Slow-light in Ring-in-ring Structure Based on Coupled-resonator-induced Transparency*

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Abstract: A ring-in-ring structure of two rings with different diameters was designed to yield coupled-resonator-induced transparency. According to iterative approach, a theoretical analytic model was established. Explicit expression of the group index at resonance was derived and discussed, and the theoretic result agree with numerical result. The influences of the reflection coefficients on the group index and the CRIT linewidth were fully studied. With proper choice of the parameters, the group index could reach 10^2~10^4.

Key words: Optical communication; Slow light; CRIT; Group index
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0 Introduction

Since slow light was discovered in Electromagnetically induced transparency (EIT)[1], the precise control of group velocity has made many progresses[2-15]. EIT is a phenomenon that can occur in atomic systems as a result of the destructive interference between excitation pathways to the upper level. Recently, Similar effects are established in coupled optical resonators. Smith et al.[11] presented that the coupled-resonator-induced transparency(CRIT) effect can be established in directly coupled optical resonators due to mode splitting and classical destructive interference. Yanik et al.[12] pointed out that the existence of a classical analogue of the EIT in coupled optical resonators is crucial for on-chip coherent manipulation of light at room temperatures, including the capabilities of stopping, storing and time reversing an incident optical pulse. Xu et al.[13,14] reported the realization of an on chip experimental all-optical analogue to EIT. Totsuka et al.[15] reported the slow light in the double ultrahigh-Q microsphere system.

In this paper, we proposed a ring-in-ring structure to achieve CRIT, and theoretically analyzed the control of group index of light in the ring-in-ring structure. We will prove that the group index of light can be controlled by the reflection coefficients r_1 and r_2. With proper choice of the parameters, the group index can reach to 10^2~10^4 orders of magnitude.

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1 Iterative approach for theoretical analysis of the ring-in-ring structure

The ring-in-ring structure is depicted in Fig. 1. It is constructed by a waveguide coupled with ring-in-ring resonator. Next, we use the iterative method[16] to deduce the transmission and dispersion of the proposed structure. For the first ring, the reflected and transmitted electric fields are:

$$E_2 = r_1E_0 + \mathrm{i}t_1E_1 \quad (1a)$$

$$E_3 = \mathrm{i}t_1E_0 + rE_1 \quad (1b)$$

where r_1, t_1 are the reflection and transmission coefficient of the first ring. Propagation of E_3 around the first ring of length L_1 is:

$$E_1 = a_1\mathrm{e}^{\mathrm{i}\phi_1}E_3 \quad (2)$$

where $\phi_1=\beta_1L_1$ and $a_1=\exp(-\alpha_1L_1/2)$ are the phase shift and attenuation factor in the first ring, respectively. According to Eq.(1a), Eq.(1b) and Eq.(2), the ratio of the output field to the input field, i.e., the transmission (E_2/E_0) across ring 1 can be deduced as:

$$\tilde{\tau}_1(\phi_1) = \frac{r_1 - a_1\mathrm{e}^{\mathrm{i}\phi_1}}{1 - r_1a_1\mathrm{e}^{\mathrm{i}\phi_1}} \quad (3)$$

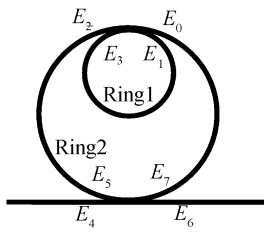

Fig.1 Illustration of ring-in-ring structure

For the second ring, the reflected and transmitted electric fields are

$$E_6 = r_2E_4 + \mathrm{i}t_2E_5 \quad (4a)$$

$$E_7 = \mathrm{i}t_2E_4 + r_2E_5 \quad (4b)$$

where r_2, t_2 are the reflection and transmission coefficient of the second ring. Propagation of E_2, E_5, and E_7 around the second ring of length L_2 are

$$E_5 = a_2 e^{i\phi_2} \tilde{\tau}_1 E_7 \tag{5}$$

Where $\phi_2=\beta_2 L_2$, $a_2=\exp(-\alpha_2 L_2/2)$ are the phase shift and attenuation factor in the second ring, respectively. According to Eq.(4a), Eq.(4b) and Eq.(5), the ratio of the output field to the input field, i.e., the transmission (E_6/E_4) across the whole structure can be deduced as

$$\tilde{\tau}_2(\phi_2,\phi_1) = \frac{r_2 - a_2\tilde{\tau}_1 e^{i\phi_2}}{1 - r_2 a_2 \tilde{\tau}_1 e^{i\phi_2}} \tag{6}$$

2 CRIT in ring-in-ring structure

The transmittance T_2 of the whole structure can be got by $T_2 = \left|\tilde{\tau}_2\right|^2$, the absorptance of the whole structure can be got by $A_2=1-T_2$:

$$A_2(\phi_1,\phi_2) = \frac{\left(1-r_2^2\right)\left(1-a_2^2\left|\tilde{\tau}_1\right|^2\right)}{\left(1-r_2 a_2\left|\tilde{\tau}_1\right|\right)^2 + 4 r_2 a_2 \left|\tilde{\tau}_1\right| \sin^2\left(\frac{\phi_1^{\text{eff}}+\phi_2}{2}\right)} \tag{7}$$

where $\phi_1^{\text{eff}}(\phi_1) \equiv \arg(\tilde{\tau}_1)$ is the effective phase shift of the first ring.

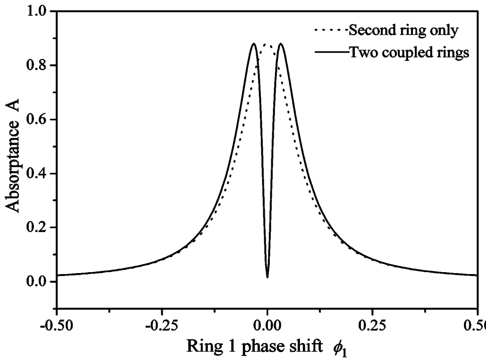

Fig.2 The Absorptance for ring-in-ring structure versus the phase shift (ϕ_1) of first ring

For a ring-in-ring structure, assuming the radiuses of the two rings in the ring-out-ring structure, R_1 and R_2, satisfy the relation of $R_2 = 2R_1$. According to $\phi_j=\beta_j L_j$, we can be obtained the relation of $\phi_2=2\phi_1$. Fig. 2 shows the absorptance A of the whole structure versus the first ring phase shift phase shift ϕ_1. The solid line represents the whole absorption of the system in a ring-in-ring structure with two rings. Dotted line represents the absorptance A for the second ring alone. We can note that the coupling of the first ring induce a splitting of the large absorption of the second ring in two absorption peaks at resonance, and then a narrow transparency window appears between the split resonances. The split is originates from coherent coupling between rings, analogous to ac-Stark shift and quantum interference that occurs in three level atomic systems.

Note that Eq.(6) can also be written as $\tilde{\tau}_2 = \left|\tilde{\tau}_2\right| \exp(i\phi_2^{\text{eff}})$, where ϕ_2^{eff} is the effective phase shift of the whole structure, it is defined as

$$\phi_2^{\text{eff}}(\phi_1,\phi_2) = \arg(\tilde{\tau}_2) \tag{8}$$

In figure 3 we plotted the effective phase shift ϕ^{eff} as a function of the first ring phase shift phase shift ϕ_1 in the ring-in-ring structure. As it was the case for absorption A, we also represented in dot line dispersion properties of the second ring alone. The coupling between the two rings induces a sharp positive phase variation which leads to a positive dispersion. The strong dispersion results in considerably slow light, for the slow light induced by CRIT effect, the group delay of the whole structure is $\tau(\omega) = \mathrm{d}\phi_2^{\text{eff}}/\mathrm{d}\omega$; the corresponding group velocity is $\upsilon_g = L/\tau(\omega)$, where L is the length of the first ring. Thus, the group index can be obtained as:

$$n_g = n_0 \frac{\mathrm{d}\phi_2^{\text{eff}}}{\mathrm{d}\phi_1} \tag{9}$$

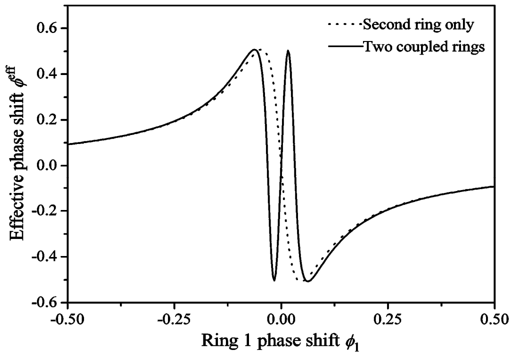

Fig.3 Effective phase shift for ring-in-ring structure versus the phase shift (ϕ_1) of first ring

In the ring-in-ring structure, both a good transmission and a large delay at resonance induced the CRIT effect, which originates from coherent coupling between rings, analogous to the EIT effect occurs in three level atomic systems.

As an example, we can consider the ring-in-ring structure is fabricated on a SiO_2 substrate whose refractive index is 1.5. For a 1550nm tunable light source, the numerical group index of the ring-in-ring structure is shown in Fig. 4, which is calculated using Eq.(6), (8) and (9). One can see that, in the narrow transparence window at resonance region, the group index increases significantly at resonance where the CRIT effect is greatest.

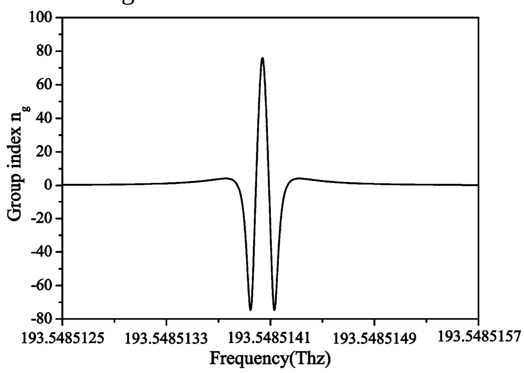

Fig.4 The group index n_g versus the first ring phase shift for the ring-in-ring structure with a_1=0.9999, a_2=0.9, r_1=0.999, r_2=0.95.

3 Analysis and discussion

As analyzed in the above, the group index n_g is hightly dependent on the parameters of the ring-in-ring structure. In order to fully analyze the role of the parameters of the ring-in-ring structure in the slowdown of light propagation in the structure, the theoretical group index of the structure is deduced. Based on Eq. (6) and the relation of $\phi_2 = 2\phi_1$, the derivative of the effective phase shift is

$$\frac{\mathrm{d}\phi_2^{\mathrm{eff}}}{\mathrm{d}\phi_1} = \frac{1}{|\tau_2|^2}\left(\mathrm{Re}\,\tau_2 \frac{\mathrm{d}\,\mathrm{Im}\,\tau_2}{\mathrm{d}\phi_1} - \mathrm{Im}\,\tau_2 \frac{\mathrm{d}\,\mathrm{Re}\,\tau_2}{\mathrm{d}\phi_1}\right) \quad (10)$$

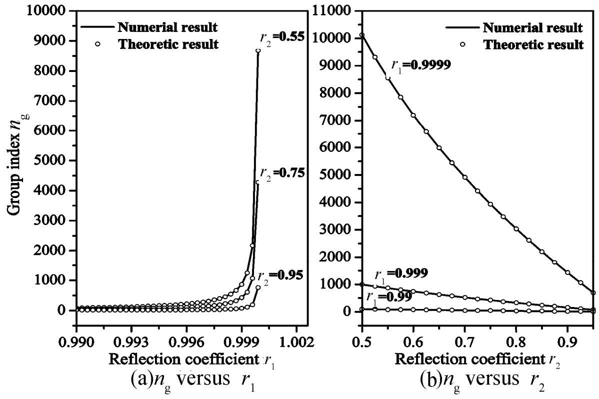

Fig.5 The group index n_g versus the reflection coefficients r_1. and versus the reflection coefficients r_2

When resonance occurs, $\mathrm{d}\,\mathrm{Re}\,\tau_2/\mathrm{d}\phi_2 = 0$. As a result, Eq.(10) can be simplified at resonance as follows

$$n_g = \frac{n_0 a_2\left(1-r_2^2\right)\left(3a_1 + a_1 r_1^2 - 2r_1 - 2r_1 a_1^2\right)}{\left[\left(1-r_1 a_1\right) + r_2 a_2\left(a_1 - r_1\right)\right]\left[r_2\left(1-r_1 a_1\right) - a_2\left(r_1 - a_1\right)\right]} \quad (11)$$

The group index n_g at resonance is shown in Fig. 5(a) which describe the cases of r_2=0.95, 0.75, 0.55 for different r_1, and in Fig. 5(b) which describe the cases of r_1=0.9999, 0.999, 0.99 for different r_2. The dotted line shows the numerical group index calculated using Eq. (6) and (8), the solid line shows the theoretical group index calculated using Eq. (11). One can see that the theoretical result agree well with the numerical result. In addition, n_g increases with increasing r_1 and decreasing r_2, and n_g increases significantly when r_1 closes to 1. Furthermore, we can know from the figure that the group index can reach to 10^2 to 10^4 orders of magnitude by using proper parameters.

As well known, for atoms in EIT, larger group index is associated with smaller EIT linewidths. The same is true for ring-in-ring structure in CRIT. In addition, it is well accepted that the narrower the linewidth is, the more difficult the detection is. Therefore, the full width at half maximum (FWHM) linewidth of the ring-in-ring structure should be considered. For the size of the second ring is two times of the size of the first ring, the theoretical linewidth of this structure is deduced. From Eq. (7), the FWHM linewidth can be obtained as

$$\delta_{\mathrm{FWHM}} = \left[2c\left(1-r_1\right)\sqrt{2r_2 a_2/r_1}\right] / \left[\left(1-r_2 a_2\right) n_0 L_1\right] \quad (12)$$

where L_1 is the length of the first ring, c is the speed of light in vacuum. As shown in Fig. 6(a) which describe the cases of r_2=0.95, 0.75, 0.55 for different r_1, and in Fig. 6(b) which describe the cases of r_1=0.999 9, 0.999, 0.99 for different r_2. One can see that the FWHM linewidth increases with r_2 and decreases with r_1, which is corresponding to the results of Fig. 5.

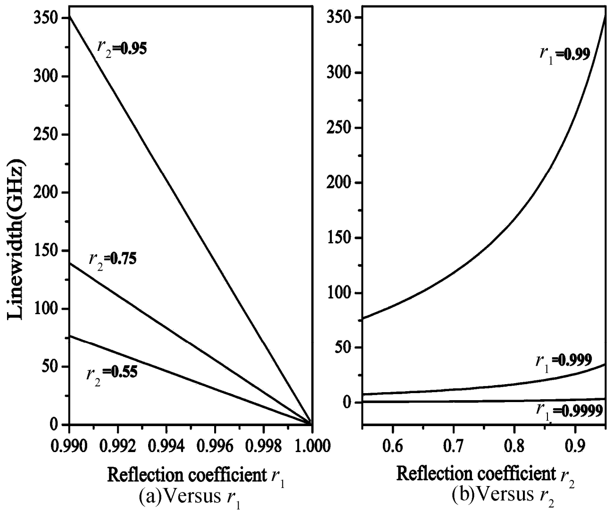

Fig.6 The FWHM linewidth versus the reflection coefficients r_1.and r_2.

4 Conclusions

In this letter, we theoretically analyzed highly dispersive ring-in-ring structure with CRIT property, and demonstrated that the slow light in CRIT is analogous to slow light in EIT. From the theoretical calculation, the group index of the whole structure is tunable due to the variety of reflection coefficients r_1 and r_2. Furthermore, we confirm that group index can reach to 10^2~10^4 orders of magnitude with proper choice of the parameters.

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基于耦合谐振透明效应的环中环结构中的光速减慢

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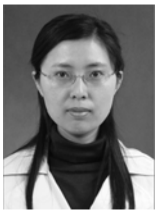
摘 要: 理论分析了环中环结构中的耦合谐振透明.利用迭代法建立了理论模型．推导了谐振情况下群折射率的精确表达式．理论结果与数值模拟结果符合得很好．深入讨论了反射系数对群折射率和CRIT线宽的影响．通过选择合适的参数，群折射率可高达10^2~10^4.

关键词：光通信；慢光；耦合谐振透明；群折射率



WANG Nan was born in 1980. Her research interests include quantum optics and nonlinear optics. Currently she is devoting to the slow light in optical resonators based on Coupled-Resonator-Induced Transparency (CRIT) effect.