一类带有时变时滞的模糊双线性系统的稳定控制

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摘　要：该文研究了一类输入和状态都带有时变时滞的模糊双线性系统的稳定控制问题。基于并行分布补偿算法(PDC)和自由权值矩阵，得到了系统时滞相关的渐近稳定的充分条件，并把这些条件转换成线性矩阵不等式(LMI)的形式，模糊控制器可以由一组线性矩阵不等式的解得到。最后，通过仿真数例验证了所提方法的有效性。

关键词：模糊控制；模糊双线性系统；时滞相关；线性矩阵不等式(LMI)
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Stability Control for a Class of Fuzzy Bilinear System with Time-Varying Delay

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Abstract: The stability control problem is considered for a class of Fuzzy Bilinear Systems (FBS) with time-varying delay in both state and input. Based on the Parallel Distributed Compensation (PDC) method and free-weighting matrices, some sufficient conditions are derived to guarantee the global asymptotically stability of the overall fuzzy system. The stabilization conditions are further formulated into Linear Matrix Inequalities (LMIs) so that the desired controller can be easily obtained by using the Matlab LMI toolbox. A numerical example is given to show the effectiveness of the presented approach.

Key words: Fuzzy control; Fuzzy Bilinear System(FBS); Delay-dependent; Linear Matrix Inequality(LMI)

1　引言

众所周知，基于 T-S 模型的模糊控制是研究非线性系统比较成功的方法之一，在稳定性分析和控制器设计方面已有很多成果面世[1−6]。时滞经常存在于许多实际系统中，其存在往往会影响系统控制性能，对非线性时滞系统的研究一直是研究的热点问题[2−6]。双线性系统是一类比较特殊的非线性系统。双线性系统模型的结构及动态特性比非线性系统简单，描述对象的近似程度比线性系统要高[7,8]。对于一些实际系统，当用线性系统模型不能描述时，往往可以用双线性系统模型来描述。

考虑 T-S 模型的有效性及双线性系统的特点，对 T-S 模糊双线性系统(FBS)的研究引起了学者的关注[9,10]。和常见的 T-S 模型不同，FBS 的模糊规则的后件部分由一个双线性模型表示。文献[9]研究了一类连续 FBS 系统的稳定性问题，并把结果推广到了带有时滞的 FBS 中[10]。但是文献[10]中只考虑了状态带有时滞，而没有考虑输入带有时滞的情况。

本文研究一类输入和状态都带有时变时滞的模糊双线性系统(FBS)的稳定控制问题。引入自由权值矩阵变量，得到了系统时滞相关的渐近稳定条件，给出了反馈控制器的设计。控制器可由一组线性矩阵不等式的解给出。本文和文献[10]相比，不同之处在于：(1)状态和输入均含有时滞项；(2)控制器的增益可以通过线性矩阵不等式直接解出；(3)自由权值矩阵的引入，去掉了时滞项 $\dot{d}<1$ 的约束。

在本文中，R^n 表示 n 维 Euclidean 空间，$\boldsymbol{P}>0$ ($\boldsymbol{P}\geq 0$) 表示 $\boldsymbol{P}$ 是一个正定(正半定)实对称矩阵。在矩阵表达式中：$\varDelta$ 表示矩阵中的对称项，diag{} 表示对角阵。$\sum_{i,j=1}^{s}$ 表示 $\sum_{i=1}^{s}\sum_{j=1}^{s}$。如不做特别说明，矩阵均表示合适维数的矩阵。

2　系统的模型描述

由T-S模型描述的时滞FBS，它的第 i 条规则可描述如下：

$$
\begin{aligned}
R^i\quad & \text{if}\ \ \xi_1(t)\ \text{is}\ F_1^i\ \text{and}\ \cdots\ \text{and}\ \xi_v(t)\ \text{is}\ F_v^i \\
\text{then}\ \ & \dot{x}(t)=\boldsymbol{A}_i x(t)+\boldsymbol{A}_{di}x(t-d)+\boldsymbol{N}_i x(t)u(t) \\
& \quad +\boldsymbol{N}_{di}x(t-d)u(t-d)+\boldsymbol{B}_i u(t)+\boldsymbol{B}_{di}u(t-d) \\
& x(t)=\phi(t),\ t\in[-\tau,0],\ i=1,2,\cdots,s
\end{aligned}\tag{1}
$$

其中 $F_j^i, j=1,2,\cdots,v$ 是模糊集合，s 是模糊规则的数目。$\xi_v(t), j=1,2,\cdots,v$ 是前提变量。$x(t)\in R^n, u(t)$

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$\in R^1$ 分别是状态变量和控制输入，$\boldsymbol{A}_i,\boldsymbol{A}_{di},\boldsymbol{N}_i,\boldsymbol{N}_{di}\in R^{n\times n},\boldsymbol{B}_i,\boldsymbol{B}_{di}\in R^{n\times 1}$ 是已知的系统矩阵。$\phi(t)$ 是系统的初始状态，时滞项 d 是时变可微函数且满足：$0\le d\le\tau,\ \dot d\le\sigma$，这里 τ 和 σ 是已知常数。

通过单点模糊化，乘积推理和中心平均反模糊化方法，模糊系统的总体模型为

$$\begin{aligned}\dot x(t)=\sum\nolimits_{i=1}^{s}h_i(\xi(t))[&\boldsymbol{A}_ix(t)+\boldsymbol{A}_{di}x(t-d)\\&+\boldsymbol{N}_ix(t)u(t)+\boldsymbol{N}_{di}x(t-d)u(t-d)\\&+\boldsymbol{B}_iu(t)+\boldsymbol{B}_{di}u(t-d)]\end{aligned}\tag{2}$$

其中 $h_i(\xi(t))=\dfrac{\omega_i(\xi(t))}{\sum_{i=1}^{s}\omega_i(\xi(t))},\omega_i(\xi(t))=\prod_{j=1}^{v}\mu_{ij}(\xi(t))$，$\mu_{ij}(\xi(t))$ 是 $\xi_j(t)$ 在 F_j^i 中隶属度函数。文中假设 $\omega_i(\xi(t))\ge 0,\sum_{i=1}^{s}\omega_i(\xi(t))>0$。由 $h_i(\xi(t))$ 的定义可知：$h_i(\xi(t))\ge 0,\ \sum_{i=1}^{s}h_i(\xi(t))=1$。以下在不引起混淆的情况下分别记 $h_i(\xi(t)),x(t-d)$ 为 $h_i,x_d(t)$。

根据文献[10]的设计思想及并行分布补偿算法，设计模糊控制器：

R^i if $\xi_1(t)$ is F_1^i and $\cdots$ and $\xi_v(t)$ is F_v^i

$$\text{then}\ \ u(t)=\frac{\rho\boldsymbol{D}_ix(t)}{\sqrt{1+x^{\mathrm T}(t)\boldsymbol{D}_i^{\mathrm T}\boldsymbol{D}_ix(t)}},\ i=1,2,\cdots,s\tag{3}$$

这里 $\boldsymbol{D}_i\in R^{1\times n}$ 是待求的控制器增益，$\rho>0$ 是待定的标量。

则整个系统的状态反馈控制律可表示为

$$\left.\begin{aligned}u(t)&=\sum\nolimits_{i=1}^{s}h_i\frac{\rho\boldsymbol{D}_ix(t)}{\sqrt{1+x^{\mathrm T}(t)\boldsymbol{D}_i^{\mathrm T}\boldsymbol{D}_ix(t)}}\\&=\sum\nolimits_{i=1}^{s}h_i\rho\sin\theta_i=\sum\nolimits_{i=1}^{s}h_i\rho\cos\theta_i\boldsymbol{D}_ix(t)\\u(t-d)&=\sum\nolimits_{i=1}^{s}h_i\frac{\rho\boldsymbol{D}_ix_d(t)}{\sqrt{1+x_d^{\mathrm T}(t)\boldsymbol{D}_i^{\mathrm T}\boldsymbol{D}_ix_d(t)}}\\&=\sum\nolimits_{i=1}^{s}h_i\rho\sin\varphi_i=\sum\nolimits_{i=1}^{s}h_i\rho\cos\varphi_i\boldsymbol{D}_ix_d(t)\end{aligned}\right\}\tag{4}$$

这里 $\sin\theta_i=\dfrac{\boldsymbol{D}_ix(t)}{\sqrt{1+x^{\mathrm T}(t)\boldsymbol{D}_i^{\mathrm T}\boldsymbol{D}_ix(t)}}$，$\cos\theta_i=\dfrac{1}{\sqrt{1+x^{\mathrm T}(t)\boldsymbol{D}_i^{\mathrm T}\boldsymbol{D}_ix(t)}}$；$\sin\varphi_i=\dfrac{\boldsymbol{D}_ix_d(t)}{\sqrt{1+x_d^{\mathrm T}(t)\boldsymbol{D}_i^{\mathrm T}\boldsymbol{D}_ix_d(t)}}$，$\cos\varphi_i=\dfrac{1}{\sqrt{1+x_d^{\mathrm T}(t)\boldsymbol{D}_i^{\mathrm T}\boldsymbol{D}_ix_d(t)}},\theta_i\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right],\varphi_i\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right],i=1,2,\cdots,s$。

在控制律式(4)的作用下，整个系统的方程可表示为

$$\begin{aligned}\dot x(t)=\sum\nolimits_{i,j=1}^{s}h_ih_j[&(\boldsymbol{A}_i+\rho\sin\theta_j\boldsymbol{N}_i+\rho\cos\theta_j\boldsymbol{B}_i\boldsymbol{D}_j)x(t)\\&+(\boldsymbol{A}_{di}+\rho\sin\varphi_j\boldsymbol{N}_{di}+\rho\cos\varphi_j\boldsymbol{B}_{di}\boldsymbol{D}_j)x_d(t)]\end{aligned}\tag{5}$$

本文目标：设计一反馈控制律式(4)，使得模糊双线性时滞系统式(5)渐近稳定。

下面给出在证明中要用到的引理：

引理 1[11]　设 $\boldsymbol{M},\boldsymbol{N}$ 和 $\boldsymbol{F}(t)$ 是维数适合的实矩阵且满足 $\boldsymbol{F}^{\mathrm T}(t)\boldsymbol{F}(t)\le\boldsymbol{I}$，则对于标量 $\varepsilon>0$ 有不等式 $\boldsymbol{M}^{\mathrm T}\boldsymbol{F}(t)\boldsymbol{N}+\boldsymbol{N}^{\mathrm T}\boldsymbol{F}^{\mathrm T}(t)\boldsymbol{M}\le\varepsilon\boldsymbol{M}^{\mathrm T}\boldsymbol{M}+\varepsilon^{-1}\boldsymbol{N}^{\mathrm T}\boldsymbol{N}$ 成立。

3　稳定性分析和控制器设计

考虑选取如下 Lyapunov-Krasovkii 泛函：

$$\begin{aligned}V(x(t))=x^{\mathrm T}(t)\boldsymbol{P}x(t)+\int_{t-d}^{t}x^{\mathrm T}(s)\boldsymbol{Q}x(s)\mathrm ds\\+\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot x^{\mathrm T}(s)\boldsymbol{R}\dot x(s)\mathrm ds\mathrm d\theta\end{aligned}\tag{6}$$

引入自由权值矩阵：

$$\boldsymbol{X}=\begin{bmatrix}\boldsymbol{X}_1\\\boldsymbol{X}_2\\\boldsymbol{X}_3\end{bmatrix},\ \boldsymbol{Y}=\begin{bmatrix}\boldsymbol{Y}_1\\\boldsymbol{Y}_2\\\boldsymbol{Y}_3\end{bmatrix}\tag{7}$$

其中 $\boldsymbol{P}>0,\boldsymbol{Q}\ge 0,\boldsymbol{R}>0$，$\boldsymbol{X}_1,\boldsymbol{X}_2,\boldsymbol{X}_3,\boldsymbol{Y}_1,\boldsymbol{Y}_2,\boldsymbol{Y}_3$ 是待求的合适维数的常数矩阵。

3.1　稳定性分析

定理 1　对于给定的正常数 ρ,τ,σ，如果对于常数 $\varepsilon_m>0,m=1,2,\cdots,6$ 存在着矩阵 $\boldsymbol{P}>0,\boldsymbol{Q}\ge 0,\boldsymbol{R}>0$ $\boldsymbol{X}_k,\boldsymbol{Y}_k,k=1,2,3$ 满足矩阵不等式(8)，则时滞 FBS 式(5)是渐近稳定。

$$\left.\begin{aligned}&\begin{bmatrix}\boldsymbol{T}_{ii}&\tau\boldsymbol{X}\\\varDelta&-\tau\boldsymbol{R}\end{bmatrix}<0,\qquad i=1,2\cdots,s\\&\begin{bmatrix}\boldsymbol{T}_{ij}+\boldsymbol{T}_{ji}&2\tau\boldsymbol{X}\\\varDelta&-2\tau\boldsymbol{R}\end{bmatrix}<0,\ 1=i<j=s\end{aligned}\right\}\tag{8}$$

其中

$$\boldsymbol{T}_{ij}=\begin{bmatrix}\boldsymbol{t}_{11,ij}&\boldsymbol{t}_{12,i}&\boldsymbol{t}_{13,i}\\\varDelta&\boldsymbol{t}_{22,ij}&\boldsymbol{t}_{23,i}\\\varDelta&\varDelta&\boldsymbol{t}_{33}\end{bmatrix}$$

$$\begin{aligned}\boldsymbol{t}_{11,ij}&=\boldsymbol{Q}+\boldsymbol{X}_1+\boldsymbol{X}_1^{\mathrm T}+\boldsymbol{Y}_1\boldsymbol{A}_i+\boldsymbol{A}_i^{\mathrm T}\boldsymbol{Y}_1^{\mathrm T}\\&\quad+a_1\boldsymbol{Y}_1\boldsymbol{Y}_1^{\mathrm T}+b_1(\boldsymbol{D}_j^{\mathrm T}\boldsymbol{B}_i^{\mathrm T}\boldsymbol{B}_i\boldsymbol{D}_j+\boldsymbol{N}_i^{\mathrm T}\boldsymbol{N}_i)\\\boldsymbol{t}_{12,i}&=-\boldsymbol{X}_1+\boldsymbol{X}_2^{\mathrm T}+\boldsymbol{Y}_1\boldsymbol{A}_{di}+\boldsymbol{A}_i^{\mathrm T}\boldsymbol{Y}_2^{\mathrm T}\\\boldsymbol{t}_{13,i}&=\boldsymbol{P}+\boldsymbol{X}_3^{\mathrm T}+\boldsymbol{A}_i^{\mathrm T}\boldsymbol{Y}_3^{\mathrm T}-\boldsymbol{Y}_1\\\boldsymbol{t}_{22,ij}&=-(1-\sigma)\boldsymbol{Q}-\boldsymbol{X}_2-\boldsymbol{X}_2^{\mathrm T}+\boldsymbol{Y}_2\boldsymbol{A}_{di}+\boldsymbol{A}_{di}^{\mathrm T}\boldsymbol{Y}_2^{\mathrm T}\\&\quad+a_2\boldsymbol{Y}_2\boldsymbol{Y}_2^{\mathrm T}+b_2(\boldsymbol{D}_j^{\mathrm T}\boldsymbol{B}_{di}^{\mathrm T}\boldsymbol{B}_{di}\boldsymbol{D}_j+\boldsymbol{N}_{di}^{\mathrm T}\boldsymbol{N}_{di})\\\boldsymbol{t}_{23,i}&=-\boldsymbol{X}_3^{\mathrm T}+\boldsymbol{A}_{di}^{\mathrm T}\boldsymbol{Y}_3^{\mathrm T}-\boldsymbol{Y}_2\\\boldsymbol{t}_{33}&=\tau R-\boldsymbol{Y}_3-\boldsymbol{Y}_3^{\mathrm T}+a_3\boldsymbol{Y}_3\boldsymbol{Y}_3^{\mathrm T}\\a_1&=\varepsilon_1+\varepsilon_2,\ a_2=\varepsilon_3+\varepsilon_4,\ a_3=\varepsilon_5+\varepsilon_6\\b_1&=\varepsilon_1^{-1}+\varepsilon_3^{-1}+\varepsilon_5^{-1},\ b_2=\varepsilon_2^{-1}+\varepsilon_4^{-1}+\varepsilon_6^{-1}\end{aligned}$$

证明 沿着系统式(5)的轨线，对$V(x(t))$求导，可得到

$$\dot V(x(t)) = 2x^{\mathrm T}(t)\boldsymbol{P}\dot x(t) + x^{\mathrm T}(t)\boldsymbol{Q}x(t) - (1-\dot d) \cdot x_d^{\mathrm T}(t)\boldsymbol{Q}x_d(t) + \tau\dot x^{\mathrm T}(t)\boldsymbol{R}(t)\dot x(t) - \int_{t-\tau}^{t}\dot x^{\mathrm T}(s)\boldsymbol{R}\dot x(s)\mathrm ds \tag{9}$$

引入 Leibniz-Newton 公式及系统方程式(5)，由式(9)可以得到

$$\begin{aligned}\dot V(x(t)) \le{}& 2x^{\mathrm T}(t)\boldsymbol{P}\dot x(t) + x^{\mathrm T}(t)\boldsymbol{Q}x(t) - (1-\sigma) \cdot x_d^{\mathrm T}(t)\boldsymbol{Q}x_d(t) + \tau\dot x^{\mathrm T}(t)\boldsymbol{R}\dot x(t)\\ &- \int_{t-d}^{t}\dot x^{\mathrm T}(s)\boldsymbol{R}\dot x(s)\mathrm ds + 2\eta^{\mathrm T}(t)\boldsymbol{X}[x(t) - x_d(t) - \int_{t-d}^{t}\dot x(s)\mathrm ds)] + 2\eta^{\mathrm T}(t)\boldsymbol{Y}\\ &\cdot \sum_{i,j=1}^{s} h_i h_j[(\boldsymbol{A}_i + \rho\cos\theta_j\boldsymbol{B}_i\boldsymbol{D}_j + \rho\sin\theta_j\boldsymbol{N}_i)x(t) + (\boldsymbol{A}_{di} + \rho\sin\varphi_j\boldsymbol{N}_{di}\\ &+ \rho\cos\varphi_j\boldsymbol{B}_{di}\boldsymbol{D}_j)x_d(t) - \dot x(t)]\end{aligned} \tag{10}$$

由引理 1 可知，存在着正常数$\varepsilon_1>0,\varepsilon_2>0$使得下面的矩阵不等式成立：

$$\left.\begin{aligned}&2x^{\mathrm T}(t)\boldsymbol{Y}_1\boldsymbol{B}_i\rho\boldsymbol{D}_j\cos\theta_j x(t) \le \varepsilon_1\rho^2 x^{\mathrm T}(t)\boldsymbol{Y}_1\boldsymbol{Y}_1^{\mathrm T} \cdot\cos^2\theta_j x(t) + \varepsilon_1^{-1}x^{\mathrm T}(t)\boldsymbol{D}_j^{\mathrm T}\boldsymbol{B}_i^{\mathrm T}\boldsymbol{B}_i\boldsymbol{D}_j x(t)\\ &2x^{\mathrm T}(t)\boldsymbol{Y}_1\boldsymbol{N}_i\rho\sin\theta_j x(t) \le \varepsilon_1\rho^2 x^{\mathrm T}(t)\boldsymbol{Y}_1\boldsymbol{Y}_1^{\mathrm T} \cdot\sin^2\theta_j x(t) + \varepsilon_1^{-1}x^{\mathrm T}(t)\boldsymbol{N}_i^{\mathrm T}\boldsymbol{N}_i x(t)\\ &2x^{\mathrm T}(t)\boldsymbol{Y}_1\boldsymbol{B}_{di}\boldsymbol{D}_j\rho\cos\varphi_j x_d(t) \le \varepsilon_2\rho^2 x^{\mathrm T}(t)\boldsymbol{Y}_1\boldsymbol{Y}_1^{\mathrm T} \cdot\cos^2\varphi_j x(t) + \varepsilon_2^{-1}x_d^{\mathrm T}(t)\boldsymbol{D}_j^{\mathrm T}\boldsymbol{B}_{di}^{\mathrm T}\boldsymbol{B}_{di}\boldsymbol{D}_j x_d(t)\\ &2x^{\mathrm T}(t)\boldsymbol{Y}_1\boldsymbol{N}_{di}\rho\sin\varphi_j x_d(t) \le \varepsilon_2\rho^2 x^{\mathrm T}(t)\boldsymbol{Y}_1\boldsymbol{Y}_1^{\mathrm T} \cdot\sin^2\varphi_j x(t) + \varepsilon_2^{-1}x_d^{\mathrm T}(t)\boldsymbol{N}_{di}^{\mathrm T}\boldsymbol{N}_{di}x_d(t)\end{aligned}\right\} \tag{11}$$

依此类推，$2x_d^{\mathrm T}(t)\boldsymbol{Y}_2, 2\dot x^{\mathrm T}(t)\boldsymbol{Y}_3$分别和系统方程相乘，可得到与$\varepsilon_k>0, k=3,4,5,6$有关的类似矩阵不等式，这里不再写出。由上所述，可知：

$$\begin{aligned}\dot V(x(t)) \le{}& \sum_{i,j=1}^{s} h_i h_j\eta^{\mathrm T}(t)\boldsymbol{T}_{ij}\eta(t) - \int_{t-d}^{t}\dot x^{\mathrm T}(s)\boldsymbol{R}\dot x(s)\mathrm ds - 2\eta^{\mathrm T}(t)\boldsymbol{X}\int_{t-d}^{t}\dot x(s)\mathrm ds\\ \le{}& \sum_{i,j=1}^{s} h_i h_j\eta^{\mathrm T}(t)(\boldsymbol{T}_{ij} + \tau\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{X}^{\mathrm T})\eta(t) - \int_{t-d}^{t}(\eta^{\mathrm T}(t)\boldsymbol{X} + \dot x^{\mathrm T}(s)\boldsymbol{R})\boldsymbol{R}^{-1}(\eta^{\mathrm T}(t)\boldsymbol{X} + \dot x^{\mathrm T}(s)\boldsymbol{R})^{\mathrm T}\mathrm ds\\ ={}& \sum_{i=1}^{s} h_i^2\eta^{\mathrm T}(t)(\boldsymbol{T}_{ii} + \tau\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{X}^{\mathrm T})\eta(t) + \sum_{1=i<j}^{s} h_i h_j\eta^{\mathrm T}(t)(\boldsymbol{T}_{ij} + \boldsymbol{T}_{ji} + 2\tau\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{X}^{\mathrm T})\eta(t)\\ &- \int_{t-d}^{t}(\eta^{\mathrm T}(t)\boldsymbol{X} + \dot x^{\mathrm T}(s)\boldsymbol{R})\boldsymbol{R}^{-1}(\eta^{\mathrm T}(t)\boldsymbol{X} + \dot x^{\mathrm T}(s)\boldsymbol{R})^{\mathrm T}\mathrm ds\end{aligned} \tag{12}$$

这里$\eta^{\mathrm T}(t) = [x^{\mathrm T}(t)\ x_d^{\mathrm T}(t)\ \dot x^{\mathrm T}(t)]$。根据 Schur 补定理，由式(9)可得到

$\boldsymbol{T}_{ii} + \tau\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{X}^{\mathrm T} < 0, \boldsymbol{T}_{ij} + \boldsymbol{T}_{ji} + 2\tau\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{X}^{\mathrm T} < 0$，由于$\boldsymbol{R}>0$，可知式(12)最后一项是非正项，所以$\dot V(x(t))<0$，则 FBS(5)是渐近稳定。

3.2 控制器设计

为求解控制器，假设$\boldsymbol{Y}_k^{-\mathrm T} = \lambda_k\boldsymbol{Z}, \boldsymbol{Z} = \boldsymbol{P}^{-1}, \lambda_k > 0, k=1,2,3$，可知$\boldsymbol{Y}_k$是非奇异矩阵。定义：$\varTheta = \mathrm{diag}\{\boldsymbol{Y}_1^{-1}, \boldsymbol{Y}_2^{-1}, \boldsymbol{Y}_3^{-1}, \boldsymbol{Y}_3^{-1}\}$，对定理 1 中的式(9)分别左乘，右乘$\varTheta$, $\varTheta^{\mathrm T}$，并记：

$\bar{\boldsymbol{Q}} = \boldsymbol{Y}_1^{-1}\boldsymbol{Q}\boldsymbol{Y}_1^{-\mathrm T}, \bar{\boldsymbol{R}} = \boldsymbol{Y}_3^{-1}\boldsymbol{R}\boldsymbol{Y}_3^{-\mathrm T}, \bar{\boldsymbol{X}}_k = \boldsymbol{Y}_k^{-1}\boldsymbol{X}_k\boldsymbol{Y}_k^{-\mathrm T}$, $k=1,2,3, \boldsymbol{D}_i\boldsymbol{Z} = \boldsymbol{M}_i, i=1,2,\cdots,s$，则可以得到

$$\left.\begin{aligned}&\begin{bmatrix}\boldsymbol{t}_{11,ij}^{(1)} & \boldsymbol{t}_{12,i}^{(1)} & \boldsymbol{t}_{13,i}^{(1)} & \tau\lambda_1^{-1}\lambda_3\bar{\boldsymbol{X}}_1\\ \varDelta & \boldsymbol{t}_{22,ij}^{(1)} & \boldsymbol{t}_{23,i}^{(1)} & \tau\lambda_2^{-1}\lambda_3\bar{\boldsymbol{X}}_2\\ \varDelta & \varDelta & \boldsymbol{t}_{33}^{(1)} & \tau\bar{\boldsymbol{X}}_3\\ \varDelta & \varDelta & \varDelta & -\tau\bar{\boldsymbol{R}}\end{bmatrix}<0,\quad i=1,2,\cdots,s\\ &\begin{bmatrix}\boldsymbol{t}_{11,ij}^{(1)}+\boldsymbol{t}_{11,ji}^{(1)} & \boldsymbol{t}_{12,i}^{(1)}+\boldsymbol{t}_{12,j}^{(1)} & \boldsymbol{t}_{13,i}^{(1)}+\boldsymbol{t}_{13,j}^{(1)} & 2\tau\lambda_1^{-1}\lambda_3\bar{\boldsymbol{X}}_1\\ \varDelta & \boldsymbol{t}_{22,ij}^{(1)}+\boldsymbol{t}_{22,ji}^{(1)} & \boldsymbol{t}_{23,i}^{(1)}+\boldsymbol{t}_{23,j}^{(1)} & 2\tau\lambda_2^{-1}\lambda_3\bar{\boldsymbol{X}}_2\\ \varDelta & \varDelta & 2\boldsymbol{t}_{33}^{(1)} & 2\tau\bar{\boldsymbol{X}}_3\\ \varDelta & \varDelta & \varDelta & -2\tau\bar{\boldsymbol{R}}\end{bmatrix}<0,\quad 1=i<j=s\end{aligned}\right\} \tag{13}$$

其中

$$\begin{aligned}\boldsymbol{t}_{11,ij}^{(1)} &= \bar{\boldsymbol{Q}} + \bar{\boldsymbol{X}}_1 + \bar{\boldsymbol{X}}_1^{\mathrm T} + \lambda_1(\boldsymbol{A}_i\boldsymbol{Z} + \boldsymbol{Z}\boldsymbol{A}_i^{\mathrm T}) + a_1\boldsymbol{I} + b_1\lambda_1^2(\boldsymbol{M}_j^{\mathrm T}\boldsymbol{B}_i^{\mathrm T}\boldsymbol{B}_i\boldsymbol{M}_j + \boldsymbol{Z}\boldsymbol{N}_i^{\mathrm T}\boldsymbol{N}_i\boldsymbol{Z})\\ \boldsymbol{t}_{12,i}^{(1)} &= -\lambda_1^{-1}\lambda_2\bar{\boldsymbol{X}}_1 + \lambda_1\lambda_2^{-1}\bar{\boldsymbol{X}}_2^{\mathrm T} + \lambda_2\boldsymbol{A}_{di}\boldsymbol{Z} + \lambda_1\boldsymbol{Z}\boldsymbol{A}_i^{\mathrm T},\\ \boldsymbol{t}_{13,i}^{(1)} &= \lambda_1\lambda_3\boldsymbol{Z} + \lambda_1\lambda_3^{-1}\bar{\boldsymbol{X}}_3^{\mathrm T} + \lambda_1\boldsymbol{Z}\boldsymbol{A}_i^{\mathrm T} - \lambda_3\boldsymbol{Z}\\ \boldsymbol{t}_{22,ij}^{(1)} &= -(1-\sigma)\lambda_1^{-2}\lambda_2^2\bar{\boldsymbol{Q}} - \bar{\boldsymbol{X}}_2 - \bar{\boldsymbol{X}}_2^{\mathrm T} + \lambda_2(\boldsymbol{A}_{di}\boldsymbol{Z} + \boldsymbol{Z}\boldsymbol{A}_{di}^{\mathrm T}) + a_2\boldsymbol{I} + b_2\lambda_2^2(\boldsymbol{M}_j^{\mathrm T}\boldsymbol{B}_{di}^{\mathrm T}\boldsymbol{B}_{di}\boldsymbol{M}_j + \boldsymbol{Z}\boldsymbol{N}_{di}^{\mathrm T}\boldsymbol{N}_{di}\boldsymbol{Z}))\\ \boldsymbol{t}_{23,i}^{(1)} &= -\lambda_2\lambda_3^{-1}\bar{\boldsymbol{X}}_3^{\mathrm T} + \lambda_2\boldsymbol{Z}\boldsymbol{A}_{di}^{\mathrm T} - \lambda_3\boldsymbol{Z}\\ \boldsymbol{t}_{33}^{(1)} &= \tau\bar R - 2\lambda_3\boldsymbol{Z} + a_3\boldsymbol{I}\end{aligned}$$

根据 Schur 补定理，式(13)可等价于

$$
\begin{cases}
\begin{bmatrix}
\boldsymbol{t}_{11,i}^{(2)} & \boldsymbol{t}_{12,i}^{(1)} & \boldsymbol{t}_{13,i}^{(1)} & \tau\lambda_1^{-1}\lambda_3\bar{\boldsymbol{X}}_1 & \boldsymbol{t}_{15,ii}^{(1)} & 0 \\
\varDelta & \boldsymbol{t}_{22,i}^{(2)} & \boldsymbol{t}_{23,i}^{(1)} & \tau\lambda_2^{-1}\lambda_3\bar{\boldsymbol{X}}_2 & 0 & \boldsymbol{t}_{26,ii}^{(1)} \\
\varDelta & \varDelta & \boldsymbol{t}_{33}^{(1)} & \tau\bar{\boldsymbol{X}}_3 & 0 & 0 \\
\varDelta & \varDelta & \varDelta & -\tau\bar{\boldsymbol{R}} & 0 & 0 \\
\varDelta & \varDelta & \varDelta & \varDelta & \boldsymbol{t}_{55}^{(1)} & 0 \\
\varDelta & \varDelta & \varDelta & \varDelta & 0 & \boldsymbol{t}_{66}^{(1)}
\end{bmatrix} < 0 \\
\qquad i = 1,2,\cdots,s \\
\begin{bmatrix}
\boldsymbol{t}_{11,i}^{(2)}+\boldsymbol{t}_{11,j}^{(2)} & \boldsymbol{t}_{12,i}^{(1)}+\boldsymbol{t}_{12,j}^{(1)} & \boldsymbol{t}_{13,i}^{(1)}+\boldsymbol{t}_{13,j}^{(1)} & 2\tau\lambda_1^{-1}\lambda_3\bar{\boldsymbol{X}}_1 & \boldsymbol{t}_{15,ij}^{(2)} & 0 \\
\varDelta & \boldsymbol{t}_{22,i}^{(2)}+\boldsymbol{t}_{22,j}^{(2)} & \boldsymbol{t}_{23,i}^{(1)}+\boldsymbol{t}_{23,j}^{(1)} & 2\tau\lambda_2^{-1}\lambda_3\bar{\boldsymbol{X}}_2 & 0 & \boldsymbol{t}_{26,ij}^{(2)} \\
\varDelta & \varDelta & 2\boldsymbol{t}_{33}^{(1)} & 2\tau\bar{\boldsymbol{X}}_3 & 0 & 0 \\
\varDelta & \varDelta & \varDelta & -2\tau\bar{\boldsymbol{R}} & 0 & 0 \\
\varDelta & \varDelta & \varDelta & \varDelta & \boldsymbol{t}_{55}^{(2)} & 0 \\
\varDelta & \varDelta & \varDelta & \varDelta & 0 & \boldsymbol{t}_{66}^{(2)}
\end{bmatrix} \\
\qquad < 0, \quad 1 = i < j = s
\end{cases}
\tag{14}
$$

其中

$$\boldsymbol{t}_{11,i}^{(2)} = \bar{\boldsymbol{Q}} + \bar{\boldsymbol{X}}_1 + \bar{\boldsymbol{X}}_1^{\mathrm{T}} + \lambda_1(\boldsymbol{A}_i\boldsymbol{Z} + \boldsymbol{Z}\boldsymbol{A}_i^{\mathrm{T}}) + a_1\boldsymbol{I}$$

$$\boldsymbol{t}_{22,i}^{(2)} = -(1-\sigma)\lambda_1^{-2}\lambda_2^2\bar{\boldsymbol{Q}} - \bar{\boldsymbol{X}}_2 - \bar{\boldsymbol{X}}_2^{\mathrm{T}} + \lambda_2(\boldsymbol{A}_{di}\boldsymbol{Z} + \boldsymbol{Z}\boldsymbol{A}_{di}^{\mathrm{T}}) + a_2\boldsymbol{I}$$

$$\boldsymbol{t}_{15,ii}^{(1)} = \begin{bmatrix}\lambda_1\boldsymbol{M}_i^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}} & \lambda_1\boldsymbol{Z}\boldsymbol{N}_i^{\mathrm{T}}\end{bmatrix}$$

$$\boldsymbol{t}_{26,ii}^{(1)} = \begin{bmatrix}\lambda_2\boldsymbol{M}_i^{\mathrm{T}}\boldsymbol{B}_{di}^{\mathrm{T}} & \lambda_2\boldsymbol{Z}\boldsymbol{N}_{di}^{\mathrm{T}}\end{bmatrix}$$

$$\boldsymbol{t}_{55}^{(1)} = \mathrm{diag}\{-b_1^{-1}\boldsymbol{I}, -b_1^{-1}\boldsymbol{I}\}$$

$$\boldsymbol{t}_{66}^{(1)} = \mathrm{diag}\{-b_2^{-1}\boldsymbol{I}, -b_2^{-1}\boldsymbol{I}\}$$

$$\boldsymbol{t}_{15,ij}^{(2)} = \begin{bmatrix}\lambda_1\boldsymbol{M}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}} & \lambda_1\boldsymbol{M}_i^{\mathrm{T}}\boldsymbol{B}_j^{\mathrm{T}} & \lambda_1\boldsymbol{Z}\boldsymbol{N}_i^{\mathrm{T}} & \lambda_1\boldsymbol{Z}\boldsymbol{N}_j^{\mathrm{T}}\end{bmatrix}$$

$$\boldsymbol{t}_{26,ij}^{(2)} = \begin{bmatrix}\lambda_2\boldsymbol{M}_j^{\mathrm{T}}\boldsymbol{B}_{di}^{\mathrm{T}} & \lambda_2\boldsymbol{M}_i^{\mathrm{T}}\boldsymbol{B}_{dj}^{\mathrm{T}} & \lambda_2\boldsymbol{Z}\boldsymbol{N}_{di}^{\mathrm{T}} & \lambda_2\boldsymbol{Z}\boldsymbol{N}_{dj}^{\mathrm{T}}\end{bmatrix}$$

$$\boldsymbol{t}_{55}^{(2)} = \mathrm{diag}\{-b_1^{-1}\boldsymbol{I}, -b_1^{-1}\boldsymbol{I}, -b_1^{-1}\boldsymbol{I}, -b_1^{-1}\boldsymbol{I}\}$$

$$\boldsymbol{t}_{66}^{(2)} = \mathrm{diag}\{-b_2^{-1}\boldsymbol{I}, -b_2^{-1}\boldsymbol{I}, -b_2^{-1}\boldsymbol{I}, -b_2^{-1}\boldsymbol{I}\}$$

可知，若存在矩阵 $\boldsymbol{Z}, \bar{\boldsymbol{Q}}, \bar{\boldsymbol{R}}, \boldsymbol{M}_i, \bar{\boldsymbol{X}}_1, \bar{\boldsymbol{X}}_2, \bar{\boldsymbol{X}}_3$ 使得式(14)成立，则系统式(5)是渐近稳定。

定理 2　对于给定的常数 $\rho > 0$，如果对于常数 $\varepsilon_k > 0, k = 1,2,\cdots,6$ 及常数 $\lambda_l > 0,\ l = 1,2,3$ 存在着矩阵 $\boldsymbol{Z} > 0, \bar{\boldsymbol{Q}} \ge 0, \bar{\boldsymbol{R}} > 0,\ \bar{\boldsymbol{X}}_1, \bar{\boldsymbol{X}}_2, \bar{\boldsymbol{X}}_3, \boldsymbol{M}_i, i = 1,2,\cdots,s$ 满足 LMI(14)，则闭环 FBS(5)是渐近稳定的，且反馈控制器为：$\boldsymbol{D}_i = \boldsymbol{M}_i\boldsymbol{Z}^{-1}, i = 1,2,\cdots,s$ 。

4　算例分析

为了进一步阐述前面的方法和结论，考虑如下带不确定的模糊双线性时滞系统：

R^i : if x_i is L_i

then $\dot{x}(t) = \boldsymbol{A}_i x(t) + \boldsymbol{A}_{di}x_d + \boldsymbol{N}_i x(t)u(t) + \boldsymbol{N}_{di}x_d(t) \cdot u_d(t) + \boldsymbol{B}_i u(t) + \boldsymbol{B}_{di}u_d(t),\ i = 1,2$

其中

$$\boldsymbol{A}_1 = \begin{bmatrix}-95.24 & 7.79 \\ 35 & -97\end{bmatrix}, \boldsymbol{A}_2 = \begin{bmatrix}-82.65 & 9.62 \\ 30 & -108\end{bmatrix},$$

$$\boldsymbol{N}_1 = \boldsymbol{N}_2 = \begin{bmatrix}-3 & 0 \\ 0 & -3\end{bmatrix}, \boldsymbol{B}_1 = \begin{bmatrix}3 \\ 0\end{bmatrix}; \boldsymbol{B}_2 = \begin{bmatrix}1 \\ 1\end{bmatrix},$$

$$\boldsymbol{A}_{d1} = \begin{bmatrix}10 & 0 \\ 5 & 20\end{bmatrix}, \boldsymbol{A}_{d2} = \begin{bmatrix}15 & 0 \\ 0 & 25\end{bmatrix}, \boldsymbol{N}_{d1} = \begin{bmatrix}0.5 & 0 \\ 0 & 0.2\end{bmatrix},$$

$$\boldsymbol{N}_{d2} = \begin{bmatrix}0.3 & 0 \\ 0 & 0.5\end{bmatrix}, \boldsymbol{B}_{d1} = \boldsymbol{B}_{d2} = \begin{bmatrix}-1 \\ 0\end{bmatrix}$$

选取隶属度函数：$\mu_{L_1}(x_1) = \dfrac{1}{1+\exp(-2x_1(t))}$ ，$\mu_{L_2}(x_1) = 1 - \mu_{L_1}(x_1)$ 。选取：$\tau = 1, \sigma = 0,\ \ \rho = 0.486$, $\varepsilon_1 = 0.055, \varepsilon_2 = 0.005, \varepsilon_3 = 0.03, \varepsilon_4 = 0.048, \varepsilon_5 = 0.19$, $\varepsilon_6 = 0.15, \lambda_1 = 0.453$, $\lambda_2 = 0.547, \lambda_3 = 0.001$ 。根据定理 2 通过 Matlab 求解相应的 LMI，可得到对称矩阵及控制器增益分别为

$$\boldsymbol{Z} = \begin{bmatrix}85.0398 & -12.2805 \\ -12.2805 & 76.9411\end{bmatrix},$$

$$\bar{\boldsymbol{Q}} = \begin{bmatrix}57.6751 & -11.3233 \\ -11.3233 & 6.2621\end{bmatrix}, \bar{\boldsymbol{R}} = \begin{bmatrix}0.1178 & 0.0237 \\ 0.0237 & 0.2299\end{bmatrix}$$

$$\boldsymbol{D}_1 = \begin{bmatrix}-5.7809 & -0.2175\end{bmatrix}, \boldsymbol{D}_2 = \begin{bmatrix}-2.7118 & -46.5131\end{bmatrix}$$

选取初始值分别为 $\phi_1(t) = [-2.0\ 1.3]^{\mathrm{T}}, \phi_2(t) = [1.8\ \ -0.8]^{\mathrm{T}}, t \in [-1, 0]$，根据上面所得到的控制器增益，由 MATLAB 仿真可得到如下图形。图 1 是系统在不同状态初始下的响应曲线，图 2 是系统控制曲线。由仿真结果可以看出，在所设计的控制器下，闭环系统是渐近稳定。

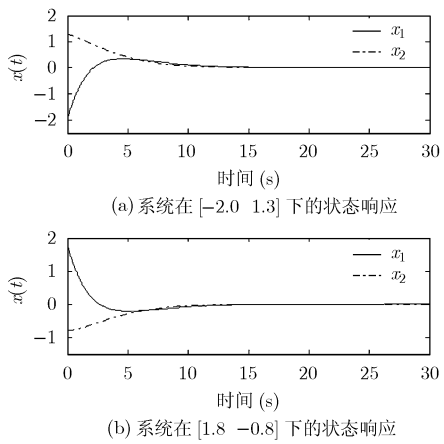

图 1　系统的状态响应

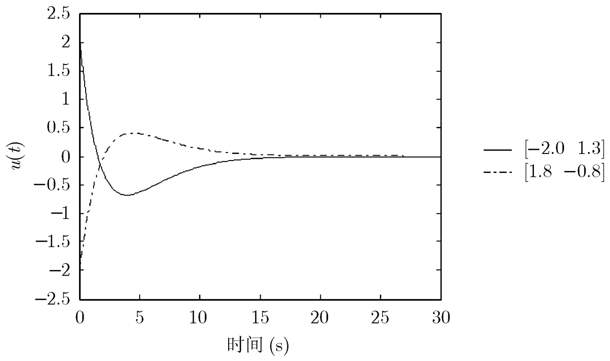

图 2 系统的控制曲线

5 结论

本文研究了一类状态和输入都带有时变时滞的模糊双线性系统的渐近稳定问题，根据 PDC 算法和引入自由权值矩阵，得到了系统时滞相关的渐近稳定条件并设计了模糊控制器，控制器的设计可通过求解一组线性矩阵不等式获得。最后由数例验证了结果的有效性。

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