《摄影测量学》（上）第五章

解析法绝对定向

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主要内容

一、绝对定向元素

二、物空间坐标近似坐标变换

三、三维空间相似变换原理

四、相似变换参数计算

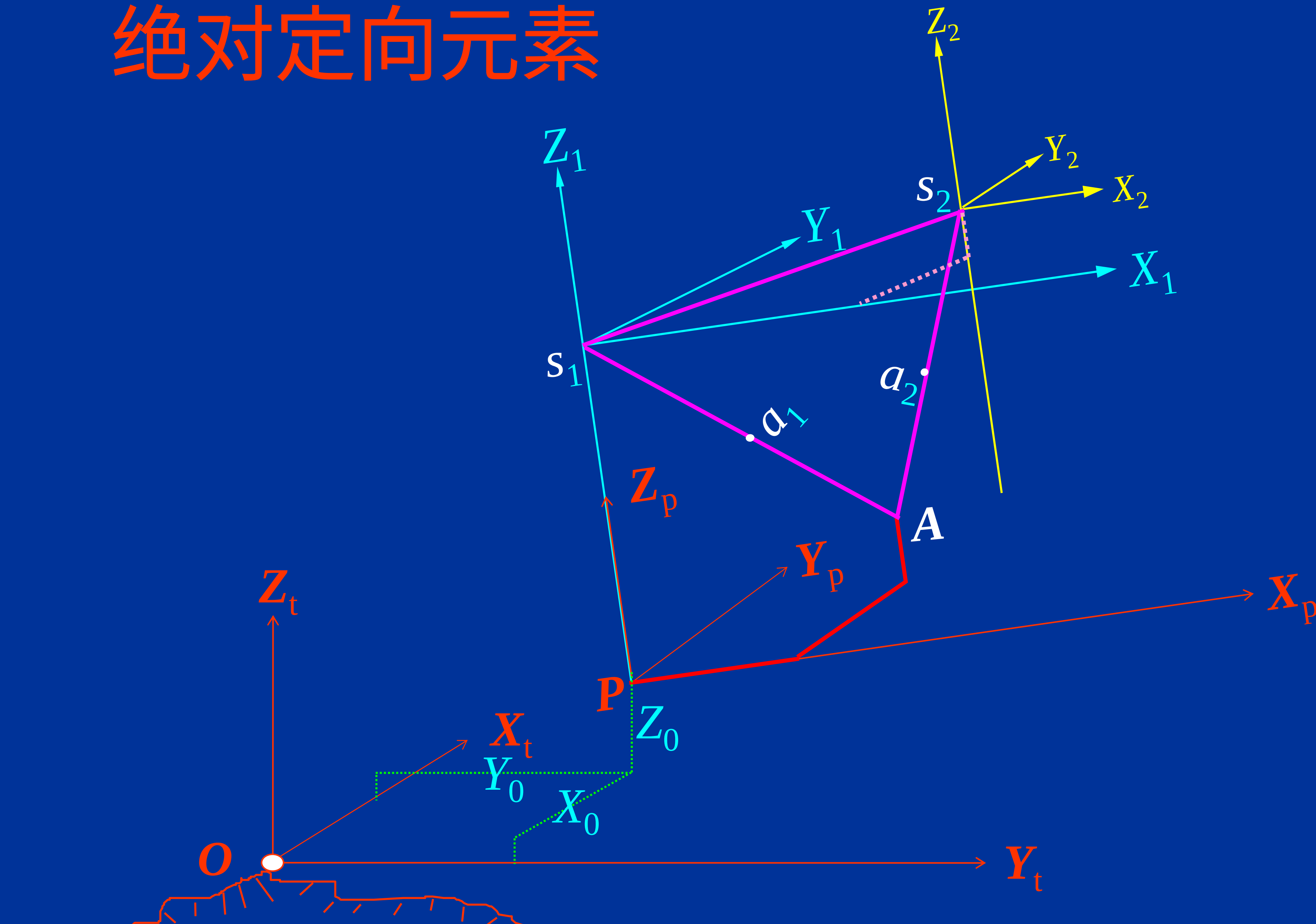
五、地面坐标计算

一、绝对定向元素

描述立体像对在摄影瞬间的绝对位置和姿态的参数称～

通过将相对定向建立的立体模型进行缩放、旋转和平移，使其达到绝对位置

绝对定向元素



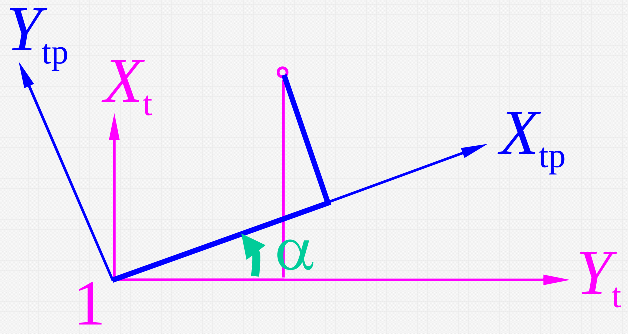
绝对定向元素：λ，X_0，Y_0，Z_0，Φ，Ω，K

二、物空间坐标近似坐标变换 (平面)

• **正变换**（由大地测量坐标系到地面摄影测量坐标系的坐标变换）

$$\begin{bmatrix} \Delta X_{tp} \\ \Delta Y_{tp} \end{bmatrix} = \lambda \begin{bmatrix} \sin\alpha & \cos\alpha \\ \cos\alpha & -\sin\alpha \end{bmatrix} \begin{bmatrix} \Delta X_t \\ \Delta Y_t \end{bmatrix} = \begin{bmatrix} b & a \\ a & -b \end{bmatrix} \begin{bmatrix} \Delta X_t \\ \Delta Y_t \end{bmatrix}$$

$$Z_{tp} = \lambda Z_t$$



$$a = \frac{\Delta X_{tp}\Delta Y_t + \Delta Y_{tp}\Delta X_t}{\Delta X_t^2 + \Delta Y_t^2}$$

$$b = \frac{\Delta X_{tp}\Delta X_t - \Delta Y_{tp}\Delta Y_t}{\Delta X_t^2 + \Delta Y_t^2}$$

$$\lambda = \sqrt{a^2 + b^2} = \sqrt{\frac{\Delta X_{tp}^2 + \Delta Y_{tp}^2}{\Delta X_t^2 + \Delta Y_t^2}}$$

二、物空间坐标近似坐标变换 (平面)

• 逆变换（由地面摄影测量坐标系到大地测量坐标系的坐标变换）

$$\begin{bmatrix} \Delta X_t \\ \Delta Y_t \end{bmatrix} = \frac{1}{\lambda^2} \begin{bmatrix} b & a \\ a & -b \end{bmatrix} \begin{bmatrix} \Delta X_{tp} \\ \Delta Y_{tp} \end{bmatrix}$$

$$Z_t = \frac{1}{\lambda} Z_{tp}$$

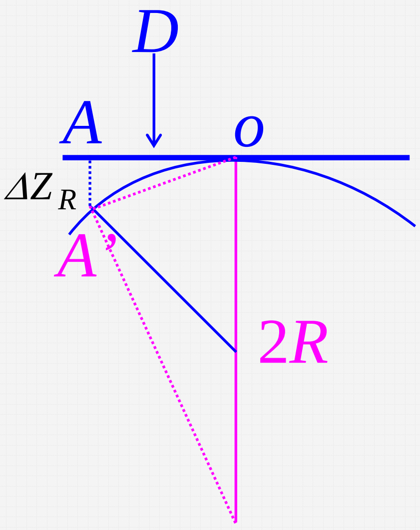
二、物空间坐标近似坐标变换 (高程)

- 地球曲率引起的高程差

$$\frac{\Delta Z_R}{A'O} = \frac{A'O}{2R}$$

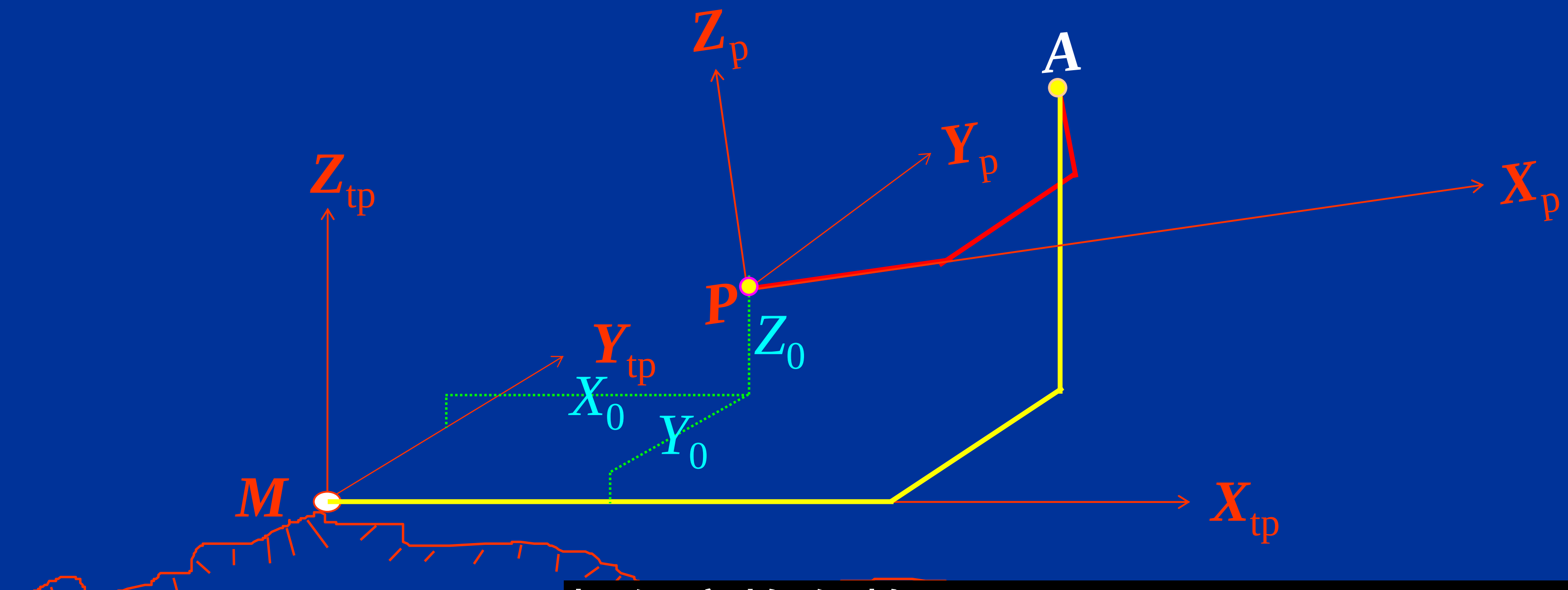
$$A'O^2 = D^2 + \Delta Z_R^2 \approx D^2$$

$$\Delta Z_R = \frac{D^2}{2R} = \frac{X_{tp}^2 + Y_{tp}^2}{2R}$$



三、三维空间相似变换原理

$$\begin{bmatrix} X_{tp} \\ Y_{tp} \\ Z_{tp} \end{bmatrix} = \lambda \boldsymbol{R} \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}$$



相似变换参数：λ，X_0，Y_0，Z_0，Φ，Ω，K

三维空间相似变换误差方程

记 $$F=\begin{bmatrix} X_{tp} \\ Y_{tp} \\ Z_{tp} \end{bmatrix}=\lambda \boldsymbol{R}\begin{bmatrix} X_{p} \\ Y_{p} \\ Z_{p} \end{bmatrix}+\begin{bmatrix} X_{0} \\ Y_{0} \\ Z_{0} \end{bmatrix}$$

$$F=F^{0}+\frac{\partial F}{\partial \lambda}\Delta\lambda+\frac{\partial F}{\partial \Phi}\Delta\Phi+\frac{\partial F}{\partial \Omega}\Delta\Omega+\frac{\partial F}{\partial K}\Delta K+\frac{\partial F}{\partial X_{0}}\Delta X_{0}+\frac{\partial F}{\partial Y_{0}}\Delta Y_{0}+\frac{\partial F}{\partial Z_{0}}\Delta Z_{0}$$

偏导数

$$\frac{\partial F}{\partial X_0} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{\partial F}{\partial Y_0} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\frac{\partial F}{\partial Z_0} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{\partial F}{\partial \lambda} = \boldsymbol{R} \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}$$

$$\frac{\partial F}{\partial \Phi} = \begin{bmatrix} -\lambda Z' \\ 0 \\ \lambda X' \end{bmatrix}$$

$$\frac{\partial F}{\partial \Omega} = \begin{bmatrix} -\lambda Y' \sin\Phi \\ \lambda X' \sin\Phi - \lambda Z' \cos\Phi \\ \lambda Y' \cos\Phi \end{bmatrix}$$

$$\frac{\partial F}{\partial K} = \begin{bmatrix} -\lambda Y' \cos\Phi \cos\Omega - \lambda Z' \sin\Omega \\ \lambda X' \cos\Phi \cos\Omega + \lambda Z' \sin\Phi \cos\Omega \\ \lambda X' \sin\Omega - \lambda Y' \sin\Phi \cos\Omega \end{bmatrix}$$

常数项

$$\boldsymbol{l} = F - F_0$$

$$\begin{bmatrix} l_X \\ l_Y \\ l_Z \end{bmatrix} = \begin{bmatrix} X_{tp} \\ Y_{tp} \\ Z_{tp} \end{bmatrix} - \lambda^0 \boldsymbol{R}^0 \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} - \begin{bmatrix} X_0^0 \\ Y_0^0 \\ Z_0^0 \end{bmatrix}$$

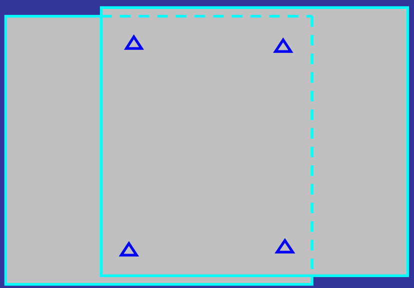
三维空间相似变换误差方程

设 $\lambda^0 = 1, \quad \Phi^0 = \Omega^0 = K^0 = 0$ 则

$$\begin{bmatrix} v_X \\ v_Y \\ v_Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X' & -Z' & 0 & -Y' \\ 0 & 1 & 0 & Y' & 0 & -Z' & X' \\ 0 & 0 & 1 & Z' & X' & Y' & 0 \end{bmatrix} \begin{bmatrix} \Delta X_0 \\ \Delta Y_0 \\ \Delta Z_0 \\ \Delta\lambda \\ \Delta\Phi \\ \Delta\Omega \\ \Delta K \end{bmatrix} - \begin{bmatrix} l_X \\ l_Y \\ l_Z \end{bmatrix}$$

法方程的建立与求解

量测 2 个平高和 1 个高程以上的控制点可以按最小二乘平差原理求绝对定向元素



$$\boldsymbol{V} = \boldsymbol{A}\boldsymbol{x} - \boldsymbol{l}, \qquad \boldsymbol{P}$$

$$\boldsymbol{x} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{l})$$

$$\sigma_0 = \sqrt{\frac{\boldsymbol{V}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{V}}{3n-7}}$$

$$\boldsymbol{Q}_{xx} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}$$

$$m_i = \sigma_0 \sqrt{\boldsymbol{Q}_{xx}^{ii}}$$

四、相似变换参数的计算

- 获取控制点的两套坐标 $X_p, Y_p, Z_p, X_{tp}, Y_{tp}, Z_{tp}$
- 给定相似变换参数的初值 $\lambda=1,\ \Phi=\Omega=K=0,\ \boldsymbol{X}_0, \boldsymbol{Y}_0, \boldsymbol{Z}_0$
- 计算重心化坐标
- 计算误差方程式的系数和常数项
- 解法方程，求相似变换参数改正数
- 计算相似变换参数的新值
- 判断迭代是否收敛

重心坐标	重心化坐标	目的

$$X_{pg}=\frac{\sum_{i=1}^{n}X_{p_i}}{n}\qquad Y_{pg}=\frac{\sum_{i=1}^{n}Y_{p_i}}{n}\qquad Z_{pg}=\frac{\sum_{i=1}^{n}Z_{p_i}}{n}$$

$$\overline{X}_{p_i}=X_{p_i}-X_{pg}$$
$$\overline{Y}_{p_i}=Y_{p_i}-Y_{pg}$$
$$\overline{Z}_{p_i}=Z_{p_i}-Z_{pg}$$

$$X_{tpg}=\frac{\sum_{i=1}^{n}X_{tp_i}}{n}\qquad Y_{tpg}=\frac{\sum_{i=1}^{n}Y_{tp_i}}{n}\qquad Z_{tpg}=\frac{\sum_{i=1}^{n}Z_{tp_i}}{n}$$

$$\overline{X}_{tp_i}=X_{tp_i}-X_{tpg}$$
$$\overline{Y}_{tp_i}=Y_{tp_i}-Y_{tpg}$$
$$\overline{Z}_{tp_i}=Z_{tp_i}-Z_{tpg}$$

- 减少模型点坐标在计算过程中的有效位数，以保证计算的精度
- 使法方程的系数简化，个别项数值变为零，以提高计算速度

相似变换的误差方程

设 $\lambda^0 = 1,\quad \Phi^0 = \Omega^0 = K^0 = 0$ 则

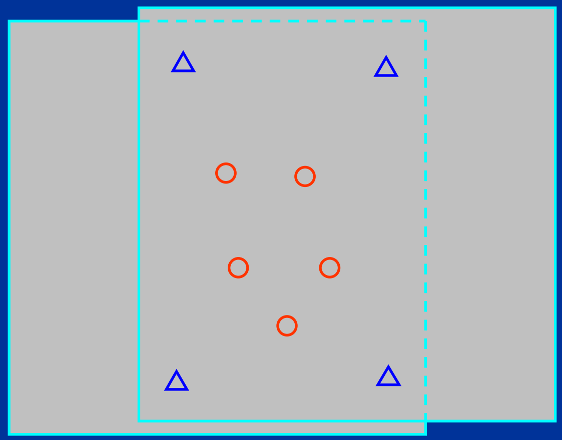
$$\begin{bmatrix} v_X \\ v_Y \\ v_Z \end{bmatrix}_i = \begin{bmatrix} 1 & 0 & 0 & \bar{X}_p & -\bar{Z}_p & 0 & -\bar{Y}_p \\ 0 & 1 & 0 & \bar{Y}_p & 0 & -\bar{Z}_p & \bar{X}_p \\ 0 & 0 & 1 & \bar{Z}_p & \bar{X}_p & \bar{Y}_p & 0 \end{bmatrix}_i \begin{bmatrix} \Delta X_0 \\ \Delta Y_0 \\ \Delta Z_0 \\ \Delta\lambda \\ \Delta\Phi \\ \Delta\Omega \\ \Delta K \end{bmatrix} - \begin{bmatrix} l_X \\ l_Y \\ l_Z \end{bmatrix}_i$$

$$\begin{bmatrix} l_X \\ l_Y \\ l_Z \end{bmatrix}_i = \begin{bmatrix} \bar{X}_{tp} \\ \bar{Y}_{tp} \\ \bar{Z}_{tp} \end{bmatrix}_i - \lambda^0 \boldsymbol{R}^0 \begin{bmatrix} \bar{X}_p \\ \bar{Y}_p \\ \bar{Z}_p \end{bmatrix}_i - \begin{bmatrix} X_0^0 \\ Y_0^0 \\ Z_0^0 \end{bmatrix}$$

相似变换的法方程

$$
\begin{bmatrix}
n_X & 0 & 0 & \sum\bar{X} & -\sum\bar{Z} & 0 & \sum\bar{Y} \\
0 & n_Y & 0 & \sum\bar{Y} & 0 & -\sum\bar{Z} & \sum\bar{X} \\
0 & 0 & n_Z & \sum\bar{Z} & \sum\bar{X} & \sum\bar{Y} & 0 \\
\sum\bar{X} & \sum\bar{Y} & \sum\bar{Z} & \sum(\bar{X}^2+\bar{Y}^2+\bar{Z}^2) & 0 & 0 & 0 \\
-\sum\bar{Z} & 0 & \sum\bar{X} & 0 & \sum(\bar{X}^2+\bar{Z}^2) & \sum\bar{X}\bar{Y} & \sum\bar{Y}\bar{Z} \\
0 & -\sum\bar{Z} & \sum\bar{Y} & 0 & \sum\bar{X}\bar{Y} & \sum(\bar{Y}^2+\bar{Z}^2) & -\sum\bar{X}\bar{Z} \\
\sum\bar{Y} & \sum\bar{X} & 0 & 0 & \sum\bar{Y}\bar{Z} & -\sum\bar{X}\bar{Z} & \sum(\bar{X}^2+\bar{Y}^2)
\end{bmatrix}
\begin{bmatrix}
\Delta X_0 \\ \Delta Y_0 \\ \Delta Z_0 \\ \Delta\lambda \\ \Delta\Phi \\ \Delta\Omega \\ \Delta K
\end{bmatrix}
=
\begin{bmatrix}
\sum l_X \\
\sum l_Y \\
\sum l_Z \\
\sum(\bar{X}l_X+\bar{Y}l_Y+\bar{Z}l_Z) \\
\sum(\bar{X}l_Z-\bar{Z}l_X) \\
\sum(\bar{Y}l_Z-\bar{Z}l_Y) \\
\sum(\bar{X}l_Y-\bar{Y}l_X)
\end{bmatrix}
$$

五、地面点坐标计算



$$\begin{bmatrix} \overline{X}_{tp} \\ \overline{Y}_{tp} \\ \overline{Z}_{tp} \end{bmatrix} = \lambda \boldsymbol{R} \begin{bmatrix} \overline{X}_{p} \\ \overline{Y}_{p} \\ \overline{Z}_{p} \end{bmatrix} + \begin{bmatrix} X_{0} \\ Y_{0} \\ Z_{0} \end{bmatrix}$$

$$\begin{bmatrix} X_{tp} \\ Y_{tp} \\ Z_{tp} \end{bmatrix} = \begin{bmatrix} \overline{X}_{tp} \\ \overline{Y}_{tp} \\ \overline{Z}_{tp} \end{bmatrix} + \begin{bmatrix} X_{tpg} \\ Y_{tpg} \\ Z_{tpg} \end{bmatrix}$$

$$\begin{bmatrix} X_{t} \\ Y_{t} \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} b & a \\ a & -b \end{bmatrix} \begin{bmatrix} X_{tp} \\ Y_{tp} \end{bmatrix}$$

$$Z_{t} = \frac{1}{\lambda} Z_{tp} - \Delta Z_{R}$$

本讲参考资料

教材

作业：

PP.40，第18、20题

张剑清，潘励，王树根 编著，《摄影测量学》，武汉大学出版社

参考书

1、李德仁，周月琴 等编，《摄影测量与遥感概论》，测绘出版社

2、李德仁，郑肇葆 编著，《解析摄影测量学》，测绘出版社