\alpha}_1 z_2 g_2^{-1} - z_2\dot{\alpha}_1\int_0^1 P_2(\vartheta z_2 + \alpha_1)\mathrm{d}\vartheta \\ &\quad + z_2^2\dot{x}_1\int_0^1 \vartheta\tfrac{\partial P_2(\vartheta z_2+\alpha_1)}{\partial x_1}\mathrm{d}\vartheta + \boldsymbol{\theta}^{2T}\Gamma_{\theta^2}^{-1}\dot{\hat{\boldsymbol{\theta}}}^2 + \tilde{\boldsymbol{c}}^{2T}\Gamma_{c^2}^{-1}\dot{\hat{\boldsymbol{c}}}^2 \\ &\quad + \tilde{\boldsymbol{\sigma}}^{2T}\Gamma_{\sigma^2}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^2 + \tfrac{1}{\gamma_2}\tilde{\delta}_2\dot{\hat{\delta}}_2 \\ &= \dot{V}_1 + z_2(x_3 + \Delta f_2) + \tilde{\boldsymbol{\theta}}^{2T}\Gamma_{\theta^2}^{-1}\dot{\hat{\boldsymbol{\theta}}}^2 + \tilde{\boldsymbol{c}}^{2T}\Gamma_{c^2}^{-1}\dot{\hat{\boldsymbol{c}}}^2 \\ &\quad + \tilde{\boldsymbol{\sigma}}^{2T}\Gamma_{\sigma^2}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^2 + \tfrac{1}{\gamma_2}\tilde{\delta}_2\dot{\hat{\delta}}_2\end{aligned} \tag{20}$$

where $\Delta f_2 = g_2^{-1}(\bar{\boldsymbol{x}}_2)f_2(\bar{\boldsymbol{x}}_2) - \dot{\alpha}_1\int_0^1 P_2(\vartheta z_2 + \alpha_1)\mathrm{d}\vartheta + z_2\dot{x}_1\int_0^1 \vartheta\frac{\partial P_2(\vartheta z_2+\alpha_1)}{\partial x_1}d\vartheta$. As in step 1, Δf_2 is approximated by a fuzzy logic system, and following similar manipulation as (13), one can get

$$\begin{aligned}&\hat{\boldsymbol{\theta}}^{2T}\hat{\boldsymbol{\xi}}^2 - \boldsymbol{\theta}^{2*T}\boldsymbol{\xi}^{2*} \\ &\le \tilde{\boldsymbol{\theta}}^{2T}(\hat{\boldsymbol{\xi}}^2 - \hat{\xi}'_{c^2}\tilde{\boldsymbol{c}}^2 - \hat{\xi}'_{\sigma^2}\hat{\boldsymbol{\sigma}}^2) + \hat{\boldsymbol{\theta}}^{2T}\hat{\xi}'_{c^2}\tilde{\boldsymbol{c}}^2 + \hat{\boldsymbol{\theta}}^{2T}\hat{\xi}'_{\sigma^2}\tilde{\boldsymbol{\sigma}}^2 + \|\hat{\boldsymbol{\theta}}^{2T} \\ &\quad \hat{\xi}'_{c^2}\|_1\bar{c}^2 + \|\hat{\boldsymbol{\theta}}^{2T}\hat{\xi}'_{\sigma^2}\|_1\bar{\sigma}^2 + \|\hat{\xi}'_{c^2}\hat{\boldsymbol{c}}^2 + \hat{\xi}'_{\sigma^2}\hat{\boldsymbol{\sigma}}^2\|_1\bar{\theta}^2 + \|\boldsymbol{\theta}^{2*}\|_1\end{aligned} \tag{21}$$

δ_2^*, $\hat{\xi}'_{c^2}$, $\hat{\xi}'_{\sigma^2}$ and ω_2 are defined similarly to δ_1^*, $\hat{\xi}'_{c^1}$, $\hat{\xi}'_{\sigma^1}$ and ω_1 respectively, with subscript 2 instead of 1. Design

$$\alpha_2 = -q_2 z_2 - z_1 - \hat{\boldsymbol{\theta}}^{2T}\hat{\boldsymbol{\xi}}^2 - \omega_2 tanh(\frac{z_2\omega_2}{\pi_2}) - \hat{\delta}_2 tanh\frac{z_2\hat{\delta}_2}{\tau_2} \tag{22}$$

$$\begin{aligned}\dot{\hat{\boldsymbol{\theta}}}^2 &= Proj[\Gamma_{\theta^2}z_2(\hat{\boldsymbol{\xi}}^2 - \hat{\xi}'_{c^2}\hat{\boldsymbol{c}}^2 - \hat{\xi}'_{\sigma^2}\hat{\boldsymbol{\sigma}}^2) - R_{\theta^2}\hat{\boldsymbol{\theta}}^2] \\ \dot{\hat{\boldsymbol{c}}}^2 &= Proj[\Gamma_{c^2}z_2\hat{\xi}'^{\mathrm{T}}_{c^2}\hat{\boldsymbol{\theta}}^2 - R_{c^2}\hat{\boldsymbol{c}}^2] \\ \dot{\hat{\boldsymbol{\sigma}}}^2 &= Proj[\Gamma_{\sigma^2}z_2\hat{\xi}'^{\mathrm{T}}_{\sigma^2}\hat{\boldsymbol{\theta}}^2 - R_{\sigma^2}\hat{\boldsymbol{\sigma}}^2] \\ \dot{\hat{\delta}}_2 &= \gamma_2 z_2 - r_2\hat{\delta}_2\end{aligned} \tag{23}$$

Then (20) can be rewritten as

$$\begin{aligned}\dot{V}_2 \le \sum_{j=1}^{2}[&-q_j z_j^2 - \tfrac{\lambda_{\theta j}^{min}}{2}\tilde{\boldsymbol{\theta}}^{jT}\Gamma_{\theta^j}^{-1}\tilde{\boldsymbol{\theta}}^j - \tfrac{\lambda_{cj}^{min}}{2}\tilde{\boldsymbol{c}}^{jT}\Gamma_{c^j}^{-1}\tilde{\boldsymbol{c}}^j \\ &- \tfrac{\lambda_{\sigma j}^{min}}{2}\tilde{\boldsymbol{\sigma}}^{jT}\Gamma_{\sigma^j}^{-1}\tilde{\boldsymbol{\sigma}}^j - \tfrac{1}{2\gamma_j}\tilde{\delta}_j^2 + \kappa(\pi_j + \tau_j) \\ &+ \tfrac{1}{2}\boldsymbol{\theta}^{j*T}\Gamma_{\theta^j}^{-1}R_{\theta j}\boldsymbol{\theta}^{j*} + \tfrac{1}{2}\boldsymbol{c}^{j*T}\Gamma_{c^j}^{-1}R_{cj}\boldsymbol{c}^{j*} \\ &+ \tfrac{1}{2}\boldsymbol{\sigma}^{j*T}\Gamma_{\sigma^j}^{-1}R_{\sigma j}\boldsymbol{\sigma}^{j*} + \tfrac{1}{2\gamma_j}\delta_j^{*2}] + z_2 z_3\end{aligned} \tag{24}$$

3)**Step i.** (3≤ i≤ n-1) Let $z_i = x_i - \alpha_{i-1}$, and design

$$\alpha_i = -q_i z_i - z_{i-1} - \hat{\boldsymbol{\theta}}^{iT}\hat{\boldsymbol{\xi}}^i - \omega_i tanh\frac{z_i\omega_i}{\pi_i} - \hat{\delta}_i tanh\frac{z_i\hat{\delta}_i}{\tau_i} \tag{25}$$

$$\begin{aligned}\dot{\hat{\boldsymbol{\theta}}}^i &= Proj[\Gamma_{\theta^i}z_i(\hat{\boldsymbol{\xi}}^i - \hat{\xi}'_{c^i}\hat{\boldsymbol{c}}^i - \hat{\xi}'_{\sigma^i}\hat{\boldsymbol{\sigma}}^i) - R_{\theta^i}\hat{\boldsymbol{\theta}}^i] \\ \dot{\hat{\boldsymbol{c}}}^i &= Proj[\Gamma_{c^i}z_i\hat{\xi}'^{\mathrm{T}}_{c^i}\hat{\boldsymbol{\theta}}^i - R_{c^i}\hat{\boldsymbol{c}}^i] \\ \dot{\hat{\boldsymbol{\sigma}}}^i &= Proj[\Gamma_{\sigma^i}z_i\hat{\xi}'^{\mathrm{T}}_{\sigma^i}\hat{\boldsymbol{\theta}}^i - R_{\sigma^i}\hat{\boldsymbol{\sigma}}^i] \\ \dot{\hat{\delta}}_i &= \gamma_i z_i - r_i\hat{\delta}_i\end{aligned} \tag{26}$$

Then the Lyapunov function

$$V_i = V_{i-1} + \int_0^{z_i} \rho P_i(\bar{\boldsymbol{x}}_{i-1}, \rho + \alpha_{i-1})\mathrm{d}\rho + \tfrac{1}{2}\tilde{\boldsymbol{\theta}}^{iT}\Gamma_{\theta^i}^{-1}\tilde{\boldsymbol{\theta}}^i + \tfrac{1}{2}\tilde{\boldsymbol{c}}^{iT}\Gamma_{c^i}^{-1}\tilde{\boldsymbol{c}}^i + \tfrac{1}{2}\tilde{\boldsymbol{\sigma}}^{iT}\Gamma_{\sigma^i}^{-1}\tilde{\boldsymbol{\sigma}}^i + \tfrac{1}{2\gamma_i}\tilde{\delta}_i^2 \tag{27}$$

satisfies the following inequality when taking its derivative

$$\begin{aligned}\dot{V}_i \le \sum_{j=1}^{i}[&-q_j z_j^2 - \tfrac{\lambda_{\theta j}^{min}}{2}\tilde{\boldsymbol{\theta}}^{jT}\Gamma_{\theta^j}^{-1}\tilde{\boldsymbol{\theta}}^j - \tfrac{\lambda_{cj}^{min}}{2}\tilde{\boldsymbol{c}}^{jT}\Gamma_{c^j}^{-1}\tilde{\boldsymbol{c}}^j - \\ &\tfrac{\lambda_{\sigma j}^{min}}{2}\tilde{\boldsymbol{\sigma}}^{jT}\Gamma_{\sigma^j}^{-1}\tilde{\boldsymbol{\sigma}}^j - \tfrac{1}{2\gamma_j}\tilde{\delta}_j^2 + \kappa(\pi_j + \tau_j) + \\ &\tfrac{1}{2}\boldsymbol{\theta}^{j*T}\Gamma_{\theta^j}^{-1}R_{\theta j}\boldsymbol{\theta}^{j*} + \tfrac{1}{2}\boldsymbol{c}^{j*T}\Gamma_{c^j}^{-1}R_{cj}\boldsymbol{c}^{j*} + \\ &\tfrac{1}{2}\boldsymbol{\sigma}^{j*T}\Gamma_{\sigma^j}^{-1}R_{\sigma j}\boldsymbol{\sigma}^{j*} + \tfrac{1}{2\gamma_j}\delta_j^{*2}] + z_i z_{i+1}\end{aligned} \tag{28}$$

4)**Step n.** Let $z_n = x_n - \alpha_{n-1}$, $\dot{z}_n$ can be written as

$$\begin{aligned}\dot{z}_n = f_n(\boldsymbol{x}) &+ g_n(\boldsymbol{x})[m(t) + \phi(\boldsymbol{x})]u - g_n(\boldsymbol{x})[m(t) + \phi(\boldsymbol{x})] \\ &btanh(\tfrac{u}{b}) + g_n(\boldsymbol{x})[m(t) + \phi(\boldsymbol{x})]\tfrac{\psi(\boldsymbol{x})}{m(t)+\phi(\boldsymbol{x})} - \dot{\alpha}_{n-1}\end{aligned} \tag{29}$$

Choose $P_n(m, \bar{\boldsymbol{x}}_{n-1}, \rho + \alpha_{n-1}) = g_n^{-1}(\bar{\boldsymbol{x}}_{n-1}, \rho + \alpha_{n-1})[m(t) + \phi(\bar{\boldsymbol{x}}_{n-1}, \rho + \alpha_{n-1})]^{-1}$ and the Lyapunov function candidate

$$\begin{aligned}V_n = V_{n-1} &+ \int_0^{z_n} \rho P_n(m, \bar{\boldsymbol{x}}_{n-1}, \rho + \alpha_{n-1})\mathrm{d}\rho + \tfrac{1}{2}\tilde{\boldsymbol{\theta}}^{nT}\Gamma_{\theta^n}^{-1} \\ &\tilde{\boldsymbol{\theta}}^n + \tfrac{1}{2}\tilde{\boldsymbol{c}}^{nT}\Gamma_{c^n}^{-1}\tilde{\boldsymbol{c}}^n + \tfrac{1}{2}\tilde{\boldsymbol{\sigma}}^{nT}\Gamma_{\sigma^n}^{-1}\tilde{\boldsymbol{\sigma}}^n + \tfrac{1}{2\gamma_n}\tilde{\delta}_n^2 + \tfrac{1}{2\gamma_b}\tilde{b}^2\end{aligned} \tag{30}$$

where Γ_{θ^n}, Γ_{c^n} and Γ_{σ^n} are positive definite matrices, γ_n and γ_b are positive constants. $\tilde{b} = \hat{b} - b$ with $\hat{b}$ the estimate of b. Define

$$\begin{aligned}\Delta f_n = g_n^{-1}(\boldsymbol{x})[m(t) + \phi(\boldsymbol{x})]^{-1}f_n(\boldsymbol{x}) &- \dot{\alpha}_{n-1}\int_0^1 P_n(\vartheta z_n \\ + \alpha_{n-1})\mathrm{d}\vartheta + z_n\sum_{j=1}^{n-1}\dot{x}_j\int_0^1 &\vartheta\tfrac{\partial P_n(\vartheta z_n+\alpha_{n-1})}{\partial x_j}\mathrm{d}\vartheta \\ + z_n\dot{m}\int_0^1 \vartheta\tfrac{\partial P_n(\vartheta z_n+\alpha_{n-1})}{\partial m}\mathrm{d}\vartheta &+ btanh(u/b)\end{aligned}$$

Then we can get

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + z_n[u + \Delta f_n + \tfrac{\psi(\boldsymbol{x})}{m(t)+\phi(\boldsymbol{x})}] + \tilde{\boldsymbol{\theta}}^{nT}\Gamma_{\theta^n}^{-1}\dot{\hat{\boldsymbol{\theta}}}^n \\ &\quad + \tilde{\boldsymbol{c}}^{nT}\Gamma_{c^n}^{-1}\dot{\hat{\boldsymbol{c}}}^n + \tilde{\boldsymbol{\sigma}}^{nT}\Gamma_{\sigma^n}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^n + \tfrac{1}{\gamma_n}\tilde{\delta}_n\dot{\hat{\delta}}_n + \tfrac{1}{\gamma_b}\tilde{b}\dot{\hat{b}} \\ &\le \dot{V}_{n-1} + z_n\{u + \Delta f_n + sgn(z_n)b[1 - tanh(1)]\} \\ &\quad + \tilde{\boldsymbol{\theta}}^{nT}\Gamma_{\theta^n}^{-1}\dot{\hat{\boldsymbol{\theta}}}^n + \tilde{\boldsymbol{c}}^{nT}\Gamma_{c^n}^{-1}\dot{\hat{\boldsymbol{c}}}^n + \tilde{\boldsymbol{\sigma}}^{nT}\Gamma_{\sigma^n}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^n \\ &\quad + \tfrac{1}{\gamma_n}\tilde{\delta}_n\dot{\hat{\delta}}_n + \tfrac{1}{\gamma_b}\tilde{b}\dot{\hat{b}}\end{aligned} \tag{31}$$

where the boundary of $\psi(\boldsymbol{x})$ described in (6) has been used. By choosing control signal

$$\begin{aligned}u = -q_n z_n - z_{n-1} &- \hat{\boldsymbol{\theta}}^{nT}\hat{\boldsymbol{\xi}}^n - \omega_n tanh\tfrac{z_n\omega_n}{\pi_n} - \hat{\delta}_n tanh\tfrac{z_n\hat{\delta}_n}{\tau_n} \\ &- sgn(z_n)\hat{b}[1 - tanh(1)]\end{aligned} \tag{32}$$

with q_n, π_n and τ_n being some positive constants, $\hat{\delta}_n$ is the estimate of $\delta_n^* = \varepsilon_n + \|\boldsymbol{\theta}^{n*}\|_1$, $\omega_n = \|\hat{\boldsymbol{\theta}}^{nT}\hat{\xi}'_{c^n}\|_1\bar{c}^n +$

$\|\hat{\boldsymbol{\theta}}^{nT}\hat{\xi}'_{\sigma^n}\|_1\bar{\sigma}^n+\|\hat{\xi}'_{c^n}\hat{\boldsymbol{c}}^n+\hat{\xi}'_{\sigma^n}\hat{\boldsymbol{\sigma}}^n\|_1\bar{\theta}^n$, and the adaptive laws are

$$\begin{aligned}&\dot{\hat{\boldsymbol{\theta}}}^n=Proj[\Gamma_{\theta^n}z_n(\hat{\boldsymbol{\xi}}^n-\hat{\xi}'_{c^n}\hat{\boldsymbol{c}}^n-\hat{\xi}'_{\sigma^n}\hat{\boldsymbol{\sigma}}^n)-R_{\theta^n}\hat{\boldsymbol{\theta}}^n]\\&\dot{\hat{\boldsymbol{c}}}^n=Proj[\Gamma_{c^n}z_n\hat{\xi}'^{\rm T}_{c^n}\hat{\boldsymbol{\theta}}^n-R_{c^n}\hat{\boldsymbol{c}}^n]\\&\dot{\hat{\boldsymbol{\sigma}}}^n=Proj[\Gamma_{\sigma^n}z_n\hat{\xi}'^{\rm T}_{\sigma^n}\hat{\boldsymbol{\theta}}^n-R_{\sigma^n}\hat{\boldsymbol{\sigma}}^n]\\&\dot{\hat{\delta}}_n=\gamma_nz_n-r_n\hat{\delta}_n\\&\dot{\hat{b}}=\gamma_b[1-tanh(1)]|z_n|-r_b\hat{b}\end{aligned}\tag{33}$$

with R_{θ^n}, R_{c^n} and R_{σ^n} being positive matrices with proper dimensions, r_n and r_b are positive constants, and taking (28) into account with $i=n-1$, (31) can be rewritten as

$$\begin{aligned}\dot{V}_n\le&\sum_{j=1}^{n}(-q_jz_j^2-\tfrac{\lambda_{\theta j}^{min}}{2}\tilde{\boldsymbol{\theta}}^{jT}\Gamma_{\theta^j}^{-1}\tilde{\boldsymbol{\theta}}^j-\tfrac{\lambda_{cj}^{min}}{2}\tilde{\boldsymbol{c}}^{jT}\Gamma_{c^j}^{-1}\tilde{\boldsymbol{c}}^j\\&-\tfrac{\lambda_{\sigma j}^{min}}{2}\tilde{\boldsymbol{\sigma}}^{jT}\Gamma_{\sigma^j}^{-1}\tilde{\boldsymbol{\sigma}}^j-\tfrac{1}{2\gamma_j}\tilde{\delta}_j^2-\tfrac{1}{2\gamma_b}\tilde{b}^2)+\sum_{j=1}^{n}[\kappa(\pi_j\\&+\tau_j)+\tfrac12\boldsymbol{\theta}^{j*T}\Gamma_{\theta^j}^{-1}R_{\theta j}\tilde{\boldsymbol{\theta}}^{j*}+\tfrac12\boldsymbol{c}^{j*T}\Gamma_{c^j}^{-1}R_{cj}\boldsymbol{c}^{j*}\\&+\tfrac12\boldsymbol{\sigma}^{j*T}\Gamma_{\sigma^j}^{-1}R_{\sigma j}\boldsymbol{\sigma}^{j*}+\tfrac{1}{2\gamma_j}\delta_j^{*2}+\tfrac{1}{2\gamma_b}b^2]\end{aligned}\tag{34}$$

with $\lambda_{\theta j}^{min}$, λ_{cj}^{min} and $\lambda_{\sigma j}^{min}$ being the minimal eigenvalues of $R_{\theta j}$, R_{cj} and $R_{\sigma j}$, $1\le j\le n$, respectively. Then it is ready to give the main result next.

2.3 Main Result

The main result is summarized in the following theorem.

Theorem 1. Consider the unknown nonlinear system (1) which satisfies Assumptions 1-3, the designed control law (32) and the adaptive laws (33), together with the intermediate variables (15), (22), (25) and the parameter updating laws (16), (23), (26) in the design steps can ensure all signals in the closed-loop system remain bounded. Furthermore, for any given value $\epsilon_0>0$, the tracking error z_1 meets $\lim_{t\to\infty}\|z_1\|^2\le\epsilon_0^2$

Proof. Let $\underline{g}_i=g_{il}$ and $\bar{g}_i=g_{iu}$ for $1\le i\le n-1$, $\underline{g}_n=\underline{m}g_{nl}$ and $\bar{g}_n=\bar{m}g_{nu}$, then from Assumption 1 and Assumption 3, one can get $\bar{g}_i^{-1}\le g_i^{-1}(\bar{\boldsymbol{x}}_i)\le\underline{g}_i^{-1}$ for $1\le i\le n-1$ and $\bar{g}_n^{-1}\le g_n^{-1}(\boldsymbol{x})(m(t)+\phi(\boldsymbol{x}))^{-1}\le\underline{g}_n^{-1}$. Then, from Lemma 2 it follows

$$-\frac{1}{2\underline{g}_i}z_i^2\le-\int_0^{z_i}\rho P_i(\bar{\boldsymbol{x}}_{i-1},\rho+\alpha_{i-1})\mathrm{d}\rho\quad 1\le i\le n\tag{35}$$

It can be concluded from (34) and (35) that

$$\begin{aligned}\dot{V}_n\le&\sum_{j=1}^{n}[-2\underline{g}_jq_j\textstyle\int_0^{z_j}\rho P_j(\bar{\boldsymbol{x}}_{j-1},\rho+\alpha_{j-1})\mathrm{d}\rho-\tfrac{\lambda_{\theta j}^{min}}{2}\tilde{\boldsymbol{\theta}}^{jT}\\&\Gamma_{\theta^j}^{-1}\tilde{\boldsymbol{\theta}}^j-\tfrac{\lambda_{cj}^{min}}{2}\tilde{\boldsymbol{c}}^{jT}\Gamma_{c^j}^{-1}\tilde{\boldsymbol{c}}^j-\tfrac{\lambda_{\sigma j}^{min}}{2}\tilde{\boldsymbol{\sigma}}^{jT}\Gamma_{\sigma^j}^{-1}\tilde{\boldsymbol{\sigma}}^j-\tfrac{1}{2\gamma_j}\tilde{\delta}_j^2\\&-\tfrac{1}{2\gamma_b}\tilde{b}^2]+\sum_{j=1}^{n}[\kappa(\pi_j+\tau_j)+\tfrac12\boldsymbol{\theta}^{j*T}\Gamma_{\theta^j}^{-1}R_{\theta j}\tilde{\boldsymbol{\theta}}^{j*}\\&+\tfrac12\boldsymbol{c}^{j*T}\Gamma_{c^j}^{-1}R_{cj}\boldsymbol{c}^{j*}+\tfrac12\boldsymbol{\sigma}^{j*T}\Gamma_{\sigma^j}^{-1}R_{\sigma j}\boldsymbol{\sigma}^{j*}\\&+\tfrac{1}{2\gamma_j}\delta_j^{*2}+\tfrac{1}{2\gamma_b}b^2]\\\le&-\mu V_n+\beta\end{aligned}\tag{36}$$

where $\mu=min\{2\underline{g}_jq_j,\ \lambda_{\theta j}^{min},\ \lambda_{cj}^{min},\ \lambda_{\sigma j}^{min}\}$ and $\beta=\sum_{j=1}^{n}[\kappa(\pi_j+\tau_j)+\frac12\boldsymbol{\theta}^{j*T}\Gamma_{\theta^j}^{-1}R_{\theta j}\tilde{\boldsymbol{\theta}}^{j*}+\frac12\boldsymbol{c}^{j*T}\Gamma_{c^j}^{-1}R_{cj}\boldsymbol{c}^{j*}+\frac12\boldsymbol{\sigma}^{j*T}\Gamma_{\sigma^j}^{-1}R_{\sigma j}\boldsymbol{\sigma}^{j*}+\frac{1}{2\gamma_j}\delta_j^{*2}+\frac{1}{2\gamma_b}b^2]$. Then for $t>0$

$$V_n\le[V_n(0)-\frac{\beta}{\mu}e^{-\mu t}]+\frac{\beta}{\mu}\tag{37}$$

From Assumption 2, b is a nonnegative bounded constant, besides, π_j, τ_j, $\Gamma_{\theta^j}^{-1}R_{\theta j}$, $\Gamma_{c^j}^{-1}R_{cj}$, $\Gamma_{\sigma^j}^{-1}R_{\sigma j}$, γ_j and γ_b are all determined by the designer, so β is bounded and can be designed as small as possible to obtain the desired tracking performance. It can be seen from (37) that z_i, $\tilde{\boldsymbol{\theta}}^i$, $\tilde{\boldsymbol{c}}^i$, $\tilde{\boldsymbol{\sigma}}^i$, $\tilde{\delta}_i$ and $\tilde{b}$ are bounded by the set $\Omega_s=\{(z_i,\tilde{\boldsymbol{\theta}}^i,\tilde{\boldsymbol{c}}^i,\tilde{\boldsymbol{\sigma}}^i,\tilde{\delta}_i,\tilde{b})|V_n\le max(V_n(0),\frac{\beta}{\mu})\}$. Thus it can be deduced that x_i $\hat{\boldsymbol{\theta}}^i$, $\hat{\boldsymbol{c}}^i$, $\hat{\boldsymbol{\sigma}}^i$, $\hat{\delta}_i$ and $\hat{b}$ remain bounded for bounded $V_n(0)$.

Note that μ and β can be tuned by choosing different design parameters, then one can always select appropriate parameters such that for any $\epsilon_0>0$, the inequality $\frac{\beta}{\mu}\le\epsilon_0^2/(2\bar{g}_1)$ is true. Then according to Assumption 3 and Lemma 2, the following inequalities can be obtained.

$$\frac{1}{2\bar{g}_1}\parallel z_1^2\parallel\le\int_0^{z_1}\rho P_1(\rho+y_r)\mathrm{d}\rho\le V_1\tag{38}$$

From this, we can further get that

$$\lim_{t\to\infty}\parallel z_1\parallel^2\le\lim_{t\to\infty}2\bar{g}_1V_1\le2\bar{g}_1\frac{\beta}{\mu}\le\epsilon_0^2\tag{39}$$

This proves that the tracking error can be made as small as possible by appropriately choosing the design parameters. So far Theorem 1 has been proved. □

Remark 2. Compared with the existing methods [8-12], the control design presented in this paper has the following advantages:

1) Unlike [8-12] where η is considered as a disturbance-like term, we further treat η as the sum of a nonlinear term and a disturbance-like term $\psi(x)$, where the nonlinear term can be approximated by fuzzy logic system together with the unknown system functions, and $\psi(x)$ has smaller upper bound than η, so the control effort needed to eliminate the disturbance-like term will be smaller than the one that treats η totally as a disturbance-like term;

2) The design of robust control against the disturbance-like term can be designed with neither the bound of $m(t)+\phi(x)$ nor the bound of b, while the exiting methods need at least one of them;

3) The considered system (1) is more general than in [8-12], since there are unknown nonlinear functions in the dynamics of each x_i $1\le i\le n$. Furthermore, the considered dead-zone model in this paper is with time-varying and perturbed slope which the existing methods cannot be applicable.

3 Simulation Example

We consider a dead-zone nonlinear system as follows.

$$\begin{aligned}&\dot{x}_1=f_1(x_1)+g_1(x_1)x_2\\&\dot{x}_2=f_2(\boldsymbol{x})+g_2(\boldsymbol{x})\mathscr{D}(u)\\&y=x_1\end{aligned}\tag{40}$$

where $f_1(x_1)=0.5x_1^2$, $f_2(\boldsymbol{x})=x_1x_2-2$, $g_1(x_1)=1+0.1x_1^2$, $g_2(\boldsymbol{x})=2+\cos(x_1x_2)$. $\mathscr{D}(u)$ is defined as (2) with $m(t)=1.25\mathrm{e}^{(-0.01t)}$, $\phi(\boldsymbol{x})=0.1\sin(x_1)$, and $b=10$. The reference signals are generated from the following system:

$$\begin{aligned}&\dot{x}_{r1}=x_{r2}\\&\dot{x}_{r2}=-x_{r1}+0.001(1-x_{r1}^2)x_{r2}\\&y_r=x_{r1},\qquad i=1,2\end{aligned}\tag{41}$$

The initial conditions are chosen as $\boldsymbol{x}_r(0)=(1.5,0.8)^T$, $\boldsymbol{x}(0)=(0.5,2)^T$. Two fuzzy logic systems with 11 fuzzy rules for each on are used as approximators in the backstepping design. The initial estimate values are $\hat{\boldsymbol{\theta}}^1(0) = \hat{\boldsymbol{\theta}}^2(0) = \mathbf{0} \in \mathbf{R}^{11}$, $\hat{\boldsymbol{c}}^1(0) = \hat{\boldsymbol{c}}^2(0) = (-10,-8,-6,-4,-2,0,2,4,6,8,10)^T$, $\hat{\boldsymbol{\sigma}}^1(0) = \hat{\boldsymbol{\sigma}}^2(0) = 0.5\boldsymbol{I}_{11}$ with $\boldsymbol{I}_{11}$ a unit column vector in $\mathbf{R}^{11}$, $\hat{\delta}_1(0) = \hat{\delta}_2(0) = 0$, $\hat{b}(0) = 1$. The design parameters are chosen as $\bar{\theta}^i = 1$, $\bar{c}^i = 10$, $\bar{\sigma}^i = 0.5$, $q_i = 1.5$, $\Gamma_{\theta^i} = 1.5I_{11\times 11}$, $\Gamma_{c^i} = 1.5I_{11\times 11}$, $\Gamma_{\sigma^i} = 1.5I_{11\times 11}$, $R_{\theta^i} = 0.1I_{11\times 11}$, $R_{c^i} = 0.1I_{11\times 11}$, $R_{\sigma^i} = 0.1I_{11\times 11}$, where $I_{11\times 11}$ is the unit matrix, $\gamma_i = 1.5$, $r_i = 0.1$, $\pi_i = 0.5$, $\tau_i = 0.5$, for $i = 1,2$, $\gamma_b = 1.5$, $r_b = 0.1$.

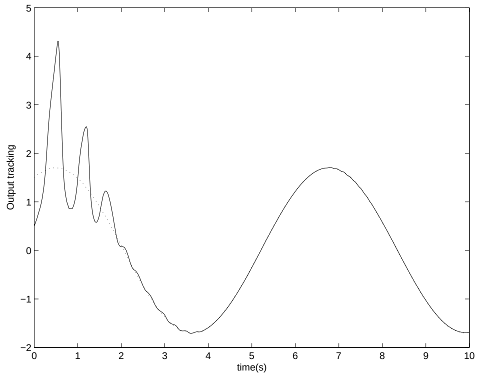

Fig. 1 The output tracking curves of the dead-zone control: y_r(dash), y(solid)

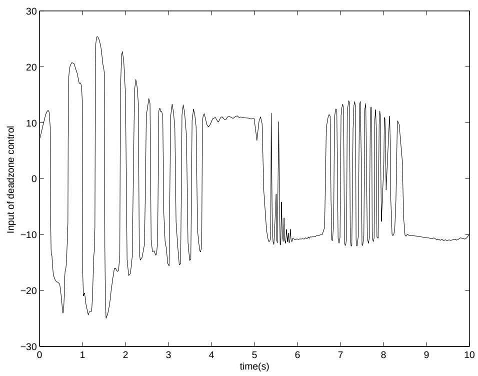

Fig. 2 The control input of the dead-zone actuator u

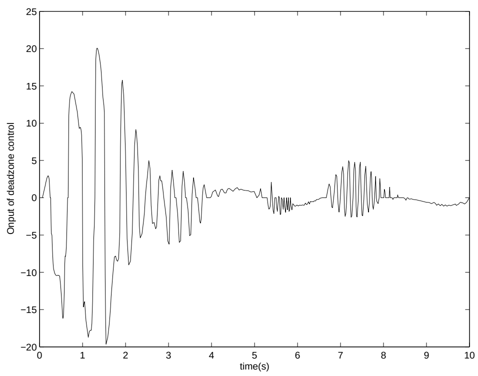

Fig. 3 The output of the dead-zone actuator $\mathscr{D}(u)$

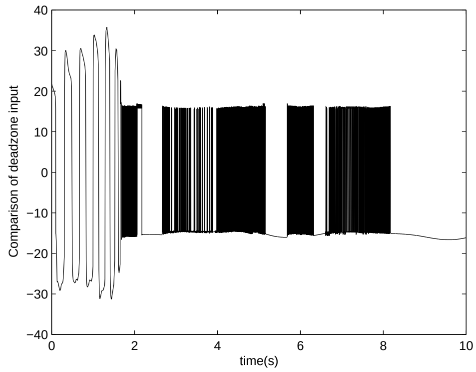

Fig. 4 The input of the dead-zone actuator u from the controller which is designed by viewing η as a disturbance-like term totally

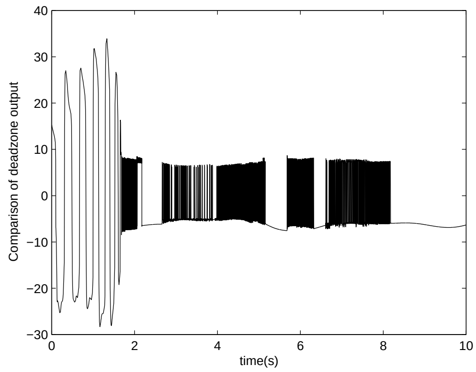

Fig. 5 The output of the dead-zone actuator $\mathscr{D}(u)$ driven by the controller which is designed by viewing η as a disturbance-like term totally

The simulation results are showed in Figs. 1-3 where the output tracking, control output of the controller and the control input of the plant are plotted respectively. In order to show that the proposed scheme is less conservative by specially treat the dead-zone nonlinearity, we also design a controller where η is viewed as a disturbance-like term totally, and the input and output of the dead-zone actuator are plotted in Figs. 4-5 respectively, it can be seen obviously that the control is more conservative than the ones in Figs. 2-3.

Remark 3. The proposed control approach needs a fuzzy approximator in each design step, so the computation burden may be heavy when the controlled system is high-order and the fuzzy logic systems have lots of rules. Generally speaking, the more fuzzy rules can lead the more exact approximation to the unknown functions. Therefore, the number of the fuzzy rules should be made as small as possible on the premise that the desired control can be obtained. As additional control is employed to compensate approximation error, few rules are needed for each fuzzy approximator to get a good control performance. Then the online computation burden can be reduced so that the controller can be achieved timely for application. We have tested the time needed for realizing the controller of Example 1 in matlab, it is 0.16s. The computer we have is with Pentium 4, 2.93 GHz CPU and 512 MB RAM.

4 Conclusion

In this paper, a class of unknown nonlinear systems with time-varying and perturbed dead-zone inputs has been successfully controlled by an adaptive fuzzy control scheme. Since the system considered are not restricted to be feedback linearizable, backstepping technique is employed to obtain the controller step by step. In each step, a nonlinearly parameterized fuzzy logic system is used to approximate the packaged unknown function because there is no much apriori knowledge about the fuzzy membership functions. Adaptive laws are given based on Lyapunov stability to update the parameters online, so that the tracking error can be made as small as possible. The dead-zone width is estimated explicitly, thus the control scheme has the ability to adapt the width of the dead-zone actuator. By specially treating the dead-zone characteristic as a perturbed linear-like term, a nonlinear term and a disturbance-like term, the robustness of the system can be achieved by less control efforts, thus the control law obtained is less conservative. It is proved in theory and showed in simulation that the closed-loop system is stable and the output tracks the given reference signal satisfactorily.

References

1 Tao G, Kokotovic P V. Adaptive control of plants with unknown dead-zones. *IEEE Transactions on Automatic Control*, 1994, **39**(1): 59 - 68

2 Zhou J, Wen C Y, Zhang Y. Adaptive output control of nonlinear systems with uncertain dead-zone nonlinearity. *IEEE Transactions on Automatic Control*, 2006, **51**(3): 504 - 511

3 Lewis F L, Woo K T, Wang L Z, Li Z X. Deadzone compensation in motion control systems using adaptive fuzzy logic control. *IEEE Transactions on Control Systems Technology*, 1999, **7**(6): 731 - 742

4 Jang J O. A deadzone compensator of a DC motor system using fuzzy logic control. *IEEE Transactions on Man, Systems and Cybernetics, Part C: Applications and Reviews*, 2001, **31**(1): 42 - 48

5 Wang X S, Su C Y, Hong H. Robust adaptive control of a class of nonlinear systems with unknown dead-zone. *Automatica*, 2004, **40**(3): 407-413

6 Ibrir S, Xie W F, Su C Y. Adaptive tracking of nonlinear systems with non-symmetric dead-zone input. *Automatica*, 2007, **43**(3): 522-530

7 Mei J D, Zhang T P, Wang Q. Adaptive neural network control with unknown dead-zone and gain sign. In: Proceedings of Chinese Control Conference. Zhangjiajie, China: IEEE, 2007. 299 - 303

8 Zhang T P, Yang Y. Adaptive fuzzy control for a class of MIMO nonlinear systems with unknown dead-zones. *Acta Automatica Sinica*, 2007, **33**(1): 96-100

9 Zhang T P, Ge S S. Adaptive neural control of MIMO nonlinear state time-varying delay systems with unknown dead-zones and gain signs.*Automatica* , 2007, **43**(6): 1021-1033

10 Wang L X, Mendel J M. Fuzzy basis functions, universal approximation, and orthogonal least-squares learning. *IEEE Transactions on Neural Networks*, 1992, **3**(5): 807 - 814

11 Wang L X. Adaptive Fuzzy Systems and Control: Design and Stability Analysis. Upper Saddle River, NJ: Prenctice-Hall, 1994.

12 Chen B S, Lee C H, Chang Y C. H_∞ tracking design of uncertain nonlinear SISO systems: adaptive fuzzy approach. *IEEE Transactions on Fuzzy Systems*, 1996, **4**(1): 32 -43

13 Han H G, Su C Y, Stepanenko Y. Adaptive control of a class of nonlinear systems with nonlinearly parameterized fuzzy approximators. *IEEE Transactions on Fuzzy Systems*, 2001, **9**(2): 315 - 323

14 Hojati M, Gazor S. Hybrid adaptive fuzzy identification and control of nonlinear systems. *IEEE Transactions on Fuzzy Systems*, 2002, **10**(2): 198 - 210

15 Li H X, Tong S C. A hybrid adaptive fuzzy control for a class of nonlinear MIMO systems. *IEEE Transactions on Fuzzy Systems*, 2003, **11**(1): 24-34

16 Yang Y S, Feng G, Ren J S. A combined backstepping and small-gain approach to robust adaptive fuzzy control for strict-feedback nonlinear systems. *IEEE Transactions on Systems, Man and Cybernetics, Part A*, 2004, **34**(3): 406 - 420

17 Wang M, Chen B, Liu X P, Shi P. Adaptive fuzzy tracking control for a class of perturbed strict-feedback nonlinear time-delay systems. *Fuzzy Sets and Systems*, 2008, **159**(8): 949-967

18 Chen B, Liu X P, Tong S C. Adaptive fuzzy output tracking control of MIMO nonlinear uncertain systems. *IEEE Transactions on Fuzzy Systems*, 2007, **15**(2): 287 - 300

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