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非线性弹性梁的动力模型

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摘要: 通过弹性力学变分原理建立了大挠度非线性梁的控制微分方程组. 该理论含有4个独立的自由度,能更精确描述非线性弹性梁的的力学行为.

关键词: 弹性梁; 非线性; 动力方程; Hamilton 原理
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Governing Motion Equation of Nonlinear-Elastic Beam

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Abstract: From the elasticity variational principle, the governing dynamic differential equations of the geometric non-linear beam with large deflection is deduced. The theory consists of four displacement variables, which will exactly describe the dynamic response of geometrical non-linear beam.

Key words: elastic beam; geometrical non-linear; dynamic equation; Hamilton's principle

0 引 言

线弹性的 Timoshenko 梁,无论是静力还是动力以及粘弹性问题都已经被进行了细致的研究[1,2]. 大挠度、大变形弹性直梁在工程领域有广泛的应用. 现有文献对该类问题均采用几何非线性、物理线性的 Timoshenko 梁理论进行分析. 文献[3]研究了大挠度的 Timoshenko 梁,并给出了静力控制微分方程组. 文献[4]对非线性粘弹性 Timoshenko 梁的动力行为进行了分析. 但由于 Timoshenko 梁理论在分析过程中采用了适用于小变形的三变量位移场,这对于分析大变形非线性梁的力学行为是不恰当. 本文采用能更好反应大变形的四变量位移场及 Green 应变张量,通过 Hamilton 变分原理得到了新的非线性大变形的动力控制方程组. 该理论能更好地描述梁的非线性力学行为.

1 基本假设

本文采用以下几个假设:

① 梁的变形由伸长、剪切和弯曲共同引起;

② 梁的横截面为剪心与形心重合的的双对称截面,且变形后仍为平面;

③ 假定应力分量 σ_y, τ_{yz} 对梁的影响可忽略,即令 $\sigma_y = \tau_{yz} = 0$

④ 采用广义胡克定律作为本构关 $\sigma_{ij} = \dfrac{E}{1+v}\varepsilon_{ij} + \dfrac{v}{1+v}\Theta\delta_{ij}$,其中 E 为弹性模量,v 为泊松比,$\Theta = \sigma_x + \sigma_y + \sigma_z$,$\delta_{ij}$ 为 Kronecker 符号,$i,j = x,y,z$.

2 几何方程

设梁的受力和几何约束皆在 xz 平面内. 外力 $\boldsymbol{P}(x,t)$ 沿梁纵向分布,梁右端作用集中力 $\boldsymbol{N}(t)$ 及力矩

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$\boldsymbol{M}(t)$. 力学模型如图1所示.

其中

$$\begin{cases}\boldsymbol{P}(x,t) = p_1(x,t)i - p_2(x,t)k \\ \boldsymbol{N}(t) = -n(t)i \\ \boldsymbol{M}(t) = -m(t)j\end{cases} \tag{1}$$

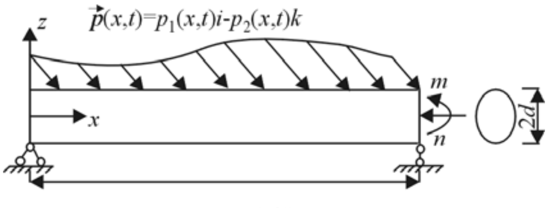

图1 力学模型
Fig.1 Mechanics model

由于位移场$\boldsymbol{u}$是(x,y,z,t)的函数[5],且$\boldsymbol{u} = \boldsymbol{U}i + \boldsymbol{V}j + \boldsymbol{W}k$. 其中$i,j,k$为$x,y,z$方向的单位向量. $U,\boldsymbol{v},W$为x,y,z方向的位移. 把u在$z=0$展开为Taylor级数:

$$\boldsymbol{u}(x,y,z,t) = \begin{bmatrix} U(x,y,z,t) \\ \boldsymbol{v}(x,y,z,t) \\ W(x,y,z,t)\end{bmatrix} = \boldsymbol{u}(x,y,0,t) + \left(\frac{\partial \boldsymbol{u}}{\partial z}\right)_{z=0} z + \frac{1}{2!}\left(\frac{\partial \boldsymbol{u}}{\partial z}\right)_{z=0} I^2 + \cdots \tag{2}$$

包括剪切变形的位移表达式可取前两项给出

$$\boldsymbol{u} = \boldsymbol{u}_0 + z\boldsymbol{u}_1 \tag{3}$$

由假设2,$\boldsymbol{u}_0$,$\boldsymbol{u}_1$定义如下:

$$\boldsymbol{u}_0 = \boldsymbol{u}(x,y,0,t) = u(x,t)i + w(x,t)k, \boldsymbol{u}_1 = \left(\frac{\partial \boldsymbol{u}}{\partial z}\right)_{z=0} = u_1(x,t)i + w_1(x,t)k \tag{4}$$

从(4)式可知,位移场包括4个自由度. 可把位移场表示为如下分量形式:

$$U = u + zu_1, \boldsymbol{v} = 0, W = w + zw_1 \tag{5}$$

为了反映梁的大变形,使用Green张量来表达应变. 其分量如下:

$$\begin{cases}\varepsilon_x = \dfrac{\partial U}{\partial x} + \dfrac{1}{2}\left(\left(\dfrac{\partial U}{\partial x}\right)^2 + \left(\dfrac{\partial \boldsymbol{v}}{\partial y}\right)^2 + \left(\dfrac{\partial W}{\partial x}\right)^2\right) = \dfrac{\partial u}{\partial x} + z\dfrac{\partial u_1}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial u}{\partial x}\right)^2 + \\ z\dfrac{\partial u}{\partial x}\dfrac{\partial u_1}{\partial x} + \dfrac{1}{2}z^2\left(\dfrac{\partial u_1}{\partial x}\right)^2 + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 + z\dfrac{\partial w}{\partial x}\dfrac{\partial w_1}{\partial x} + \dfrac{1}{2}z^2\left(\dfrac{\partial w_1}{\partial x}\right)^2 \\ \gamma_{xz} = \dfrac{1}{2}\left(\dfrac{\partial U}{\partial z} + \dfrac{\partial W}{\partial x}\right) + \dfrac{1}{2}\left[\left(\dfrac{\partial U}{\partial z}\dfrac{\partial U}{\partial x}\right) + \left(\dfrac{\partial \boldsymbol{v}}{\partial z}\dfrac{\partial \boldsymbol{v}}{\partial x}\right) + \left(\dfrac{\partial W}{\partial z}\dfrac{\partial W}{\partial x}\right)\right] = \\ \dfrac{1}{2}\left(u_1 + \dfrac{\partial w}{\partial x} + z\dfrac{\partial w_1}{\partial x}\right) + \dfrac{1}{2}u_1\left(\dfrac{\partial u}{\partial x} + z\dfrac{\partial u_1}{\partial x}\right) + \dfrac{1}{2}w_1\left(\dfrac{\partial w}{\partial x} + z\dfrac{\partial w_1}{\partial x}\right) \\ \varepsilon_z = \dfrac{\partial W}{\partial z} + \dfrac{1}{2}\left[\left(\dfrac{\partial U}{\partial z}\right)^2 + \left(\dfrac{\partial \boldsymbol{v}}{\partial z}\right)^2 + \left(\dfrac{\partial W}{\partial z}\right)^2\right] = w_1 + \dfrac{1}{2}u_1^2 + \dfrac{1}{2}w_1^2\end{cases} \tag{6}$$

由假设3,4及应变分量的表达式可得$\varepsilon_y = 0, \gamma_{xy} = \gamma_{yz} = 0$. 对经典的Timoshenko梁由于假设$\varepsilon_z = 0$,由$\varepsilon_z$的表达式有:

$$\varepsilon_z = \frac{\partial W}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial \boldsymbol{v}}{\partial z}\right)^2 + \left(\frac{\partial W}{\partial z}\right)^2\right] = w_1 + \frac{1}{2}u_1^2 + \frac{1}{2}w_1^2 = 0 \tag{7}$$

在小位移假设的条件下,把ε_z线性化可得$w_1 = 0$,从而得到经典的Timoshenko梁的三变量位移场. 文献[3,4]以该三变量位移场及Green应变张量做为讨论非线性弹性梁的基础,得到了一些结果. 但是在大变形情况下,由于u_1^2,w_1^2不能忽略,若仍采用Timoshenko梁的假设显然不适合描述大位移、大变形的情况.

3 变分原理

为了得到大变形 Timoshenko 梁的控制方程, 本文借助弹性动力学 Hamilton 变分原理. 原理表述如下[6]:

在满足位移边界条件、几何方程和物理方程的情况下,给定 t_0 及 t_1 时的位移$\boldsymbol{u}_{oi}$,$\boldsymbol{u}_{1i}$($i = x,y,z$). 则泛函

$$\Pi_H = \int_{t_0}^{t_1}(T - \Pi)\,\mathrm{d}t \tag{8}$$

为极值($\delta\Pi_H = 0$) 的$\boldsymbol{u}_i$ 必导出问题的精确解,即必导出满足动力方程和边界条件的$\boldsymbol{u}_i$.
其中, T 为动能,Π 为系统的总势能. 本文中

$$T = \iiint_\tau \frac{1}{2}\rho\frac{\partial u_i}{\partial t}\frac{\partial u_i}{\partial t}\mathrm{d}\tau = \iiint_\tau \frac{1}{2}\rho\left[\left(\frac{\partial u}{\partial t} + z\frac{\partial u_1}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t} + z\frac{\partial w_1}{\partial t}\right)^2\right]\mathrm{d}\tau \tag{9}$$

$$\Pi = \Pi_1 + \Pi_2 + \Pi_3 + \Pi_4 \tag{10}$$

其中,

$$\Pi_1 = \frac{1}{2}\iiint_\tau \sigma_x\varepsilon_x\mathrm{d}\tau = \frac{1}{2}\iiint_\tau \sigma_x\left[\frac{\partial u}{\partial x} + z\frac{\partial u_1}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + z\frac{\partial u}{\partial x}\frac{\partial u_1}{\partial x} + \frac{1}{2}z^2\left(\frac{\partial u_1}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + z\frac{\partial w}{\partial x}\frac{\partial w_1}{\partial x} + \frac{1}{2}z^2\left(\frac{\partial w_1}{\partial x}\right)^2\right]\mathrm{d}\tau \tag{11}$$

$$\Pi_2 = \iiint_\tau k\tau_{xz}\gamma_{xz}\mathrm{d}\tau = \iiint_\tau k\tau_{xz}\left[\frac{1}{2}\left(u_1 + \frac{\partial w}{\partial x} + z\frac{\partial w_1}{\partial x}\right) + \frac{1}{2}u_1\left(\frac{\partial u}{\partial x} + z\frac{\partial u_1}{\partial x}\right) + \frac{1}{2}w_1\left(\frac{\partial w}{\partial x} + z\frac{\partial w_1}{\partial x}\right)\right]\mathrm{d}x \tag{12}$$

$$\Pi_3 = \frac{1}{2}\iiint_\tau \chi\sigma_z\left(w_1 + \frac{1}{2}u_1^2 + \frac{1}{2}w_1^2\right)\mathrm{d}\tau \tag{13}$$

$$\Pi_4 = \int_0^l p_1(u + u_1 d)\mathrm{d}x - \int_0^l p_2(w + w_1\mathrm{d})\mathrm{d}x - nu(l) - mu_1(l) \tag{14}$$

在此处ρ 为材料的密度. d 为对称轴至荷载$\boldsymbol{p}(x,t)$ 作用点处的距离. κ 为考虑γ_{xz} 的不均匀及γ_{xz} 的影响引入的常数,其数值可由最小余能原理并结合振动或波的传播来确定[5]. 对矩形截面可取 5/6. χ 为考虑ε_z 沿 z 轴分布不均匀性引入的常数,其数值可用求κ 的方法得到. 对 T,Π_1,Π_2,Π_3,Π_4,进行变分,由泛函取极值的必要条件.

$$\delta\Pi_H = \delta\int_{t_1}^{t_2}(T - \Pi)\mathrm{d}t = \int_{t_0}^{t_2}\delta T\mathrm{d}t - \int_{t_0}^{t_1}\delta\Pi\mathrm{d}t = 0 \tag{15}$$

注意到在 t_0 及 t_1 时的位移 $\boldsymbol{u}_{0i}$,$\boldsymbol{u}_{li}$,($i = x,y,z$) 是给定的,有

$$\begin{cases}\delta u|_{t_0} = \delta u|_{t_1} = \delta w|_{t_0} = \delta w|_{t_0} = 0\\ \delta u_1|_{t_0} = \delta u_1|_{t_1} = \delta w_1|_{t_0} = \delta w_1|_{t_1} = 0\end{cases} \tag{16}$$

同时,由于奇函数在对称区间的积分为零,经整理理终可得

$$\delta T = -\int_{t_0}^{t_1}\left\{\int_l^0\left[\left(m\frac{\partial^2 u}{\partial t^2}\right)\delta u + \left(I_m\frac{\partial^2 u_1}{\partial t^2}\right)\delta u_1 + \left(m\frac{\partial^2 w}{\partial t^2}\right)\delta w + \left(I_m\frac{\partial^2 w_1}{\partial t^2}\right)\delta w_1\right]\mathrm{d}x\right\}\mathrm{d}x \tag{17}$$

$$\delta\Pi_1 = \int_{t_0}^{t_1}\iiint_v \sigma_x\delta\varepsilon_x\,dv\mathrm{d}x = \int_{t_{01}}^{t_1}\left\{\left[N + N\frac{\partial u}{\partial x} + M\frac{\partial u_1}{\partial x}\right]_0^l\delta u + \left[M + M\frac{\partial u}{\partial x} + \Phi\frac{\partial u_1}{\partial x}\right]_0^1\delta u_1 + \right.$$

$$\left[N\frac{\partial w}{\partial x} + M\frac{\partial w_1}{\partial x}\right]_0^1 \delta w + \left[M\frac{\partial w}{\partial z} + \Phi\frac{\partial w_1}{\partial x}\right]_0^l \delta w_1 \Big\}\mathrm{d}t -$$

$$\int_{t_0}^{t_1}\int_0^l \Big\{ \left[\frac{\partial N}{\partial x} + \frac{\partial N}{\partial x}\frac{\partial u}{\partial x} + N\frac{\partial^2 u}{\partial x^2} + \frac{\partial M}{\partial x}\frac{\partial u_1}{\partial x} + M\frac{\partial^2 u_1}{\partial x^2}\right]\delta u +$$

$$\left[\frac{\partial M}{\partial x} + \frac{\partial M}{\partial x}\frac{\partial u}{\partial x} + M\frac{\partial^2 u}{\partial x^2} + \frac{\partial \Phi}{\partial x}\frac{\partial u_1}{\partial x} + \Phi\frac{\partial^2 u_1}{\partial x^2}\right]\delta u_1 +$$

$$\left[\frac{\partial N}{\partial x}\frac{\partial w}{\partial x} + N\frac{\partial^2 w}{\partial x^2} + \frac{\partial M}{\partial x}\frac{\partial w_1}{\partial x} + M\frac{\partial^2 w_1}{\partial x^2}\right]\delta w +$$

$$\left[\frac{\partial M}{\partial x}\frac{\partial w}{\partial x} + M\frac{\partial^2 w}{\partial x^2} + \frac{\partial \Phi}{\partial x}\frac{\partial w_1}{\partial x} + M\frac{\partial^2 w_1}{\partial x^2}\right]\delta w_1 \Big\}\mathrm{d}x\mathrm{d}t \tag{18}$$

$$\delta\Pi_2 = 2\int_{t_0}^{t_1}\iiint_v kI_{xz}\delta\gamma_{xz}\mathrm{d}v\mathrm{d}x = k\int_{t_0}^{t_1}\Big\{\left[Qu_1\delta u + Q\delta w + Qw_1\delta w\right]_0^l\Big\}\mathrm{d}t -$$

$$k\int_{t_0}^{t_1}\int_0^l\Big\{\left[Q\frac{\partial u_1}{\partial x} + u_1\frac{\partial Q}{\partial x}\right]\delta u - \left[Q + Q\frac{\partial u}{\partial x}\right]\delta u_1 +$$

$$\left[\frac{\partial Q}{\partial x} + Q\frac{\partial w_1}{\partial x} + w_1\frac{\partial Q}{\partial x}\right]\delta w - Q\frac{\partial w}{\partial x}\delta w_1\Big\}\mathrm{d}x\mathrm{d}t \tag{19}$$

$$\delta\Pi_3 = \int_{t_0}^{t_1}\iiint_v \chi\sigma_z\delta\varepsilon_z\mathrm{d}v\mathrm{d}x = \chi\int_{t_0}^{t_1}\int_0^l\left[\Omega\delta w_1 + \Omega u_1\delta u_1 + \Omega w_1\delta w_1\right]\mathrm{d}x\mathrm{d}t \tag{20}$$

$$\delta\Pi_4 = \int_{t_0}^{t_1}\Big\{\left[\int_0^1 p_1\delta u\mathrm{d}x + \int_0^l dp_1\delta u_1\mathrm{d}x - \int_0^1 p_2\delta w\mathrm{d}x - \int_0^1 dp_2 w_1\mathrm{d}x\right]\Big\}\mathrm{d}t$$

$$-\int_{t_0}^{t_1}\Big(n\delta u(l) + m\delta u_1(l)\Big)\mathrm{d}t \tag{21}$$

其中,N,M,Q,I_m,Φ,m,Ω 定义如下:

$$\iint_s \sigma_x\mathrm{d}y\mathrm{d}z = N \qquad \iint_s \delta_x z\mathrm{d}y\mathrm{d}z = M$$

$$\iint_s \tau_{xz}\mathrm{d}y\mathrm{d}z = Q \qquad \iint_s \rho z^2\mathrm{d}y\mathrm{d}z = I_m$$

$$\iint_s \sigma_x z^2\mathrm{d}y\mathrm{d}z = \Phi \qquad \iint_s \rho\mathrm{d}y\mathrm{d}z = m$$

$$\iint_s \sigma_z\mathrm{d}y\mathrm{d}z = \Omega \tag{22}$$

在(22)式中,积分区域s为垂直z轴的平截面变形后所成的倾斜截面. 从以上可以看出N,Q,M即截面轴力、剪力、弯矩. I_m 是截面质量惯性矩. m 为沿 x 方向单位长度的质量. Ω 为变形后斜截面上 z 方向的合力. Φ 是为了运算简便而引入的形式化定义.

由(15) ~ (22) 式,并注意到$\delta u,\delta u_1,\delta w,\delta w$ 的任意性可得控制方程(23) 式及边界条件(24) 式.

$$\begin{cases} m\dfrac{\partial^2 u}{\partial t^2} = \left(\dfrac{\partial N}{\partial x} + \dfrac{\partial N}{\partial x}\dfrac{\partial u}{\partial x} + N\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial M}{\partial x}\dfrac{\partial u_1}{\partial x} + M\dfrac{\partial^2 u_1}{\partial x^2} + kQ\dfrac{\partial u_1}{\partial x} + ku_1\dfrac{\partial Q}{\partial x} - p_1\right) \\ I_m\dfrac{\partial^2 u_1}{\partial t^2} = \left(\dfrac{\partial M}{\partial x} + \dfrac{\partial M}{\partial x}\dfrac{\partial u}{\partial x} + M\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial \Phi}{\partial x}\dfrac{\partial u_1}{\partial x} + \Phi\dfrac{\partial^2 u_1}{\partial x^2} - kQ - kQ\dfrac{\partial u}{\partial x} - \chi\Omega u_1 - dp_1\right) \\ m\dfrac{\partial^2 w}{\partial t^2} = \left(\dfrac{\partial N}{\partial x}\dfrac{\partial w}{\partial x} + N\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial M}{\partial x}\dfrac{\partial w_1}{\partial x} + M\dfrac{\partial^2 w_1}{\partial x^2} + k\dfrac{\partial Q}{\partial x} + kQ\dfrac{\partial w_1}{\partial x} + kw_1\dfrac{\partial Q}{\partial x} + p_2\right) \\ I_m\dfrac{\partial^2 w_1}{\partial t^2} = \left(\dfrac{\partial M}{\partial x}\dfrac{\partial w}{\partial x} + M\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial \Phi}{\partial x}\dfrac{\partial w_1}{\partial x} + M\dfrac{\partial^2 w_1}{\partial x^2} - kQ\dfrac{\partial w}{\partial x} - \chi\Omega w_1 - \chi\Omega + dp_2\right) \end{cases} \tag{23}$$

$$\begin{cases}\left[N+N\frac{\partial u}{\partial x}+M\frac{\partial u_1}{\partial x}\right]\delta u\big|_0^l+kQu_1\delta u\big|_0^l+n\delta u(l)=0\\ \left[M+M\frac{\partial u}{\partial x}+\Phi\frac{\partial u_1}{\partial x}\right]\delta u_1\big|_0^l+m\delta u_1(l)=0\\ \left[N\frac{\partial w}{\partial x}+M\frac{\partial w_1}{\partial x}\right]\delta w\big|_0^l+kQ\delta w\big|_1^l=0\\ \left[M\frac{\partial w}{\partial x}+\Phi\frac{\partial w_1}{\partial x}\right]\delta w_1\big|_0^l+kQw_1\delta w_1\big|_0^l=0\end{cases}\tag{24}$$

从(23)式,(24)式可见方程及边界条件中出现了力及其导数与位移及其导数的耦合项,这比线性 Timoshenko 梁控制微分方程及边界条件复杂的多. 同时由假设④及(22)式可得

$$\begin{cases}N=\frac{E+\upsilon}{1+\upsilon}\left[A\frac{\partial u}{\partial x}+\frac{1}{2}A\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}I_y\left(\frac{\partial u_1}{\partial x}\right)^2+\frac{1}{2}A\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}I_y\left(\frac{\partial w_1}{\partial x}\right)^2\right]\\ \quad\frac{A}{1+\upsilon}\left(w_1+\frac{1}{2}u_1^2+\frac{1}{2}w_1^2\right)\\ M=\frac{E+\upsilon}{1+\upsilon}I_y\left(\frac{\partial u_1}{\partial x}+\frac{\partial u}{\partial x}\frac{\partial u_1}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w_1}{\partial x}\right)\\ Q=\frac{EA}{2(1+\upsilon)}\left(u_1+\frac{\partial w}{\partial x}+u_1\frac{\partial u}{\partial x}+w_1\frac{\partial w}{\partial x}\right)\\ \Omega=\frac{\upsilon}{1+\upsilon}\left[A\frac{\partial u}{\partial x}+\frac{1}{2}A\left(\frac{\partial u}{\partial x}\right)^2\frac{1}{2}I_y\left(\frac{\partial u_1}{\partial x}\right)^2+\frac{1}{2}A\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}I_y\left(\frac{\partial w_1}{\partial x}\right)^2\right]\\ \quad\frac{E+\upsilon}{1+\upsilon}A\left(w_1+\frac{1}{2}u_1^2+\frac{1}{2}w_1^2\right)\\ \Phi=\frac{E+\upsilon}{1+\upsilon}\left[I_y\frac{\partial u}{\partial x}+\frac{1}{2}I_y\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}K\left(\frac{\partial u_1}{\partial x}\right)^2+\frac{1}{2}I_y\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}K\left(\frac{\partial w_1}{\partial x}\right)^2\right]\\ \quad\frac{\upsilon}{\upsilon+1}I_y\left(w_1+\frac{1}{2}u_1^2+\frac{1}{2}w_1^2\right)\end{cases}\tag{25}$$

其中,

$$I_y=\iint_s z^2\mathrm{d}y\mathrm{d}z,\qquad K=\iint_s z^4\mathrm{d}y\mathrm{d}z,\qquad A=\iint_s\mathrm{d}y\mathrm{d}z\tag{26}$$

把(25)式代入(23)式,(24)式即可得用位移表示的控制方程及边界条件. 但进行实际问题的求解时,如是静定问题可由静力平衡方程求 N,M,Q 的表达式代入(23)式,(24)式求解. 在特殊的边界条件下可进一步简化. 例如在两端是固定支座时由

$$\begin{cases}u(l)=\delta u(0)=0,\quad \delta u_1(l)=\delta u_1(0)=0\\ \delta w(l)=\delta w_1(0)=0,\quad \delta w_1(l)=\delta w_1(0)=0\end{cases}\tag{27}$$

可得边界条件(24)式自动满足. 对一般的梁应联立(23)式,(24)式,(25)式及初值条件求解.

4 结 论

本文通过采用4变量的位移场,应用 Green 应变张量描述梁的大变形,通过变分法求得控制微分方程组及边界条件. 若把(25)式代入(23)式,(24)式,同时略去动力项,并令

$$w_1=0,\quad \frac{\partial w_1}{\partial x}=0,\quad \Omega=0\tag{28}$$

则可得文献[3]中不计以基础弹性介质性能有关项(以 a 有关)的静力控制方程组及边界件.

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试验结果表明,该高速公路抗滑表层和粗粒式沥青混凝土具有很强的渗透性,中粒式沥青混凝土渗透性差,水泥稳定碎石层几乎不透水.材料的渗透性除抗滑表层沿路面分布较均匀外,其余3种材料表现出明显的不均衡.

中粒式沥青混凝土采用Ⅰ型密集配时可起到隔水作用,但其上的抗滑表层及Ⅱ型粗粒式沥青混凝土并不能阻止水分渗入沥青混合料面层内部;由于水稳层渗透系数很低,渗入面层的水分将不会下渗至垫层或路基,可确保路基不受路面入渗雨水的影响.

参考文献:

[1] 电力工业标准汇编.水电卷.勘测(下册)-土工试验规程(SD 128—84)[S].北京:水利电力出版社,1995.83-9.
[2]交通部标准.公路土工试验规程(JTJ 051—93)[S].北京:人民交通出版社,1993.118-127.
[3]中华人民共和国行业标准.土工试验规程(SL237—1999)[S].北京:中国水利电力出版社,1999.
[4]交通部标准.公路路基路面现场测试规程(JTJ059—95)[S].北京:人民交通出版社,2000.1-107.

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若令

$$\begin{cases} w_1 = 0, \quad \frac{\partial w_1}{\partial x} = 0, \quad \Omega = 0, \quad \Phi = 0 \\ \frac{\partial u}{\partial x}\frac{\partial u_1}{\partial x} = \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u_1}{\partial x^2} = \frac{\partial w}{\partial x}\frac{\partial w_1}{\partial x} = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w_1}{\partial x^2} = 0 \end{cases} \tag{29}$$

即,略去 N,M,Q,Ω,Φ 及其导数与位移及其导数的耦合项则方程及边界条件简化为经典的Timoshenko梁控制微分方程及边界条件[5].若略去(23)式的动力项,可得非线性梁的静力控制方程及边界条件.从本文中的结果可以看出,非线性的梁理论如采用4变量的位移场将使得问题变得极其复杂,求解更为困难.同时,该理论中含有的考虑的不均匀性引入的常数参数 χ,该参数物理意义及数值需进行进一步讨论.

参考文献:

[1] 邢誉峰,钱志英.Hamilton体系中Timoshenko梁冲击问题的描述和求解[J].振动工程学报,2005,18(3):266-271.
[2] 程昌钧,卢华勇.粘弹性Timoshenko梁的变分原理和静动力行为分析[J].固体力学学报,2002,23(2):190-195.
[3] 张燕,卢华勇.线弹性Timoshenko梁的曲屈与分叉[J].东北师范大学学报,2004,36(3):42-51.
[4] 李国根,朱正右,程昌钧.非线性粘弹性Timoshenko梁的动力行为的分析[J].力学季刊,2001,22(3):347-351.
[5] 鹫津久一郎著.弹性和塑性力学的变分原理[M].老亮译.北京:科学出版社,1984.
[6] 李卓球,董文堂.非线性弹性理论基础[M].北京:科学出版社,2004:88-110.