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# 第四节 定位算子



# 主要内容

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- Wong-Trinder圆点定位算子
- Forstner 定位算子
- 高精度角点与直线定位算子

# 内定向

IO Parameter Display / Modify

NO	X	Y	
0:	295.562	150.592	0.0020 0.0078
1:	199.998	8618.103	-0.0020 -0.0078
2:	8669.973	8714.658	0.0020 0.0078
3:	8765.714	245.670	-0.0020 -0.0078

Interior Orientation Parameters:  
xo= 4483.131 yo= 4432.264

Transformation Matrix:  
1.001102      0.011308  
-0.011327     1.001264

MX = 0.001995 (mm)  
MY = 0.007759 (mm)

Automatic       Manual

Approximate      Save/Exit      Quit      Left      Right      Up      Down



# 1. Wong-Trinder园点定位算子

- 利用二值图像重心对圆点进行定位
- 利用阈值 $T = (\text{最小灰度值} + \text{平均灰度值}) / 2$ 将窗口中的影像二值化
- 计算目标重心坐标 $(x, y)$ 与园度

# • 园度的计算公式

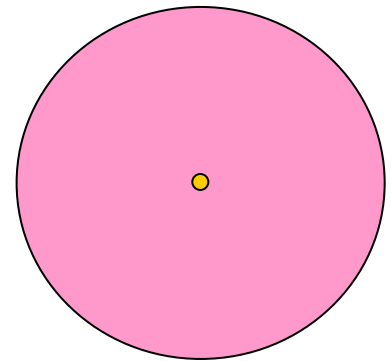
$$x = m_{10} / m_{00}$$

$$y = m_{01} / m_{00}$$

$$\gamma = M'_x / M'_y$$

$$M'_x = \frac{M_{20} + M_{02}}{2} + \sqrt{\left(\frac{M_{20} - M_{02}}{2}\right)^2 + M_{11}^2}$$

$$M'_y = \frac{M_{20} + M_{02}}{2} - \sqrt{\left(\frac{M_{20} - M_{02}}{2}\right)^2 + M_{11}^2}$$



# Wong-Trinder园点定位算子

$$m_{pq} = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} i^p j^q g_{ij} \quad (p, q = 0, 1, 2, \dots)$$

$$M_{pq} = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} (i-x)^p (j-y)^q g_{ij} \quad (p, q = 0, 1, 2, \dots)$$

**p + q阶原点矩与中心矩**

当r小于阈值时，目标不是园；否则园心为 (x, y)

## 2.Trinder 改进算子

算子受二值化影响，误差可达0.5像素

$$\left. \begin{aligned} x &= \frac{1}{M} \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} i g_{ij} W_{ij} \\ y &= \frac{1}{M} \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} j g_{ij} W_{ij} \end{aligned} \right\}$$

$$M = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} g_{ij} W_{ij}$$

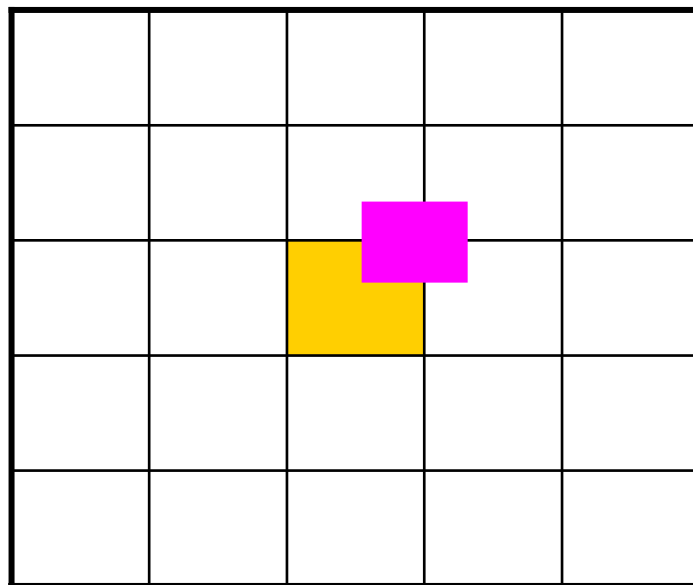
原始灰度

定位精度可达0.01像素

# 3. Forstner定位算子

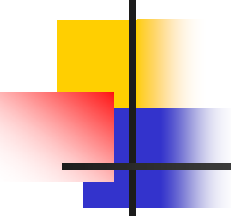
最佳窗口选择

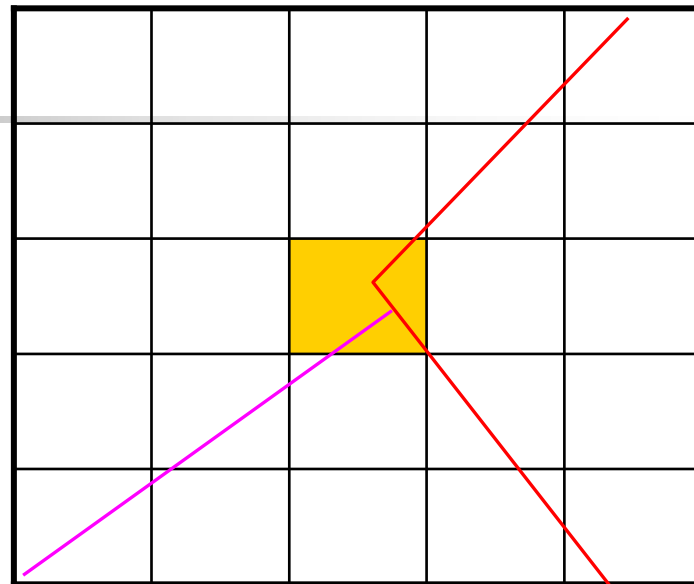
最佳窗口内加权  
重心化



窗口内像元的加权重心



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- 以原点到窗口内边缘直线的距离为观测值，梯度模之平方为权



$$\left. \begin{aligned} v &= x_0 \cos\theta + y_0 \sin\theta - (x \cos\theta + y \sin\theta) \\ \omega(x, y) &= |\nabla g|^2 = g_x^2 + g_y^2 \end{aligned} \right\}$$

## 4.高精度角点与直线定位算子

- 数学模型

$$g(x) = \int_{-\infty}^x S(x) dx$$

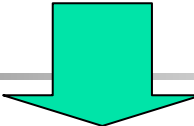
刀刃曲线

线扩散函数

影像的梯度

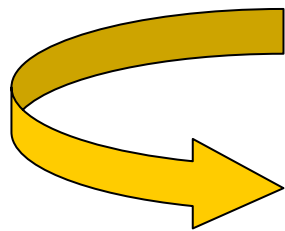
$$\nabla g(x) = \frac{d}{dx} g(x) = \frac{d}{dx} \int_{-\infty}^x S(x) dx = S(x)$$

# 理想的线扩散函数服从高斯分布


$$S(x, y) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2\sigma^2} (x \cos \theta + y \sin \theta - \rho)^2\right]$$

影像的梯度


$$\nabla g(x, y) = \alpha \cdot \exp\left[-k(x \cos \theta + y \sin \theta - \rho)^2\right]$$



$$v(x, y) = c_0 d\alpha + c_1 dk + c_2 d\rho + c_3 d\theta + c_4$$



## 线性化误差方程式中的系数

$$c_0 = \exp[-k_0(x \cos \theta_0 + y \sin \theta_0 - \rho_0)^2]$$

$$c_1 = -a_0 c_0 (x \cos \theta_0 + y \sin \theta_0 - \rho_0)^2$$

$$c_2 = 2a_0 k_0 c_0 (x \cos \theta_0 + y \sin \theta_0 - \rho_0)$$

$$c_3 = c_2 (x \sin \theta_0 - y \cos \theta_0)$$

$$c_4 = a_0 \exp[-k(x \cos \theta_0 + y \sin \theta_0 - \rho_0)^2] - \nabla g(x, y)$$

$a_0, k_0, \rho_0$ 与 $\theta_0$ 为参数的近似值

# 采用梯度的模为观测值

## Roberts梯度

$$\nabla g(i, j) = \sqrt{(g_{i+1, j+1} - g_{i, j})^2 + (g_{i+1, j} - g_{i, j+1})^2}$$

$$d\nabla g = -\cos \beta dg_{i, j} + \sin \beta dg_{i+1, j} - \sin \beta dg_{i, j+1} + \cos \beta dg_{i+1, j+1}$$

## 梯度角

$$\begin{aligned} m_{\nabla g}^2 &= \cos^2 \beta \cdot m^2 + \sin^2 \beta \cdot m^2 + \sin^2 \beta \cdot m^2 + \cos^2 \beta \cdot m^2 \\ &= 2m^2 \end{aligned}$$

## 误差

## 噪声误差

单位权中误差为

$$m_0 = \sqrt{2} m$$



## •初值确定

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Hough变换确定直线参数初值 $\rho_0$  ,  $\theta_0$ 。

$$a_0 = \max\{\nabla g(x, y)\}$$

$a_0$ 是梯度的最大值

$$k_0 = - \frac{\ln \nabla g(x_0, y_0) - \ln a_0}{(x_0 \cos \theta_0 + y_0 \sin \theta_0 - \rho_0)^2}$$

$(x_0, y_0)$ 为直线附近任一点的坐标



## ●粗差的剔除

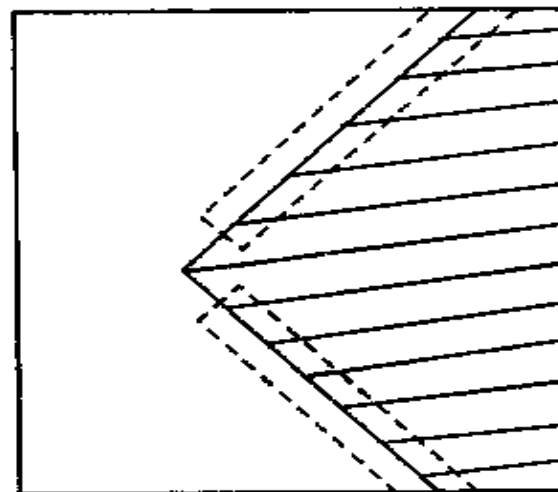
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$$W_{i,j} = \begin{cases} 1, & \sigma_0^2 < \sigma_n^2 \text{ OR } \sigma_0^2 / v_{ij}^2 > 1 \\ \sigma_0^2 / v_{ij}^2 & \end{cases}$$

采用选权迭代法，使粗差在平差的过程中自动地被逐渐剔除

## • 窗口的选择

精确定位窗口在粗定位  
矩形窗口中确定



## • 角点定位

$$\left. \begin{aligned} \rho_1 &= x \cos \theta_1 + y \sin \theta_1 \\ \rho_2 &= x \cos \theta_2 + y \sin \theta_2 \end{aligned} \right\}$$



$$\left. \begin{aligned} x_c &= \frac{\rho_1 \sin \theta_2 - \rho_2 \sin \theta_1}{\sin(\theta_2 - \theta_1)} \\ y_c &= \frac{\rho_2 \cos \theta_1 - \rho_1 \cos \theta_2}{\sin(\theta_2 - \theta_1)} \end{aligned} \right\}$$



# • 理论精度

$$\sigma_0 = \sqrt{\frac{\sum v^2}{n-4}}, n \text{ 为观测值个数}$$

$$\begin{bmatrix} q_{\rho\rho} & q_{\rho\theta} \\ q_{\theta\rho} & q_{\theta\theta} \end{bmatrix}$$

单位权中误差

直线参数  $\rho$ ,  $\theta$  的协因素阵

$$D = \begin{bmatrix} \sigma_{01}^2 q_{\rho_1\rho_1} & \sigma_{01}^2 q_{\rho_1\theta_1} & 0 & 0 \\ \sigma_{01}^2 q_{\rho_1\theta_1} & \sigma_{01}^2 q_{\theta_1\theta_1} & 0 & 0 \\ 0 & 0 & \sigma_{02}^2 q_{\rho_2\rho_2} & \sigma_{02}^2 q_{\rho_2\theta_2} \\ 0 & 0 & \sigma_{02}^2 q_{\rho_2\theta_2} & \sigma_{02}^2 q_{\theta_2\theta_2} \end{bmatrix}$$

两直线参数的  
协方差阵

$$\sigma_P = \sqrt{\sigma_x^2 + \sigma_y^2}$$

理论定位精度  
为0.02像素