Delay-dependent H_∞ Control for T-S Fuzzy Systems Based on a Switching Fuzzy Model and Piecewise Lyapunov Function

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Abstract This paper studies the problem of H_∞ control for discrete-time Takagi-Sugeno (T-S) fuzzy systems with time delays. The T-S fuzzy system is transformed to an equivalent switching fuzzy system. Consequently, the delay-dependent stabilization criteria with H_∞ performance are derived for the switching fuzzy systems based on the piecewise Lyapunov function. The proposed conditions are given in terms of linear matrix inequalities (LMIs). The interactions among the fuzzy subsystems are considered in each subregion, and accordingly the proposed conditions are less conservative than the previous results. Finally, a design example is given to show the validity of the proposed method.

Key words Delay-dependent, fuzzy systems, linear matrix inequalities (LMIs), piecewise Lyapunov function

The well-known Takagi-Sugeno (T-S) fuzzy model[1] has recognized as a popular and powerful tool in approximating complex nonlinear systems. As a consequence, the study of T-S fuzzy systems has attracted an increasing interest in the past decades. In view of time delays frequently occurring in practical dynamic systems, T-S fuzzy model was first used to deal with the stability analysis and control synthesis of nonlinear time delay systems in [2]. Afterwards, many people devoted a great deal of effort to both theoretical research and implementation techniques for T-S fuzzy systems with time delays[3−4]. These results rely on the existence of a common positive definite matrix P for all linear models, which in general leads to a conservative result. To reduce this conservatism, the piecewise Lyapunov function approach[5−6] and the fuzzy Lyapunov function approach[7−8] have been proposed. However, all the aforementioned results have not considered the interactions among the fuzzy subsystems. Based on the switching fuzzy model, the interactions among the fuzzy subsystems in each subregion were presented in [9] for system without time delays. However, the result in [9] does not include any performance criteria in the design of the control law. In addition, all matrices $X_{jkl}, 1 \le k < l \le \beta(j)$ are required to be symmetric. In [10], for system without time delays, each Z_{ji} was not required to be symmetric. However, [10] only addressed the common Lyapunov function approach which typically led to conservative results. It is well known that the delay-dependent results are generally less conservative than delay-independent ones, especially when the size of the delay is small[5−8].

In this paper, two new delay-dependent stabilization criteria with H_∞ performance for discrete-time T-S fuzzy systems with time delays are dealt with by using a switching fuzzy model and piecewise Lyapunov function. To obtain the relaxed LMI conditions, in each subregion Ω_j, the interactions among the fuzzy subsystems in that subregion are presented by one matrix Z_j. Furthermore, the decoupling technique by the introduction of an auxiliary slack variable is applied, such that there does not exist any product terms of the Lyapunov matrix variables and system dynamic matrices in the LMI constraints. Since only a set of LMIs is involved, the controller design is quite simple and numerically tractable. An example is given to illustrate the validity of the proposed method.

Notation. For a real matrix S, $H_e\{S\}$ denotes $S+S^{\rm T}$. The symmetric elements of the symmetric matrix will be denoted by $*$.

Now we introduce the following lemma that will play an important role in our main results.

Lemma 1. For real matrices P_1, P_2, P_3, P_4, A, A_d, B, $X_j (j = 1, \cdots, 5)$, and $D_i (i = 1, \cdots, 10)$ with compatible dimensions, the inequalities shown at the bottom of the next page are equivalent, where U is an extra slack nonsingular matrix.

Proof. As in [5–6, 11], we can rewrite the inequality (b) as

$$\begin{bmatrix}\Sigma_3\\ \Sigma_1\end{bmatrix}^{\rm T}\begin{bmatrix}\Sigma_0 & \Sigma_2\\ \Sigma_2^{\rm T} & 0\end{bmatrix}\begin{bmatrix}\Sigma_3\\ \Sigma_1\end{bmatrix} < 0 \tag{1}$$

where

$$\Sigma_1 = \begin{bmatrix}-I & A & A_d & 0 & B & 0\end{bmatrix}, \Sigma_3 = {\rm diag}\{I, I, I, I, I, I\},$$

$$\Sigma_2 = \begin{bmatrix}U^{\rm T}\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}, \quad \Sigma_0 = \begin{bmatrix}0 & P_1 & P_2 & P_3 & P_4 & 0\\ * & D_1 & D_2 & D_3 & D_4 & X_1\\ * & * & D_5 & D_6 & D_7 & X_2\\ * & * & * & D_8 & D_9 & X_3\\ * & * & * & * & D_{10} & X_4\\ * & * & * & * & * & X_5\end{bmatrix}$$

Then we choose the orthogonal complement of Σ_1 as

$$\Sigma_{1\perp} = \begin{bmatrix}A & A_d & 0 & B & 0\\ I & 0 & 0 & 0 & 0\\ 0 & I & 0 & 0 & 0\\ 0 & 0 & I & 0 & 0\\ 0 & 0 & 0 & I & 0\\ 0 & 0 & 0 & 0 & I\end{bmatrix}$$

which satisfies $\Sigma_1\Sigma_{1\perp} = 0$. Moreover, $[\Sigma_1^{\rm T}, \Sigma_{1\perp}]$ is of column full rank. Then it follows that (1) is equivalent to the following matrix inequality:

$$\Sigma_{1\perp}^{\rm T}\begin{bmatrix}\Sigma_3\\ \Sigma_1\end{bmatrix}^{\rm T}\begin{bmatrix}\Sigma_0 & \Sigma_2\\ \Sigma_2^{\rm T} & 0\end{bmatrix}\begin{bmatrix}\Sigma_3\\ \Sigma_1\end{bmatrix}\Sigma_{1\perp} < 0$$

which can be further reduced to

$$\Sigma_{1\perp}^{\rm T}\Sigma_0\Sigma_{1\perp} < 0 \tag{2}$$

Thus, we have shown that the inequality (b) is equivalent to (2).

It is also easily seen that matrix inequality (a) can also be rewriten as (2). □

1 Problem formulation

Consider the following T-S fuzzy model with state delay:
Rule i: IF $\theta_1(t)$ is μ_{i1}, $\theta_2(t)$ is $\mu_{i2}, \cdots, \theta_p(t)$ is μ_{ip} then

$$\begin{aligned}\boldsymbol{x}(t+1) &= A_i\boldsymbol{x}(t) + A_{di}\boldsymbol{x}(t-\tau) + B_i\boldsymbol{u}(t) + B_{1i}\boldsymbol{d}(t)\\ \boldsymbol{z}(t) &= C_i\boldsymbol{x}(t) + C_{di}\boldsymbol{x}(t-\tau) + D_i\boldsymbol{u}(t) + D_{1i}\boldsymbol{d}(t)\\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t), \quad t = -\tau, \cdots, -1, 0\end{aligned} \tag{3}$$

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where $i \in I = \{1, 2, \cdots, r\}$, r is the number of if-then rules, $\theta_1(t) \sim \theta_p(t)$ are the premise variables, μ_{ij} is the fuzzy set, $\boldsymbol{x}(t) \in \mathbf{R}^n$, $\boldsymbol{z}(t) \in \mathbf{R}^{n_1}$, $\boldsymbol{u}(t) \in \mathbf{R}^m$, $\boldsymbol{d}(t) \in \mathbf{R}^{m_1}$, and $\boldsymbol{\phi}(t)$ are respectively the state, the controlled output, the control input, the disturbance, and the initial condition. $\tau > 0$ is an integer denoting the known constant time delay.

As illustrated in [5−6], we will define open subregions as $\Omega_p (p = 1, \cdots, s)$ in the state-space. The corresponding close subregions are defined as $\bar{\Omega}_p$, which satisfy

$$\bar{\Omega}_p \cap \bar{\Omega}_q = \bar{\Omega}_i^v, \quad p \neq q, \quad p, q = 1, \cdots, s, \quad i \in I$$

where $\bar{\Omega}_i^v = \{\boldsymbol{\theta} | h_i(\boldsymbol{\theta}) = 1, 0 \leq h_i(\boldsymbol{\theta} + \boldsymbol{\epsilon}) < 1, \forall 0 < |\boldsymbol{\epsilon}| \ll 1\}$. v is the set of face indexes of the polyhedral hull $\bar{\Omega}_i = \cup \bar{\Omega}_i^v, i \in I$. $h_i(\boldsymbol{\theta}) = \omega_i(\boldsymbol{\theta}(t)) / \sum_{i=1}^r \omega_i(\boldsymbol{\theta}(t))$, $\omega_i(\boldsymbol{\theta}(t)) = \Pi_{j=1}^p \mu_{ij}(\theta_j(t))$, and $\boldsymbol{\theta}(t) = [\theta_1(t), \cdots, \theta_p(t)]$.

Then, we will follow the idea of [9] to rewrite system (3) to be an equivalent discrete-time switching fuzzy system as the following form:

Region rule j:
IF $\boldsymbol{x}(t) \in \Omega_j$, then
Local plant rule k
IF $\theta_1(t)$ is $\mu_{jk1}, \cdots, \theta_p(t)$ is μ_{jkp}, then

$$\begin{aligned}
\boldsymbol{x}(t+1) &= A_{jk}\boldsymbol{x}(t) + A_{jdk}\boldsymbol{x}_\tau(t) + B_{jk}\boldsymbol{u}(t) + B_{j1k}\boldsymbol{d}(t) \\
\boldsymbol{z}(t) &= C_{jk}\boldsymbol{x}(t) + C_{jdk}\boldsymbol{x}_\tau(t) + D_{jk}\boldsymbol{u}(t) + D_{j1k}\boldsymbol{d}(t) \\
\boldsymbol{x}(t) &= \boldsymbol{\phi}(t), \quad t = -\tau, \cdots, -1, 0 \\
& k = 1, 2, \cdots, \beta(j), \quad j = 1, 2, \cdots, s
\end{aligned} \tag{4}$$

where $\boldsymbol{x}_\tau(t) = \boldsymbol{x}(t - \tau)$, Ω_j denotes the j-th subregion, s is the number of subregions partitioned on the state space, and $\beta(j)$ is the number of rules in the subregion Ω_j. The final output of the switching fuzzy system is inferred as

$$\begin{aligned}
\boldsymbol{x}(t+1) &= \sum_{k=1}^{\beta(j)} h_{jk}\{A_{jk}\boldsymbol{x}(t) + A_{jdk}\boldsymbol{x}_\tau(t) + B_{jk}\boldsymbol{u}(t) + B_{j1k}\boldsymbol{d}(t)\} \\
\boldsymbol{z}(t) &= \sum_{k=1}^{\beta(j)} h_{jk}\{C_{jk}\boldsymbol{x}(t) + C_{jdk}\boldsymbol{x}_\tau(t) + D_{jk}\boldsymbol{u}(t) + D_{j1k}\boldsymbol{d}(t)\} \\
\boldsymbol{x}(t) &= \boldsymbol{\phi}(t), \quad t = -\tau, \cdots, -1, 0,
\end{aligned} \tag{5}$$

where $\boldsymbol{x}(t) \in \Omega_j$,

$$h_{jk} = h_{jk}(\boldsymbol{\theta}(t)) = \frac{\prod_{l=1}^{p} \mu_{jkl}(\theta_l(t))}{\sum_{k=1}^{\beta(j)} \prod_{l=1}^{p} \mu_{jkl}(\theta_l(t))}$$

In this paper, two less conservative results for the controller design can be obtained by considering a delayed feedback switching fuzzy controller of the following form:

$$\boldsymbol{u}(t) = -\sum_{k=1}^{\beta(j)} h_{jk}\big[F_{jk}\boldsymbol{x}(t) + F_{jdk}\boldsymbol{x}_\tau(t)\big], \quad \boldsymbol{x}(t) \in \Omega_j \tag{6}$$

By substituting (6) into (5), the closed-loop switching fuzzy system can be represented as

$$\begin{aligned}
\boldsymbol{x}(t+1) &= \hat{A}_{jkl}\boldsymbol{x}(t) + \hat{A}_{jdkl}\boldsymbol{x}_\tau(t) + \hat{B}_{j1k}\boldsymbol{d}(t) \\
\boldsymbol{z}(t) &= \hat{C}_{jkl}\boldsymbol{x}(t) + \hat{C}_{jdkl}\boldsymbol{x}_\tau(t) + \hat{D}_{j1k}\boldsymbol{d}(t)
\end{aligned} \tag{7}$$

where $\boldsymbol{x}(t) \in \Omega_j$,

$$\begin{bmatrix} \hat{A}_{jkl} & \hat{A}_{jdkl} & \hat{B}_{j1k} \\ \hat{C}_{jkl} & \hat{C}_{jdkl} & \hat{D}_{j1k} \end{bmatrix} = \sum_{k=1}^{\beta(j)} \sum_{l=1}^{\beta(j)} h_{jk}h_{jl} \begin{bmatrix} A_{jk} - B_{jk}F_{jl} & A_{jdk} - B_{jk}F_{jdl} & B_{j1k} \\ C_{jk} - D_{jk}F_{jl} & C_{jdk} - D_{jk}F_{jdl} & D_{j1k} \end{bmatrix}$$

The objective of this paper is to design a suitable controller for system (5) with a guaranteed performance in the H_∞ sense that given a constant $\gamma > 0$, find a controller (6), such that the following two requirements are satisfied:

1) The disturbance-free system (7) is globally asymptotically stable;

2) Subject to assumption of zero initial conditions, the controlled output satisfies $\|\boldsymbol{z}\|_2 < \gamma \|\boldsymbol{d}\|_2$ for any nonzero $\boldsymbol{d} \in l_2$.

2 Main results

Let the subregion transition from Ω_j to Ω_i be denoted by $\Omega = \{(j, i) | \boldsymbol{x}(t) \in \Omega_j, \boldsymbol{x}(t+1) \in \Omega_i\}$. Here, i may be equal to j in Ω, when $\boldsymbol{x}(t)$ and $\boldsymbol{x}(t+1)$ are in the same subregion. Consequently, we have the following result.

Theorem 1. Given a constant $\gamma > 0$, the closed-loop discrete-time switching fuzzy system (7) is globally stable with H_∞ performance γ, if there exist a set of positive-definite matrices P_j, Q_1, Q_2, the nonsingular matrix F, and matrices X_{j1}, X_{j2}, X_{j3}, X_{j4}, N_{j1}, N_{j2}, N_{j3}, N_{j4}, M_{jk}, M_{jdk}, $j = 1, \cdots, s$, $k = 1, 2, \cdots, \beta(j)$, such that the following LMIs are satisfied:

$$\Pi_{ijkk} < 0, \; k = 1, \cdots, \beta(j), \; (j, i) \in \Omega \tag{8}$$

$$\Pi_{ijkl} + \Pi_{ijlk} < 0, \; 0 < k < l \leq \beta(j), \; (j, i) \in \Omega \tag{9}$$

$$(a) \begin{bmatrix} H_e\{P_1^{\mathrm{T}}A\} + D_1 & P_1^{\mathrm{T}}A_d + A^{\mathrm{T}}P_2 + D_2 & A^{\mathrm{T}}P_3 + D_3 & A^{\mathrm{T}}P_4 + P_1^{\mathrm{T}}B + D_4 & X_1 \\ * & H_e\{P_2^{\mathrm{T}}A_d\} + D_5 & A_d^{\mathrm{T}}P_3 + D_6 & A_d^{\mathrm{T}}P_4 + P_2^{\mathrm{T}}B + D_7 & X_2 \\ * & * & D_8 & P_3^{\mathrm{T}}B + D_9 & X_3 \\ * & * & * & H_e\{B^{\mathrm{T}}P_4\} + D_{10} & X_4 \\ * & * & * & * & X_5 \end{bmatrix} < 0$$

$$(b) \begin{bmatrix} -H_e\{U\} & P_1 + U^{\mathrm{T}}A & P_2 + U^{\mathrm{T}}A_d & P_3 & P_4 + U^{\mathrm{T}}B & 0 \\ * & D_1 & D_2 & D_3 & D_4 & X_1 \\ * & * & D_5 & D_6 & D_7 & X_2 \\ * & * & * & D_8 & D_9 & X_3 \\ * & * & * & * & D_{10} & X_4 \\ * & * & * & * & * & X_5 \end{bmatrix} < 0$$

where $\Pi_{ijkl} =$

$$\begin{bmatrix} -H_e\{F\} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & 0 & 0 \\ * & D_{1j} & D_{2j} & D_{3j} & D_{4j} & X_{j1} & \Pi_{27} \\ * & * & D_{5j} & D_{6j} & D_{7j} & X_{j2} & \Pi_{37} \\ * & * & * & D_{8j} & D_{9j} & X_{j3} & 0 \\ * & * & * & * & D_{10} & X_{j4} & D_{j1k}^{\rm T} \\ * & * & * & * & * & -\tau^{-1}Q_2 & 0 \\ * & * & * & * & * & * & -I \end{bmatrix}$$

with

$\Pi_{12} = P_i + N_{j1} + A_{jk}F - B_{jk}M_{jl}$, $\Pi_{13} = N_{j2} + A_{jdk}F - B_{jk}M_{jdl}$, $\Pi_{14} = P_i + N_{j3}$, $\Pi_{15} = N_{j4} + B_{j1k}$, $\Pi_{27} = (C_{jk}F - D_{jk}M_{jl})^{\rm T}$, $\Pi_{37} = (C_{jdk}F - D_{jk}M_{jdl})^{\rm T}$, $D_{1j} = Q_1 - 2P_j + H_e\{X_{j1} - N_{j1}\}$, $D_{2j} = -X_{j1} + X_{j2}^{\rm T} - N_{j2}$, $D_{3j} = X_{j3}^{\rm T} - N_{j1}^{\rm T} - N_{j3}$, $D_{4j} = X_{j4}^{\rm T} - N_{j4}$, $D_{5j} = -Q_1 - H_e\{X_{j2}\}$, $D_{6j} = -N_{j2}^{\rm T} - X_{j3}^{\rm T}$, $D_{7j} = -X_{j4}^{\rm T}$, $D_{8j} = \tau Q_2 - H_e\{N_{j3}\}$, $D_{9j} = -N_{j4}$, $D_{10} = -\gamma^2 I$

Moreover, the control law is given by

$$F_{jk} = M_{jk}F^{-1},\ F_{jdk} = M_{jdk}F^{-1},\ k = 1, \cdots, \beta(j)$$

Proof. Let

$$\bar{X} = F^{-\rm T}XF^{-1}$$

where X stands for $P_j, Q_1, Q_2, N_{j1}, N_{j2}, N_{j3}, X_{j1}, X_{j2}, X_{j3}, D_{1j}, D_{2j}, D_{3j}, D_{5j}, D_{6j}$, and $D_{8j} (j = 1, \cdots, s)$.

Choose the following piecewise Lyapunov function:

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

$$V_1(t) = 2\boldsymbol{x}^{\rm T}(t)\bar{P}_j\boldsymbol{x}(t), V_2(t) = \sum_{m=t-\tau}^{t-1}\boldsymbol{x}^{\rm T}(m)\bar{Q}_1\boldsymbol{x}(m)$$

$$V_3(t) = \sum_{\theta=-\tau}^{-1}\sum_{m=t+\theta}^{t-1}\boldsymbol{\zeta}^{\rm T}(m)\bar{Q}_2\boldsymbol{\zeta}(m), \quad \boldsymbol{x}(t) \in \Omega_j$$

where $\boldsymbol{\zeta}(t) = \boldsymbol{x}(t+1) - \boldsymbol{x}(t)$. Define $\Delta V(t) = V(t+1) - V(t)$ then along the solution (7), we have

$$\Delta V_1(t) = 2[\hat{A}_{jkl}\boldsymbol{x}(t) + \hat{A}_{jdkl}\boldsymbol{x}_\tau(t) + \hat{B}_{j1k}\boldsymbol{d}(t)]^{\rm T}\bar{P}_i \times [\boldsymbol{\zeta}(t) + \boldsymbol{x}(t)] - 2\boldsymbol{x}^{\rm T}(t)\bar{P}_j\boldsymbol{x}(t) \tag{10}$$

$$\Delta V_2(t) = \boldsymbol{x}^{\rm T}(t)\bar{Q}_1\boldsymbol{x}(t) - \boldsymbol{x}_\tau^{\rm T}(t)\bar{Q}_1\boldsymbol{x}_\tau(t) \tag{11}$$

$$\Delta V_3(t) = \tau\boldsymbol{\zeta}^{\rm T}(t)\bar{Q}_2\boldsymbol{\zeta}(t) - \sum_{m=t-\tau}^{t-1}\boldsymbol{\zeta}^{\rm T}(m)\bar{Q}_2\boldsymbol{\zeta}(m) \tag{12}$$

Observing the definition of $\boldsymbol{\zeta}(t)$ and system (7), we can get the following equations:

$$\mathscr{M}_1 = 2[\boldsymbol{x}^{\rm T}(t)\bar{X}_{j1} + \boldsymbol{x}_\tau^{\rm T}(t)\bar{X}_{j2} + \boldsymbol{\zeta}^{\rm T}(t)\bar{X}_{j3} + \boldsymbol{d}^{\rm T}(t)X_{j4}U] \times \Big[\boldsymbol{x}(t) - \boldsymbol{x}_\tau(t) - \sum_{m=t-\tau}^{t-1}\boldsymbol{\zeta}(m)\Big] = 0 \tag{13}$$

$$\mathscr{M}_2 = 2[\boldsymbol{x}^{\rm T}(t)\bar{N}_{j1}^{\rm T} + \boldsymbol{x}_\tau^{\rm T}(t)\bar{N}_{j2}^{\rm T} + \boldsymbol{\zeta}^{\rm T}(t)\bar{N}_{j3}^{\rm T} + \boldsymbol{d}^{\rm T}(t)N_{j4}^{\rm T}U] \times [(\hat{A}_{jkl} - I)\boldsymbol{x}(t) + \hat{A}_{jdkl}\boldsymbol{x}_\tau(t) + \hat{B}_{j1k}\boldsymbol{d}(t) - \boldsymbol{\zeta}(t)] = 0 \tag{14}$$

Since $\pm 2\boldsymbol{a}^{\rm T}\boldsymbol{b} \le \boldsymbol{a}^{\rm T}M\boldsymbol{a} + \boldsymbol{b}^{\rm T}M^{-1}\boldsymbol{b}$ holds for compatible vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, and any compatible matrix $M > 0$, we have

$$-2[\boldsymbol{x}^{\rm T}(t)\bar{X}_{j1} + \boldsymbol{x}_\tau^{\rm T}(t)\bar{X}_{j2} + \boldsymbol{\zeta}^{\rm T}(t)\bar{X}_{j3} + \boldsymbol{d}^{\rm T}(t)X_{j4}U] \times \sum_{m=t-\tau}^{t-1}\boldsymbol{\zeta}(m) \le \tau\boldsymbol{\xi}^{\rm T}(t)\begin{bmatrix}\bar{X}_{j1}\\ \bar{X}_{j2}\\ \bar{X}_{j3}\\ X_{j4}U\end{bmatrix}\bar{Q}_2^{-1}\begin{bmatrix}\bar{X}_{j1}\\ \bar{X}_{j2}\\ \bar{X}_{j3}\\ X_{j4}U\end{bmatrix}^{\rm T}\boldsymbol{\xi}(t) + \sum_{m=t-\tau}^{t-1}\boldsymbol{\zeta}^{\rm T}(m)\bar{Q}_2\boldsymbol{\zeta}(m) \tag{15}$$

Then, from (10) ~ (15) and considering (7), we can obtain that

$$\Delta V(t) + \boldsymbol{z}^{\rm T}(t)\boldsymbol{z}(t) - \gamma^2\boldsymbol{d}^{\rm T}(t)\boldsymbol{d}(t) \le \boldsymbol{\xi}^{\rm T}(t)\Xi_{ijkl}\boldsymbol{\xi}(t) \tag{16}$$

where $\Xi_{ijkl} =$

$$\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14}\\ * & \Xi_{22} & \Xi_{23} & \Xi_{24}\\ * & * & \Xi_{33} & \Xi_{34}\\ * & * & * & \Xi_{44}\end{bmatrix} + \tau\begin{bmatrix}\bar{X}_{j1}\\ \bar{X}_{j2}\\ \bar{X}_{j3}\\ X_{j4}U\end{bmatrix}\bar{Q}_2^{-1}\begin{bmatrix}\bar{X}_{j1}\\ \bar{X}_{j2}\\ \bar{X}_{j3}\\ X_{j4}U\end{bmatrix}^{\rm T} + \begin{bmatrix}\hat{C}_{jkl} & \hat{C}_{jdkl} & 0 & \hat{D}_{j1k}\end{bmatrix}^{\rm T}\begin{bmatrix}\hat{C}_{jkl} & \hat{C}_{jdkl} & 0 & \hat{D}_{j1k}\end{bmatrix}$$

with

$\Xi_{11} = H_e\{(\bar{P}_i + \bar{N}_{j1})^{\rm T}\hat{A}_{jkl}\} + \bar{D}_{1j}$, $\Xi_{12} = (\bar{P}_i + \bar{N}_{j1})^{\rm T} \times \hat{A}_{jdkl} + \hat{A}^{\rm T}T_{jkl}\bar{N}_{j2} + \bar{D}_{2j}$, $\Xi_{13} = \hat{A}_{jkl}^{\rm T}(\bar{P}_i + \bar{N}_{j3}) + \bar{D}_{3j}$, $\Xi_{14} = (\bar{P}_i + \bar{N}_{j1})^{\rm T}\hat{B}_{j1k} + \hat{A}_{jkl}^{\rm T}U^{\rm T}N_{j4} + U^{\rm T}D_{4j}$, $\Xi_{22} = \bar{D}_{5j} + H_e\{\bar{N}_{j2}^{\rm T}\hat{A}_{jdkl}\}$, $\Xi_{23} = \hat{A}_{jdkl}^{\rm T}(\bar{P}_i + \bar{N}_{j3}) + \bar{D}_{6j}$, $\Xi_{33} = \bar{D}_{8j}$, $\Xi_{24} = \hat{A}_{jdkl}^{\rm T}U^{\rm T}N_{j4} + \bar{N}_{j2}^{\rm T}\hat{B}_{j1k} + U^{\rm T}D_{7j}$, $\Xi_{34} = U^{\rm T}D_{9j} + (\bar{P}_i + \bar{N}_{j3})^{\rm T}\hat{B}_{j1k}$, $\Xi_{44} = H_e\{\hat{B}_{j1k}^{\rm T}U^{\rm T}N_{j4}\} + D_{10}$

Then

$$\Delta V(t) + \boldsymbol{z}^{\rm T}(t)\boldsymbol{z}(t) - \gamma^2\boldsymbol{d}^{\rm T}(t)\boldsymbol{d}(t) < 0 \tag{17}$$

If

$$\Xi_{ijkl} < 0 \tag{18}$$

by Schur complement, (18) is equivalent to

$$\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \bar{X}_{j1} & \hat{C}_{jkl}^{\rm T}\\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \bar{X}_{j2} & \hat{C}_{jdkl}^{\rm T}\\ * & * & \Xi_{33} & \Xi_{34} & \bar{X}_{j3} & 0\\ * & * & * & \Xi_{44} & X_{j4}U & \hat{D}_{j1k}^{\rm T}\\ * & * & * & * & -\tau^{-1}\bar{Q}_2 & 0\\ * & * & * & * & * & -I\end{bmatrix} < 0 \tag{19}$$

Using Lemma 1, (19) is equivalent to (20), as shown at the bottom of the next page.

Let $U = F^{-1}$, $F_{jl} = M_{jl}U$, $F_{jdl} = M_{jdl}U$, and $G = {\rm diag}\{F, F, F, F, I, F, I\}$. Pre- and post-multiplying the left-hand side of (20) by $G^{\rm T}$ and G, respectively, we have the following equation

$$\Pi = \sum_{k=1}^{\beta(j)}h_{jk}^2\Pi_{ijkk} + \sum_{k<l}^{\beta(j)}h_{jk}h_{jl}(\Pi_{ijkl} + \Pi_{ijlk}) \tag{21}$$

If (8) and (9) hold, $\Pi < 0$, which implies that (17) holds.

Therefore, when assuming the zero disturbance input, from (10) $\sim$ (15), we can obtain that

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \mathscr{M}_1 + \mathscr{M}_2 \le \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}_\tau(t) \\ \boldsymbol{\zeta}(t) \end{bmatrix}^{\mathrm{T}} \left\{ \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix} + \tau \begin{bmatrix} \bar{X}_{j1} \\ \bar{X}_{j2} \\ \bar{X}_{j3} \end{bmatrix} \bar{Q}_2^{-1} \begin{bmatrix} \bar{X}_{j1} \\ \bar{X}_{j2} \\ \bar{X}_{j3} \end{bmatrix}^{\mathrm{T}} \right\} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}_\tau(t) \\ \boldsymbol{\zeta}(t) \end{bmatrix}$$

By Schur complement, LMI (19) implies $\Delta V < 0$. We can conclude that the closed-loop system (7) with $\boldsymbol{d}(t) = \boldsymbol{0}$ is asymptotically stable.

Now, to establish the H_∞ performance for the closed loop system (7), asssume zero-initial condition, and consider the following index:

$$J = \sum_{t=0}^{\infty} [\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t) - \gamma^2 \boldsymbol{d}^{\mathrm{T}}(t)\boldsymbol{d}(t)] \tag{22}$$

Under zero initial condition and (17), we have $J \le \sum_{t=0}^{\infty}[-\Delta V(t)] = -V(\infty) + V(0) < 0$, which means that $\|\boldsymbol{z}\|_2 < \gamma \|\boldsymbol{d}\|_2$. □

To reduce the stabilization criterion further, the interactions among the fuzzy subsystems in each subregion Ω_j will be considered. In addition, all matrices Z_{jkl}, $1 \le k < l \le \beta(j)$, are not required to be symmetric. Consequently, we have the following result.

Theorem 2. Given a constant $\gamma > 0$, the closed-loop discrete-time switching fuzzy system (7) is globally stable with H_∞ performance γ, if there exist a set of positive-definite matrices P_j, Q_1, Q_2, the nonsingular matrix F, and matrices X_{j1}, X_{j2}, X_{j3}, X_{j4}, N_{j1}, N_{j2}, N_{j3}, N_{j4}, M_{jk}, M_{jdk}, $Z_{jlk} = Z_{jkl}^{\mathrm{T}}$, $j = 1, \cdots, s$, $k, l = 1, \cdots, \beta(j)$, such that the following LMIs are satisfied:

$$\Pi_{ijkk} < Z_{jkk},\ k = 1, \cdots, \beta(j),\ (j, i) \in \Omega \tag{23}$$

$$\Pi_{ijkl} + \Pi_{ijlk} < Z_{jkl} + Z_{jkl}^{\mathrm{T}}, 0 < k < l \le \beta(j), (j, i) \in \Omega \tag{24}$$

$$Z_j = \begin{bmatrix} Z_{j11} & Z_{j12} & \cdots & Z_{j1\beta(j)} \\ Z_{j21} & Z_{j22} & \cdots & Z_{j2\beta(j)} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{j\beta(j)1} & Z_{j\beta(j)2} & \cdots & Z_{j\beta(j)\beta(j)} \end{bmatrix} < 0 \tag{25}$$

Moreover, the control law is given by

$$F_{jk} = M_{jk}F^{-1},\ F_{jdk} = M_{jdk}F^{-1},\ k = 1, \cdots, \beta(j)$$

Proof. If (23) and (24) are feasible

$$\Pi < \sum_{k=1}^{\beta(j)} h_{jk}^2 Z_{jkk} + \sum_{k<l}^{\beta(j)} h_{jk}h_{jl}(Z_{jkl} + Z_{jkl}^{\mathrm{T}}) = \mathbb{H}_j^{\mathrm{T}} Z_j \mathbb{H}_j$$

where $\mathbb{H}_j \equiv \begin{bmatrix} h_{j1}I & h_{j2}I & \cdots & h_{j\beta(j)}I \end{bmatrix}^{\mathrm{T}}$.

If (25) holds, $\Pi < 0$. Then, we have (17). Then, the Theorem 2 can be proved by following the same lines as in the proof of Theorem 1. □

3 Numerical example

Example 1. Consider a local communication network system which is borrowed from [6] to show the effectiveness and advantage of the results in this paper.

The membership functions μ_{i1} are

$$\mu_{11} = \begin{cases} 1, & -5 \le x_1 < -2 \\ 0.5(1 - 0.5x_1), & -2 \le x_1 < 2 \\ 0, & 2 \le x_1 \le 5 \end{cases}$$

$$\mu_{21} = \begin{cases} 0, & -5 \le x_1 < -2 \\ 0.5(1 + 0.5x_1), & -2 \le x_1 < 2 \\ 1 - (1 - \exp\{-5(x_1 - 3.5)\})^{-1}, & 2 < x_1 \le 5 \end{cases}$$

$$\mu_{31} = \begin{cases} 0, & -5 \le x_1 \le 2 \\ (1 + \exp\{-5(x_1 - 3.5)\})^{-1}, & 2 < x_1 \le 5 \end{cases}$$

The system matrices are

$$A_1 = \begin{bmatrix} 1 & 2.5 \\ 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix},\ A_3 = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0.015 & 0 \\ 1 & 0 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.005 & 0 \\ 0.33 & 0 \end{bmatrix}, A_{d3} = \begin{bmatrix} 0.009 & 0 \\ 0.6 & 0 \end{bmatrix}$$

$$\boldsymbol{B}_1 = \boldsymbol{B}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}}, \boldsymbol{B}_3 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}^{\mathrm{T}}, \boldsymbol{B}_{11} = \begin{bmatrix} 0 & 0.02 \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{B}_{12} = \begin{bmatrix} 0.02 & 0 \end{bmatrix}^{\mathrm{T}},\ \boldsymbol{B}_{13} = \begin{bmatrix} 0.05 & 0.05 \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{C}_1 = \boldsymbol{C}_2 = \begin{bmatrix} 0.02 & -0.03 \end{bmatrix},\ \boldsymbol{C}_3 = \begin{bmatrix} 0.01 & -0.02 \end{bmatrix}$$

$$\boldsymbol{C}_{d1} = \boldsymbol{C}_{d2} = \boldsymbol{C}_{d3} = \begin{bmatrix} 0.03 & -0.01 \end{bmatrix},$$

$$D_1 = D_2 = D_3 = 0.1,\ D_{11} = D_{12} = D_{13} = 0.05$$

The parameter α is adjusted to compare the relaxation of Theorems 1 and 2. For $\gamma = 0.2$, Table 1 shows the allowable values of α. Divide the state space into three subregions. The membership functions μ_{ji1} and partition of subregions are shown in Fig. 1. The system matrices are $A_{11} = A_{21} = A_1$, $A_{1d1} = A_{2d1} = A_{d1}$, $\boldsymbol{B}_{11} = \boldsymbol{B}_{21} = \boldsymbol{B}_1$, $\boldsymbol{B}_{111} = \boldsymbol{B}_{211} = \boldsymbol{B}_{11}$, $\boldsymbol{C}_{11} = \boldsymbol{C}_{21} = \boldsymbol{C}_1$, $\boldsymbol{C}_{1d1} = \boldsymbol{C}_{2d1} = \boldsymbol{C}_{d1}$, $D_{11} = D_{21} = D_1$, $D_{111} = D_{211} = D_{11}$, for $\mu_{11} = \mu_{111} \cup \mu_{211}$, $A_{22} = A_{31} = A_2$, $A_{2d2} = A_{3d1} = A_{d2}$, $\boldsymbol{B}_{22} = \boldsymbol{B}_{31} = \boldsymbol{B}_2$, $\boldsymbol{B}_{212} = \boldsymbol{B}_{311} = \boldsymbol{B}_{12}$, $\boldsymbol{C}_{22} = \boldsymbol{C}_{31} = \boldsymbol{C}_2$, $\boldsymbol{C}_{2d2} = \boldsymbol{C}_{3d1} = \boldsymbol{C}_{d2}$, $D_{22} = D_{31} = D_2$, $D_{212} = D_{311} = D_{12}$, for $\mu_{21} = \mu_{221} \cup \mu_{311}$, $A_{32} = A_3$, $A_{3d2} = A_{d3}$, $\boldsymbol{B}_{32} = \boldsymbol{B}_3$, $\boldsymbol{B}_{312} = \boldsymbol{B}_{13}$, $\boldsymbol{C}_{32} = \boldsymbol{C}_3$, $\boldsymbol{C}_{3d2} = \boldsymbol{C}_{d3}$, $D_{32} = D_3$, $D_{312} = D_{13}$, for $\mu_{31} = \mu_{321}$.

Table 1 Allowable values of α

Methods	Theorem 1	Theorem 2
a	[0.412, 2.186]	[0.411, 3.172]

$$\sum_{k=1}^{\beta(j)} \sum_{l=1}^{\beta(j)} h_{jk}h_{jl} \begin{bmatrix} -H_e\{U\} & \bar{P}_i + \bar{N}_{j1} + U^{\mathrm{T}}A_{jkl} & \bar{N}_{j2} + U^{\mathrm{T}}A_{jdkl} & \bar{P}_i + \bar{N}_{j3} & U^{\mathrm{T}}(N_{j4} + B_{j1k}) & 0 & 0 \\ * & \bar{D}_{1j} & \bar{D}_{2j} & \bar{D}_{3j} & U^{\mathrm{T}}D_{4j} & \bar{X}_{j1} & C_{jkl}^{\mathrm{T}} \\ * & * & \bar{D}_{5j} & \bar{D}_{6j} & U^{\mathrm{T}}D_{7j} & \bar{X}_{j2} & C_{jdkl}^{\mathrm{T}} \\ * & * & * & \bar{D}_{8j} & U^{\mathrm{T}}D_{9j} & \bar{X}_{j3} & 0 \\ * & * & * & * & D_{10} & X_{j4}U & D_{j1k}^{\mathrm{T}} \\ * & * & * & * & * & -\tau^{-1}\bar{Q}_2 & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{20}$$

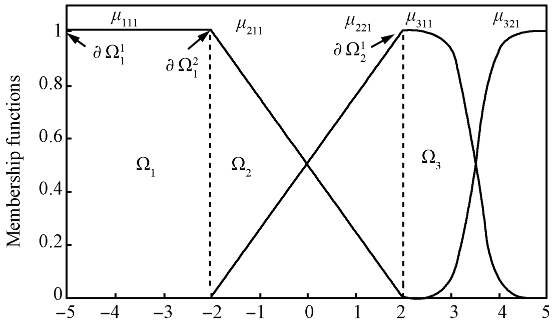

Fig. 1 Membership functions and partition of subspaces

Assume $\alpha = 3$ and $d(t) = \mathrm{e}^{-0.2t}\sin t$. References [5] and [6] fail to find a feasible solution. In contrast, by Theorem 2, we obtain

$$P_1 = \begin{bmatrix} 0.9154 & -0.7736 \\ -0.7736 & 1.0807 \end{bmatrix},\ P_2 = \begin{bmatrix} 1.3193 & -0.9962 \\ -0.9962 & 1.2010 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 1.3188 & -0.9973 \\ -0.9973 & 1.2025 \end{bmatrix},\ F = \begin{bmatrix} 2.5980 & -2.0094 \\ -1.9773 & 2.3448 \end{bmatrix}$$

$$F_{11} = \begin{bmatrix} 0.4517 & 1.6895 \end{bmatrix},\ F_{21} = \begin{bmatrix} 0.4450 & 1.6864 \end{bmatrix}$$

$$F_{22} = \begin{bmatrix} 0.4463 & 0.7129 \end{bmatrix},\ F_{31} = \begin{bmatrix} 0.1074 & 0.2879 \end{bmatrix}$$

$$F_{32} = \begin{bmatrix} 1.1942 & 3.9106 \end{bmatrix},\ F_{1d1} = \begin{bmatrix} 0.5198 & -0.0003 \end{bmatrix}$$

$$F_{2d1} = \begin{bmatrix} 0.5198 & -0.0003 \end{bmatrix},\ F_{2d2} = \begin{bmatrix} 0.1723 & -0.0003 \end{bmatrix}$$

$$F_{3d1} = \begin{bmatrix} 0.2146 & -0.0022 \end{bmatrix},\ F_{3d2} = \begin{bmatrix} 0.4508 & 0.0015 \end{bmatrix}$$

Fig. 2 shows the state responses of the closed-loop system (7) with initial conditions $\boldsymbol{\phi}(t) = [-3.5\mathrm{e}^{-t/3}, t+4]^{\mathrm{T}}, t = -3, \cdots, 0$.

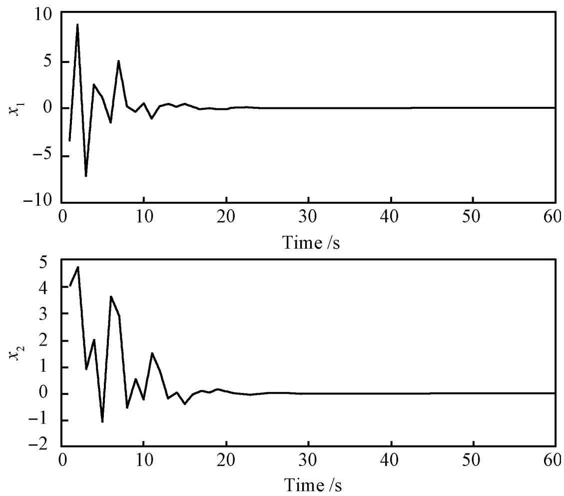

Fig. 2 The state responses

4 Conclusion

Based on the switching fuzzy system, piecewise Lyapunov function, and state transitions between all possible subregions, two new stabilization conditions with H_∞ performance for discrete-time fuzzy system have been proposed. The new condition has reduced the conservatism of the previous works. If the conditions are feasible, the state feedback controllers can be easily constructed by solving a set of LMIs. An example was presented to demonstrate the advantage of the proposed approach.

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