Krein Space-based H_∞ Fault Estimation for Linear Discrete Time-varying Systems

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Abstract This paper deals with the problem of H_∞ fault estimation for a class of linear discrete time-varying systems with L_2-norm bounded unknown input. The main contribution is the development of a new Krein space-based approach to H_∞ fault estimation. The problem of H_∞ fault estimation is firstly equated to the minimum of a scalar quadratic form. Then, by introducing a corresponding system in Krein space, a sufficient and necessary condition on the existence of an H_∞ fault estimator is derived and a solution to its parameter matrices is obtained in terms of matrix Riccati equation. Finally, two numerical examples are given to demonstrate the efficiency of the proposed method.

Key words H_∞ fault estimation, Krein space, Kalman filtering, matrix Riccati equation

During the past ten years, H_∞ filtering has been an important approach to robust fault detection and many achievements have been obtained for linear time invariant (LTI) systems[1−4]. For linear time varying (LTV) systems, however, few contributions have been published in the literatures[5−6]. An adaptive observer was proposed to generate residual signal[5], and a parity space-based fault detection method was developed in [6]. For general linear discrete time-varying (LDTV) systems with L_2-norm bounded unknown input, research on H_∞ fault estimation is still open and challenging, which motivates the present study.

In contrast, some recent researches on H_∞ filtering for LDTV systems have led to an interesting connection with Kalman filtering in Krein space[7−11]. It has been shown that a finite horizon linear estimation problem for LDTV systems can be cast into a problem of calculating the minimum point of a certain quadratic form and, by applying linear estimation in Krein space, one can calculate recursively the minimum point via Riccati equation. Comparing with linear estimation approaches in Hilbert space, the ones in Krein space are much more computationally attractive and, because of the existence of necessary and sufficient conditions, less conservative.

The main purpose of this paper is to deal with the problem of H_∞ fault estimation for LDTV systems. The main idea is to build a relationship between H_∞ fault estimation and Kalman filtering in Krein space. It will be shown that the problem of H_∞ fault estimation for an LDTV system can be equated to that of linear estimation associated with a corresponding Krein space stochastic system and an H_∞ fault estimator can be obtained by solving a Kalman filtering problem in Krein space.

Notations. Elements in Krein space will be denoted by boldface letters, and elements in the Euclidean space of complex numbers will be denoted by normal letters. Whenever the Krein space elements and the Euclidean space elements satisfy the same set of constraints, we will denote them by the same letters, with the former ones being boldface and the latter ones being normal. The superscripts "−1" and "T" stand for the inverse and transpose of a matrix, respectively. $\delta_{ij}=0$ for $i\neq j$ and $\delta_{ii}=1$. I is the identity matrix with appropriate dimensions. For a real symmetric matrix P, $P>0$ (respectively, $P<0$) means that P is a real positive definite (respectively, negative definite) matrix. $\hat{\theta}(i|j)$ stands for the Krein space projection of $\theta(i)$ onto linear space $L\left\{\boldsymbol{y}_f(0),\cdots,\boldsymbol{y}_f(j-1),\boldsymbol{y}(j)\right\}$. diag$\{A_1,A_2,\cdots,A_m\}$ denotes a block-diagonal matrix, where $A_i\ (i=1,\cdots,m)$ is the element in diagonal.

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1 Problem statement

Consider the following LDTV system

$$\left\{\begin{array}{l} x(k+1)=\Phi(k)x(k)+\Gamma(k)d(k)+R_1(k)f(k)\\ y(k)=H(k)x(k)+v(k)+R_2(k)f(k)\\ x(0)=x_0\end{array}\right. \tag{1}$$

where $x(k)\in\mathbf{R}^n$, $y(k)\in\mathbf{R}^q$, $d(k)\in\mathbf{R}^p$, $v(k)\in\mathbf{R}^q$, and $f(k)\in\mathbf{R}^r$ represent the state, output measurement, unknown input, measurement noise, and fault to be estimated, respectively. $d(k)$, $v(k)$, and $f(k)$ are $l_2[0,N]$ bounded, and N is an integer. $\Phi(k)$, $\Gamma(k)$, $R_1(k)$, $H(k)$, and $R_2(k)$ are known matrices with appropriate dimensions. Without loss of generality, it is assumed that $[\Phi(k)\quad \Gamma(k)]$ has full row rank for all k.

Remark 1. For a system described by state space representation, there exist a number of ways to model faults and, because of the way how they affect the system dynamics, the faults are usually divided into two categories, i.e., additive faults and multiplicative faults. Typical additive faults met in practice are offset in sensors and actuators or drift in sensors, while changes in model parameters caused by malfunctions in the process or in the sensors and actuators are often called multiplicative faults. As summarized in [2], the multiplicative faults can be modelled as additive faults sometime. In this paper, we focus our study on dealing with additive faults.

The H_∞ fault estimation problem under investigation is stated as: given a scalar $\gamma>0$, to find a fault estimator such that

$$\frac{\displaystyle\sum_{i=0}^{N} r_f^{\mathrm{T}}(k)r_f(k)}{x_0^{\mathrm{T}}P_0^{-1}x_0+\displaystyle\sum_{i=0}^{N} w^{\mathrm{T}}(k)w(k)}<\gamma^2 \tag{2}$$

where

$$w(k)=\left\{\begin{array}{ll}\left[\begin{array}{ccc} d^{\mathrm{T}}(k) & f^{\mathrm{T}}(k) & v^{\mathrm{T}}(k)\end{array}\right]^{\mathrm{T}} & \text{for } k=0,1,\cdots,N-1\\ \left[\begin{array}{cc} f^{\mathrm{T}}(k) & v^{\mathrm{T}}(k)\end{array}\right]^{\mathrm{T}} & \text{for } k=N\end{array}\right.$$

$$r_f(k)=\breve{f}(k)-f(k),\quad k=0,1,\cdots,N$$

$\breve{f}(k)$ denotes the estimate of $f(k)$, and P_0 is a given positive definite weighting matrix.

Remark 2. Under zero initial condition, the fault estimation on the basis of (2) can be regarded as a finite horizon version of H_∞ filtering formulation in [3], i.e., to find a fault estimate such that the $l_2[0, N]$-induced gain from unknown input to fault estimate error is less than γ.

Define

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ f(k) \end{bmatrix}, \quad \bar{d}(k) = \begin{bmatrix} d(k) \\ f(k+1) \end{bmatrix}$$

And rewrite (1) into the following augmented system

$$\begin{cases} \bar{x}(k+1) = \bar{\Phi}(k)\bar{x}(k) + \bar{\Gamma}(k)\bar{d}(k) \\ y(k) = \bar{H}(k)\bar{x}(k) + v(k) \\ \breve{f}(k) = \bar{L}(k)\bar{x}(k) + r_f(k) \\ \bar{x}(0) = \begin{bmatrix} x_0^{\rm T} & 0 \end{bmatrix}^{\rm T} \end{cases} \tag{3}$$

where

$$\begin{aligned} \bar{\Phi}(k) &= \begin{bmatrix} \Phi(k) & R_1(k) \\ 0 & 0 \end{bmatrix} \\ \bar{\Gamma}(k) &= \begin{bmatrix} \Gamma(k) & 0 \\ 0 & I \end{bmatrix} \\ \bar{H}(k) &= \begin{bmatrix} H(k) & R_2(k) \end{bmatrix} \\ \bar{L}(k) &= \begin{bmatrix} 0 & I \end{bmatrix} \end{aligned}$$

Then, we consider the following fault estimator

$$\begin{cases} \hat{\bar{x}}(k+1) = F(k)\hat{\bar{x}}(k) + G(k)y(k) \\ \breve{f}(k) = M_1(k)\hat{\bar{x}}(k) + M_2(k)y(k) \\ \hat{\bar{x}}(0) = 0 \end{cases} \tag{4}$$

Subsequently, the H_∞ fault estimation problem can be reformulated as to find $F(k)$, $G(k)$, $M_1(k)$, and $M_2(k)$, such that (2) is satisfied.

Let

$$J_N = \begin{bmatrix} \bar{x}_0 \\ \bar{d}_{N-1} \\ v_N \\ r_{f,N} \end{bmatrix}^{\rm T} \begin{bmatrix} \bar{P}_0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}^{-1} \begin{bmatrix} \bar{x}_0 \\ \bar{d}_{N-1} \\ v_N \\ r_{f,N} \end{bmatrix}$$

where

$$\begin{aligned} \bar{P}_0 &= {\rm diag}\{P_0, I\} \\ v_N &= \begin{bmatrix} v^{\rm T}(0) & v^{\rm T}(1) & \cdots & v^{\rm T}(N) \end{bmatrix}^{\rm T} \\ \bar{d}_{N-1} &= \begin{bmatrix} \bar{d}^{\rm T}(0) & \bar{d}^{\rm T}(1) & & \bar{d}^{\rm T}(N-1) \end{bmatrix}^{\rm T} \\ r_{f,N} &= \begin{bmatrix} r_f^{\rm T}(0) & r_f^{\rm T}(1) & \cdots & r_f^{\rm T}(N) \end{bmatrix}^{\rm T} \end{aligned}$$

Now, the H_∞ fault estimation problem is equivalent to:

1) Solving the minimum problem of J_N with respect to $\bar{x}_0$, v_N and $\bar{d}_{N-1}$;

2) Choosing $\breve{f}(k)$ such that the value of J_N at its minimum is positive.

According to [9−10], the minimum of J_N over $\bar{x}_0$, v_N, and $\bar{d}_{N-1}$ can be derived by using Kalman filtering theory in Krein space. In the following, we will first define an associated Krein space stochastic system. Then, Kalman filtering theory in Krein space can be applied to solve the minimum problem of J_N. Finally, a solution to parameter matrices $F(k)$, $G(k)$, $M_1(k)$, and $M_2(k)$ will be derived such that $J_N > 0$.

2 Design of an H_∞ fault estimator

First, we introduce the following Krein space stochastic system corresponding to (3)

$$\begin{cases} \boldsymbol{x}(k+1) = \bar{\Phi}(k)\boldsymbol{x}(k) + \bar{\Gamma}(k)\boldsymbol{d}(k) \\ \boldsymbol{y}(k) = \bar{H}(k)\boldsymbol{x}(k) + \boldsymbol{v}(k) \\ \boldsymbol{z}(k) = \bar{L}(k)\boldsymbol{x}(k) + \boldsymbol{r}_f(k) \\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \end{cases} \tag{5}$$

where $\bar{\Phi}(k)$, $\bar{\Gamma}(k)$, $\bar{H}(k)$, and $\bar{L}(k)$ are the same as in (3); $\boldsymbol{x}(k)$ is a state vector; $\boldsymbol{d}(k)$, $\boldsymbol{v}(k)$, and $\boldsymbol{r}_f(k)$ are input vectors; $\boldsymbol{y}(k)$ and $\boldsymbol{z}(k)$ are output vectors; and $\boldsymbol{x}_0$, $\boldsymbol{d}(k)$, $\boldsymbol{v}(k)$, and $\boldsymbol{r}_f(k)$ are uncorrelated random vectors with zero means and the following covariance matrix

$$\left\langle \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{d}(i) \\ \boldsymbol{v}(i) \\ \boldsymbol{r}_f(i) \end{bmatrix}, \begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{d}(j) \\ \boldsymbol{v}(j) \\ \boldsymbol{r}_f(j) \end{bmatrix} \right\rangle = \begin{bmatrix} \bar{P}_0 & 0 & 0 & 0 \\ 0 & I\delta_{ij} & 0 & 0 \\ 0 & 0 & I\delta_{ij} & 0 \\ 0 & 0 & 0 & -\gamma^2 I\delta_{ij} \end{bmatrix} \tag{6}$$

Let

$$\boldsymbol{y}_f(k) = \begin{bmatrix} \boldsymbol{y}(k) \\ \boldsymbol{z}(k) \end{bmatrix}, \quad \boldsymbol{v}_{yf}(k) = \begin{bmatrix} \boldsymbol{v}(k) \\ \boldsymbol{r}_f(k) \end{bmatrix}$$

where

$$\langle \boldsymbol{v}_{yf}(i), \boldsymbol{v}_{yf}(j) \rangle = R_{yf}(i)\delta_{ij}, \quad R_{yf}(i) = {\rm diag}\left\{I, -\gamma^2 I\right\} \tag{7}$$

It follows from (5) that

$$\begin{cases} \boldsymbol{x}(k+1) = \bar{\Phi}(k)\boldsymbol{x}(k) + \bar{\Gamma}(k)\boldsymbol{d}(k) \\ \boldsymbol{y}_f(k) = \begin{bmatrix} \bar{H}(k) \\ \bar{L}(k) \end{bmatrix} \boldsymbol{x}(k) + \boldsymbol{v}_{yf}(k) \\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \end{cases} \tag{8}$$

Denote

$$\begin{aligned} \boldsymbol{e}(k) &= \boldsymbol{y}_f(k) - \hat{\boldsymbol{y}}_f(k|k-1) \\ R_e(k) &= \langle \boldsymbol{e}(k), \boldsymbol{e}(k) \rangle \\ \boldsymbol{e}_k &= \begin{bmatrix} \boldsymbol{e}^{\rm T}(0) & \boldsymbol{e}^{\rm T}(1) & \cdots & \boldsymbol{e}^{\rm T}(k) \end{bmatrix}^{\rm T} \\ R_{ek} &= \langle \boldsymbol{e}_k, \boldsymbol{e}_k \rangle \\ \tilde{\boldsymbol{x}}(k|k-1) &= \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k|k-1) \\ P(k) &= \langle \tilde{\boldsymbol{x}}(k|k-1), \tilde{\boldsymbol{x}}(k|k-1) \rangle \\ P(0) &= \bar{P}_0 \end{aligned}$$

where

$$\hat{\boldsymbol{y}}_f(k|k-1) = \begin{bmatrix} \bar{H}(k) \\ \bar{L}(k) \end{bmatrix} \hat{\boldsymbol{x}}(k|k-1)$$

Applying Kalman filtering theory in Krein space[7−8], the following result can be obtained.

Lemma 1. Given a scalar $\gamma > 0$, the minimum problem of J_N subject to (3) is solvable if and only if $R_e(k)$ and $R_{yf}(k)$ have the same inertia for all $k = 0, 1, \cdots, N$. If this is the case, the minimum of J_N is

$$\min_{\bar{x}_0, \bar{d}(k)} J_N = e_N^{\rm T} R_{eN}^{-1} e_N \tag{9}$$

where

$$\begin{cases} e_N = \begin{bmatrix} e^{\rm T}(0) & e^{\rm T}(1) & \cdots & e^{\rm T}(N) \end{bmatrix}^{\rm T} \\ e(k) = \begin{bmatrix} e_y^{\rm T}(k) & e_f^{\rm T}(k) \end{bmatrix}^{\rm T} \\ e_y(k) = y(k) - \bar{H}(k)\hat{\bar{x}}(k|k-1) \\ e_f(k) = \breve{f}(k) - \bar{L}(k)\hat{\bar{x}}(k|k-1) \\ \hat{\bar{x}}(0|-1) = 0, \quad k = 0, 1, \cdots, N \end{cases}$$

and $\hat{\breve{x}}(k|k-1)$ is given by

$$\begin{cases} \hat{\breve{x}}(k+1|k) = \bar\Phi(k)\hat{\breve{x}}(k|k-1) - \\ \qquad K_p(k)\left(\begin{bmatrix} y(k) \\ \breve{f}(k)\end{bmatrix} - \begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}\hat{\breve{x}}(k|k-1)\right) \\ K_p(k) = \bar\Phi(k)P(k)\begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}^{\rm T} R_e^{-1}(k)\end{cases} \quad (10)$$

$P(k)$ is calculated by the following Riccati equation

$$\begin{cases} P(k+1) = \bar\Phi(k)P(k)\bar\Phi^{\rm T}(k) - \bar\Phi(k)P(k)\begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}^{\rm T}\times \\ \qquad R_e^{-1}(k)\begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}P(k)\bar\Phi^{\rm T}(k) + \bar\Gamma(k)\bar\Gamma^{\rm T}(k) \\ P(0) = \bar P_0\end{cases} \quad (11)$$

□

From the definition of $\boldsymbol{e}(k)$ and its product, we have

$$\boldsymbol{e}(k) = \begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}\tilde{\boldsymbol{x}}(k|k-1) + \boldsymbol{v}_{yf}(k)$$

$$R_e(k) = R_{yf}(k) + \begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}P(k)\begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}^{\rm T} = \begin{bmatrix} I + \bar H(k)P(k)\bar H^{\rm T}(k) & \bar H(k)P(k)\bar L^{\rm T}(k) \\ \bar L(k)P(k)\bar H^{\rm T}(k) & -\gamma^2 I + \bar L(k)P(k)\bar L^{\rm T}(k)\end{bmatrix}$$

Define $\bar e_f(k) = \breve f(k) - \hat f(k|k)$ with

$$\hat f(k|k) = \hat f(k|k-1) + \bar L(k)P(k)\bar H^{\rm T}(k)\times (\bar H(k)P(k)\bar H^{\rm T}(k) + I)^{-1}e_y(k) \quad (12)$$

Then, (9) can be re-expressed as

$$\min_{\bar x_0, \bar d(k)} J_N = \sum_{i=0}^N \begin{bmatrix} e_y(k)\\ e_f(k)\end{bmatrix}^{\rm T} R_e^{-1}(k)\begin{bmatrix} e_y(k)\\ e_f(k)\end{bmatrix} = \sum_{i=0}^N \begin{bmatrix} e_y(k)\\ \bar e_f(k)\end{bmatrix}^{\rm T} \bar R_e^{-1}(k)\begin{bmatrix} e_y(k)\\ \bar e_f(k)\end{bmatrix} \quad (13)$$

where

$$\bar R_e(k) = {\rm diag}\left\{\bar H(k)P(k)\bar H^{\rm T}(k) + I, \ \bar L(k)\left(P^{-1}(k) + \bar H^{\rm T}(k)\bar H(k)\right)^{-1}\bar L^{\rm T}(k) - \gamma^2 I\right\}$$

$R_e(k)$ and $\bar R_e(k)$ have the same inertia. Therefore, J_N has the minimum if and only if $\bar R_e(k)$ and $R_{yf}(k)$ have the same inertia for all $k = 0, 1, \cdots, N$.

Moreover, from (11), we have

$$P(k+1) = \begin{bmatrix}\bar\Phi(k) & \bar\Gamma(k)\end{bmatrix}\begin{bmatrix} P(k)\phi(k) & 0 \\ 0 & I\end{bmatrix}\begin{bmatrix}\bar\Phi^{\rm T}(k)\\ \bar\Gamma^{\rm T}(k)\end{bmatrix} = \begin{bmatrix}\bar\Phi(k) & \bar\Gamma(k)\end{bmatrix}\begin{bmatrix}\varphi^{-1}(k) & 0 \\ 0 & I\end{bmatrix}\begin{bmatrix}\bar\Phi^{\rm T}(k)\\ \bar\Gamma^{\rm T}(k)\end{bmatrix}$$

where

$$\phi(k) = I - \begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}^{\rm T} R_e^{-1}(k)\begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}P(k)$$

$$\varphi(k) = P^{-1}(k) + \begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}^{\rm T} R_{yf}^{-1}(k)\begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}$$

Under the assumption that $\begin{bmatrix}\Phi(k) & \Gamma(k)\end{bmatrix}$ is of full row rank, $\begin{bmatrix}\bar\Phi(k) & \bar\Gamma(k)\end{bmatrix}$ is of full row rank too. Therefore, $P(k+1) > 0$ if and only if

$$P^{-1}(k) + \begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}^{\rm T} R_{yf}^{-1}(k)\begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix} > 0$$

i.e.,

$$P^{-1}(k) + \bar H^{\rm T}(k)\bar H(k) - \gamma^{-2}\bar L^{\rm T}(k)\bar L(k) > 0 \quad (14)$$

In this case,

$$\bar L(k)(P^{-1}(k) + \bar H^{\rm T}(k)\bar H(k))^{-1}\bar L^{\rm T}(k) - \gamma^2 I < 0$$

$$\bar H(k)P(k)\bar H^{\rm T}(k) + I > 0$$

So, $\bar R_e(k)$ and $R_{yf}(k)$ have the same inertia if and only if (14) is satisfied.

Let $\bar e_f(k) = 0$, i.e.,

$$\breve f(k) = \hat f(k|k-1) + \bar L(k)P(k)\bar H^{\rm T}(k) (\bar H(k)P(k)\bar H^{\rm T}(k) + I)^{-1}e_y(k) \quad (15)$$

It follows from (13) that

$$\min_{\bar x_0, \bar d(k)} J_N = \sum_{k=0}^N e_y^{\rm T}(k)(\bar H(k)P(k)\bar H^{\rm T}(k) + I)^{-1}e_y(k) > 0$$

which means that the H_∞ fault estimation problem is solvable.

Let $\hat{\breve{x}}(k) = \hat{\breve{x}}(k|k-1)$. Then, the parameter matrices $F(k)$, $G(k)$, $M_1(k)$, and $M_2(k)$ can be derived from (10) and (15) as

$$\begin{cases} G(k) = \bar\Phi(k) - F(k)\bar H(k) \\ F(k) = \bar\Phi(k)P(k)\begin{bmatrix}\bar H(k)\\ \bar L(k)\end{bmatrix}^{\rm T} R_e^{-1}(k)\begin{bmatrix} I \\ M_2(k)\end{bmatrix} \\ M_1(k) = \bar L(k)\left(I + P(k)\bar H^{\rm T}(k)\bar H(k)\right)^{-1} \\ M_2(k) = M_1(k)P(k)\bar H^{\rm T}(k)\end{cases} \quad (16)$$

We now conclude the following result.

Theorem 1. Given a scalar $\gamma > 0$, J_N has a minimum and its value at its minimum is positive if and only if (14) is satisfied. In this case, the fault estimator achieving (2) can be obtained from (4) and (16), where $P(k)$ is calculated recursively from matrix Riccati equation (11).

Remark 3. It should be pointed out that parameter estimation is a widely used approach to multiplicative fault estimation. Theorem 1 provides us an Krein space-based approach to additive fault estimation for LDTV systems with L_2-norm bounded unknown input. In comparison, the former one can achieve more accurate fault estimation but more complicated and computationally time-consuming.

3 Numerical examples

Example 1. Considering system (1) with the following parameters

$$\Phi(k) = \begin{bmatrix} 0.2 & 1 + {\rm e}^{-k} & -0.2\sin(10k) \\ 0 & 0.7 & 0.1 \\ 0 & 0 & 0.3\end{bmatrix}$$

$$\Gamma(k) = \begin{bmatrix} -\dfrac{1.6059}{k} \\ 0.57867 \\ -0.3569\end{bmatrix}, \quad R_1(k) = \begin{bmatrix} -\dfrac{1.6059}{k} \\ 0.57867 \\ -0.3569\end{bmatrix}$$

$$H(k) = \begin{bmatrix} -1.2193 & 1.8369 & 0.4253 \end{bmatrix}, \quad R_2(k) = 8$$

set $x_0 = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}^{\mathrm{T}}$, $P_0 = I$ and $\gamma = 0.1$. We design an H_∞ fault estimator using Theorem 1. Suppose that the unknown input $d(k)$ and the measurement noise $v(k)$ are simulated as in Figs. 1 and 2, respectively. A fault signal (dashed lines) and its estimation (solid lines) are shown in Fig. 3, where we see that a satisfied fault estimation has been obtained.

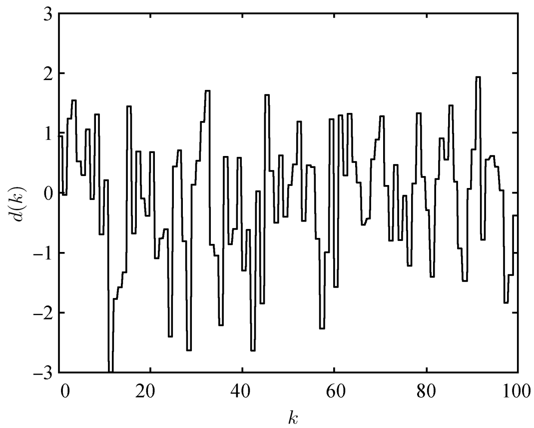

Fig. 1 The unknown input $d(k)$

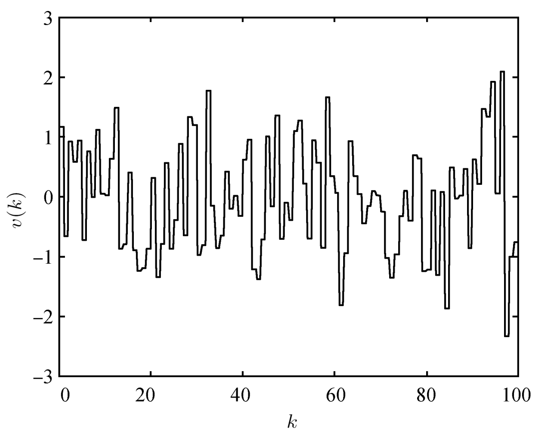

Fig. 2 The measurement noise $v(k)$

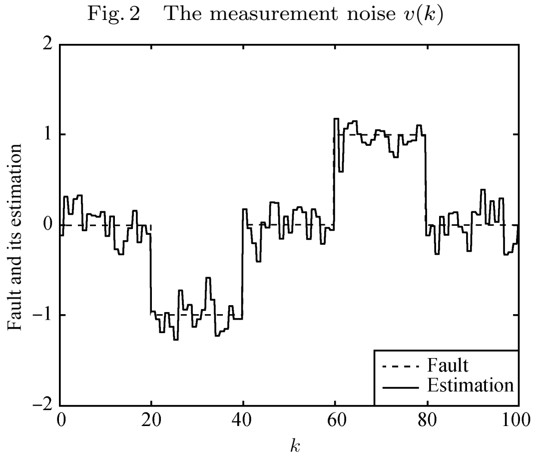

Fig. 3 The fault and its estimation

Example 2. Considering a linear periodic system in [12] which is described by (1) with period 2 and the following parameters

$$\Phi(0) = \begin{bmatrix} 0.25 & 0.25 & 0.1 & -0.1 \\ 0.5 & 0.1 & 0.1 & 0.5 \\ 0.5 & -0.2 & 0.2 & 0.25 \\ 0.1 & 0 & 0.25 & 0.1 \end{bmatrix}$$

$$\Gamma(0) = \begin{bmatrix} 1.3 \\ 1.8 \\ 1.6 \\ 0.32 \end{bmatrix}, \quad R_1(0) = \begin{bmatrix} 0.1 \\ -1 \\ 0.2 \\ 0.1 \end{bmatrix}$$

$$H(0) = \begin{bmatrix} 0.25 & 0.1 & 0.2 & 0.1 \\ -0.1 & 0.5 & 0.2 & 0.5 \\ 0.25 & 0.5 & -0.1 & 0.1 \end{bmatrix}$$

$$R_2(0) = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.4 \end{bmatrix}, \quad R_2(1) = \begin{bmatrix} -0.2 \\ -0.1 \\ 0.3 \end{bmatrix}$$

$$\Phi(1) = \begin{bmatrix} 0.1 & 0.2 & 0.1 & -0.1 \\ -0.1 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.1 & 0.25 \\ 0 & 0.1 & 0.1 & 0.25 \end{bmatrix}$$

$$\Gamma(1) = \begin{bmatrix} 3.2 \\ 2 \\ -1 \\ -2 \end{bmatrix}, \quad R_1(1) = \begin{bmatrix} 0.1 \\ -1 \\ 0.2 \\ 0.1 \end{bmatrix}$$

$$H(1) = \begin{bmatrix} 0.1 & 0.25 & 0.1 & -0.1 \\ 0.25 & 0.1 & 0.2 & 0.1 \\ 0.1 & 0.25 & -0.2 & 0.5 \end{bmatrix}$$

set $x_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$, $P_0 = I$, and $\gamma = 0.3$. We design an H_∞ fault estimator applying with Theorem 1. The unknown input $d(k) = \sin(0.01\pi k)$ is shown in Fig. 4. A fault and its estimation are shown in Fig. 5. When the fault in Fig. 5 occurs, the residual signal generated by parity space- and observer-based approach in [12] are shown in Figs. 6 and 7, respectively. It is seen from the simulation results that fully unknown input decoupling in Fig. 5 is achieved and the obtained fault estimation is almost the same as the fault.

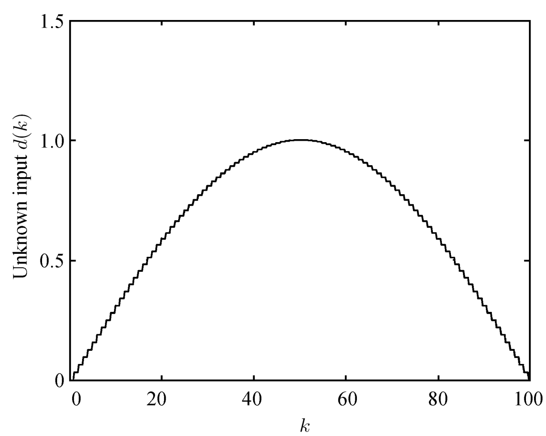

Fig. 4 The unknown input $d(k)$

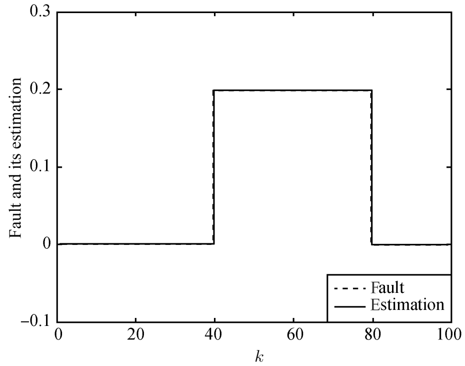

Fig. 5 The fault and its estimation

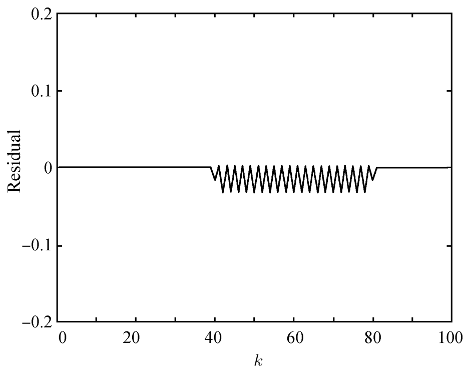

Fig. 6 The residual using parity space approach

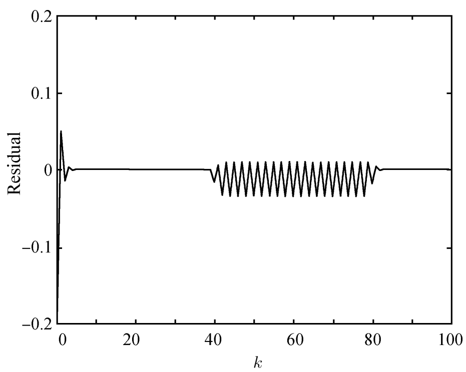

Fig. 7 The residual using observer approach

4 Conclusion

In this paper, the problem of finite-horizon H_∞ fault estimation have been investigated for LDTV systems. A relationship between H_∞ fault estimation and Kalman filtering in Krein space has been built and, based on this, a new approach to H_∞ fault estimation has been proposed for LDTV systems. The problem of H_∞ fault estimation has been equated to a minimization problem of a scalar quadratic form subject to an augmented system firstly. A corresponding Krein space stochastic system has introduced and, by applying Kalman filtering theory in Krein space, a sufficient and necessary solvable condition on the existence of the minimum and a solution to an H_∞ fault estimator have been obtained in terms of Riccati equations. Two numerical examples have been given to show the effectiveness of the proposed method.

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