Conservatism Reduction in Stabilizing Two-level Control of Discrete Large Scale Systems

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Abstract **—** The aim of this paper is to propose a new algorithm for multilevel stabilizer of large scale systems. In two level stabilizer method, a set of locally stabilizers for the individual subsystems in a completely decentralized environment is designed. The solution of the control problem involves designing of a global controller on a higher hierarchical level that provides corrective signals to account for interconnections effect. The principle feature of this paper is to reduce conservativeness in global controller design. Here the key point is to reduce the effect of interactions instead of neutralized them. In fact our idea unlike prior methods does not ignore the possible beneficial aspects of the interactions and does not try to neutralize them.

I. INTRODUCTION

There are different methods for control of large scale systems such as two-level method, decentralized and centralized control and etc. In this paper we have focused on the two-level method. The idea of this method that introduced by Siljack [1] is shown in Fig. 1

In present method, each control signal consists of two segments: local and global controller signals. Local controllers are used to control each subsystem, ignoring the interactions. A global controller may be applied to minimize the effect of interactions and improve the performance of the overall system.

Multilevel control offers advantages in the following situations.

1) A decentralized approach will be effective if the large scale system is composed of weakly coupled subsystems [2]-[4], or a decomposition

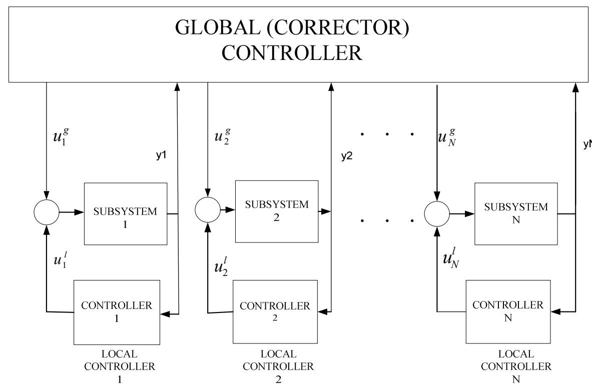


Fig. 1. A multi-level structure.

and rearrangement of variables are used to achieve weak coupling as is the case of sparse systems [5]. In situations when the subsystems are strongly connected and cannot be simply reconstituted to reduce the strength of the couplings, the assumption of weak coupling may produce gross inaccuracies of the obtained results.

2) Since a large-scale system will invariably be an interconnection of several subunits, one of the important phenomena that must be accounted for in the design of controllers and estimators is the occurrence of structural perturbations, i.e., changes in the interconnection pattern within the system during operation [6]. When a system is expected to undergo structural perturbations, the classical control techniques do not provide a satisfactory solution of the control problem and may results in a closed-loop system which is unstable.

3) Dealing with large systems, centralized controller design methods are either uneconomical because of an excessive computation time required, or impossible due to excessive computer storage needed.

4) Hierarchical coordinating methods, though conceptually very simple, require iterative solution procedures, which often lead to convergence difficulties [7].

Most of the researches adopting the philosophy that interactions are non beneficial (all of the interactions are toward deteriorating system objectives). Then the global controller is designed to neutralize all of them [8], [9]. Indeed it is assumed that:

$$H - BM = 0 \Rightarrow M = (B^T B)^{-1} B^T H$$

where H, B and M are representing interaction matrix, system input and global controller respectively. By selecting this structure for global controller there will be some problems.

- The ideal effect of two level control (neutralizing the effect of interactions) can't be obtained unless the rank of the composite matrix $[B \mid H]$ is equal to the rank of the matrix B itself. In case of inequality some performance criteria may not be satisfied.
- Even in case of rank equality, is it necessary to neutralize the effect of interactions completely? Is there a way not to neutralize the interactions? Are all of them non beneficial? Maybe there is an interaction that is helping system through its objectives. And maybe we can achieve the goals and desired performances or also improving them by assuming another structure for matrix M (see e.g. [10]).
- Also not trying to neutralize all of the interactions would lead to less conservativeness in global controller design.

As we mentioned although the idea of multi-level control appeared very early, no significant development has been made in this research area for a long time. Recently, Duan *et al.* presented a series of significant results on cooperative control of linear and nonlinear systems [11-14], and shown that unstable subsystems can form a stable large-scale system through appropriate interaction and cooperation feedback controllers.

In this paper, two new algorithms to design a global controller for large scale system are developed. We shall make use of different idea to develop an alternative multilevel scheme, which unlike prior methods, does not ignore the possible beneficial aspects of the interconnections and does not try to neutralize them. Therefore the design will not be much conservative. Furthermore all of these problems which are explained earlier will be solved by using these algorithms. Also it could be possible to use this idea for innovation in control design for systems which is including subsystems interaction.

This paper is organized as follows. In section II system description, required definition and lemmas are described. Algorithms for designing two-level stabilizer are introduced in section III. The application of design methods in three-region energy resources system and simulation results will be presented in section IV. Finally Concluding remarks follow in Section V.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Let a discrete large scale system that contains two subsystems be described as follow:

$$x_{i+1} = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} u_i, \qquad (1)$$

(A_1, B_1) and (A_2, B_2) are respectively state-space matrices of subsystems (1) and (2), A_{12} and A_{21} are their interaction matrices.

Throughout this paper, following assumptions are made.

Assumption 1. Both subsystems are controllable.

Assumption 2. Each subsystem is stabilized with a local controller (i.e. A_1 and A_2 are stable). We will stabilize the system using a state-feedback (i.e. $u_i = k_g x$).

We will refer to existing several methods to design local stabilizers and do not go further discus on this subject.

The objective of this work is to design the two-level stabilizer such that:

Objective 1. Overall closed loop system is stable. According to assumptions, we know two above subsystems are stable but the system (1) may be unstable because of being interacted.

Objective 2. Despite of prior methods, we don't want the global controller neutralize the effect of interactions and to consider the possible beneficial aspects of the interconnections effect.

Definition1: Polynomial:

$$P(z) = d_0 + d_1 z + d_2 z^2 + \ldots + d_n z^n, \qquad (2)$$

is Schur stable, if and only if [15]

$$
\begin{aligned}
&P(1) > 0,\\
&(-1)^n P(-1) > 0,\\
&\det(F(d)) > 0.
\end{aligned}
\tag{3}
$$

Where $F(d)$ is a $(n-1)\times(n-1)$ matrix such as:

$$
F(d) = \begin{bmatrix} d_n & d_{n-1} & \dots & d_3 & d_2 - d_0 \\ 0 & d_n & \dots & d_4 - d_0 & d_3 - d_1 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & -d_0 & \dots & d_n - d_{n-4} & d_{n-1} - d_{n-3} \\ -d_0 & -d_1 & \dots & -d_{n-3} & d_n - d_{n-2} \end{bmatrix}. \tag{4}
$$

Lemma 1: Given any real matrices A, B, C and D of appropriate dimensions. Then the following inequality holds

$$
\begin{aligned}
\det\begin{bmatrix} A & B \\ C & D \end{bmatrix} &= \det(A)\times\det(D - CA^{-1}B)\\
&= \det(D)\times\det(A - BD^{-1}C).
\end{aligned}
\tag{5}
$$

Lemma 2. Given a real matrix A. if A is Schur stable then [14]

$$
\det(I - A) > 0. \tag{6}
$$

Lemma 3. Let $A = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix}$. If there are two matrixes such A'_{12} and A'_{21} in order to decompose A_{12} and A_{21} like below

$$
A_{12} = A'_{12}(I - A_2), \tag{7}
$$

$$
A_{21} = A'_{21}(I - A_1), \tag{8}
$$

Then the below equality is hold:

$$
\begin{aligned}
&\det(I - A)\\
&= \det(I - A_1)\times\det(I - A_2)\times\det(I - A'_{21}A'_{12}).
\end{aligned}
\tag{9}
$$

Proof: considering lemma 1, the following equation is obtained:

$\det(I - A) = \det(I - A_1)\times\det(I - A_2 - A_{21}(I - A_1)^{-1}A_{12}),$

by substituting (7) and (8) in above equation we have:

$\det(I - A) = \det(I - A_1)\times$

$\det(I - A_2 - A'_{21}(I - A_1)(I - A_1)^{-1}A'_{12}(I - A_2)),$

From simple calculation:

$\det(I - A)$

$= \det(I - A_1)\times\det(I - A_2)\times\det(I - A'_{21}A'_{12}),$

That the proof is achieved completely.

III. MAIN RESULTS

According to lemma 3 if large scale system (1) is Schur stable, then $\det(I - A) > 0$. From assumption 2 the two subsystems A_1 and A_2 are stabilized therefore $\det(I - A_1) > 0$ and $\det(I - A_2) > 0$. Thus, it should be $\det(I - A'_{21}A'_{12}) > 0$ to guaranty stability of (1) logically. This condition is a necessary but insufficient condition.

Now by replacing nonzero interactions (nonzero elements of A_{12} and A_{21} matrices) with parameters such as $a_1, a_2, \ldots$, and using (7) and (8), we have the following equation:

$$
\varphi(a_i) = \det(I - A'_{21}(a_i)A'_{12}(a_i)) > 0,\ i = 1, 2, \ldots \tag{10}
$$

By solving equation (10), some constraints on interactions can be obtained. In this step, in order to satisfy attained constraints and finally stabilizing closed loop system, we have to design an suitable global controller.

The global controller is obtained such as:

$$
BK_g = F\ ,\ F = \tilde{H} - H. \tag{11}
$$

Where K_g is the global controller gain matrix, $B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$ is the global input matrix, $H = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}$ represents interaction matrix and $\tilde{H}$ is the modified interaction matrix. In fact, $\tilde{H}$ is obtained with replacing the nonzero elements of H with values calculated from (10). We can calculate the global controller from the below set of LMIs:

$$
\begin{bmatrix} \beta S & (BK_g - F)^T \\ (BK_g - F) & I \end{bmatrix} > 0, \tag{12}
$$

$$\begin{bmatrix} I & S \\ S & I \end{bmatrix} > 0, \tag{13}$$

Where β is a sufficiently small positive scalar and S is a positive definite matrix.

As a conclusion, we can use following algorithm for designing the global controller.

Algorithm 1: *Designing the global controller for time-discontinuous large scale systems.*

Step 1. Replace the nonzero elements of A_{12} and A_{21} matrices with some parameters such as $a_1, a_2, \ldots$

Step 2. Calculate A'_{12} and A'_{21} Matrices using (7) and (8), in form of a function of $a_1, a_2, \ldots$

Step 3. Identify $a_1, a_2, \ldots$ in coordinate with equation (10).

Step 4. Compute the global controller from (11).

Step 5. Make closed loop system by replacing (11) with (1) and then test its stability.

Remark 1. Lemma 3 introduces a necessary but insufficient condition; in fact if $\det(I - A) > 0$ we can't result stability, because this determinant is larger than zero when all system eigenvalues are in the unit circle or even number of eigenvalues are out of the unit circle. Therefore, due to being algorithm 1 in basis of this lemma, it is always impossible to attain solution from that and we must apply it when large scale system has odd number of unstable poles before designing global controller.

Remark 2. A'_{12} and A'_{21} exist if $(I - A_1)$ and $(I - A_2)$ are nonsingular.

Remark 3. Algorithm 1 has been proposed for the large scale systems with two subsystems. This algorithm is applicable for systems with more than two subsystems considering of special structure for main system. For instance, in case of following three subsystems and every similar system with larger number of subsystems.

$$x_{i+1} = \begin{bmatrix} A_1 & A_{12} & 0 \\ A_{21} & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} u_i. \tag{14}$$

In fact if the main system has two dependent and some independent subsystems, we can apply this algorithm.

Remark 4. Let's now consider the system

$$x_{i+1} = \begin{bmatrix} A_1 & A_{12} & A_{13} \\ A_{21} & A_2 & A_{23} \\ A_{31} & A_{32} & A_3 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} u_i, \tag{15}$$

To stabilize it, one can use algorithm 2. The aim is to design a global controller to stabilize the closed-loop system.

Algorithm 2

Step 1: consider subsystem

$$\tilde{x}_{i+1} = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix} \tilde{x}_i + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \tilde{u}_i, \tag{16}$$

From algorithm 1 calculate the necessary conditions for the stability of subsystem (16).

Step 2: Calculate the modified interaction matrices $\tilde{H}_{12}$ and $\tilde{H}_{21}$ for subsystem (16) and create below system.

$$x_{i+1} = \begin{bmatrix} A_1 & \tilde{H}_{12} & A_{13} \\ \tilde{H}_{21} & A_2 & A_{23} \\ A_{31} & A_{32} & A_3 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} u_i, \tag{17}$$

Assume that system (17) is composed of two independent subsystems like:

$$x^1_{i+1} = \begin{bmatrix} A_1 & \tilde{H}_{12} \\ \tilde{H}_{21} & A_2 \end{bmatrix} x^1_i + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} u^1_i, \tag{18}$$

$$x^2_{i+1} = A_3\, x^2_i + B_3 u^2_i, \tag{19}$$

Systems (18) and (19) are stable, then any instability in (17) is because of A_{13}, A_{23}, A_{31} and A_{32} interaction matrices. Go to step 3.

Step 3: calculate stability conditions on A_{13}, A_{23}, A_{31} and A_{32} using algorithm 1.

Step 4: Compose the global controller from modified interactions as in the following:

$$BK_g = F \;\;,\;\; F = \tilde{H} - H$$
$$= \begin{bmatrix} 0 & \tilde{H}_{12} & \tilde{H}_{13} \\ \tilde{H}_{21} & 0 & \tilde{H}_{23} \\ \tilde{H}_{31} & \tilde{H}_{32} & 0 \end{bmatrix} - \begin{bmatrix} 0 & A_{12} & A_{13} \\ A_{21} & 0 & A_{23} \\ A_{31} & A_{32} & 0 \end{bmatrix}. \tag{20}$$

Now test the stability of closed-loop global system.

IV. SIMULATION RESULTS

In this section the algorithm 1 is simulated to demonstrate its performance. Consider a three-region energy resources system as shown in Fig 2 [16].

This system can be described by following discrete state space:

$$x_{i+1} = \begin{bmatrix} A_1 & A_{12} & 0 \\ A_{21} & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} u_i, \tag{21}$$

where

$$\begin{gathered}
\mathrm{A}_1 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & .5 & .75 \\ .75 & .5 & 1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \\
A_2 = \begin{bmatrix} 1 & .5 & .25 & 0 \\ 0 & .5 & 0 & .2 \\ .5 & 0 & .5 & .25 \\ 0 & .2 & .25 & .25 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \\
\mathrm{A}_3 = \begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.5 & 0.25 & 0.2 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \\
A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 1.5 & 0 & 0 & 0 \end{bmatrix}, A_{21} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & .5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\end{gathered} \tag{22}$$

It's composed of three coupled subsystems. Here the aim is to stabilize it, using proposed algorithm 1.

Step 1: By computing eigenvalues of each subsystem, we can observe that all of them are unstable. First we should set interactions to zero and stabilize every subsystem with local controllers. This part is not presented in algorithm 1 because of stability assumption 2.

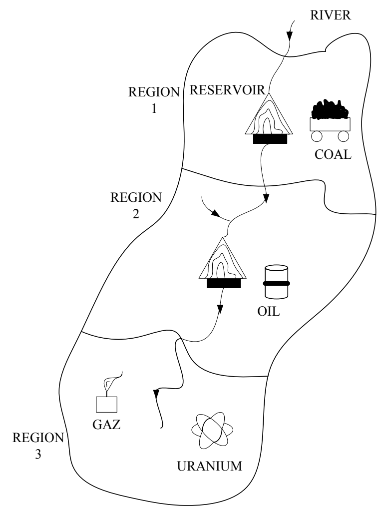


Fig. 2. A three-region energy resources system.

Stabilizing subsystems

The open loop eigenvalues of Subsystems are:

$$\begin{aligned}
\lambda_i^1 &= 2.1622, 0.1689 \pm 0.3406i, \\
\lambda_i^2 &= 1.2207, \text{-}0.0324, 0.5309 \pm 0.1364i, \\
\lambda_i^3 &= 1.3618\;, 0.3975, 0.0092,
\end{aligned} \tag{23}$$

where λ_i^j represents eigenvalues of j^{th} subsystem. Since they are unstable, we can stabilize them with static state feedback:

$$\begin{aligned}
u_1 &= K_1 x = [0.7419 \quad 0.7715 \quad 1.2425]x, \\
u_2 &= K_2 x = [0.7381 \quad 0.5462 \quad 0.3468 \quad 0.22]x, \\
u_3 &= K_3 x = [\,1.1280 \quad 0.5911 \quad 0.5055]x.
\end{aligned} \tag{24}$$

Now substitute A_1, A_2 and A_3 for each subsystem closed-loop matrices $A_{1cl}, A_{2_{cl}}$ and $A_{3_{cl}}$ and compose below matrix:

$$\begin{bmatrix} A_{1_{cl}} & A_{12} & 0 \\ A_{21} & A_{2_{cl}} & 0 \\ 0 & 0 & A_{3_{cl}} \end{bmatrix}, \tag{25}$$

Where $A_{j_{cl}}$ is the closed-loop matrix of each subsystem i.e. $A_{j_{cl}} = A_j - B_j K_j$ and $\left|\lambda_i(A_{j_{cl}})\right| < 1$.

The eigenvalues of the new system (25) are:

$$\lambda_i = 1.5889\ ,\text{-}0.7902\ ,0.7795\ ,\text{-}0.3920\ ,0.2960\ ,$$
$$0.2539, 0.1364\ ,0.6284, \text{-}0.0098\ ,0.0348$$

where we can infer that it is unstable. Therefore source of instability is interactions. Since there is no connection between subsystem (3) and set of subsystems (1) and (2), this subsystem doesn't affect the overall stability and we can use Algorithm 1.

Step 2: Replace the nonzero elements of A_{12} and A_{21} matrices with some parameters such as $a_1, a_2, \ldots$:

$$A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 \\ a_2 & 0 & 0 & 0 \end{bmatrix},\ A_{21} = \begin{bmatrix} 0 & 0 & a_3 \\ 0 & a_4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad (26)$$

Step 3: Calculate A'_{12} and A'_{21} Matrices using (7) and (8), in form of a function of $a_1, a_2, \ldots$:

$$A'_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -2.3193a_1 & 0.2588a_1 & -0.8022a_1 & -0.039a_1 \\ 2.9984a_2 & 1.234a_2 & 0.3745a_2 & -0.0179a_2 \end{bmatrix} \qquad (27)$$

$$A'_{21} = \begin{bmatrix} .0066a_3 & .3263a_3 & .6066a_3 \\ .0099a_4 & 1.5105a_4 & .9098a_4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad (28)$$

Step 4: Identify a_1, a_2,… in coordinate with equation (10). To satisfy equation (10), first we should calculate:

$$\begin{aligned} &\det(I - A'_{21}A'_{12}) = \\ &1 - 0.3909a_1a_4 - 1.1227a_2a_4 - 0.7566a_1a_3 \\ &-1.8188a_2a_3 + 4.4128a_1a_2a_3a_4 + 0.0001a_1^2a_4a_3 \\ &+.0002a_2^2a_3a_4, \end{aligned} \qquad (29)$$

There are different choices to meet (10). One of them e.g. is $a_1 = a_2 = a_4 = 0.5, a_3 = 1$. Then we can form $\tilde{H}$ as:

$$\tilde{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (30)$$

Step 5: Computing the global controller from (11). Where:

$$F = \tilde{H} - H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (31)$$

Step 6: We can now form closed-loop system and then test its stability.

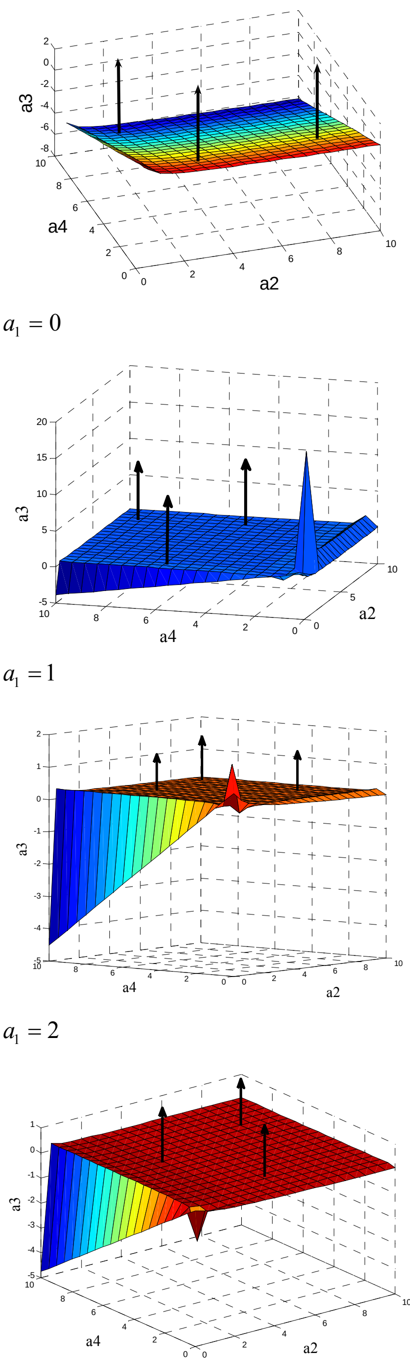
Note that all of its eigenvalues are in unit circle.

As we expected this controller reduces conservativeness. Other previous methods ideally lead to a controller with 4 nonzero elements in their F matrix as it is illustrated in the following matrix:

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (32)$$

It leads to neutralizing all the interactions. It is worth mentioning that selected values for

a_i, $i = 1,...,4$ are not optimal. As it is mentioned later there are different choices for $a_1, a_2, ...$. Fig. 3, shows possible values of a_2, a_3 and a_4 by initializing parameter a_1 with several values.



a) $a_1 = 0$

b) $a_1 = 1$

c) $a_1 = 2$

d) $a_1 = 3$

Fig. 3. Different choices of a_2, a_3, a_4 in return of a_1.

V. CONCLUSION

In this paper, we addressed stabilizing two-level control for discrete time linear large scale systems. Then we developed two new algorithms to design a global controller for these systems. This approach led to conservatism reduction. As it is stated, there is different solution for algorithms. One can select a solution by optimality criteria that are not presented in this paper. We exploit one of these solutions and applied to a three-region energy resources system. The effectiveness of the design was shown in design results. Our future work intends considering conservatism reduction in H_∞ control of large scale systems.

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