Assigning People to Jobs at Ajax Auto: A Case that Applies the Linear Assignment Algorithm to a Common HR Problem

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ABSTRACT

This paper presents a case that has been successfully used more than 30 times to teach the application of the linear assignment algorithm to the human resource problem of assigning people to jobs. The case has generally been used in human resource courses in an MBA program but has occasionally been successfully used with small classes of very good undergraduates. We offer a brief summary of some of the recent literature on assigning people to jobs, the text of the case, an explanation of how to formulate and solve the problem using Excel, and teaching notes explaining how to get the most out of the case discussion and/or case write-ups.

INTRODUCTION

While human resource courses at colleges and universities do not often emphasize management science, most HR professors would acknowledge that there are some management science applications in human resources. This paper focuses on a case to teach the linear assignment algorithm, optimizing assignment to task based on ratings in a cross training program. We argue that it is important that MBA students see the application of management science methods applied to human resource problems because there is a considerable amount of work that has been done in this area. Those who are going to be managing human resource functions would do well to be aware of the potential contributions of management science to the problem of assigning people to tasks, positions, or jobs.

Human resource professionals and academics may not be familiar with the management science literature focused on the assignment problem. It is beyond the scope of this paper to provide a full literature review. We will offer only a few recent studies in this area to reinforce our point that this is a problem that management scientists, both practitioners and academics, have worked on extensively.

RECENT MANAGEMENT SCIENCE STUDIES

Most recently, work by Srour, Haas, and Morton (2006) focuses on a problem common to the construction industry in the United States and other parts of the world -- namely, shortages workers with the appropriate construction skills. They further note that there are *no human resource management* strategies for staffing construction workers at project, corporate, regional, or industry levels. Their work "presents a framework to optimize the investment in, and make the best use of, the available workforce with the intent to reduce project costs and improve schedule performance (Srour, Haas, & Morton, 2006; p. 1158)".

Amacost and James (2005) report on a linear programming assignment model for U.S. Air Force Academy graduates. Their algorithm allows measuring and balancing cadets' class standing, Air Force career field requirements, and cadets' career field preferences in order to assign each cadet to a career field in the Air Force. Their computational experiments show the superiority of their method over previous classification approaches. Their method yields more than a 10% increase in the number of cadets assigned to their top career field choice and almost a 100% reduction in the number of cadets not receiving any of their career field choices. The outcome of using such a model would likely be a cadre of Air Force Academy graduates who will be much more satisfied with their field assignments than previous classes.

Holder (2005) proposes a model designed to aid in optimizing the process of assigning sailors to jobs. This procedure attempts to achieve an increased level of sailor satisfaction by providing a list of possible jobs from which a sailor may choose. His work demonstrates that the optimal partition from an interior-point algorithm is particularly useful when designing the job lists. Once again, here is an article focused on assigning people to jobs in some optimal sense.

We could easily adduce ten more examples of management science work on the optimal assignment of people to jobs. However, articles such as these are rarely cited in human resource textbooks and even more rarely included in course work. The Ajax Auto Case presented below is brief, can be done in Excel, takes a fairly short amount of time for students to complete, and provides them with an experience that will allow them to connect their coursework in management science and human resource management.

The objectives of the Ajax Automobile Case are to teach techniques of assigning people to jobs, critical reasoning, effective written communication, clear communication of quantitative analysis, and spreadsheet skills using Excel and Solver.

THE AJAX AUTO COMPANY CASE

To combat rising labor costs in Germany and to get around certain import duties, BMW, among other car makers, imports unassembled automobiles into Mexico. In Puebla, as well as other cities around the country, they assemble the cars and sell them in the Mexican market. In order to make this strategy work, it is vital to produce high quality automobiles -- and the finish of the automobiles contributes considerably to consumer perceptions of quality. Hence, it is important to assign workers to those tasks for which they are best suited in order to produce a high quality product. This is especially true in Mexico where highly skilled workers are sometimes quite hard to find.

Juan Gonzalez De La Borda is the plant manager of the plant in Mexico that assembles BMW models imported from Germany. His plant in Puebla is a model of efficiency and effectiveness. As a result of recent Business Process Reengineering training, he has workers who have been cross-trained in all the main tasks of the plant. Moreover, he has the training ratings of each worker in each task. These ratings are prepared by the training supervisor on the last day of each training program. The ratings are shown in Table 1.

Table 1

Ratings of the Workers in the Cross-Training Program

	Sheet Metal Finishing	Tires, Brakes, Steering	Engine and Trans.	Windshield and Windows	Trunk and Bumpers	Electrical Harness	Seats and Carpeting	Coachwork and Headliner	Lights and Wipers	Quality Coord.
Ceron	6	9	4	6	7	4	5	4	8	7
Gonzales	3	5	2	4	8	4	4	5	6	6
Laborde	8	10	5	5	9	7	5	7	8	8
Muñoz	6	7	4	5	6	6	7	7	5	8
Ortiz	4	5	5	4	9	4	2	4	7	5
Penazola	6	9	4	8	5	5	5	6	7	8
Pisano	7	4	5	5	5	9	4	2	4	7
Ramirez	6	2	5	10	4	4	4	7	8	6
Sanchez	5	2	3	4	7	3	4	7	6	5
Villareal	4	6	5	6	8	4	3	7	9	8

The extensive cross-training done at this plant means that any worker can be assigned to any of these ten jobs. However, such an assignment is often less than optimal. As you can see, workers are simply better at some jobs than others. Juan has called you in to use your extensive knowledge of management science to calculate the ideal assignment of workers to jobs[1].

Using your knowledge of "solver" in Excel, set up and solve this problem. In the Ideal Assignment Form below, indicate which worker should be assigned to which job. Show all your work.

Ideal Assignment

1. Sheet metal: _______________
2. Tires, brakes, steering: _______________
3. Engine and transmission: _______________
4. Windshield and windows: _______________
5. Trunk and bumpers: _______________
6. Electrical harness: _______________
7. Seats and carpeting: _______________
8. Coachwork and headliner: _______________
9. Lights and wipers: _______________
10. Quality coordination: _______________

THE QUANTITATIVE MODEL

It is useful to think of decision making situations in terms of the ABCs of Optimization (Savage, 1993), which is a convenient way to organize all the relevant information. The ABCs stand for the questions: (1) What is "Adjustable?" (2) What is "Best?" and (3) What "Constrains" us? The Adjustable quantities are the decision variables, quantities whose best values are to be chosen. These define the essence of the choice to be made, a list of numbers to be determined. Specifying what is Best defines the objective of the decision, i.e., a way to measure how good each possible decision is. The objective in business applications is usually either to maximize the profit or to minimize the cost resulting from the decision. The analyst defines an objective function, which is a formula that computes the value of the objective for any set of values of the decision variables. Finally, the Constraints describe all aspects of the decision making environment that limit the decision in some way, such as resource availability. Constraints are defined as relationships among the decision variables that must be satisfied if the

[1] In order to make this problem readily soluble for people with limited management science backgrounds, this assignment problem is a simplification of the actual problem.

decision is to be a feasible one. These relationships may be expressed as equations or inequalities.

Once the decision making situation has been formulated into the ABCs of Optimization, the resulting mathematical model can be solved to find the values of the decision variables that give the best value of the objective function among all decisions that satisfy the constraints. There are many software packages available to solve problems of this form, including the Solver tool of Microsoft Excel.

The Ajax Auto Assignment problem may be cast in the ABCs format as follows.

Assign workers to tasks, so as to:

Maximize the sum of all worker ratings for the tasks to which they have been

assigned, subject to:

The total number of workers assigned to each task = 1,
The total number of tasks assigned to each worker = 1.

In order to translate the problem statement into mathematical form we first label the workers and tasks with indices from 1 to 10, and define the decision variables

$$x_{ij} = \begin{cases} 1, \text{ if worker "i" is assigned to task "j" and} \\ 0, \text{ if worker "i" is not assigned to task "j".} \end{cases}$$

Then we define r_{ij} as the rating of worker "i" for task "j", and our mathematical formulation takes the following form:

$$\text{Maximize } \sum_{i,j} x_{ij}\, r_{ij}$$

subject to:

$$\sum_i x_{ij} = 1, \text{ for } j = 1,\ldots, 10.$$

$$\sum_j x_{ij} = 1, \text{ for } i = 1,\ldots, 10.$$

$$x_{ij} = 0 \text{ or } 1, \text{ for } i = 1,\ldots, 10 \text{ and } j = 1,\ldots, 10.$$

A mathematical problem of this form is called a linear optimization model or a linear program. This particular problem is of a special form called an assignment problem (Albright, et al, 2006). As we noted earlier, assignment problems have been studied in great depth in the management science literature. We will briefly discuss the literature in the Model Variants section below.

The Excel Worksheet

We proceed by showing how Microsoft Excel's Solver tool can be used to analyze the assignment problem formulated above. The Solver tool is a standard feature of Excel, but it must be added to the default Excel configuration through the *Tools/Add-Ins…* menu selection.

To begin with, we must have our assignment model formulated in a proper spreadsheet layout. This means that the worksheet must have:

1. a cell for each decision variable
2. a cell with a formula that calculates the objective function
3. a cell with a formula that calculates the left-hand side of each constraint, and
4. a cell with the numerical value of each constraint's right-hand side.

Our assignment model does not require the fourth element, because the right-hand side of each constraint is 1. We will never want to change this value, so we will enter it directly into the Solver tool. The worksheet pictured in Figure 1 is designed to implement the assignment model for the Ajax Auto Assignment problem. Notice several features of the worksheet design.

The worksheet is clearly divided into four separate sections, each reserved for a different aspect of the model: a model overview, input data cells, decision variable cells and calculations. The organization accomplishes a number of goals. First, by providing an overview and separating the cells into clearly labeled sections, the worksheet is easily understood by users other than the worksheet designer. Second, by listing input data only in the designated cells and never entering them directly into formulas, the model parameters are clear and are easily changed when desired. Calculation cells contain no numbers, only cell references. Third, by assigning names to the most important ranges of cells, formulas are easily entered and understood. The named ranges are Ratings (the 10 by 10 table of worker-task ratings in the Input Data section), Assignments (the 10 by 10 table of decision cells), Score (the calculated objective value), Tasks (the row of 10 cells displaying the calculated number of tasks assigned to each worker) and Workers (the row of 10 cells displaying the calculated number of workers assigned to each task).

To see how to create names for cells or ranges of cells, refer to Figure 2, where the upper-left corner of the Excel window is displayed. Rows 2 through 42 are hidden, and cell C45 has been selected. The name "Score" is assigned to cell C45 by clicking in the *Name Box* (below the left end of the *Toolbar*) and entering the name. To name a range of cells follow the same procedure after first selecting the range of interest. More options for creating and managing named ranges are available by selecting *Name* under the *Insert* menu.

The cell contents are easily described. The input data cells are the ratings of each worker for each task. The decision cells must all be either 0 or 1 for a feasible decision, but the problem can be solved with any numbers in these cells initially. There are only three types of calculations. The formula in the Total Score cell (named Score, Figure 2) is =SUMPRODUCT (Assignments, Ratings). The Sumproduct function operates on cell ranges of equal dimensions, computes the products of cells in equivalent range positions, and adds all the products together. It is a very useful function that easily performs 100 multiplications and 99 additions for us here.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	***Model Overview***											
3												
4	Assign workers to tasks so as to								**Named Ranges**			
5		Maximize the sum of all worker ratings for the tasks to which they have been assigned,							Assignments	$C$32:$L$41		
6		subject to:							Ratings	$C$16:$L$25		
7			Total number of workers assigned to each task = 1						Score	$C$45		
8			Total number of taks assigned to each worker = 1						Tasks	$C$51:$L$51		
9									Workers	$C$48:$L$48		
10												
11	***Input Data***											
12												
13		**Ratings:**										
14												
15			**Sheet Metal Finishing**	**Tires, Brakes, Steering**	**Engine and Trans.**	**Windshield and Windows**	**Trunk and Bumpers**	**Electrical Harness**	**Seats and Carpeting**	**Coachwork and Headliner**	**Lights and Wipers**	**Quality Coord.**
16		**Ceron**	6	9	4	6	7	4	5	4	8	7
17		**Gonzales**	3	5	2	4	8	4	4	5	6	6
18		**Laborde**	8	10	5	5	9	7	5	7	8	8
19		**Munoz**	6	7	4	5	6	6	7	7	5	8
20		**Ortiz**	4	5	5	4	9	4	2	4	7	5
21		**Penazola**	6	9	4	8	5	5	5	6	7	8
22		**Pisano**	7	4	5	5	5	9	4	2	4	7
23		**Ramirez**	6	2	5	10	4	4	4	7	8	6
24		**Sanchez**	5	2	3	4	7	3	4	7	6	5
25		**Villareal**	4	6	5	6	8	4	3	7	9	8
26												
27	***Decisions***											
28												
29		**Assignments:**										
30												
31			**Sheet Metal Finishing**	**Tires, Brakes, Steering**	**Engine and Trans.**	**Windshield and Windows**	**Trunk and Bumpers**	**Electrical Harness**	**Seats and Carpeting**	**Coachwork and Headliner**	**Lights and Wipers**	**Quality Coord.**
32		**Ceron**	0	1	0	0	0	0	0	0	0	0
33		**Gonzales**	0	0	0	0	1	0	0	0	0	0
34		**Laborde**	1	0	0	0	0	0	0	0	0	0
35		**Munoz**	0	0	0	0	0	0	1	0	0	0
36		**Ortiz**	0	0	1	0	0	0	0	0	0	0
37		**Penazola**	0	0	0	0	0	0	0	0	0	1
38		**Pisano**	0	0	0	0	0	1	0	0	0	0
39		**Ramirez**	0	0	0	1	0	0	0	0	0	0
40		**Sanchez**	0	0	0	0	0	0	0	1	0	0
41		**Villareal**	0	0	0	0	0	0	0	0	1	0
42												
43	***Calculations***											
44												
45		**Total Score:**	80									
46												
47		**Task:**	**Sheet Metal Finishing**	**Tires, Brakes, Steering**	**Engine and Trans.**	**Windshield and Windows**	**Trunk and Bumpers**	**Electrical Harness**	**Seats and Carpeting**	**Coachwork and Headliner**	**Lights and Wipers**	**Quality Coord.**
48	**Total Assignments:**		1	1	1	1	1	1	1	1	1	1
49												
50		**Person:**	**Ceron**	**Gonzales**	**Laborde**	**Munoz**	**Ortiz**	**Penazola**	**Pisano**	**Ramirez**	**Sanchez**	**Villareal**
51	**Total Assignments:**		1	1	1	1	1	1	1	1	1	1
52												

Figure 1. Assignment model worksheet

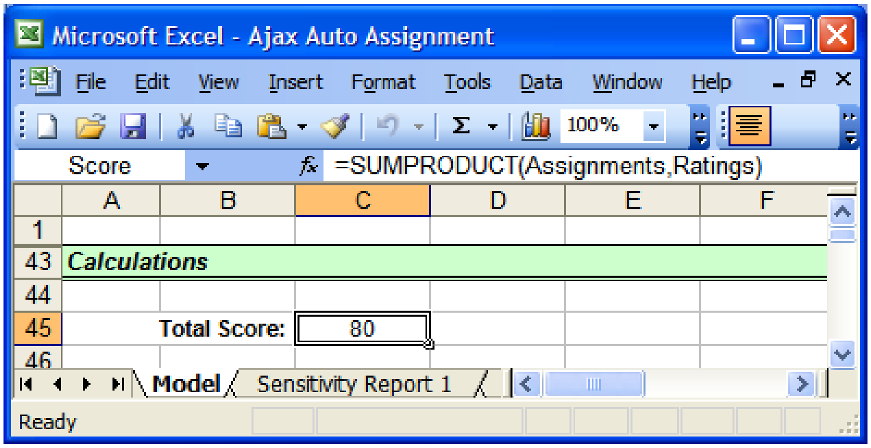

Figure 2. Cell C45's name and contents.

Three rows below the Total Score calculation is a row of 10 cells named Workers. Each of these cells is the sum of one column of the Assignments range, and displays the number of workers assigned to the task for that column. Three rows further down is a row of 10 cells named Tasks. Each of these cells is the sum of one row of the Assignments range, and displays the number of tasks assigned to the worker for that row.

Optimization

To get access to the Solver tool, we click on *Solver* under the *Tools* menu. A dialog window like the one pictured below in Figure 3 appears. This picture already includes the proper entries for our problem.

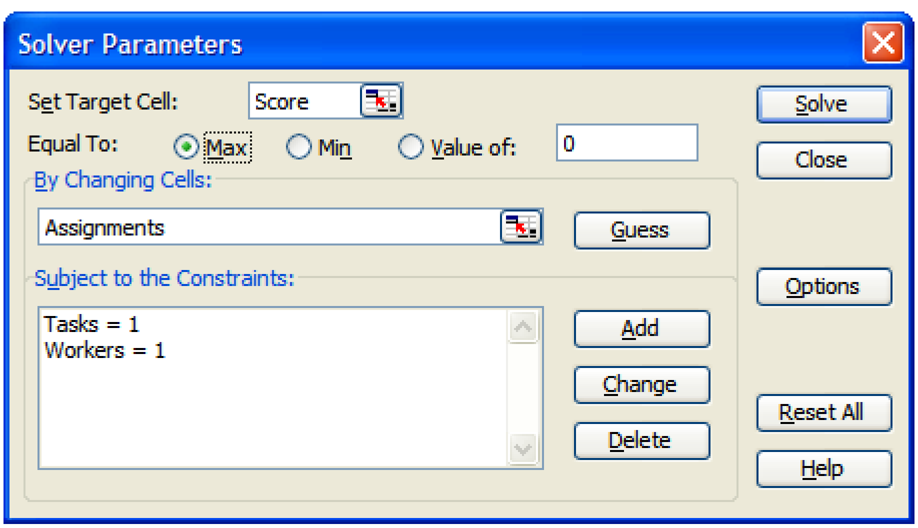

Figure 3. The Solver Parameters dialog window.

Notice that the address of the cell with the objective function formula has been entered in the *Set Target Cell* box, the *Max* radio button has been clicked, and the range of cells containing the decision variable values has been entered in the *By Changing Cells* box. This entry may be made either by keying the range name into the box or by pointing and clicking on the cells of the worksheet itself. To enter the constraint information, we first click in the *Subject to the Constraints* box, and then click *Add*. A dialog window like the one pictured below in Figure 4 will appear. Notice that the address of the cells containing the left-hand sides of the Tasks constraints have been entered in the *Cell Reference* box, the = option has been selected, and the number 1 has been entered in the *Constraint* box. This will require each of the 10 cells in the Tasks range to be constrained to the value 1. Both sets of constraints were entered in this way. Other kinds of constraints can be specified when formulating different optimization models by selecting any of the options available in the central box: = , >= , <= , *int* (for integer), or *bin* (for binary).

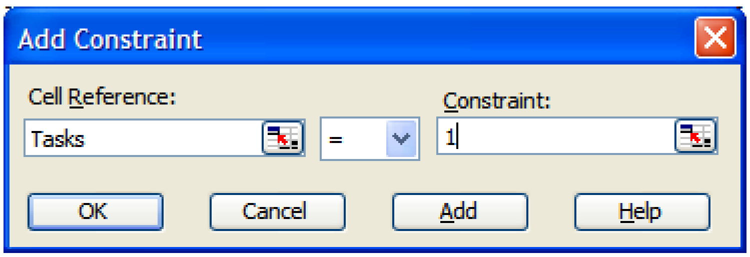

Figure 4. The Add Constraint dialog window.

After entering a constraint in the *Add Constraint* dialog window, we click on *OK*. This will return us to the original *Solver Parameters* dialog window.

To encode the 0/1 restrictions on the decision variables, we could use the constraint specification procedure described above, entering the Assignments range in the *Cell Reference* box and selecting the *bin* option instead of = in the central box. It turns out, however, that assignment models are guaranteed to return 0/1 optimal solutions without such an explicit constraint. If we merely require nonnegative decision variables the solution algorithm operates more efficiently, and we can accomplish this through the *Options* dialog window. We click on the *Options* button (Figure 3), which makes another dialog window (pictured below in Figure 5) appear. We select *Assume Linear Model* and *Assume Non-Negative*, and then click on *OK*.

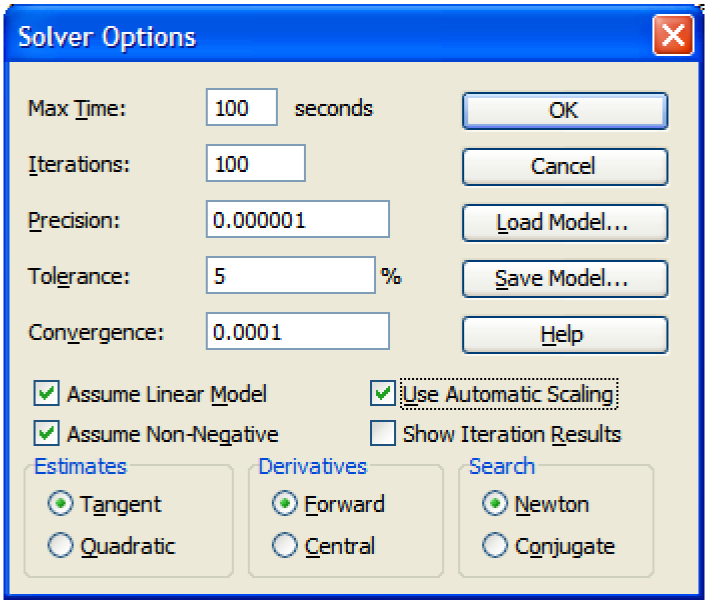

Figure 5. The Solver Options dialog window.

This takes us back to the completed *Solver Parameters* window (Figure 3), which is ready for us to click on *Solve* to start the calculations. Once calculations are complete, the *Solver Results* dialog window appears (pictured below in Figure 6). If we want our worksheet to display the optimal solution, we click on the *Keep Solver Solution* radio button and then click on *OK*.

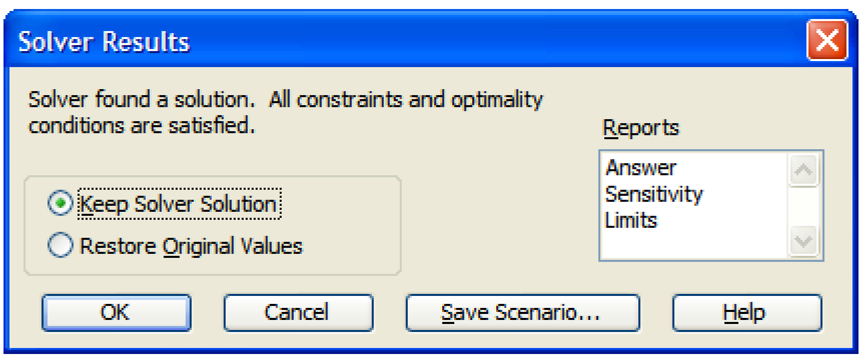

Figure 6. The Solver Results dialog window.

The original worksheet pictured in Figure 1 displays the optimal solution, with a total score of 80. Referring to the Assignments range we see that the optimal plan assigns the "Tires, Brakes, Steering" task to Ceron, "Trunks and Bumpers" to Gonzales, and so on.

Model variants

The standard form of the assignment model has equal numbers of workers and tasks. If an assignment application has more workers than tasks, then it can be put in standard form by creating a sufficient number of dummy tasks to bring the total up to the number of workers. The worker ratings for the dummy tasks can then be set to zero so that assigning a dummy task to a worker is equivalent to assigning no task at all.

Another possible model variant occurs if there is any task for which it is not permissible or desirable to assign a worker with a rating less than some minimum value. If this is the case, then any rating less than that amount can be revised to a very negative value, so that it could not be optimal to assign that worker to that task.

Finally, it might be important to consider assignments that are made repeatedly over time. In this case, it might be desirable to vary each worker's assignments so as to reduce the negative effects of boredom. It is possible to modify the assignment model to take this effect into account, but the result is a very complicated algorithm that is beyond the scope of this paper (Bhadury & Radovilsky, 2006).

TEACHING NOTE

First, we have found that this case lends itself well to both discussion and a written case analysis. In the class discussion after students have done the Excel work, it is useful to remind them what is being optimized. The measurements of performance are training ratings from a cross-training program. They are NOT performance ratings. It is likely that training ratings are correlated to performance, but they are certainly not identical to them. The students also need to examine the question of when the optimal assignment might not be optimal. That is, what are other variables that might make this assignment less than optimal in a real life setting?

Second, students need to reason from the optimal to the possible. They also need to imagine how to do this over time so that optimal solutions are produced many times. It is also helpful to discuss what other measures of performance might be better than the training ratings and what optimality might mean for such measures. Beyond this, the instructor should ask the students to imagine what circumstances might make optimal assignment critical to organizational success. In this discussion, the example of the military could be used. The military services take thousands of people in each year and must match them as effectively as possible with military jobs. To the extent that they are able to do this, they are able to deploy their human resources to the fullest.

Third, we have found that requiring students to write an analysis of the results of the optimization is helpful in confirming their understanding of the linear assignment algorithm. Writing up the case makes them describe precisely what they have done, what the quantitative results were, and what the results mean. One of the problems that most business students have is that they tend to believe that numbers speak for themselves. Describing the tables of results of this case is useful to reinforce the point that students must explain the numbers that they report.

Fourth, the students' spreadsheet skills using Excel and Solver are reinforced. The case presents an opportunity for the instructor to reintroduce Solver and to reinforce the use of it that students learned in their quantitative methods course. Oftentimes students learn skills in their quantitative methods courses that are not reinforced in subsequent coursework. This case offers an opportunity for reinforcement in the context of a course that students often view as purely qualitative.

Regardless of whether the instructor processes the case verbally, requires a written case analysis, or does both, Ajax Auto teaches valuable lessons about human resources and the application of quantitative methods to human resource problems.

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Journal of Human Resources Education
An online refereed journal sponsored by
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