

## Research Letter

# The Distribution of Path Losses for Uniformly Distributed Nodes in a Circle

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When simulating a wireless network, users/nodes are usually assumed to be distributed uniformly in space. Path losses between nodes in a simulated network are generally calculated by determining the distance between every pair of nodes and applying a suitable path loss model as a function of this distance (power of distance with an environment-specific path loss exponent) and adding a random component to represent the log-normal shadowing. A network with  $N$  nodes consists of  $N(N - 1)/2$  path loss values. In order to generate statistically significant results for system-level simulations, Monte Carlo simulations must be performed where the nodes are randomly distributed at the start of every run. This is a time-consuming operation which need not be carried out if the distribution of path losses between the nodes is known. The probability density function (pdf) of the path loss between the centre of a circle and a node distributed uniformly within a the circle is derived in this work.

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## 1. Introduction

System-level computer simulations of wireless, mobile networks are commonplace in current research in the wireless communications field. Users are oftentimes uniformly distributed in hexagonal cells in a system simulation. In order to find the signal-to-interference-and-noise ratio (SINR) on any link, path loss information must be known between the two communicating entities as well as the entity in question and all other potential interfering entities. Various empirically obtained path loss models exist which simulate varying propagation environments as shown in [1–4]. In [5], the authors derive the probability density function (pdf) of the distance between two nodes within communication range of one another (based on the path loss between them). This is then extended to calculating the number of communicable nodes in the vicinity of the node in question. However, to the best of the authors' knowledge, no work has been done to analytically model the distribution of path losses between uniformly distributed nodes in a network (irrespective of whether they can sustain communication between one another or not).

Calculating the path loss between the nodes in a cellular system is usually done by actually distributing the nodes uniformly, calculating the distances between them and applying an appropriate path loss model to these distances. This must be repeated many times in order to get a statistically significant result. This operation is computationally expensive and simulation runtimes can be drastically reduced if this step could be skipped. In order to do so, knowledge of the distribution of path losses between uniformly distributed nodes in a hexagon is essential.

In this letter, the pdf of the distribution of path losses between the centre of a circle and uniformly distributed nodes within it is derived. This novel derivation can be used to find the distribution of a whole class of path loss models. A circular scenario is preferred over a hexagonal one because the derivation of the aforementioned pdf is straightforward for the circular case. Furthermore, it is also shown in this paper that the theory derived for a circular scenario agrees with the simulations for the hexagonal case very closely and is therefore a very good approximation.

The derivation of the pdf is shown in Section 2. The comparisons between theory and simulation and discussions

thereof are shown in Section 3. Section 4 contains concluding remarks.

## 2. Distribution of Path Losses for Uniformly Distributed Users in a Circle

Path loss is generally represented as some power of distance,  $\gamma$ , plus a random variation about this power law due to shadowing [1]. Beyond some close-in distance  $d_0$ , the path loss (in dB) can be written as

$$L = a + 10\gamma \log_{10}\left(\frac{d}{d_0}\right) + \xi; \quad d \geq d_0, \quad (1)$$

where  $a$  is an intercept which is the free-space path loss at distance  $d_0$ ,  $d$  is the distance between the two points, and  $\xi$  is the shadow fading variation about the linear relationship in the log domain and is a zero-mean, normally distributed random variable with standard deviation  $\sigma$ , that is,  $\mathcal{N}(0, \sigma^2)$ . For the sake of convenience, in (1), we make the substitutions  $b = 10\gamma$  and  $X = d/d_0$ . Thus, the path loss between two points separated by  $X d_0$  meters is written as

$$L = a + b \log_{10}(X) + \xi. \quad (2)$$

For a circle of radius  $R$  having uniform node distribution, it is known that the pdf of the distance of any point from the centre,  $x$ , is [6]

$$f_X(x) = \frac{2x}{R^2}, \quad x \in [0, R]. \quad (3)$$

The angular distribution is uniform on  $[0, 2\pi]$ . Deriving the pdf of  $L$  involves the addition of two random variables and a constant. First we find the pdf of the random variable  $Y = b \log_{10}X$  using the transformation of random variables. Thus,

$$f_Y(y) = \frac{2(\ln 10)10^{2y/b}}{bR^2}, \quad y \in (-\infty, b \log_{10}R]. \quad (4)$$

Next, we calculate the pdf of

$$Z = \overbrace{b \log_{10}X}^Y + \overbrace{\mathcal{N}(0, \sigma^2)}^W. \quad (5)$$

The pdf of  $W$  is known to be

$$f_W(w) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{w^2}{2\sigma^2}\right). \quad (6)$$

Since this is an addition of two independent random variables ( $Y$  and  $W$ ), we can use the convolution integral to

find the pdf of  $Z$ , that is,  $f_Z(z) = f_Y(y) \star f_W(w)$  (where  $\star$  is the convolution operator):

$$\begin{aligned} f_Z(z) &= \int_{-\infty}^{\infty} f_W(w) f_Y(z-w) dw \\ &= \frac{2(\ln 10)}{\sqrt{2\pi}\sigma b R^2} \int_A^{\infty} \exp\left(-\frac{w^2}{2\sigma^2}\right) 10^{2(z-w)/b} dw \\ &= \frac{2(\ln 10)}{\sqrt{2\pi}\sigma b R^2} \exp\left\{\frac{2b(\ln 10)z + 2\sigma^2(\ln 10)^2}{b^2}\right\} \\ &\quad \times \int_A^{\infty} \exp\left\{-\frac{(w + (2\sigma^2(\ln 10))/b)^2}{2\sigma^2}\right\} dw, \end{aligned} \quad (7)$$

where  $A = z - b \log_{10}R$ . Let  $k = w + (2\sigma^2(\ln 10))/b$ . Using this in (7), then

$$\begin{aligned} f_Z(z) &= \frac{2(\ln 10)}{\sqrt{2\pi}\sigma b R^2} \exp\left\{\frac{2b(\ln 10)z + 2\sigma^2(\ln 10)^2}{b^2}\right\} \\ &\quad \times \int_B^{\infty} \exp\left\{-\frac{k^2}{2\sigma^2}\right\} dk, \end{aligned} \quad (8)$$

where  $B = z - b \log_{10}R + (2\sigma^2(\ln 10))/b$ . Let  $l = k/\sqrt{2}\sigma$ . Decomposing the integral into the standard form of the error function, then

$$\begin{aligned} f_Z(z) &= \frac{2(\ln 10)}{\sqrt{2\pi}\sigma b R^2} \exp\left\{\frac{2b(\ln 10)z + 2\sigma^2(\ln 10)^2}{b^2}\right\} \\ &\quad \times \int_C^{\infty} \exp\{-l^2\} \sqrt{2}\sigma dl, \end{aligned} \quad (9)$$

where

$$C = \frac{z}{\sqrt{2}\sigma} - \frac{b \log_{10}R}{\sqrt{2}\sigma} + \frac{\sqrt{2}\sigma(\ln 10)}{b}. \quad (10)$$

Using the definition of the error function as follows:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, \quad (11)$$

we get

$$\begin{aligned} f_Z(z) &= \frac{(\ln 10)}{bR} \exp\left\{\frac{2b(\ln 10)z + 2\sigma^2(\ln 10)^2}{b^2}\right\} \\ &\quad \times \{1 - \operatorname{erf}(C)\}. \end{aligned} \quad (12)$$

Finally, we solve for the pdf of

$$L = a + \overbrace{b \log_{10}X + \mathcal{N}(0, \sigma^2)}^Z. \quad (13)$$

This final step translates to a shift and the pdf of  $L$  is given as

$$\begin{aligned} f_L(\ell) &= \frac{(\ln 10)}{bR^2} \exp\left\{\frac{2b(\ln 10)(\ell - a) + 2\sigma^2(\ln 10)^2}{b^2}\right\} \\ &\quad \times \{1 - \operatorname{erf}(D)\}; \quad \forall \ell, \end{aligned} \quad (14)$$

where

$$D = \frac{\ell b - a b - b^2 \log_{10} R + 2\sigma^2 (\ln 10)}{\sqrt{2} b \sigma}. \quad (15)$$

Equation (14) gives the pdf of path loss distribution in a circular scenario of radius  $R$  as seen from the centre of the circle. Using an alternate expression for the error function as shown in [7], (14) can be written as

$$f_L(\ell) = \frac{(\ln 10)}{bR^2} \exp\left\{\frac{2b(\ln 10)(\ell - a) + 2\sigma^2(\ln 10)^2}{b^2}\right\} \\ \times \left\{1 - \left(1 - \frac{2}{\pi} \int_0^{\pi/2} \exp\left[-\frac{2D^2}{2\sin^2\theta}\right] d\theta\right) \operatorname{sgn}(D)\right\}; \\ \forall \ell, \quad (16)$$

where  $D$  holds the same meaning as defined above and  $\operatorname{sgn}(\cdot)$  is the signum function.

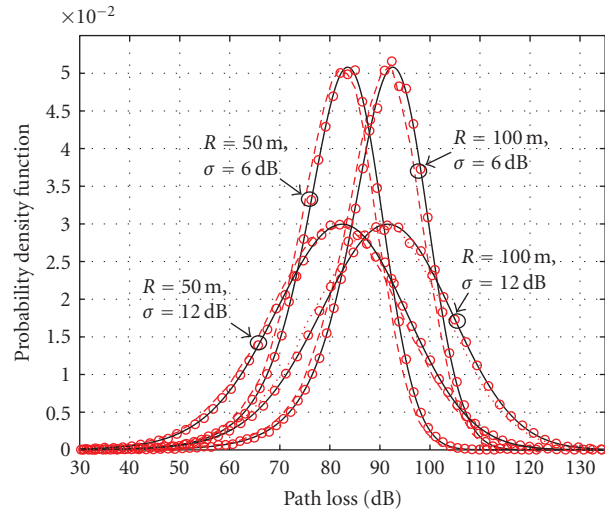
### 3. Results

The parameters specified in the UMTS path loss model [8] are used for the simulations (intercept,  $a = 37$  and  $b = 30$ ). The match between theory and simulations is investigated for varying circle radii and standard deviations for the log-normal shadowing component. Simulations are also carried out with the nodes distributed uniformly in hexagons having the same major radii as the circles to demonstrate the practicality of the derived theory.

The simulation is run by uniformly distributing 100,000 nodes in a circle or hexagon of the specified radius and then calculating the distance between the centre and each point. The histogram of these distances is then compared against theory. Figure 1(a) shows the match between the pdfs obtained through simulations and those derived in Section 2 for varying scenario radii and log-normal shadowing standard deviation values. Good agreement between theory and simulation is seen in all cases. Furthermore, it is observed that the theory for uniformly distributed nodes in a circle very closely approximates the case when the nodes are uniformly distributed in a hexagon having the same major radius as the circle. As expected, the pdf for the hexagonal case shifts to the left. This is because the major radius of the hexagon is the same as the radius of the circle, causing the hexagon to fit inside the circle. Therefore, there are a few areas not covered by nodes in the hexagonal case which causes the shift. The situation would be reversed if the minor radius of the hexagon were made equal to the radius of the circle. Figure 1(b) shows the cumulative distribution functions (cdfs) associated with the pdfs depicted in Figure 1(a).

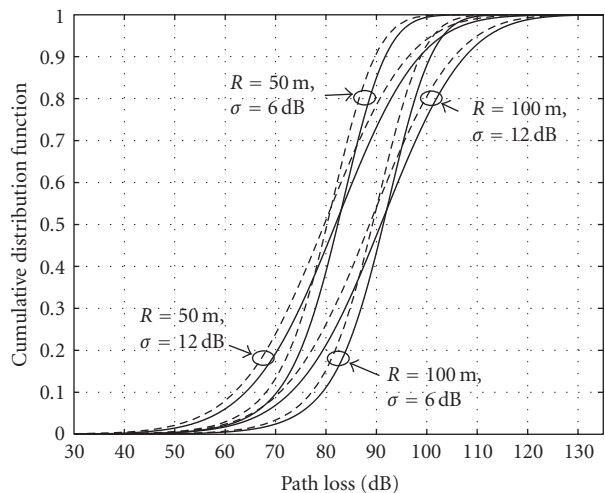
### 4. Conclusion

This paper has presented an analytical derivation of the pdf of path loss distribution between the centre of a circle and uniformly distributed nodes within it.



— Theory  
 ○ Simulation  
 - - - Hexagonal (simulated)

(a)



— Circular  
 - - - Hexagonal

(b)

FIGURE 1: (a) Pdfs of path loss distribution for uniformly distributed nodes in circles and hexagons having the same major radii. Simulation and theory are in good agreement as scenario radii and standard deviation of the log-normal component are varied. (b) The corresponding cdfs.

The derived pdf assists in calculating the carrier-to-interference ratios, interference thresholds and exclusion regions in *ad hoc* and sensor networks. In addition, it is envisaged that the result can be used to develop and assess routing and scheduling algorithms for such networks without running time-consuming Monte Carlo simulations.

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## Special Issue on Network Coding for Wireless Networks

### Call for Papers

The main idea in network coding was introduced in 2000 by Ahlswede et al. With network coding, an intermediate node in a network cannot only forward its incoming packets but also encode them. It has been shown that the use of network coding can enhance the performance of wired networks significantly. Recent work indicates that network coding can also offer significant benefits for wireless networks.

Communications over wireless channels are error-prone and unpredictable due to fading, mobility, and intermittent connectivity. Moreover, in wireless networks, transmissions are broadcast and can be overheard by neighbors, which is treated in current systems as interference. Finally, security poses new challenges in wireless networks, where both passive and active attacks have quite different premises than in wired networks. Ideas in network coding promise to help toward all these issues, allowing to gracefully add redundancy to combat errors, take advantage of the broadcast nature of the wireless medium and achieve opportunistic diversity, exploit interference rather than be limited by it, and provide secure network communication against adversarial attacks.

In this special issue, we are interested in original research articles which can carry the momentum further and take the wireless network coding research to the next level. The areas of interest include novel network code designs and algorithms, new applications of wireless network coding, network coding capacity, and performance analysis. In addition to original research articles, we are open to review articles. The following list indicates topics of interest which is by no means exhaustive:

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- Physical layer network coding
- Joint source coding and network coding
- Graph codes and network coding
- Reduced complexity decoding for network coding
- Secure network coding
- Capacity and fundamental bounds on network coding performance
- Cross-layer optimization and network coding
- Energy-efficient network coding
- TCP, routing, MAC, or scheduling algorithms for network codes

- Wireless network coding for multimedia application
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## Special Issue on Femtocell Networks

### Call for Papers

Recently, there has been a growing interest in femtocell networks both in academia and industry. They offer significant advantages for next-generation broadband wireless communication systems. For example, they eliminate the dead-spots in a macrocellular network. Moreover, due to short communication distances (on the order of tens of meters), they offer significantly better signal qualities compared to the current cellular networks. This makes high-quality voice communications and high data rate multimedia type of applications possible in indoor environments.

However, this new type of technology also comes with its own challenges, and there are significant technical problems that need to be addressed for successful deployment and operation of these networks. Standardization efforts related to femtocell networks in 3GPP (e.g., under TSG-RAN Working Group 4 and LTE-Advanced) and IEEE (e.g., under IEEE 802.16m) are already underway.

The goal of this special issue is to solicit high-quality unpublished research papers on design, evaluation, and performance analysis of femtocell networks. Suitable topics include but are not limited to the following:

- Downlink and uplink PHY/MAC design for femtocells in 3G systems, WiMAX systems, and LTE systems
- Interference analysis, avoidance, and mitigation
- Coexistence between a macrocellular network and femtocell network
- Resource allocation techniques
- Closed subscriber group (CSG) versus open-access femtocells
- Power control and power saving mechanisms (e.g., sleep/idle mode etc.)
- Mobility support and handover
- Time synchronization
- Multiple antenna techniques
- Tradeoffs between femtocells, picocells, relay networks, and antenna arrays
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## Special Issue on Advances in Quality and Performance Assessment for Future Wireless Communication Services

### Call for Papers

Wireless communication services are evolving rapidly in tandem with developments and vast growth of heterogeneous wireless access and network infrastructures and their potential. Many new, next-generation, and advanced future services are being conceived. New ideas and innovation in performance and QoS, and their assessment, are vital to the success of these developments. These should be open and transparent, with not only network-provider-driven but also service-provider-driven and especially user-driven, options on management and control to facilitate always best connected and served (ABC&S), in whatever way this is perceived by the different stake holders. To wireless communication services suppliers and users, alike the complexity and integrability of the immense, diverse, heterogeneous wireless networks' infrastructure should add real benefits and always appear as an attractive user-friendly wireless services enabler, as a wireless services performance enhancer and as a stimulant to wireless services innovation. Effecting the integration of services over a converged IP platform supported by this diverse and heterogeneous wireless infrastructure presents immense QoS and traffic engineering challenges. Within this context, a special issue is planned to address questions, advances, and innovations in quality and performance assessment in heterogeneous wireless service delivery.

Topics of interest include, but are not limited to:

- Performance evaluation and traffic modelling
- Performance assessments and techniques at system/flow level, packet level, and link level
- Multimedia and heterogeneous service integration-performance issues, tradeoffs, user-perceived QoS, and quality of experience
- Network planning; capacity; scaling; and dimensioning
- Performance assessment, management, control, and solutions: user-driven; service-provider-driven; network-provider-driven; subscriber-centric and consumer-centric business model dependency issues
- Wireless services in support of performance assessment, management, and control of multimedia service delivery
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