Delay-dependent Robust Stabilization for Uncertain Singular Systems with Multiple Input Delays

DU Zhao-Ping[1] ZHANG Qing-Ling[1, 2] LIU Li-Li[1]

Abstract In this paper, the problem of delay-dependent robust stabilization is investigated for singular systems with multiple input delays and admissible uncertainties. First, an improved delay-dependent stabilization criterion for the nominal system is established in terms of linear matrix inequalities (LMIs). Then, based on this criterion, the problem is solved via state feedback controller, which guarantees that the resultant closed-loop system is regular, impulse free, and stable for all admissible uncertainties. Numerical examples are provided to illustrate the effectiveness of the proposed method.

Key words Delay-dependent, stability, robust stabilization, singular systems, multiple input delays

Time-delays are often encountered in various dynamic systems, such as manufacturing systems, economic systems, biological systems, networked control systems, etc. They are generally regarded as a main source of instability and poor performance in such systems. In the past decades, the problems of robust stability and robust stabilization for linear time-delay systems have been investigated. Commonly, the existing results can be classified into two types: delay-independent conditions[1−2] and delay-dependent conditions[3−7]. Generally, the delay-independent case is more conservative than the delay-dependent case.

In recent years, much attention has been focused on the problems of robust stability and stabilization analysis for singular time-delay systems. Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems, or semi-state systems[8]. Many fundamental results based on standard state-space systems have been extended to singular systems. However, it is known that the problem for singular systems is much more complicated than that for standard state-space systems.

Over the last few years, the delay-independent case has been extensively studied[9−10]. Recently, the problem of delay-dependent robust stability for uncertain discrete singular time-delay systems has been considered. Ji[11] solved the problem based on the assumption that the system was regular and causal. Ma[12] and Wang[13] discussed the problem for uncertain discrete singular time-varying delay systems. In [12], the delay-dependent robust stabilization result was proposed by transforming the system into a standard state-space system. In [13], Wang investigated the problem of delay-dependent robust H_∞ control for the system based on a finite sum inequality.

For continuous singular time-delay systems, Wu[14], Zhu[15], and Bounkas[16] gave some results on delay-dependent stability and stabilization for single state-delay singular systems. References [17−20] discussed the problem of delay-dependent H_∞ control for single time-delay singular systems. However, to the best of our knowledge, the stability and stabilization problems for continuous singular systems with multiple time-delays have not yet been fully investigated. Particularly, delay-dependent sufficient conditions are few even non-existing in the published works.

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1. Institute of Systems Science, Northeastern University, Shenyang 110004, P. R. China 2. Key Laboratory of Integrated Automation of Process Industry, Ministry of Education, Northeastern University, Shenyang 110004, P. R. China
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In this paper, we consider the problem of delay-dependent robust stabilization for uncertain singular systems with multiple input delays. First, a delay-dependent stabilization criterion for the resultant nominal system is established in terms of LMIs. Then, based on this criterion, the delay-dependent robust stabilization criterion is proposed. Finally, two examples are given to show the effectiveness of the presented results.

The rest of this paper is organized as follows. In Section 2, the problem is formulated. In Section 3, the main results including results on stabilization and robust stabilization are given. In Section 4, numerical examples are presented to illustrate the proposed results effectively. Finally, the conclusion is given in Section 5.

Notations. Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is semi-positive definite (respectively, positive definite).

1 Problem statement

Consider the following uncertain singular system with multiple input delays:

$$\begin{aligned} E\dot{\boldsymbol{x}}(t) &= (A+\Delta A)\boldsymbol{x}(t) + \sum_{i=1}^{n}(B_i+\Delta B_i)\boldsymbol{u}(t-\tau_i) \\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t), \quad t\in[-\bar{\tau},0] \end{aligned} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^{\tilde{n}}$ is the state, $\boldsymbol{u}(t) \in \mathbf{R}^m$ is the control input, τ_i are constant time-delays satisfying $0 < \tau_i \leq \bar{\tau}$, $i = 1, 2, \cdots, n$, $\bar{\tau}$ is the upper bound of τ_i, and $\boldsymbol{\phi}(t)$ is a compatible vector valued initial function. The matrix $E \in \mathbf{R}^{\tilde{n}\times\tilde{n}}$ may be singular and $\text{rank}\,(E) = r \leq \tilde{n}$ is assumed. A, B_i, $i = 1, 2, \cdots, n$ are real constant matrices, and ΔA, ΔB_i, $i = 1, 2, \cdots, n$ are norm-bounded uncertain parameter matrices, and are assumed to be of the following forms:

$$\Delta A = DF(t)H, \quad \Delta B_i = D_iF_i(t)H_i \tag{2}$$

where D, H, D_i, and H_i are known real constant matrices with appropriate dimensions, and $F(t) \in \mathbf{R}^{k\times g}$, $F_i(t) \in \mathbf{R}^{k_i\times g_i}$ are unknown parameter matrices satisfying

$$F^{\mathrm{T}}(t)F(t) \leq I, \quad F_i^{\mathrm{T}}(t)F_i(t) \leq I \tag{3}$$

The parametric uncertainties ΔA and ΔB_i are said to be admissible if both (2) and (3) hold.

Without loss of generality, we can assume that the matrices E and A in (1) have the following forms:

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{4}$$

The purpose of this paper is to develop a delay-dependent robust stabilization condition for uncertain singular time-delay system (1). Then, we can design the state feedback controller

$$\boldsymbol{u}(t) = K\boldsymbol{x}(t)$$

for system (1) such that the following closed-loop system is regular, impulse free, and stable for all time-delays τ_i satisfying $0 < \tau_i \le \bar{\tau}$, $i = 1, 2, \cdots, n$.

$$\begin{aligned} E\dot{\boldsymbol{x}}(t) &= (A + \Delta A)\boldsymbol{x}(t) + \sum_{i=1}^{n}(B_i + \Delta B_i)K\boldsymbol{x}(t - \tau_i) \\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t), \quad t \in [-\bar{\tau}, 0] \end{aligned} \tag{5}$$

where K is a constant matrix to be determined.

The nominal singular time-delay system of (5) can be described as

$$\begin{aligned} E\dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + \sum_{i=1}^{n} B_i K\boldsymbol{x}(t - \tau_i) \\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t), \quad t \in [-\bar{\tau}, 0] \end{aligned} \tag{6}$$

Definition 1[8, 15, 21]**.**

1) The pair (E, A) is said to be regular if $\det(sE - A)$ is not identically zero.

2) The pair (E, A) is said to be impulse free if $\deg(\det(sE - A)) = \text{rank}(E)$.

Definition 2. For a given scalar $\bar{\tau} > 0$, the singular time-delay system (6) is said to be regular and impulse free for all constant time-delays τ_i satisfying $0 < \tau_i \le \bar{\tau}$, $i = 1, 2, \cdots, n$, if the pair (E, A) is regular and impulse free.

Definition 3. The uncertain singular time-delay system (1) is said to be robustly stabilizable, if the resultant closed-loop system (5) is regular, impulse free, and stable for all constant time-delays τ_i satisfying $0 < \tau_i \le \bar{\tau}$, $i = 1, 2, \cdots, n$ and all admissible uncertainties ΔA and ΔB_i.

The following lemmas are used in the proof of the main results.

Lemma 1[22]**.** Assume that $\boldsymbol{a}(\cdot) \in \mathbf{R}^{n_a}$, $\boldsymbol{b}(\cdot) \in \mathbf{R}^{n_b}$, and $\check{N}(\cdot) \in \mathbf{R}^{n_a \times n_b}$ are defined on the interval Ω. Then, for any matrices $X \in \mathbf{R}^{n_a \times n_a}$, $Y \in \mathbf{R}^{n_a \times n_b}$, and $Z \in \mathbf{R}^{n_b \times n_b}$, the following holds:

$$\begin{aligned} &-2\int_{\Omega} \boldsymbol{a}^{\mathrm{T}}(\alpha)\check{N}\boldsymbol{b}(\alpha)\mathrm{d}\alpha \le \\ &\int_{\Omega} \begin{bmatrix} \boldsymbol{a}(\alpha) \\ \boldsymbol{b}(\alpha) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y - \check{N} \\ Y^{\mathrm{T}} - \check{N}^{\mathrm{T}} & Z \end{bmatrix} \begin{bmatrix} \boldsymbol{a}(\alpha) \\ \boldsymbol{b}(\alpha) \end{bmatrix} \mathrm{d}\alpha \end{aligned}$$

where $\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \ge 0$.

Lemma 2[23]**.** Given a symmetric matrix Ω and matrices Γ, Ξ with appropriate dimensions, we have

$$\Omega + \Gamma\Delta\Xi + \Xi^{\mathrm{T}}\Delta^{\mathrm{T}}\Gamma^{\mathrm{T}} < 0$$

for all Δ satisfying $\Delta^{\mathrm{T}}\Delta \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Omega + \varepsilon\Gamma\Gamma^{\mathrm{T}} + \varepsilon^{-1}\Xi^{\mathrm{T}}\Xi < 0$$

Lemma 3[24]**.** For symmetric positive-definite matrix Q and matrices P and R with appropriate dimensions, matrix inequality $P^{\mathrm{T}}R + R^{\mathrm{T}}P \le R^{\mathrm{T}}QR + P^{\mathrm{T}}Q^{-1}P$ holds.

2 Main results

In this section, we discuss the condition of robust stabilization for uncertain singular time-delay system (1). First, we present a delay-dependent criterion guaranteeing system (6) to be regular, impulse free, and stable, which plays an important role in obtaining the delay-dependent condition for system (1).

Theorem 1. For a given state feedback gain K, the nominal singular time-delay system (6) is regular, impulse free, and stable for all time-delays τ_i satisfying $0 < \tau_i \le \bar{\tau}$, $i = 1, 2, \cdots, n$ if there exist matrices $Z_i > 0$, $Q_i > 0$, X_i, Y_i, $i = 1, 2, \cdots, n$ and a nonsingular matrix P, such that the following LMIs hold:

$$E^{\mathrm{T}}P = P^{\mathrm{T}}E \ge 0 \tag{7}$$

$$\begin{bmatrix} X_i & Y_i \\ Y_i^{\mathrm{T}} & E^{\mathrm{T}}Z_iE \end{bmatrix} \ge 0 \tag{8}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12}^{\mathrm{T}} & \Omega_{22} \end{bmatrix} < 0, \quad i = 1, 2, \cdots, n \tag{9}$$

where

$$\begin{aligned} \Omega_{11} &= \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^{\mathrm{T}} & \Gamma_{22} \end{bmatrix} \\ \Gamma_{11} &= P^{\mathrm{T}}A + A^{\mathrm{T}}P + \sum_{i=1}^{n}(\bar{\tau}X_i + Y_i + Y_i^{\mathrm{T}} + Q_i) \\ \Gamma_{12} &= [P^{\mathrm{T}}B_1K - Y_1, P^{\mathrm{T}}B_2K - Y_2, \cdots, P^{\mathrm{T}}B_nK - Y_n] \\ \Gamma_{22} &= -\text{diag}\{Q_1, Q_2, \cdots, Q_n\} \\ \Omega_{12} &= \bar{\tau}[A, B_1K, B_2K, \cdots, B_nK]^{\mathrm{T}}[Z_1, Z_2, \cdots, Z_n] \\ \Omega_{22} &= -\bar{\tau}\text{diag}\{Z_1, Z_2, \cdots, Z_n\} \end{aligned}$$

Proof. The proof of this theorem is divided into two parts. First, we will show that system (6) is regular and impulse free, which is equivalent to say that (E, A) is regular and impulse free. From (4), (7), and (8), it is easy to see that

$$Y_i = \begin{bmatrix} Y_{i1} & 0 \\ Y_{i2} & 0 \end{bmatrix}, \ Z_i = \begin{bmatrix} Z_{i11} & Z_{i12} \\ Z_{i12}^{\mathrm{T}} & Z_{i22} \end{bmatrix}, \ P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \tag{10}$$

From (8) and (9), we get

$$P^{\mathrm{T}}A + A^{\mathrm{T}}P + \sum_{i=1}^{n}(Y_i + Y_i^{\mathrm{T}}) < 0$$

Then, we can obtain $A_{22}^{\mathrm{T}}P_3 + P_3^{\mathrm{T}}A_{22} < 0$, which implies that A_{22} is nonsingular, and thus the pair (E, A) is regular and impulse free. Therefore, system (6) is regular and impulse free.

Next, we will show that system (6) is stable. Choose the following Lyapunov functional candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

where

$$\begin{aligned} V_1(t) &= \boldsymbol{x}^{\mathrm{T}}(t)E^{\mathrm{T}}P\boldsymbol{x}(t) \\ V_2(t) &= \sum_{i=1}^{n}\int_{-\tau_i}^{0}\int_{t+\beta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_iE\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta \\ V_3(t) &= \sum_{i=1}^{n}\int_{t-\tau_i}^{t}\boldsymbol{x}^{\mathrm{T}}(\alpha)Q_i\boldsymbol{x}(\alpha)\mathrm{d}\alpha \end{aligned}$$

where matrixes $P, Z_i > 0, Q_i > 0$ are given in Theorem 1 and satisfy (7) ∼ (9).

Using the Leibniz-Newton formula, we have $\boldsymbol{x}(t-\tau_i) = \boldsymbol{x}(t) - \int_{t-\tau_i}^{t} \dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha$. Then, system (6) can be written as

$$\begin{aligned} E\dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + \sum_{i=1}^{n} B_i K\big[\boldsymbol{x}(t) - \int_{t-\tau_i}^{t} \dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha\big] = \\ &\left(A + \sum_{i=1}^{n} B_i K\right)\boldsymbol{x}(t) - \sum_{i=1}^{n} B_i K \int_{t-\tau_i}^{t} \dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha \end{aligned}$$

Thus, the time derivative of $V_1(t)$ is given by

$$\begin{aligned} \dot{V}_1(t) &= \dot{\boldsymbol{x}}^{\mathrm{T}}(t)E^{\mathrm{T}}P\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)P^{\mathrm{T}}E\dot{\boldsymbol{x}}(t) = \\ &2\boldsymbol{x}^{\mathrm{T}}(t)P^{\mathrm{T}}\left(A + \sum_{i=1}^{n} B_i K\right)\boldsymbol{x}(t) - \\ &2\sum_{i=1}^{n} \boldsymbol{x}^{\mathrm{T}}(t)P^{\mathrm{T}}B_i K \int_{t-\tau_i}^{t} \dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha \end{aligned}$$

Defining $\boldsymbol{a}(\cdot)$, $\boldsymbol{b}(\cdot)$, and $\check{N}$ as $\boldsymbol{a}(\alpha) = \boldsymbol{x}(t)$, $\boldsymbol{b}(\alpha) = \dot{\boldsymbol{x}}(\alpha)$, and $\check{N} = P^{\mathrm{T}}B_iK$ for all $\alpha \in [t-\tau_i, t]$, and applying Lemma 1, we can get

$$\begin{aligned} \dot{V}_1(t) \le\ & 2\boldsymbol{x}^{\mathrm{T}}(t)P^{\mathrm{T}}\left(A + \sum_{i=1}^{n} B_i K\right)\boldsymbol{x}(t) + \\ &\sum_{i=1}^{n} \tau_i \boldsymbol{x}^{\mathrm{T}}(t)X_i\boldsymbol{x}(t) + \\ &\sum_{i=1}^{n} \int_{t-\tau_i}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_iE\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha + \\ &2\sum_{i=1}^{n} \boldsymbol{x}^{\mathrm{T}}(t)(Y_i - P^{\mathrm{T}}B_iK)\int_{t-\tau_i}^{t} \dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha \le \\ &\boldsymbol{x}^{\mathrm{T}}(t)\big[P^{\mathrm{T}}A + A^{\mathrm{T}}P + \sum_{i=1}^{n}(\tau_i X_i + Y_i + Y_i^{\mathrm{T}})\big]\boldsymbol{x}(t) + \\ &\sum_{i=1}^{n} \int_{t-\tau_i}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_iE\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha + \\ &2\sum_{i=1}^{n} \boldsymbol{x}^{\mathrm{T}}(t)(P^{\mathrm{T}}B_iK - Y_i)\boldsymbol{x}(t-\tau_i) \end{aligned}$$

where matrixes X_i, Y_i, and Z_i satisfy (8).

Since $\dot{V}_2(t)$ and $\dot{V}_3(t)$ yield the relation

$$\begin{aligned} \dot{V}_2(t) =\ & \sum_{i=1}^{n} \int_{-\tau_i}^{0} [\dot{\boldsymbol{x}}^{\mathrm{T}}(t)E^{\mathrm{T}}Z_iE\dot{\boldsymbol{x}}(t) - \\ &\dot{\boldsymbol{x}}^{\mathrm{T}}(t+\beta)E^{\mathrm{T}}Z_iE\dot{\boldsymbol{x}}(t+\beta)]\mathrm{d}\beta = \\ &\sum_{i=1}^{n} \tau_i\big[A\boldsymbol{x}(t) + \sum_{i=1}^{n} B_iK\boldsymbol{x}(t-\tau_i)\big]^{\mathrm{T}} Z_i\big[A\boldsymbol{x}(t) + \\ &\sum_{i=1}^{n} B_iK\boldsymbol{x}(t-\tau_i)\big] - \sum_{i=1}^{n} \int_{t-\tau_i}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_iE\dot{\boldsymbol{x}}(\alpha)\mathrm{d}\alpha \end{aligned} \tag{11}$$

$$\dot{V}_3(t) = \sum_{i=1}^{n} \{\boldsymbol{x}^{\mathrm{T}}(t)Q_i\boldsymbol{x}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\tau_i)Q_i\boldsymbol{x}(t-\tau_i)\} \tag{12}$$

we have the time derivative of $V(t)$ along the trajectory of system (6) as

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \le \boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{\Omega}\,\boldsymbol{\xi}(t) \tag{13}$$

where

$$\begin{aligned} \boldsymbol{\xi}(t) &= [\boldsymbol{x}^{\mathrm{T}}(t), \boldsymbol{x}^{\mathrm{T}}(t-\tau_1), \boldsymbol{x}^{\mathrm{T}}(t-\tau_2), \cdots, \boldsymbol{x}^{\mathrm{T}}(t-\tau_n)]^{\mathrm{T}} \\ \tilde{\Omega} &= \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} \\ \tilde{\Omega}_{12}^{\mathrm{T}} & \tilde{\Omega}_{22} \end{bmatrix} \\ \tilde{\Omega}_{11} &= P^{\mathrm{T}}A + A^{\mathrm{T}}P + \sum_{i=1}^{n}(\bar{\tau}X_i + Y_i + Y_i^{\mathrm{T}} + Q_i + \bar{\tau}A^{\mathrm{T}}Z_iA) \\ \tilde{\Omega}_{12} &= \Bigg[P^{\mathrm{T}}B_1K - Y_1 + \sum_{i=1}^{n} \bar{\tau}A^{\mathrm{T}}Z_iB_1K, \cdots, P^{\mathrm{T}}B_nK - \\ &\quad Y_n + \sum_{i=1}^{n} \bar{\tau}A^{\mathrm{T}}Z_iB_nK\Bigg] \\ \tilde{\Omega}_{22} &= -\mathrm{diag}\{Q_1, Q_2, \cdots, Q_n\} + \\ &\sum_{i=1}^{n} \begin{bmatrix} \bar{\tau}K^{\mathrm{T}}B_1^{\mathrm{T}}Z_iB_1K & \cdots & \bar{\tau}K^{\mathrm{T}}B_1^{\mathrm{T}}Z_iB_nK \\ \vdots & \ddots & \vdots \\ \bar{\tau}K^{\mathrm{T}}B_n^{\mathrm{T}}Z_iB_1K & \cdots & \bar{\tau}K^{\mathrm{T}}B_n^{\mathrm{T}}Z_iB_nK \end{bmatrix} \end{aligned}$$

It follows that the inequality $\tilde{\Omega} < 0$ guarantees $\dot{V}(t) < 0$ for all non-zero $\boldsymbol{\xi}(t)$. Hence, $\tilde{\Omega} < 0$ guarantees that system (6) is stable for all time-delays τ_i satisfying $0 < \tau_i \le \bar{\tau}$, $i = 1, 2, \cdots, n$. By Schur complement, $\tilde{\Omega} < 0$ is equivalent to LMI (9). Then, we have the desired result immediately. □

Theorem 1 presents a stabilization result for the nominal system of (1). Now, we are in a position to present the result on the problem of delay-dependent robust stabilization for system (1).

Theorem 2. For a given state feedback gain K, the uncertain singular time-delay system (1) is robustly stabilizable for all time-delays τ_i satisfying $0 < \tau_i \le \bar{\tau}$, $i = 1, 2, \cdots, n$ and all admissible uncertainties if there exist matrices $Z_i > 0$, $Q_i > 0$, X_i, Y_i, a nonsingular matrix P, and scalars $\varepsilon, \varepsilon_i, i = 1, 2, \cdots, n$, such that (7), (8), and the following LMI hold:

$$\begin{bmatrix} \hat{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} \\ \hat{\Omega}_{12}^{\mathrm{T}} & \hat{\Omega}_{22} & \hat{\Omega}_{23} \\ \hat{\Omega}_{13}^{\mathrm{T}} & \hat{\Omega}_{23}^{\mathrm{T}} & \hat{\Omega}_{33} \end{bmatrix} < 0, \quad i = 1, 2, \cdots, n \tag{14}$$

where

$$\begin{aligned} \hat{\Omega}_{11} &= \begin{bmatrix} \hat{\Gamma}_{11} & \hat{\Gamma}_{12} \\ \hat{\Gamma}_{12}^{\mathrm{T}} & \hat{\Gamma}_{22} \end{bmatrix} \\ \hat{\Gamma}_{11} &= \Gamma_{11} + \varepsilon H^{\mathrm{T}}H \\ \hat{\Gamma}_{12} &= \Gamma_{12} \\ \hat{\Gamma}_{22} &= \mathrm{diag}\{-Q_1 + \varepsilon_1 K^{\mathrm{T}}H_1^{\mathrm{T}}H_1K, -Q_2 + \\ &\quad \varepsilon_2 K^{\mathrm{T}}H_2^{\mathrm{T}}H_2K, \cdots, -Q_n + \varepsilon_n K^{\mathrm{T}}H_n^{\mathrm{T}}H_nK\} \\ \hat{\Omega}_{12} &= \Omega_{12} \\ \hat{\Omega}_{13} &= [P, \underbrace{0, \cdots, 0}_{n}]^{\mathrm{T}}[D, D_1, D_2, \cdots, D_n] \\ \hat{\Omega}_{22} &= \Omega_{22} \\ \hat{\Omega}_{23} &= \bar{\tau}[Z_1, Z_2, \cdots, Z_n]^{\mathrm{T}}[D, D_1, D_2, \cdots, D_n] \\ \hat{\Omega}_{33} &= -\mathrm{diag}\{\varepsilon I, \varepsilon_1 I, \varepsilon_2 I, \cdots, \varepsilon_n I\} \end{aligned}$$

and $\Gamma_{11}, \Gamma_{12}, \Omega_{12}, \Omega_{22}$ are defined in (9).

Proof. Replace A and $B_i, i = 1, 2, \cdots, n$ in (9) with $A+DF(t)H$ and $A_i + D_iF_i(t)H_i, i = 1, 2, \cdots, n$, respectively. Then, we have

$$\Omega + U^{\mathrm{T}}F^{\mathrm{T}}(t)V + V^{\mathrm{T}}F(t)U + \sum_{i=1}^{n} U_i^{\mathrm{T}}F_i^{\mathrm{T}}(t)V_i + \sum_{i=1}^{n} V_i^{\mathrm{T}}F_i(t)U_i < 0 \tag{15}$$

where

$$\begin{aligned} U &= [H, \underbrace{0, \cdots, 0}_{2n}] \\ V &= [D^{\mathrm{T}}P, \underbrace{0, \cdots, 0}_{n}, \underbrace{\bar\tau D^{\mathrm{T}}Z_1, \cdots, \bar\tau D^{\mathrm{T}}Z_n}_{n}] \\ U_i &= [\underbrace{0, \cdots, 0}_{i}, H_iK, \underbrace{0, \cdots, 0}_{2n-i}] \\ V_i &= [D_i^{\mathrm{T}}P, \underbrace{0, \cdots, 0}_{n}, \underbrace{\bar\tau D_i^{\mathrm{T}}Z_1, \cdots, \bar\tau D_i^{\mathrm{T}}Z_n}_{n}] \end{aligned}$$

and $\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12}^{\mathrm{T}} & \Omega_{22} \end{bmatrix}$ is defined in (9). According to Lemma 2, (15) is equivalent to

$$\Omega + \varepsilon U^{\mathrm{T}}U + \varepsilon^{-1}V^{\mathrm{T}}V + \sum_{i=1}^{n} \varepsilon_i U_i^{\mathrm{T}}U_i + \sum_{i=1}^{n} \varepsilon_i^{-1}V_i^{\mathrm{T}}V_i < 0$$

From Schur complement and Theorem 1, we can get Theorem 2 easily. □

Remark 1. As shown in Theorems 1 and 2, when B_iK and ΔB_iK are set to be A_i and ΔA_i, respectively, A_i and ΔA_i are matrices with appropriate dimensions, and ΔA_i is an admissible uncertainty, then system (5) is a singular system with multiple state delays and admissible uncertainties. That is to say, our results are valid to some singular systems with multiple state delays and admissible uncertainties.

In the following, we are in a position to present the result on the controller design for uncertain singular time-delay system (1). First, a controller design method for system (6) is given.

Theorem 3. The nominal singular time-delay system (6) is regular, impulse free, and stable for all time-delays τ_i satisfying $0 < \tau_i \leq \bar\tau$, $i = 1, 2, \cdots, n$ if there exist matrices $\tilde Z_i > 0, \tilde Q_i > 0, N, \tilde X_i, \tilde Y_i$, $i = 1, 2, \cdots, n$ and a nonsingular matrix X, such that the following LMIs hold:

$$X^{\mathrm{T}}E^{\mathrm{T}} = EX \geq 0 \tag{16}$$

$$\begin{bmatrix} \tilde X_i & \tilde Y_i \\ \tilde Y_i^{\mathrm{T}} & X^{\mathrm{T}}E^{\mathrm{T}} + EX - E^{\mathrm{T}}\tilde Z_iE \end{bmatrix} \geq 0 \tag{17}$$

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^{\mathrm{T}} & \Theta_{22} \end{bmatrix} < 0, \quad i = 1, 2, \cdots, n \tag{18}$$

In this case, a desired state feedback gain is given by

$$K = NX^{-1}$$

where

$$\begin{aligned} \Theta_{11} &= \begin{bmatrix} \check\Gamma_{11} & \check\Gamma_{12} \\ \check\Gamma_{12}^{\mathrm{T}} & \check\Gamma_{22} \end{bmatrix} \\ \check\Gamma_{11} &= AX + X^{\mathrm{T}}A^{\mathrm{T}} + \sum_{i=1}^{n}(\bar\tau\tilde X_i + \tilde Y_i + \tilde Y_i^{\mathrm{T}} + \tilde Q_i) \\ \check\Gamma_{12} &= [B_1N - \tilde Y_1, B_1N - \tilde Y_2, \cdots, B_nN - \tilde Y_n] \\ \check\Gamma_{22} &= -\mathrm{diag}\{\tilde Q_1, \tilde Q_2, \cdots, \tilde Q_n\} \\ \Theta_{12} &= \bar\tau[AX, B_1N, B_2N, \cdots, B_nN]^{\mathrm{T}}[\underbrace{I, I, \cdots, I}_{n}] \\ \Theta_{22} &= -\bar\tau\mathrm{diag}\{\tilde Z_1, \tilde Z_2, \cdots, \tilde Z_n\} \end{aligned}$$

Proof. Using Schur complement, (9) is shown to be equivalent to

$$\begin{bmatrix} \tilde\Theta_{11} & \tilde\Theta_{12} \\ \tilde\Theta_{12}^{\mathrm{T}} & \tilde\Theta_{22} \end{bmatrix} < 0 \tag{19}$$

where

$$\begin{aligned} \tilde\Theta_{11} &= \begin{bmatrix} \tilde\Gamma_{11} & \tilde\Gamma_{12} \\ \tilde\Gamma_{12}^{\mathrm{T}} & \tilde\Gamma_{22} \end{bmatrix} \\ \tilde\Gamma_{11} &= P^{\mathrm{T}}A + A^{\mathrm{T}}P + \sum_{i=1}^{n}(\bar\tau X_i + Y_i + Y_i^{\mathrm{T}} + Q_i) \\ \tilde\Gamma_{12} &= [P^{\mathrm{T}}B_1K - Y_1, P^{\mathrm{T}}B_2K - Y_2, \cdots, P^{\mathrm{T}}B_nK - Y_n] \\ \tilde\Gamma_{22} &= -\mathrm{diag}\{Q_1, Q_2, \cdots, Q_n\} \\ \tilde\Theta_{12} &= \bar\tau[A, B_1K, B_2K, \cdots, B_nK]^{\mathrm{T}}[\underbrace{I, I, \cdots, I}_{n}] \\ \tilde\Theta_{22} &= -\bar\tau\mathrm{diag}\{Z_1^{-1}, Z_2^{-1}, \cdots, Z_n^{-1}\} \end{aligned}$$

Then, pre- and post-multiplying both sides of inequality (19) by $\mathrm{diag}\{\underbrace{P^{-\mathrm{T}}, \cdots, P^{-\mathrm{T}}}_{n+1}, \underbrace{I, \cdots, I}_{n}\}$ and its transpose, and defining $X = P^{-1}$, $\tilde X_i = X^{\mathrm{T}}X_iX$, $\tilde Y_i = X^{\mathrm{T}}Y_iX$, $\tilde Q_i = X^{\mathrm{T}}Q_iX$, $\tilde Z_i = Z_i^{-1}$, $N = KX$, we can obtain inequality (18).

Pre- and post-multiplying (7) by X^{T} and X, respectively, we can get (16) easily.

Pre- and post-multiplying both sides of inequality (8) by $\mathrm{diag}\{X^{\mathrm{T}}, X^{\mathrm{T}}\}$ and its transpose, we have

$$\begin{bmatrix} \tilde X_i & \tilde Y_i \\ \tilde Y_i^{\mathrm{T}} & X^{\mathrm{T}}E^{\mathrm{T}}\tilde Z_i^{-1}EX \end{bmatrix} \geq 0, \quad i = 1, 2, \cdots, n \tag{20}$$

From (4), (10), and (20), we have

$$X^{\mathrm{T}}E^{\mathrm{T}}\tilde Z_i^{-1}EX = \begin{bmatrix} P_1^{-\mathrm{T}}Z_{i11}P_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} \geq 0 \tag{21}$$

Using Lemma 3, we can get

$$\begin{aligned} & X^{\mathrm{T}}E^{\mathrm{T}} + EX - E^{\mathrm{T}}\tilde Z_iE = \\ & \begin{bmatrix} P_1^{-\mathrm{T}} + P_1^{-1} - (Z_{i11} - Z_{i12}Z_{i22}^{-1}Z_{i12}^{\mathrm{T}})^{-1} & 0 \\ 0 & 0 \end{bmatrix} \leq \\ & \begin{bmatrix} P_1^{-\mathrm{T}}(Z_{i11} - Z_{i12}Z_{i22}^{-1}Z_{i12}^{\mathrm{T}})P_1^{-1} & 0 \\ 0 & 0 \end{bmatrix} \leq \\ & X^{\mathrm{T}}E^{\mathrm{T}}\tilde Z_i^{-1}EX \end{aligned}$$

Then, if

$$\begin{bmatrix} \tilde X_i & \tilde Y_i \\ \tilde Y_i^{\mathrm{T}} & X^{\mathrm{T}}E^{\mathrm{T}} + EX - E^{\mathrm{T}}\tilde Z_iE \end{bmatrix} \geq 0$$

we can get inequality (20).

It is easy to see that (16) ∼ (18) imply (7) ∼ (9), respectively. Therefore, according to Theorem 1, we can see for all time-delays τ_i satisfying $0 < \tau_i \leq \bar\tau$, $i = 1, 2, \cdots, n$

that system (6) is regular, impulse free, and stable, and the corresponding state feedback gain is $K = NX^{-1}$. □

Next, based on Theorem 3, we give the following theorem.

Theorem 4. The uncertain singular time-delay system (1) is robustly stabilizable for all time-delays τ_i satisfying $0 < \tau_i \leq \bar{\tau}$, $i = 1, 2, \cdots, n$ and all admissible uncertainties if there exist matrices $\tilde{Z}_i > 0, \tilde{Q}_i > 0, N, \tilde{X}_i, \tilde{Y}_i$, a nonsingular matrix X, and scalars $\varepsilon, \varepsilon_i, i = 1, 2, \cdots, n$, such that (16), (17) and the following LMI hold:

$$\begin{bmatrix} \hat{\Theta}_{11} & \hat{\Theta}_{12} & \hat{\Theta}_{13} \\ \hat{\Theta}_{12}^{\mathrm{T}} & \hat{\Theta}_{22} & \hat{\Theta}_{23} \\ \hat{\Theta}_{13}^{\mathrm{T}} & \hat{\Theta}_{23}^{\mathrm{T}} & \hat{\Theta}_{33} \end{bmatrix} < 0, \quad i = 1, 2, \cdots, n \tag{22}$$

In this case, a desired state feedback controller is given by

$$\boldsymbol{u}(t) = NX^{-1}\boldsymbol{x}(t)$$

where

$$\begin{aligned}
\hat{\Theta}_{11} &= \begin{bmatrix} \bar{\Gamma}_{11} & \bar{\Gamma}_{12} \\ \bar{\Gamma}_{12}^{\mathrm{T}} & \bar{\Gamma}_{22} \end{bmatrix} \\
\bar{\Gamma}_{11} &= AX + X^{\mathrm{T}}A^{\mathrm{T}} + \sum_{i=1}^{n}(\bar{\tau}\tilde{X}_i + \tilde{Y}_i + \tilde{Y}_i^{\mathrm{T}} + \tilde{Q}_i) + \\
&\quad \varepsilon DD^{\mathrm{T}} + \sum_{i=1}^{n}\varepsilon_i D_i D_i^{\mathrm{T}} \\
\bar{\Gamma}_{12} &= [B_1N - \tilde{Y}_1, B_2N - \tilde{Y}_2, \cdots, B_nN - \tilde{Y}_n] \\
\bar{\Gamma}_{22} &= -\mathrm{diag}\{\tilde{Q}_1, \tilde{Q}_2, \cdots, \tilde{Q}_n\} \\
\hat{\Theta}_{12} &= \bar{\tau}[AX, B_1N, B_2N, \cdots, B_nN]^{\mathrm{T}}[\underbrace{I, \cdots, I}_{n}] + \\
&\quad \varepsilon[D^{\mathrm{T}}, \underbrace{0, \cdots, 0}_{n}]^{\mathrm{T}}[\underbrace{\bar{\tau}D^{\mathrm{T}}, \cdots, \bar{\tau}D^{\mathrm{T}}}_{n}] + \\
&\quad \sum_{i=1}^{n}\varepsilon_i[D_i^{\mathrm{T}}, \underbrace{0, \cdots, 0}_{n}]^{\mathrm{T}}[\underbrace{\bar{\tau}D_i^{\mathrm{T}}, \cdots, \bar{\tau}D_i^{\mathrm{T}}}_{n}] \\
\hat{\Theta}_{22} &= -\bar{\tau}\mathrm{diag}\{\tilde{Z}_1, \tilde{Z}_2, \cdots, \tilde{Z}_n\} + \\
&\quad \varepsilon[\underbrace{\bar{\tau}D^{\mathrm{T}}, \cdots, \bar{\tau}D^{\mathrm{T}}}_{n}]^{\mathrm{T}}[\underbrace{\bar{\tau}D^{\mathrm{T}}, \cdots, \bar{\tau}D^{\mathrm{T}}}_{n}] + \\
&\quad \sum_{i=1}^{n}\varepsilon_i[\underbrace{\bar{\tau}D_i^{\mathrm{T}}, \cdots, \bar{\tau}D_i^{\mathrm{T}}}_{n}]^{\mathrm{T}}[\underbrace{\bar{\tau}D_i^{\mathrm{T}}, \cdots, \bar{\tau}D_i^{\mathrm{T}}}_{n}] \\
\hat{\Theta}_{13} &= \mathrm{diag}\{X^{\mathrm{T}}H^{\mathrm{T}}, N^{\mathrm{T}}H_1^{\mathrm{T}}, N^{\mathrm{T}}H_2^{\mathrm{T}}, \cdots, N^{\mathrm{T}}H_n^{\mathrm{T}}\} \\
\hat{\Theta}_{23} &= 0 \\
\hat{\Theta}_{33} &= -\mathrm{diag}\{\varepsilon I, \varepsilon_1 I, \varepsilon_2 I, \cdots, \varepsilon_n I\}
\end{aligned}$$

Proof. Replace A and $B_i, i = 1, 2, \cdots, n$ in (18) with $A + DF(t)H$ and $B_i + D_iF_i(t)H_i$, $i = 1, 2, \cdots, n$, respectively. Similarly to the proof of Theorem 2, we can get this theorem easily. □

Remark 2. From the proof of these theorems, we can see that all the obtained results are expressed in terms of LMIs involving no decomposition of system matrices, which makes the design method relatively simple and reliable.

3 Examples

To demonstrate the effectiveness of our method, we briefly consider the following two examples.

Example 1 demonstrates the effectiveness of the obtained criterion in Theorem 1 in that it may readily be used to find a solution to the problem proposed.

Example 1. Consider the following singular system with multiple input delays:

$$E\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B_1K\boldsymbol{x}(t - \tau_1) + B_2K\boldsymbol{x}(t - \tau_2)$$

where

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & 0 \\ 0 & -1 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} -1.1 & 1 \\ -0.2 & 0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.1 & 0.3 \\ 0.04 & 0.2 \end{bmatrix}$$
$$K = \mathrm{diag}\{1, 1\}$$

According to Theorem 1 and using Matlab LMI toolbox, it is found that this system is regular, impulse free, and stable for all time-delays τ_i satisfying $0 < \tau_i \leq 0.6432, i = 1, 2$. However, the methods in [14−16] fall short of obtaining this delay-dependent condition.

Example 2. Consider the following uncertain singular system with input delay:

$$E\dot{\boldsymbol{x}}(t) = (A + \Delta A)\boldsymbol{x}(t) + (B_1 + \Delta B_1)K\boldsymbol{x}(t - \tau_1) \tag{23}$$

where

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0.46 \\ -1.0 & -2.0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix}$$

and the uncertain matrices can be described by $\Delta A = DF(t)H$, $\Delta B_1 = D_1F_1(t)H_1$, and with $\|\triangle A\| \leq 0.2$, $\|\triangle B\| \leq 0.2$.

$$D = D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$$
$$F(t) = F_1(t) = \begin{bmatrix} \cos t & 0 \\ 0 & \sin t \end{bmatrix}$$
$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

In this case, according to Theorem 4 and using Matlab LMI toolbox, it is found $\bar{\tau} = 8.6569$. For example, when $\bar{\tau} = 1.5$, the solutions to system (23) are shown below

$$X = \begin{bmatrix} 0.6268 & 0 \\ -0.3904 & 0.4097 \end{bmatrix}, \tilde{X}_1 = \begin{bmatrix} 0.0656 & -0.0126 \\ -0.0126 & 0.2388 \end{bmatrix}$$
$$\tilde{Y}_1 = \begin{bmatrix} -0.1405 & 0 \\ -0.0797 & 0 \end{bmatrix}, \tilde{Z}_1 = \begin{bmatrix} 0.6637 & -0.1546 \\ -0.1546 & 1.9092 \end{bmatrix}$$
$$\tilde{Q}_1 = \begin{bmatrix} 0.1260 & -0.0435 \\ -0.0435 & 0.3083 \end{bmatrix}, N = \begin{bmatrix} -0.0830 & 0.0006 \end{bmatrix}$$
$$\varepsilon = 2.7945, \ \varepsilon_1 = 0.9400$$

and the corresponding state feedback controller is

$$\boldsymbol{u}(t) = \begin{bmatrix} -0.1315 & 0.0014 \end{bmatrix}\boldsymbol{x}(t)$$

However, the methods in [14−16] are not feasible to this example.

Remark 3. Theorems in this paper extend the results in [14−16], which deal with the problem of stability for single time-delay systems, to multiple time-delay systems. That is to say, our results are more general and are an improvement over the previous ones.

4 Conclusion

The problem of delay-dependent robust stabilization for singular systems with multiple input delays and admissible uncertainties has been investigated. First, a delay-dependent stabilization criterion for the nominal system is presented. Then, based on this criterion, the problem is solved via state feedback controller, thus guaranteeing the singular system with multiple input delays and admissible uncertainties to be robustly stabilizable, and an explicit expression of the desired state feedback controller is also given. The obtained results are expressed in terms of LMIs involving no decomposition of system matrices, making the design method relatively simple and reliable. Two examples are given to show the effectiveness of the proposed method and the improvement over some existing methods.

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DU Zhao-Ping Ph. D. candidate at the Institute of Systems Science, Northeastern University. He received his master degree from Northeastern University in 2005. His research interest covers singular systems and networked control systems. Corresponding author of this paper.
E-mail: duzhaoping98@163.com

ZHANG Qing-Ling Professor at Northeastern University. His research interest covers singular systems, networked control systems, and robust control.
E-mail: qlzhang@mail.neu.edu.cn

LIU Li-Li Ph. D. candidate at the Institute of Systems Science, Northeastern University. She is also a lecturer at Northeastern University. Her research interest covers nonlinear systems, networked control systems, and singular systems.
E-mail: liulili19760909@sina.com