

轴向受力梁强非线性超谐波与次谐波共振的能量迭代法

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摘要: 研究了在横向动载荷作用下存在轴向力的一类梁的强非线性振动; 建立了梁振动的二阶强非线性非自治微分方程, 并对求解强非线性自治系统的能量迭代法加以改进, 用于求解梁的强非线性非自治系统, 其方法是: 由能量法得到主共振、超谐共振和次谐共振的一次近似解的表达式; 引入牛顿迭代的思想和最小二乘法, 得到高次近似解的表达式。研究结果表明: 用改进后的能量迭代法求解强非线性非自治系统精度较高; 分析这种非线性梁的振动时, 除了要考虑其主共振外, 还要考虑超谐共振和次谐共振。

关键词: 梁; 强非线性非自治系统; 超谐共振; 次谐共振; 能量迭代法

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Energy-iterative method for solution of super-harmonic and sub-harmonic resonance of strongly non-linear vibrations with an axially forcing beam

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Abstract: Strongly non-linear vibrations of an axially forcing beam under transverse dynamic loads were investigated. The Duffing equation, the equation of strongly non-linear and non-autonomous oscillation system of the beam were set up. The energy-iteration method for strongly non-linear autonomous system was modified to deal with strongly non-linear non-autonomous system, and the modified new method was developed, the analytic expressions of solution of primary resonance and superharmonic and subharmonic resonance by the energy method were obtained, and the higher order analytic expression of solution was gotten by means of Newton iteration idea and least square technique principle. The results show that the modified energy iterative method for strongly non-linear and non-autonomous system is both effective and accurate, and that not only the primary resonance but also the superharmonic and subharmonic resonance must be considered in this sort of strongly non-linear beams.

Key words: beam; strongly non-linear and non-autonomous system; superharmonic resonance; subharmonic resonance; energy iterative method

在结构和机械振动中, 许多模型可以归结为在 横向动载荷作用下有轴向力的非线性振动, 如带锯、

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输流管道、空中索道、纸张、皮带、纺织纤维、磁带和桥梁等可归结为弦或梁的模型^[1]。A. H. Nayfeh 等^[2-4]归纳了弱非线性振动的各种摄动法, 并研究了梁的主共振以及超谐、次谐共振和多个谐波的联合共振等各种弱非线性振动, 用多尺度法研究了梁的弱非线性振动的内共振、组合谐波共振; L. Azrar 等^[5]用谐波平衡法研究梁的非线性自由振动及强迫振动; H. R. Öz^[6]用多尺度法研究了轴向速度为变速度, 且 $\varepsilon \ll 1$ 时梁的弱非线性振动。

随着科学技术的迅速发展, 梁的非线性振动尤其是强非线性振动问题日益突出。弱非线性振动分析的方法不能适用于强非线性振动问题的分析。为此, 曾庆元等^[7,8]建立了桥梁空间振动的有限元模型, 用数值方法求解桥梁的非线性振动; 黄建亮等^[9]用半解析的改进的 L-P 法, 研究了轴向力为常量时梁的强非线性振动的超谐与次谐波解^[9], 并给出了非线性项系数 $\varepsilon < 1$ 时的算例。能量—迭代法^[10,11]在应用能量法^[12-14]求解的基础上进一步迭代, 不仅能对系统进行定性分析, 而且定量求解的结果精度较高。能量迭代法^[10,11]仅能求解强非线性自治系统, 在此, 作者对其进行改进, 推广应用于求解轴向力为变量时梁的强非线性非自治振动, 求解当非线性项系数 $\varepsilon \gg 1$ 时的主共振及超谐波与次谐波共振解。

1 梁的非线性振动微分方程

图 1 所示为受横向动载荷和轴向拉力的两端简支弹性弯曲梁结构图。设梁的长度为 L , 横截面积为 A , 横截面对中性轴的惯性矩为 I , 质量密度为 ρ , 抗弯刚度为 EI , 回转半径为 $r = \sqrt{I/A}$, 梁的横向载荷为 $q(x, t)$, 其振动方程为:

$$\rho A \frac{\partial^2 y}{\partial t^2} - S \frac{\partial^2 y}{\partial x^2} - \frac{\partial S}{\partial x} \frac{\partial y}{\partial x} + EI \frac{\partial^4 y}{\partial x^4} = q(x, t) \quad (1)$$

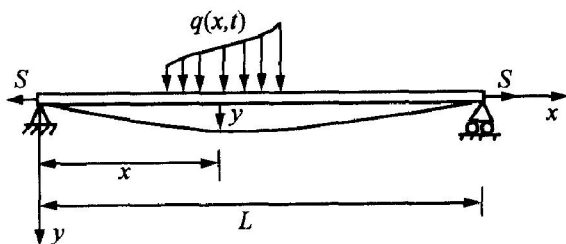


图 1 受横向动载荷和轴向拉力的梁的结构图

Fig. 1 Beam under axially force and transverse dynamic loads

其中, S 为轴力, 其一般表达式为:

$$S(x, t) = EA \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial y}{\partial x} \right)^2 \right]$$

轴力包括 2 部分: $EA \cdot (\partial u / \partial x)$ 和 $EA \cdot (\partial y / \partial x)^2 / 2$ 。当 $\partial u / \partial x = o((\partial y / \partial x)^2)$, 即轴向力主要由梁横截面的转角位移引起时, 可以略去梁轴向的应变对轴力的影响, 即 $\int_0^L \frac{\partial u}{\partial x} dx = 0$, 故

$$S(x, t) = EA (\partial y / \partial x)^2 / 2 \quad (2)$$

将式(2)代入式(1), 得:

$$\rho A \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} - \left[\frac{3EA}{2L} \int_0^L \left(\frac{\partial y}{\partial x} \right)^2 dx \right] \frac{\partial^2 y}{\partial x^2} = q(x, t) \quad (3)$$

采用基本模态 $y(x, t) = V(t) \sin \frac{\pi x}{L}$, 代入式(3), 有

$$\frac{d^2 V(t)}{dt^2} + \frac{\pi^4 EI}{L^4 \rho A} V(t) + \frac{3\pi^4 EA}{4L^4 \rho A} V^3(t) = q(x, t) / \sin \frac{\pi x}{L} \quad (4)$$

引入无量纲量:

$$b = \frac{\pi^4 EI}{L^4 \rho A}, x = \frac{V}{r}, r = \sqrt{\frac{I}{NA}} \quad (5)$$

方程(4)可以写为:

$$\frac{d^2 x}{dt^2} + bx + \frac{3b}{4} x^3 = q(x, t) / \sin \frac{\pi x}{L} \quad (6)$$

其中: 非线性项 $3bx^3/4$ 即为轴力的影响项。取 $q(x, t) = h \cos \Omega t \cdot \sin \frac{\pi x}{L}$ (其中, Ω 为动载荷变化的频率), 有

$$\frac{d^2 x}{dt^2} + bx + \frac{3b}{4} x^3 = P \cos \Omega t \quad (7)$$

其中: $P = h / (\rho A r)$ 。式(7)即梁的强非线性无阻尼受迫振动 Duffing 方程。此系统除有主共振响应外, 还可能有超谐、次谐及超次谐共振响应^[15-19]。

2 梁的主振动和超、次谐振动周期解的能量迭代解法

式(7)为二阶强非线性非自治系统, 可以表达为一般形式:

$$\ddot{x} + g(x) + f(x, \dot{x}, \Omega) = 0 \quad (8)$$

对上式两端同乘 dx 后积分, 得

$$[\dot{x}^2/2 + V(x)] - [\dot{x}_0^2/2 + V(x_0)] =$$

$$- \int f(x, \dot{x}, \Omega) \dot{x} dt. \tag{9}$$

式中: $V(x) = \int g(x) dx$, 为系统的势能函数。式(9)表明系统的机械能变化等于非有势力所做的功。因为求周期解, 系统在 1 个周期内平均能量守恒, 应有:

$$\dot{x}^2(T)/2 + V(x(T)) = \dot{x}^2(0)/2 + V(x(0)). \tag{10}$$

取 1 个周期内的平均值 \bar{x} 为其一次近似解, 有

$$\dot{x}^2/2 + V(x) = \dot{x}_0^2/2 + V(x_0). \tag{11}$$

根据式(9)和式(11), 有

$$\int f(x, \dot{x}, \Omega) \dot{x} dx = 0. \tag{12}$$

设一次近似周期解为:

$$\begin{cases} x_1 = a^* \cos \theta \\ \theta = \Omega t + \varphi \end{cases} \tag{13}$$

初始条件为:

$$\theta_0 = 0, x_{10} = a^* \cos \theta_0 = a^*, \tag{14}$$

故

$$\dot{x}_1 = -a^* \dot{\theta} \sin \theta, \dot{x}_{10} = -a^* \dot{\theta}_0 \sin \theta_0 = 0. \tag{15}$$

将式(13)和式(14)代入式(11)得:

$$\theta = \frac{d\theta}{dt} = \sqrt{b(1 + \frac{9a^{*2}}{16})(1 + \frac{3a^{*2}}{16 + 9a^{*2}} \cos 2\theta)}. \tag{16}$$

令

$$\omega = \sqrt{b(1 + 9a^{*2}/16)},$$

有

$$\frac{d\theta}{dt} \approx \omega + \frac{3ba^{*2}}{32\omega} \cos 2\theta. \tag{17}$$

2.1 梁的主振动周期解

$$\text{令 } \omega = \Omega, \tag{18}$$

$$\text{有 } \sqrt{b(1 + 9a^{*2}/16)} = \Omega.$$

由此解得:

$$a^* = \frac{4}{3} \sqrt{\frac{\Omega^2}{b} - 1}. \tag{19}$$

当 $\Omega > b$ 时, a^* 为正实数, 有周期解。

取一组数据: $E = 200 \text{ GPa}$, $\rho = 7855.7 \text{ kg/m}$, $r = \sqrt{3}/3 \times 10^{-3} \text{ m}$, $L = 3.46 \text{ m}$, $P = 1$, 则有 $b = 17.232$ 。对应于文献[9]中的 ε 本例 $\varepsilon = 3b/4 = 12.924 \gg 1$, 此时取 $\Omega = 9 > \sqrt{b} = 4.15$, 存在主振动周期解。将 Ω 和 b 代入式(19)得: $a^* = 2.56$ 。为确定式(13)第 2 式中相应于 a^* 的初相位 φ , 令式(12)的被积函数中的非周期项为零, 并将式(15)和式(17)代入式(12), 令其非周期项为零, 有

$$\left[-\frac{1}{2} P a^* \Omega + 3 P a^{*3} b / (128 \omega) \right] \sin \varphi = 0.$$

得 $\varphi = 0$ 。故主振动一次近似周期解为:

$$x_1 = a^* \cos \Omega t = 2.56 \cos 9t. \tag{20}$$

为求二次近似解 x_2 , 将式(7)写为:

$$\ddot{x}_2 + F(x_2, \dot{x}_2, \Omega) = 0. \tag{21}$$

设

$$x_2 = x_1 + w_1, \tag{22}$$

将式(21)中 $F(x_2, \dot{x}_2, \Omega)$ 在 (x_1, \dot{x}_1) 邻域内展开为泰勒级数:

$$F(x_2, \dot{x}_2, \Omega) = F(x_1, \dot{x}_1, \Omega) + F'_x(x_1, \dot{x}_1, \Omega)w_1 + F'_{\dot{x}}(x_1, \dot{x}_1, \Omega)\dot{w}_1 + \dots \tag{23}$$

将式(22)和式(23)代入式(21), 有

$$\begin{aligned} \ddot{w}_1 + F'_x(x_1, \dot{x}_1, \Omega)w_1 + F'_{\dot{x}}(x_1, \dot{x}_1, \Omega)\dot{w}_1 = \\ - [x_1 + F(x_1, \dot{x}_1, \Omega)]. \end{aligned} \tag{24}$$

对照式(7), $F(x, \dot{x}, \Omega) = b(x + 3x^3/4) - P \cos \Omega$ 。故式(24)即为

$$\begin{aligned} \ddot{w}_1 + b(1 + 9x^2/4)w_1 = \\ - [x_1 + b(x_1 + 3x_1^3/4) - P \cos \Omega]. \end{aligned} \tag{25}$$

将 $x_1 = a^* \cos \Omega t$ 代入式(25), 有

$$\begin{aligned} \ddot{w}_1 + b[1 + 9a^{*2}/8 + (9a^{*2}/8) \cos 2\Omega]w_1 = \\ (a^* \Omega^2 - ba^* - 9ba^{*3}/16 + P) \cos \Omega - \\ (3ba^{*3}/16) \cos 3\Omega. \end{aligned} \tag{26}$$

这是关于 w_1 的二阶周期系数的非齐次方程。按谐波平衡思想, 令

$$w_1 = \alpha_0 + \sum_{i=1}^N (\alpha_i \cos i\theta + \beta_i \sin i\theta), \tag{27}$$

由式(13)及 $\varphi = 0$, 有

$$\theta = \Omega t. \tag{28}$$

将式(27) (取 $N = 3$) 代入式(26), 合并 $\sin j \Omega$ 和 $\cos j \Omega$ 各项, 得:

$$A_0 + \sum_{j=1}^5 (A_j \cos j \Omega + B_j \sin j \Omega) = 0. \tag{29}$$

采用最小二乘法, 令

$$I = \frac{1}{2\pi} \int_0^{2\pi} \left[A_0 + \sum_{j=1}^5 (A_j \cos j\theta + B_j \sin j\theta) \right]^2 d\theta. \tag{30}$$

根据三角函数正交性, 有

$$\begin{aligned} I(\alpha, \alpha, \beta) = A_0^2 + \\ \frac{1}{2} \sum_{j=1}^5 [A_j^2(\alpha, \alpha, \beta) + B_j^2(\alpha, \alpha, \beta)]. \end{aligned} \tag{31}$$

令

$$\frac{\partial I}{\partial \alpha} = 0, \frac{\partial I}{\partial \alpha} = 0, \frac{\partial I}{\partial \beta} = 0. \tag{32}$$

其中: $i = 1, 2, 3$ 。

将式(32)写成矩阵方程形式:

$$[M]\{u\} = \{h\}. \tag{33}$$

其中: $[M]$ 为系数矩阵; $\{u\} = (\alpha_0, \alpha_1, \beta_1, \dots, \alpha_3, \beta_3)^T$;

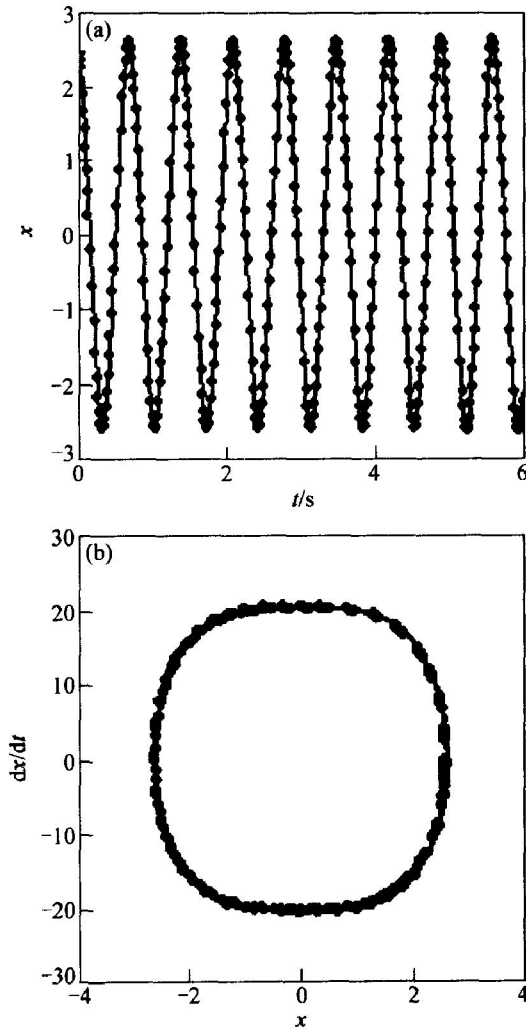
{h} 为常数阵列。解出{u}, 代入式(27) 和式(22), 即得二次近似的主振动周期解。算得

$$\{u\} = (0, -0.0385, 0, 0, 0, 0.0851, 0)^T,$$

故二次近似主振动周期解表达式为:

$$x_2 = 2.5215 \cos \Omega t + 0.0881 \cos 3 \Omega t. \quad (34)$$

用本方法求解所得结果与四阶 Runge-kutta 法数值解所得结果进行对比, 结果见图 2。



(a) 时间-位移图; (b) 相图;
实线为解析值; 黑点为计算值

图 2 Duffing 方程的主振动时间-位移图和相图

Fig. 2 Diagrams of time-displacement and phase of primary resonance of Duffing equation

2.2 梁的次谐波振动周期解

令 $\omega = \Omega/3$, 求梁的 1/3 次谐波振动周期解。据式(16), 有

$$a^* = \frac{4}{3} \sqrt{\frac{\Omega^2}{9b} - 1}.$$

故当 $\Omega^2 > 9b$ 时, a^* 有实数解。据前面的参数设置, $\Omega > 12.45$ 。取 $\Omega = 48$, 有

$$a^* = 4.963.$$

而式(7)中 x^3 的系数不随 ω 的改变, $\epsilon = 3b/4 = 12.924 \gg 1$ 。据式(12)被积函数中非周期项系数为零

的条件, 得 $\sin 3\Phi = 0$, 可取

$$\Phi = \pi/3.$$

故一次近似 1/3 次谐解为

$$x_1 = a^* \cos(\Omega t/3 - \pi/3). \quad (35)$$

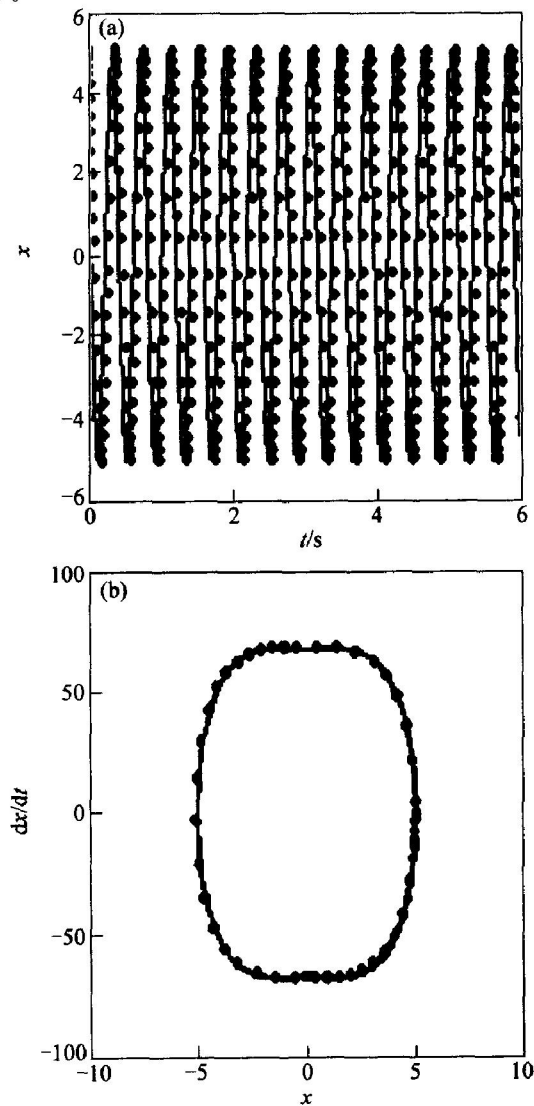
以下求二次近似的 1/3 次谐解。其方法与前面求二次近似解相同, 只是此时

$$\Theta = \Omega/3 + \pi/3. \quad (36)$$

应用式(22) ~ (33), 据式(36), 将式(29)中的 $\cos j\theta$ 和 $\sin j\theta$ 展开, 按 $\cos j\Omega$ 和 $\sin j\Omega$ 归并各项求得 $A_0, A_j, B_j (j = 1, 2, \dots, 5)$, 最后求得 $\{u\} = (0, -0.1068, 0, 0, 0, 0.2015, 0)^T$, 代入式(22), 得

$$x_2 = 4.8292 \cos(\Omega t/3 + \pi/3) + 0.2015(\Omega t + \pi).$$

该结果与四阶 Runge-kutta 法对照结果如图 3 所示。



(a) 时间-位移图; (b) 相图;
实线为解析值; 黑点为计算值

图 3 Duffing 方程 1/3 次谐波振动的时 间-位移图和相图

Fig. 3 Diagrams of time-displacement and phase of 1/3 subharmonic resonance of Duffing equation

2.3 梁的超谐振动周期解

令 $\omega = 3\Omega$, $a^* = \frac{4}{3} \sqrt{\frac{9\Omega^2}{b} - 1}$ 。当 $\Omega > 1.384$ 时,

a^* 有实数解。取 $\Omega = 6$, 有 $a^* = 5.626$, 再由式(12)中被积函数的非周期项为零确定 φ , 此时非周期项系数显然为零, 故可取 $\varphi = 0$ 。从而一次近似 $\omega = 3\Omega$ 超谐振动周期解为:

$$x_1 = a^* \cos 3\Omega t. \quad (37)$$

在求其二次近似解时, 令 $\theta = 3\Omega t$ 。为使一次近似解谐波 $\cos 3\Omega t$ 得到修正, 设 w_1 为

$$w_1 = \alpha_0 + \sum_{i=1}^3 [\alpha_i \cos(i\theta/3) + \beta_i \sin(i\theta/3)]. \quad (38)$$

按以上方法求 A_0, A_j 和 $B_j (j = 1, 2, \dots, 9)$ 时, 按 $j\Omega$ 的谐波归并各项, 应用式(22)~(33), 求得:

$$\{u\} = (0, 0.0011, 0, 0, 0, -0.0003, 0)^T.$$

即得

$$x_2 = 5.626 \cos 3\Omega t - 0.0003 \cos 9\Omega t. \quad (39)$$

与四阶 Rong-kutta 法计算结果对照, 结果见图 4。

3 结 论

a. 应用能量原理与迭代方法求出强非线性轴向受力梁的主振动和超谐与次谐振动周期解。与四阶 Rung-kutta 法结果的对照图形可知, 采用该方法可有效地求解该类梁的强非线性振动问题, 在非线性的项系数 $\epsilon \gg 1$ 时精度较高。

b. 由于考虑了轴力的影响, 方程式(7)中出现强非线性项, 使梁的振动不仅有主共振, 还出现超谐共振、次谐共振, 因此, 在这类梁的设计中, 不仅要考虑 $\omega \approx \Omega$ 的主共振, 还应考虑各种超谐共振和次谐共振的影响。

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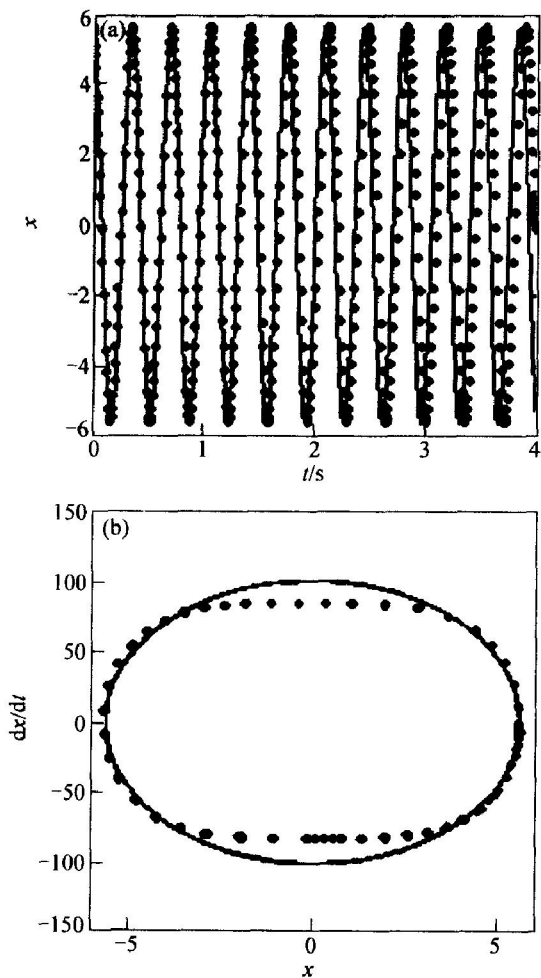
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(a) 时间-位移图; (b) 相图;
实线为解析值; 黑点为计算值

图 4 Duffing 方程在 $\omega = 3\Omega$ 时的超谐振动时间-位移图和相图

Fig. 4 Diagrams of time-displacement and phase of sub harmonic resonance of Duffing equation at $\omega = 3\Omega$

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