解析法相对定向

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主要内容

一、相对定向元素

二、解析相对定向原理

三、相对定向元素计算

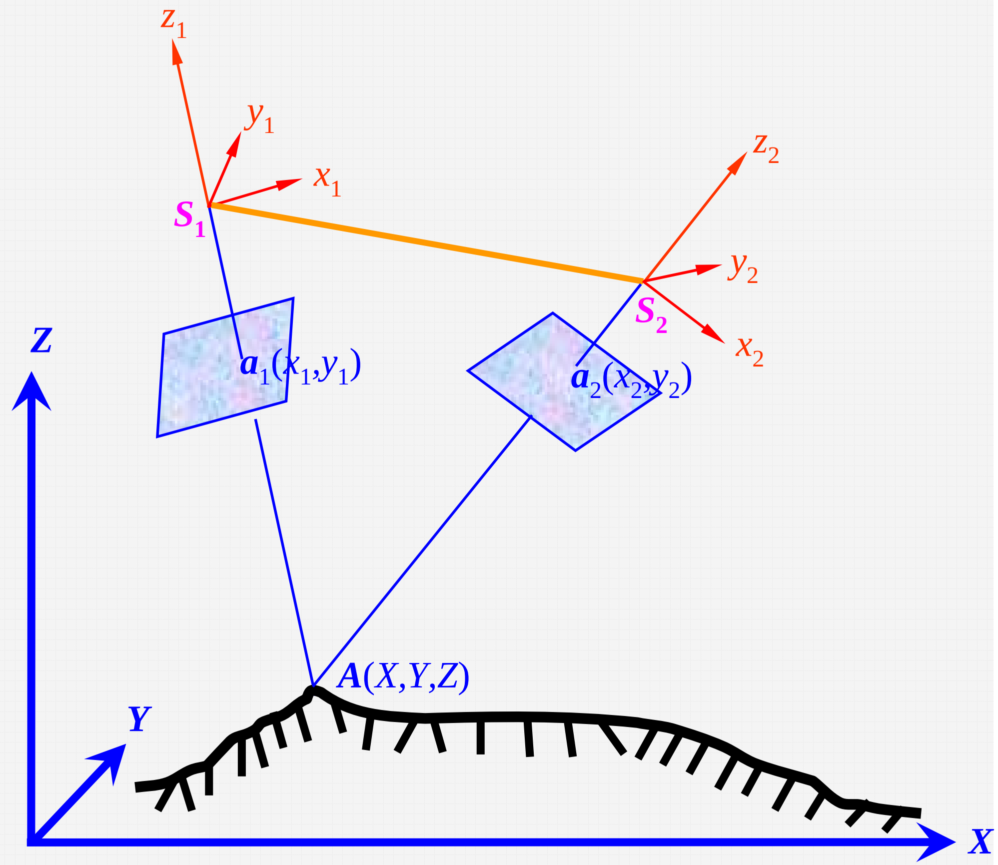
四、模型点坐标计算

一、相对定向元素

像片外方位元素：

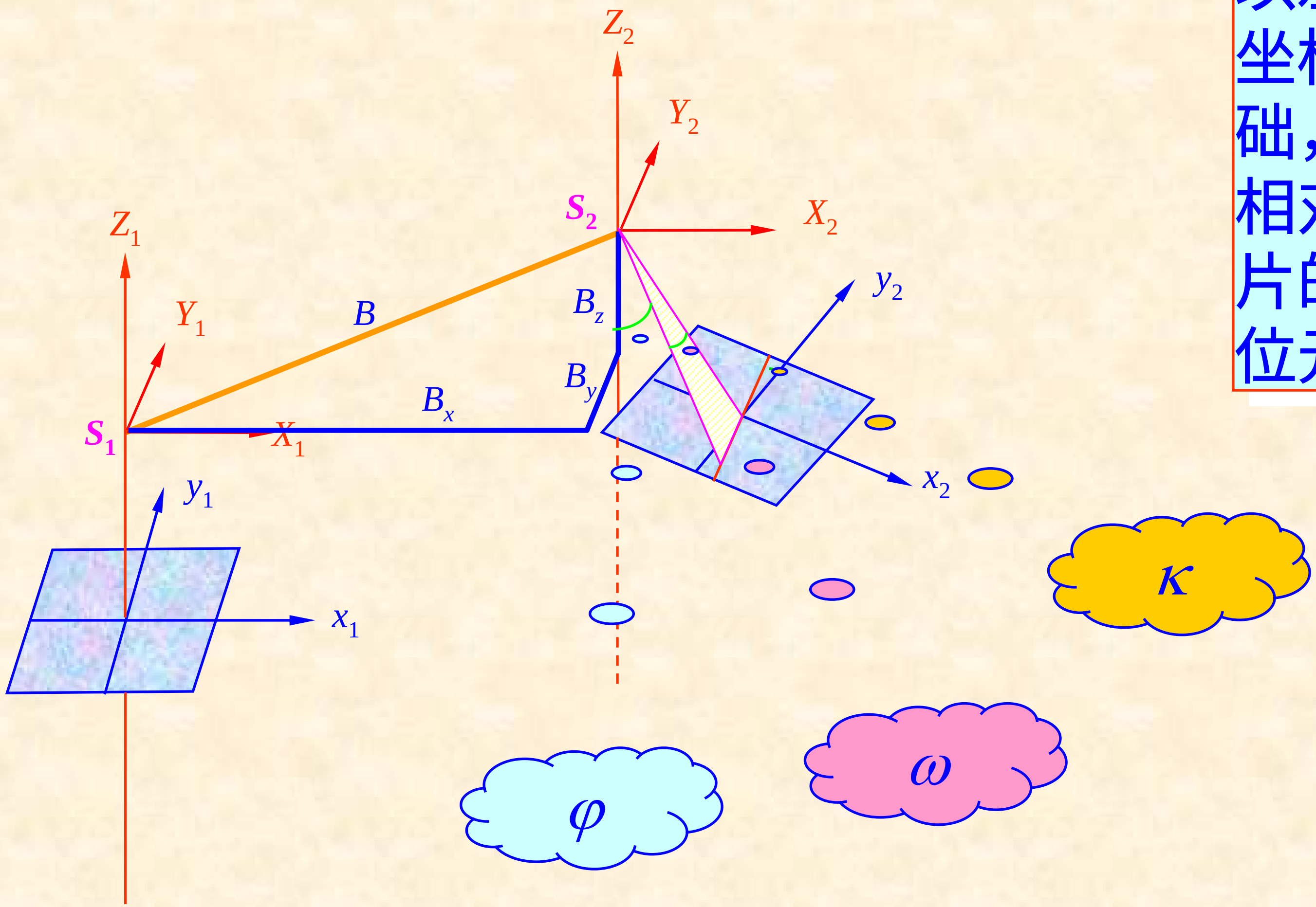
X_{s1}，Y_{s1}，Z_{s1}，φ_1，ω_1，κ_1

X_{s2}，Y_{s2}，Z_{s2}，φ_2，ω_2，κ_2



描述立体像对中两张像片相对位置和姿态关系的参数

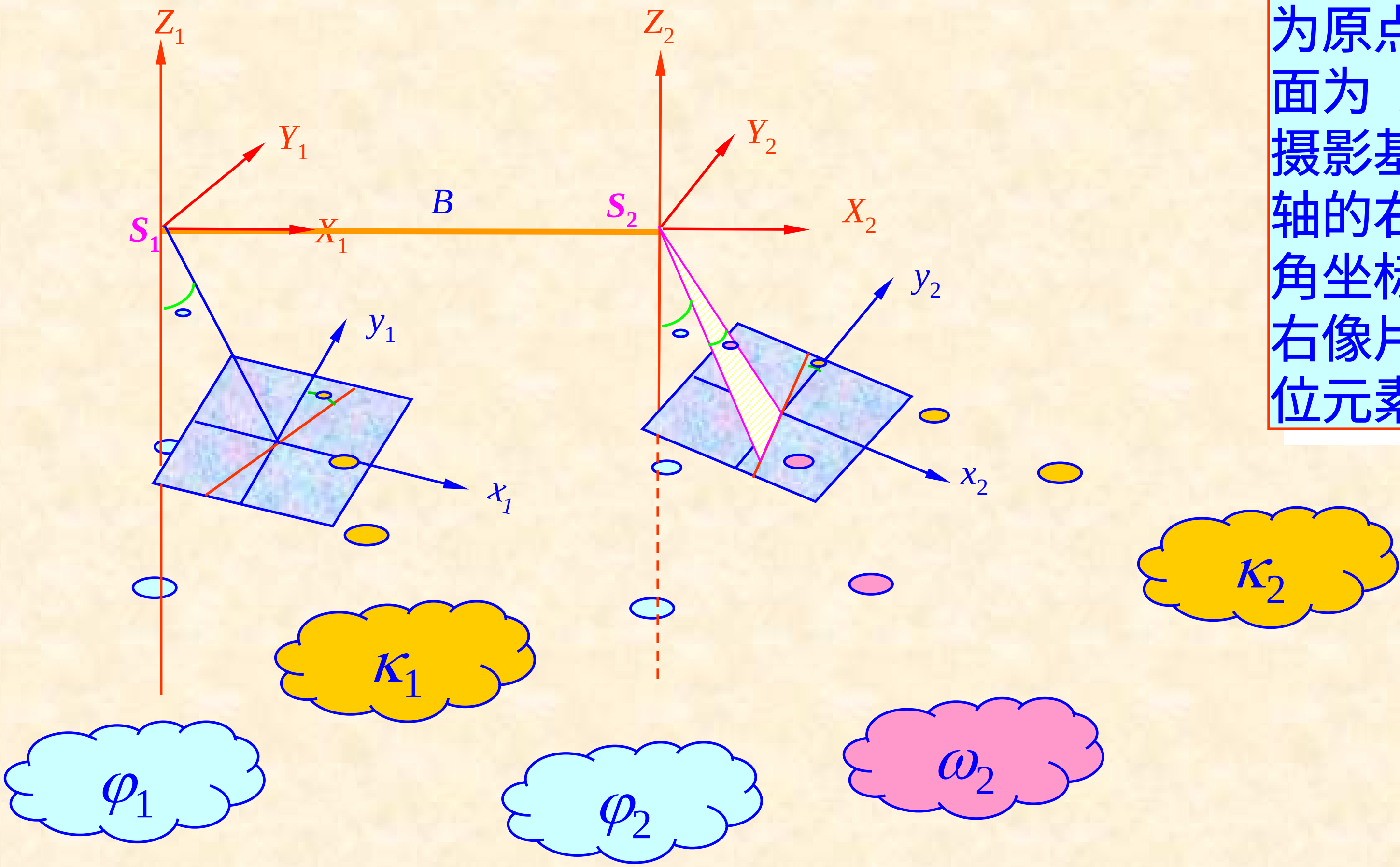
连续法相对定向元素



以左像空间坐标系为基础，右像片相对于左像片的相对方位元素

连续法相对定向元素：B_y，B_z，φ，ω，κ

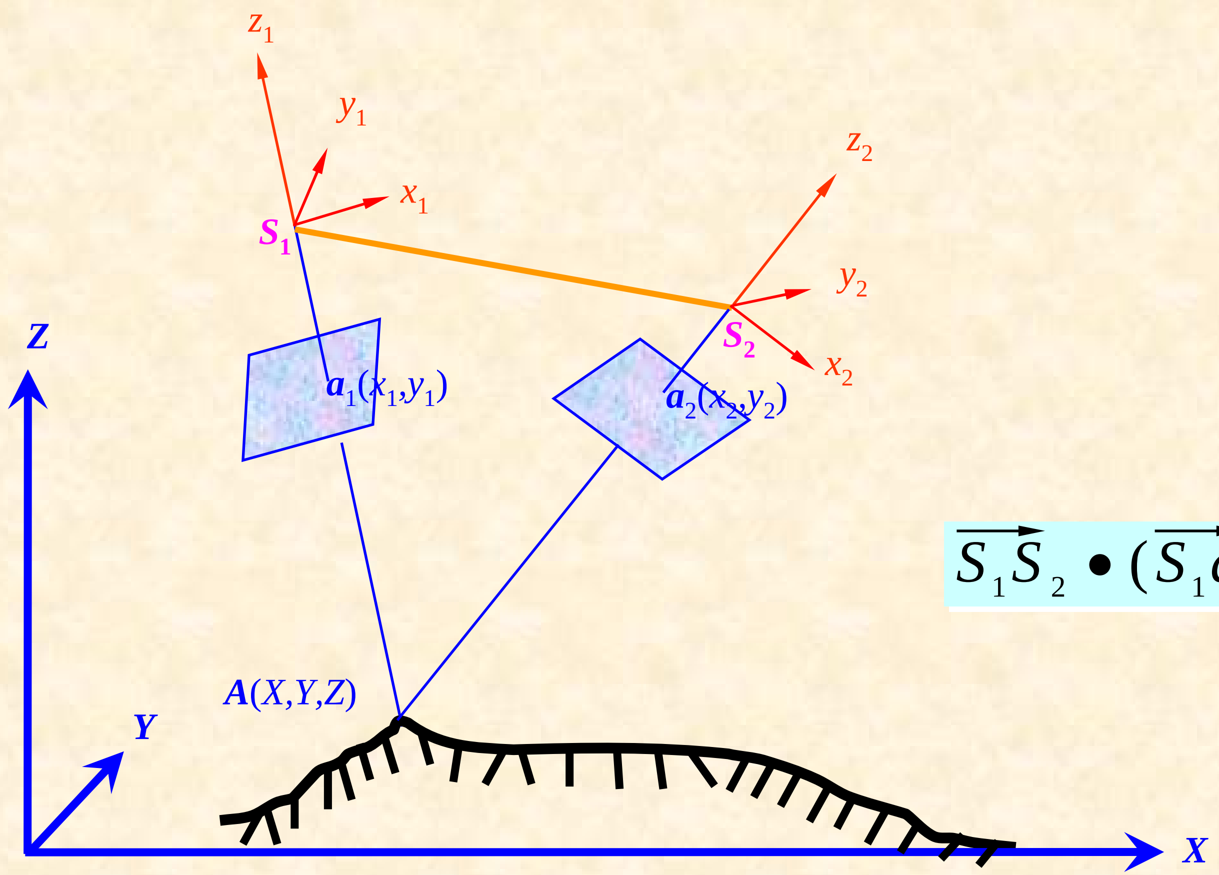
单独法相对定向元素



在以左摄影中心为原点、左主核面为 XZ 平面、摄影基线为 X 轴的右手空间直角坐标系中，左右像片的相对方位元素

单独法相对定向元素：φ_1， κ_1， φ_2， ω_2， κ_2

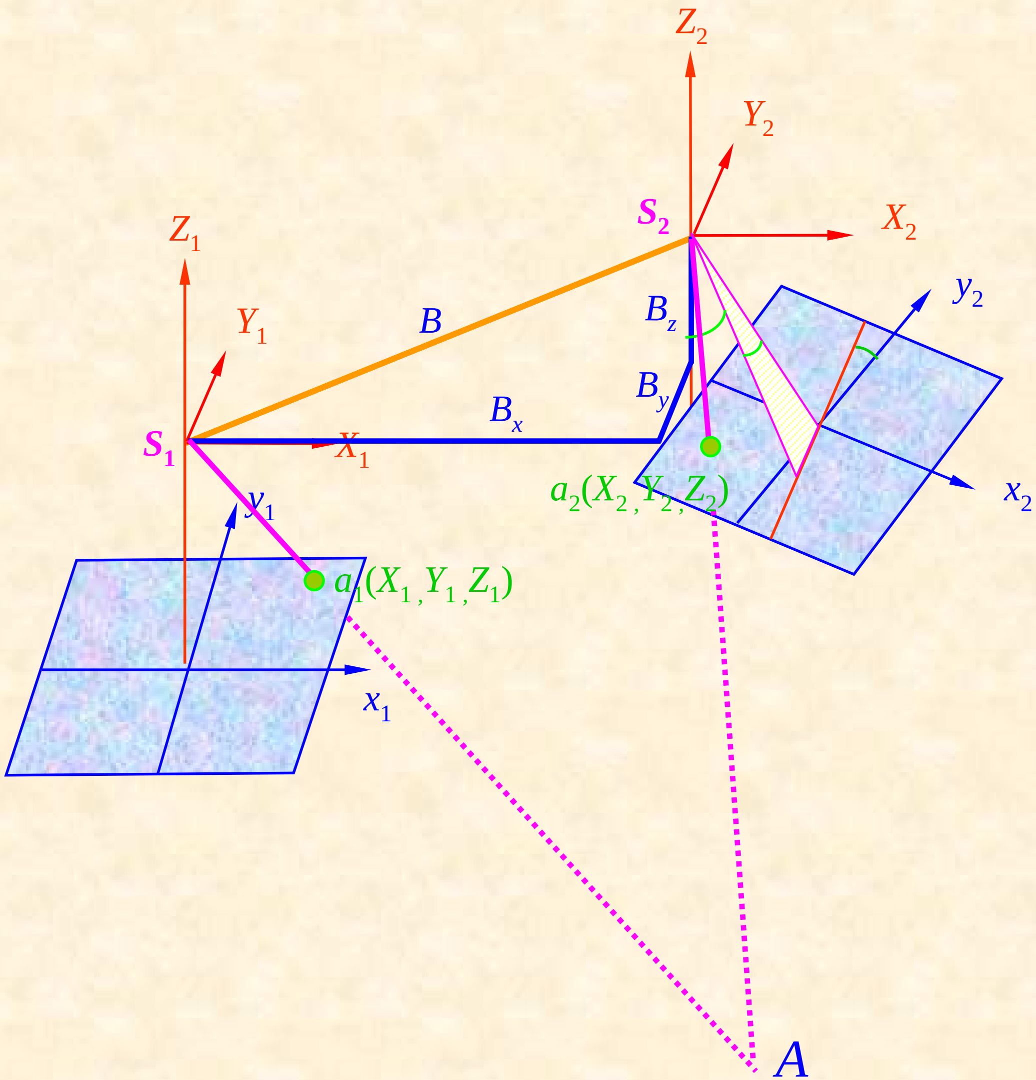
二、解析相对定向原理



同名光线
对对相交
于核面内

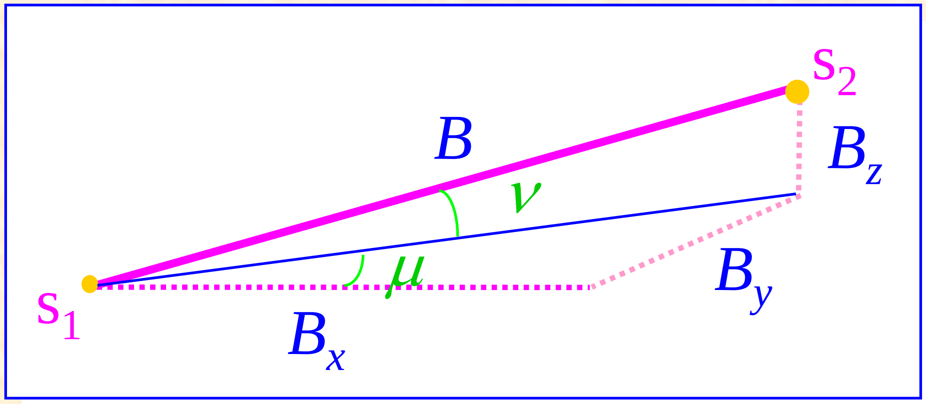
$$\overrightarrow{S_1S_2} \bullet (\overrightarrow{S_1a_1} \times \overrightarrow{S_2a_2}) = 0$$

1、连续法解析相对定向原理



$$\begin{vmatrix} B_x & B_y & B_z \\ X_1 & Y_1 & Z_1 \\ X_2 + B_x & Y_2 + B_y & Z_2 + B_z \end{vmatrix} = \begin{vmatrix} B_x & B_y & B_z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = 0$$

连续法解析相对定向原理



$$B_y = B_x \operatorname{tg}\mu \approx B_x \mu$$

$$B_z = \frac{B_x}{\cos\mu}\operatorname{tg}\nu \approx B_x \nu$$

$$\begin{vmatrix} B_x & B_y & B_z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = 0$$

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ -f \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \boldsymbol{R}\begin{bmatrix} x_2 \\ y_2 \\ -f \end{bmatrix}$$

连续法解析相对定向原理

$$F = B_x \begin{vmatrix} 1 & \mu & \nu \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = 0$$

$$F = F^0 + \frac{\partial F}{\partial \mu}\Delta\mu + \frac{\partial F}{\partial \nu}\Delta\nu + \frac{\partial F}{\partial \varphi}\Delta\varphi + \frac{\partial F}{\partial \omega}\Delta\omega + \frac{\partial F}{\partial \kappa}\Delta\kappa = 0$$

偏导数 1

$$\frac{\partial F}{\partial \mu} = B_x \begin{vmatrix} 0 & 1 & 0 \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix}$$

$$= -B_x \begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}$$

$$= B_x (X_2 Z_1 - X_1 Z_2)$$

$$\frac{\partial F}{\partial \nu} = B_x (X_1 Y_2 - X_2 Y_1)$$

偏导数 2

$$\frac{\partial F}{\partial \varphi} = B_x \begin{vmatrix} 1 & \mu & \nu \\ X_1 & Y_1 & Z_1 \\ \frac{\partial X_2}{\partial \varphi} & \frac{\partial Y_2}{\partial \varphi} & \frac{\partial Z_2}{\partial \varphi} \end{vmatrix}$$

$$\frac{\partial F}{\partial \varphi} = \begin{vmatrix} B_x & B_y & B_z \\ X_1 & Y_1 & Z_1 \\ -Z_2 & 0 & X_2 \end{vmatrix} \approx B_x Y_1 X_2$$

$$\frac{\partial F}{\partial \omega} = \begin{vmatrix} B_x & B_y & B_z \\ X_1 & Y_1 & Z_1 \\ 0 & -Z_2 & Y_2 \end{vmatrix} \approx B_x (Y_1 Y_2 + Z_1 Z_2)$$

$$\frac{\partial F}{\partial \kappa} = \begin{vmatrix} B_x & B_y & B_z \\ X_1 & Y_1 & Z_1 \\ -Y_2 & X_2 & 0 \end{vmatrix} \approx -B_x X_2 Z_1$$

偏倒数 2-1

$$\frac{\partial \boldsymbol{R}}{\partial \varphi}=\frac{\partial(\boldsymbol{R}_{\varphi}\boldsymbol{R}_{\omega}\boldsymbol{R}_{\kappa})}{\partial \varphi}=\frac{\partial \boldsymbol{R}_{\varphi}}{\partial \varphi}\boldsymbol{R}_{\varphi}^{-1}\boldsymbol{R}_{\varphi}\boldsymbol{R}_{\omega}\boldsymbol{R}_{\kappa}=\frac{\partial \boldsymbol{R}_{\varphi}}{\partial \varphi}\boldsymbol{R}_{\varphi}^{-1}\boldsymbol{R}$$

$$\frac{\partial \boldsymbol{R}_{\varphi}}{\partial \varphi}\boldsymbol{R}_{\varphi}^{-1}=\begin{bmatrix}-\sin\varphi & 0 & -\cos\varphi\\ 0 & 0 & 0\\ \cos\varphi & 0 & -\sin\varphi\end{bmatrix}\begin{bmatrix}\cos\varphi & 0 & \sin\varphi\\ 0 & 1 & 0\\ -\sin\varphi & 0 & \cos\varphi\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & -1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}$$

$$\frac{\partial\begin{bmatrix}X_2\\ Y_2\\ Z_2\end{bmatrix}}{\partial \varphi}=\frac{\partial \boldsymbol{R}_{\varphi}}{\partial \varphi}\boldsymbol{R}_{\varphi}^{-1}\boldsymbol{R}\begin{bmatrix}x_2\\ y_2\\ -f\end{bmatrix}=\begin{bmatrix}0 & 0 & -1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}X_2\\ Y_2\\ Z_2\end{bmatrix}=\begin{bmatrix}-Z_2\\ 0\\ X_2\end{bmatrix}$$

偏倒数 2-1

$$\frac{\partial \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}}{\partial \omega} = \begin{bmatrix} 0 & -\sin\varphi & 0 \\ \sin\varphi & 0 & -\cos\varphi \\ 0 & \cos\varphi & 0 \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \approx \begin{bmatrix} 0 \\ -Z_2 \\ Y_2 \end{bmatrix}$$

$$\frac{\partial \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}}{\partial \kappa} = \begin{bmatrix} 0 & -\cos\varphi\cos\omega & -\sin\omega \\ \cos\varphi\cos\omega & 0 & \sin\varphi\cos\omega \\ \sin\omega & -\sin\varphi\cos\omega & 0 \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \approx \begin{bmatrix} -Y_2 \\ X_2 \\ 0 \end{bmatrix}$$

线性化方程

$$(X_2Z_1 - X_1Z_2)B_x\Delta\mu + (X_1Y_2 - X_2Y_1)B_x\Delta\nu + X_2Y_1B_x\Delta\varphi + (Y_1Y_2 + Z_1Z_2)B_x\Delta\omega - X_2Z_1B_x\Delta\kappa + F_0 = 0$$

等式两边同时除以 $Z_1X_2 - X_1Z_2$

$$B_x\Delta\mu + \frac{X_1Y_2 - X_2Y_1}{Z_1X_2 - X_1Z_2}B_x\Delta\nu + \frac{X_2Y_1}{Z_1X_2 - X_1Z_2}B_x\Delta\varphi + \frac{Y_1Y_2 + Z_1Z_2}{Z_1X_2 - X_1Z_2}B_x\Delta\omega - \frac{X_2Z_1}{Z_1X_2 - X_1Z_2}B_x\Delta\kappa + \frac{F_0}{Z_1X_2 - X_1Z_2} = 0$$

系数约简

$$\because \quad N_2 = \frac{B_x Z_1 - B_z X_1}{X_1 Z_2 - Z_1 X_2}$$

$$\therefore \quad X_1 Z_2 - Z_1 X_2 = \frac{B_x}{N_2}\left(Z_1 - \frac{B_z}{B_x} X_1\right) \approx \frac{B_x}{N_2} Z_1$$

又 $$\frac{Y_1}{Z_1} = \frac{N_1 Y_1}{N_1 Z_1} = \frac{N_2 Y_2 + B_y}{N_2 Z_2 + B_z} \approx \frac{N_2 Y_2}{N_2 Z_2} = \frac{Y_2}{Z_2}$$

$$\therefore \quad \frac{Y_1}{Y_2} = \frac{Z_1}{Z_2}$$

$$\frac{X_1 Y_2 - X_2 Y_1}{Z_1 X_2 - X_1 Z_2} = -\frac{Y_2}{Z_2}$$

$$\frac{B_x Y_1}{Z_1 X_2 - X_1 Z_2} = -\frac{Y_2}{Z_2} N_2$$

$$\frac{B_x (Y_1 Y_2 + Z_1 Z_2)}{Z_1 X_2 - X_1 Z_2} = -\left(Z_2 + \frac{Y_2^2}{Z_2}\right) N_2$$

$$\frac{B_x X_2 Z_1}{Z_1 X_2 - X_1 Z_2} = -N_2 X_2$$

常数项约简

$$\frac{F_0}{Z_1X_2 - X_1Z_2} = \frac{\begin{vmatrix} B_x & B_y & B_z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix}}{Z_1X_2 - X_1Z_2}$$

$$= \frac{\begin{vmatrix} B_x & B_z \\ X_2 & Z_2 \end{vmatrix}}{Z_1X_2 - X_1Z_2} Y_1 - \frac{\begin{vmatrix} B_x & B_z \\ X_1 & Z_1 \end{vmatrix}}{Z_1X_2 - X_1Z_2} Y_2 - \frac{\begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}}{Z_1X_2 - X_1Z_2} B_y$$

$$= -N_1Y_1 + N_2Y_2 + B_y$$

$$= -Q$$

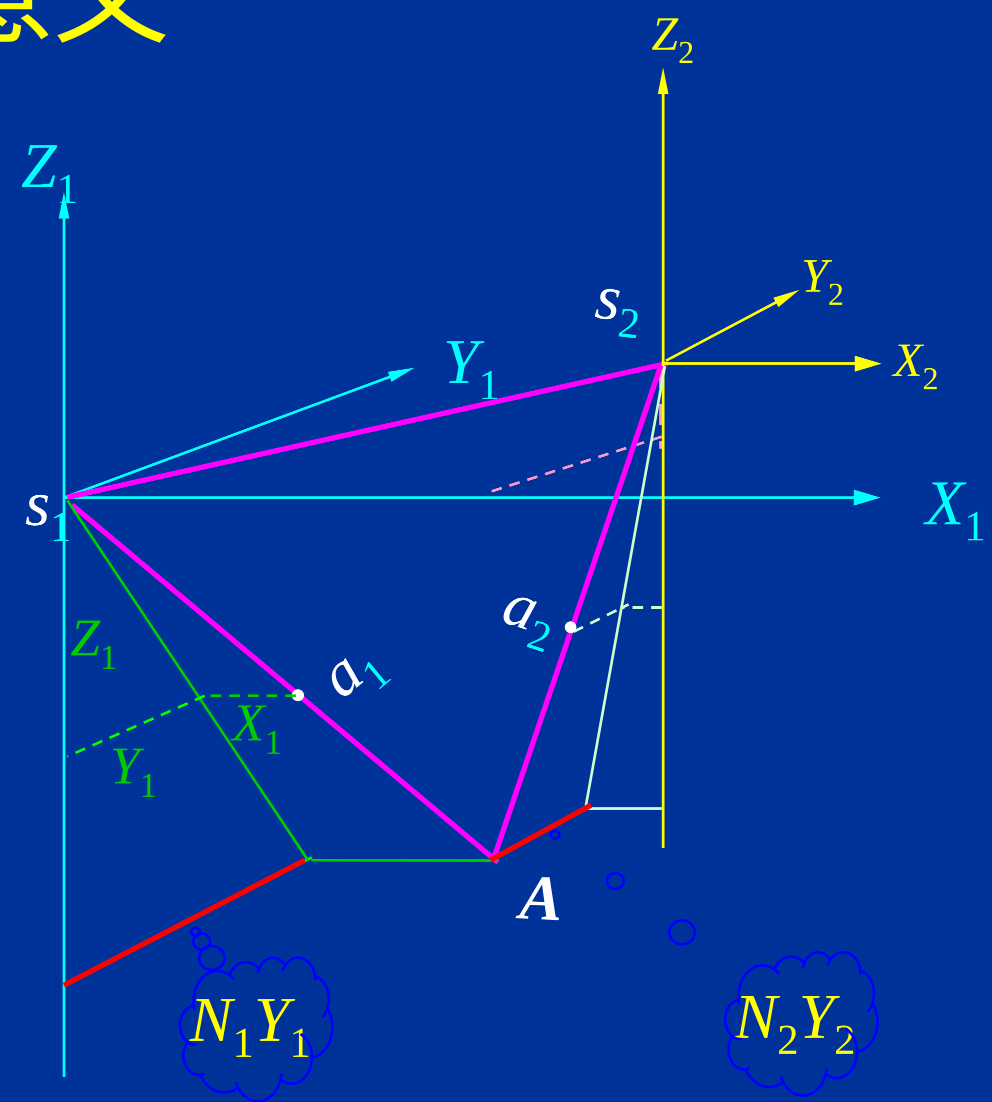
连续法相对定向中常数项的几何意义

$$Q = N_1Y_1 - (N_2Y_2 + B_y)$$

Q为定向点上模型上下视差

当一个立体像对完成相对定向， **$Q=0$**

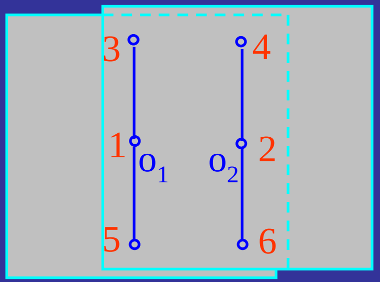
当一个立体像对未完成相对定向，即同名光线不相交， **$Q\neq0$**



误差方程及法方程的建立

$$v_Q = B_x \Delta\mu - \frac{Y_2}{Z_2} B_x \Delta\nu - \frac{X_2 Y_2}{Z_2} N_2 \Delta\varphi - \left(Z_2 + \frac{Y_2^2}{Z_2}\right) N_2 \Delta\omega + X_2 N_2 \Delta\kappa - Q$$

量测 **5** 个以上的同名点可以按最小二乘平差法求相对定向元素



$$\boldsymbol{V} = \boldsymbol{Ax} - \boldsymbol{l}, \qquad \boldsymbol{P}$$

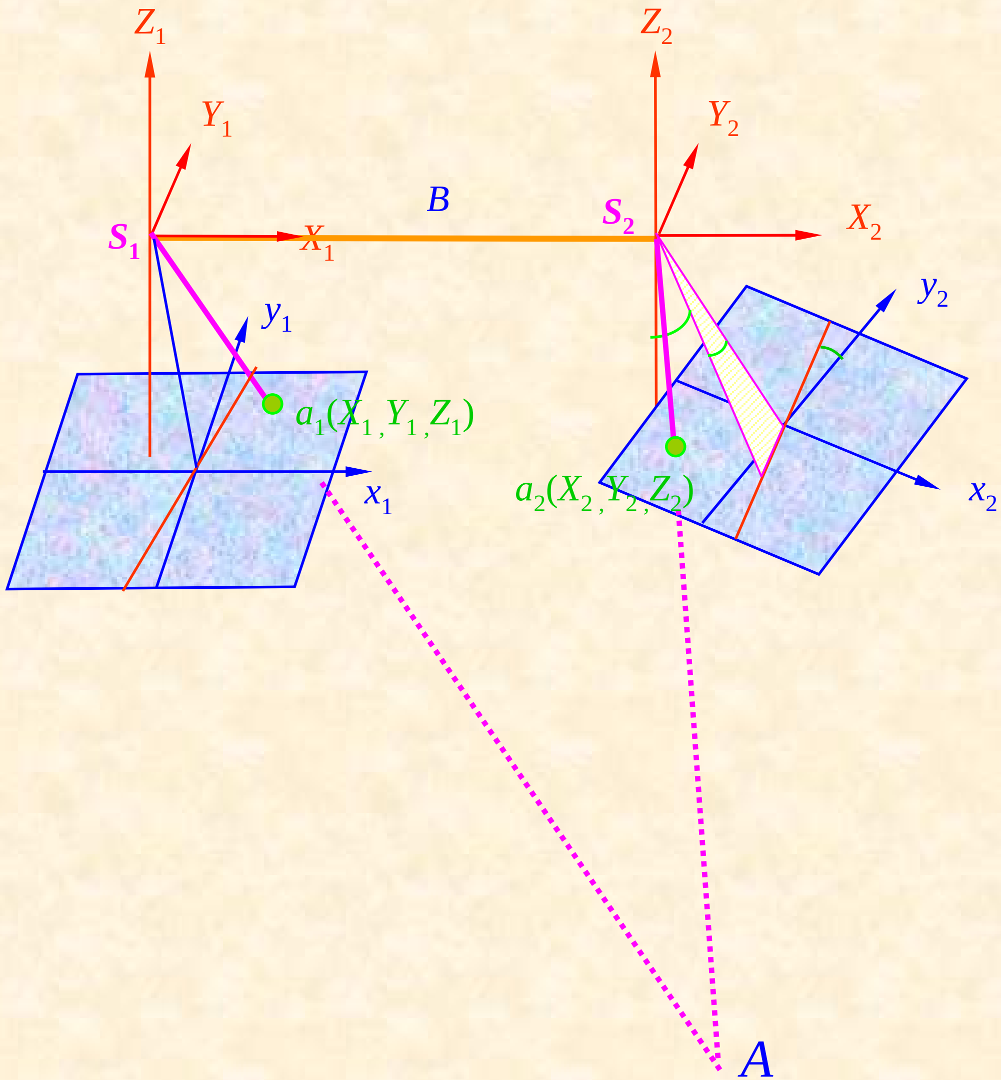
$$\boldsymbol{x} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA})^{-1}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Pl})$$

$$\sigma_0 = \sqrt{\frac{\boldsymbol{V}^{\mathrm{T}}\boldsymbol{PV}}{n-5}}$$

$$\boldsymbol{Q}_{xx} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA})^{-1}$$

$$m_i = \sigma_0 \sqrt{\boldsymbol{Q}_{xx}^{ii}}$$

2、单独法解析相对定向原理



$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \boldsymbol{R}_1 \begin{bmatrix} x_1 \\ y_1 \\ -f \end{bmatrix}$$

$$\begin{vmatrix} B & 0 & 0 \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = 0$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \boldsymbol{R}_2 \begin{bmatrix} x_2 \\ y_2 \\ -f \end{bmatrix}$$

单独法解析相对定向原理

$$F = B\begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix} = 0$$

$$F = F^0 + \frac{\partial F}{\partial \varphi_1}\Delta\varphi_1 + \frac{\partial F}{\partial \kappa_1}\Delta\kappa_1 + \frac{\partial F}{\partial \varphi_2}\Delta\varphi_2 + \frac{\partial F}{\partial \omega_2}\Delta\omega_2 + \frac{\partial F}{\partial \kappa_2}\Delta\kappa_2 = 0$$

偏倒数 3-1

$$\frac{\partial \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}}{\partial \varphi_1} = \begin{bmatrix} -Z_1 \\ 0 \\ X_1 \end{bmatrix}$$

$$\frac{\partial \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}}{\partial \kappa_1} = \begin{bmatrix} -Y_1 \\ X_1 \\ 0 \end{bmatrix}$$

$$\frac{\partial \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}}{\partial \varphi_2} = \begin{bmatrix} -Z_2 \\ 0 \\ X_2 \end{bmatrix}$$

$$\frac{\partial \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}}{\partial \omega_2} = \begin{bmatrix} 0 \\ -Z_2 \\ Y_2 \end{bmatrix}$$

$$\frac{\partial \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}}{\partial \kappa_2} = \begin{bmatrix} -Y_2 \\ X_2 \\ 0 \end{bmatrix}$$

偏导数 3-2

$$\frac{\partial F}{\partial \varphi_1} = \begin{vmatrix} B & 0 & 0 \\ -Z_1 & 0 & X_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = -BX_1Y_2$$

$$\frac{\partial F}{\partial \kappa_1} = \begin{vmatrix} B & 0 & 0 \\ -Y_1 & X_1 & 0 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = BX_1Z_2$$

$$\frac{\partial F}{\partial \varphi_2} = \begin{vmatrix} B & 0 & 0 \\ X_1 & Y_1 & Z_1 \\ -Z_2 & 0 & X_2 \end{vmatrix} = BY_1X_2$$

$$\frac{\partial F}{\partial \omega_2} = \begin{vmatrix} B & 0 & 0 \\ X_1 & Y_1 & Z_1 \\ 0 & -Z_2 & Y_2 \end{vmatrix} = B(Y_1Y_2 + Z_1Z_2)$$

$$\frac{\partial F}{\partial \kappa_2} = \begin{vmatrix} B & 0 & 0 \\ X_1 & Y_1 & Z_1 \\ -Y_2 & X_2 & 0 \end{vmatrix} = -BX_2Z_1$$

线性化方程

$$-X_1Y_2B\Delta\varphi_1 + X_1Z_2B\Delta\kappa_1 + X_2Y_1B\Delta\varphi_2 + (Y_1Y_2 + Z_1Z_2)B\Delta\omega_2 - X_2Z_1B\Delta\kappa_2 + F_0 = 0$$

等式两边同时乘以 $\dfrac{f}{BZ_1Z_2}$ 并视 $Z_1 = Z_2 = -f$

$$\frac{X_1Y_2}{Z_1}\Delta\varphi_1 - X_1\Delta\kappa_1 - \frac{X_2Y_1}{Z_1}\Delta\varphi_2 - (Z_1 + \frac{Y_1Y_2}{Z_1})\Delta\omega_2 + X_2\Delta\kappa_2 + \frac{fF_0}{BZ_1Z_2} = 0$$

常数项约简

$$
\begin{aligned}
-\frac{fF_0}{BZ_1Z_2} &= -f\,\frac{B\begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}}{BZ_1Z_2} \\
&= -f\,\frac{Y_1Z_2 - Y_2Z_1}{Z_1Z_2} \\
&= -f\,\frac{Y_1}{Z_1} + f\,\frac{Y_2}{Z_2} \\
&= y_{t_1} - y_{t_2} \\
&= q
\end{aligned}
$$

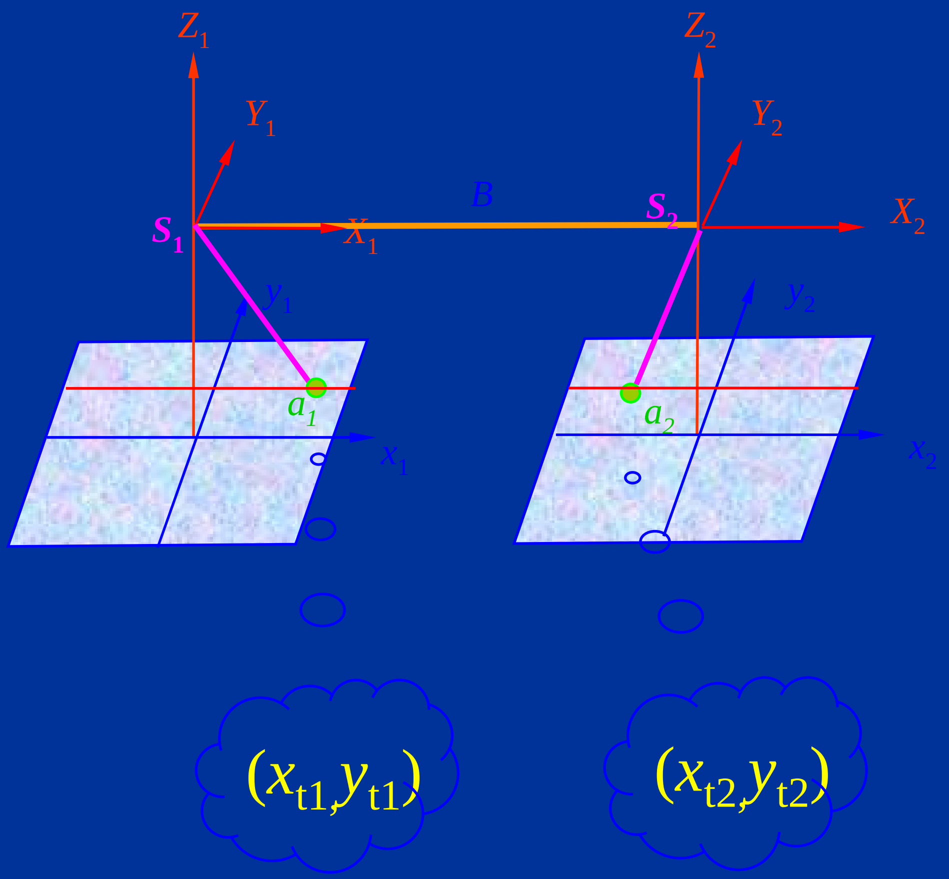
单独法相对定向中常数项的几何意义

$$q = y_{t_1} - y_{t_2}$$

q为相当于像空间辅助坐标系中一对理想像对上同名像点的上下视差

当一个立体像对完成相对定向，$q=0$

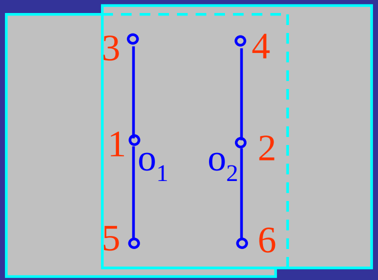
当一个立体像对未完成相对定向，即同名光线不相交， $q \neq 0$



误差方程及法方程的建立

$$v_q = \frac{X_1 Y_2}{Z_1}\Delta\varphi_1 - X_1\Delta\kappa_1 - \frac{X_2 Y_1}{Z_1}\Delta\varphi_2 - (Z_1 + \frac{Y_1 Y_2}{Z_1})\Delta\omega_2 + X_2\Delta\kappa_2 - q$$

量测 **5** 个以上的同名点可以按最小二乘平差法求相对定向元素



$$\boldsymbol{V} = \boldsymbol{A}\boldsymbol{x} - \boldsymbol{l}, \qquad \boldsymbol{P}$$

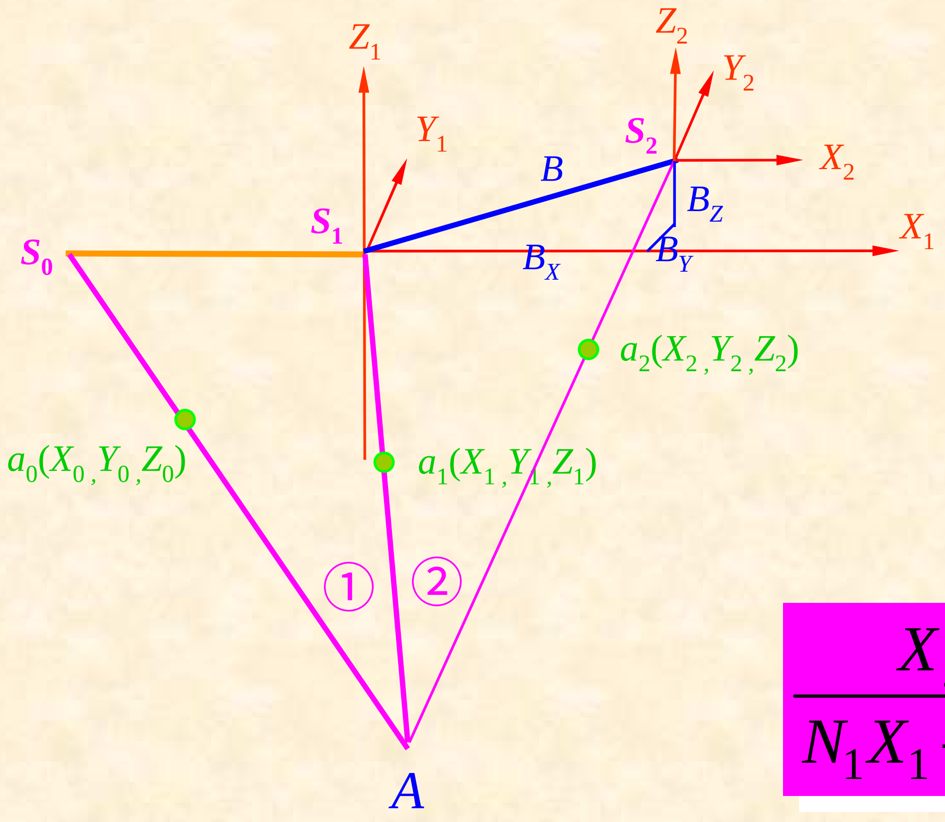
$$\boldsymbol{x} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{l})$$

$$\sigma_0 = \sqrt{\frac{\boldsymbol{V}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{V}}{n-5}}$$

$$\boldsymbol{Q}_{xx} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A})^{-1}$$

$$m_i = \sigma_0\sqrt{\boldsymbol{Q}_{xx}^{ii}}$$

3、带模型连接条件的解析相对定向原理



$$\frac{X_2}{N_1 X_1 - B_X} = \frac{Y_2}{N_1 Y_1 - B_Y} = \frac{Z_2}{N_1 Z_1 - B_Z}$$

带模型连接条件的连续法解析相对定向原理

$$F_Y = Z_2(N_1Y_1 - B_Y) - Y_2(N_1Z_1 - B_Z) = 0$$
$$F_X = Z_2(N_1X_1 - B_X) - X_2(N_1Z_1 - B_Z) = 0$$

$$B_x = B_x\lambda$$
$$B_y \approx B_x\mu$$
$$B_z \approx B_x\nu$$

线性化

$$F_Y = F_Y^0 + \frac{\partial F_Y}{\partial \varphi}\Delta\varphi + \frac{\partial F_Y}{\partial \omega}\Delta\omega + \frac{\partial F_Y}{\partial \kappa}\Delta\kappa + \frac{\partial F_Y}{\partial \lambda}\Delta\lambda + \frac{\partial F_Y}{\partial \mu}\Delta\mu + \frac{\partial F_Y}{\partial \nu}\Delta\nu = 0$$
$$F_X = F_X^0 + \frac{\partial F_X}{\partial \varphi}\Delta\varphi + \frac{\partial F_X}{\partial \omega}\Delta\omega + \frac{\partial F_X}{\partial \kappa}\Delta\kappa + \frac{\partial F_X}{\partial \lambda}\Delta\lambda + \frac{\partial F_X}{\partial \mu}\Delta\mu + \frac{\partial F_X}{\partial \nu}\Delta\nu = 0$$

偏导数

$$\frac{\partial F_Y}{\partial \varphi} = N_2 X_2 Y_2$$

$$\frac{\partial F_Y}{\partial \omega} = N_2 (Y_2^2 + Z_2^2)$$

$$\frac{\partial F_Y}{\partial \kappa} = -N_2 X_2 Z_2$$

$$\frac{\partial F_Y}{\partial \lambda} = 0$$

$$\frac{\partial F_Y}{\partial \mu} = -B_X Z_2$$

$$\frac{\partial F_Y}{\partial \nu} = B_X Y_2$$

$$\frac{\partial F_X}{\partial \varphi} = N_2 (X_2^2 + Z_2^2)$$

$$\frac{\partial F_X}{\partial \omega} = N_2 X_2 Y_2$$

$$\frac{\partial F_X}{\partial \kappa} = N_2 Y_2 Z_2$$

$$\frac{\partial F_X}{\partial \lambda} = -B_X Z_2$$

$$\frac{\partial F_X}{\partial \mu} = 0$$

$$\frac{\partial F_X}{\partial \nu} = B_X X_2$$

线性化方程

$$X_2Y_2N_2\Delta\varphi+(Z_2^2+Y_2^2)N_2\Delta\omega-X_2Z_2N_2\Delta\kappa \quad -B_XZ_2\Delta\mu+B_XY_2\Delta\nu+F_Y^0=0$$

$$(Z_2^2+X_2^2)N_2\Delta\varphi+X_2Y_2N_2\Delta\omega+Y_2Z_2N_2\Delta\kappa-B_XZ_2\Delta\lambda \quad +B_XX_2\Delta\nu+F_X^0=0$$

等式两边同时除以 Z_2 则

$$\frac{F_Y^0}{Z_2}=N_1Y_1-N_2Y_2-B_Y$$

$$\frac{F_X^0}{Z_2}=N_1X_1-N_2X_2-B_X$$

$$v_Q=-\frac{X_2Y_2}{Z_2}N_2\Delta\varphi-(Z_2+\frac{Y_2^2}{Z_2})N_2\Delta\omega+X_2N_2\Delta\kappa \quad +B_X\Delta\mu-\frac{Y_2}{Z_2}B_X\Delta\nu-Q$$

$$v_P=-(Z_2+\frac{X_2^2}{Z_2})N_2\Delta\varphi-\frac{X_2Y_2}{Z_2}N_2\Delta\omega-Y_2N_2\Delta\kappa+B_X\Delta\lambda \quad -\frac{X_2}{Z_2}B_X\Delta\nu-P$$

误差方程及法方程的建立

$$v_Q = -\frac{X_2Y_2}{Z_2}N_2\Delta\varphi - (Z_2+\frac{Y_2^2}{Z_2})N_2\Delta\omega + X_2N_2\Delta\kappa + B_X\Delta\mu - \frac{Y_2}{Z_2}B_X\Delta\nu - Q$$

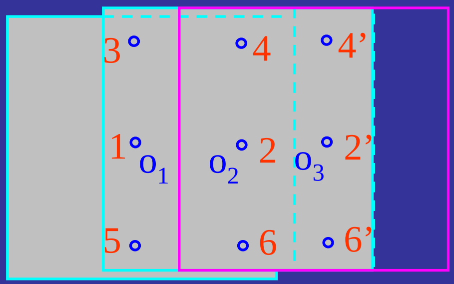
$$v_P = -(Z_2+\frac{X_2^2}{Z_2})N_2\Delta\varphi - \frac{X_2Y_2}{Z_2}N_2\Delta\omega - Y_2N_2\Delta\kappa + B_X\Delta\lambda - \frac{X_2}{Z_2}B_X\Delta\nu - P$$

$$N_2^{(2)} = \frac{N_2^{(1)}Y_2^{(1)} - B_Z}{Z_2^{(2)}}$$

$$Q = N_2^{(1)}Y_2^{(1)} - N_2^{(2)}Y_2^{(2)} - B_Y$$

$$P = N_2^{(1)}X_2^{(1)} - N_2^{(2)}X_2^{(2)} - B_X$$

量测 3 个以上的模型连接点可以按最小二乘平差法求相对定向元素



$$\boldsymbol{V} = \boldsymbol{Ax} - \boldsymbol{l}, \qquad \boldsymbol{P}$$

$$\boldsymbol{x} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA})^{-1}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Pl})$$

$$\sigma_0 = \sqrt{\frac{\boldsymbol{V}^{\mathrm{T}}\boldsymbol{PV}}{2n-6}}$$

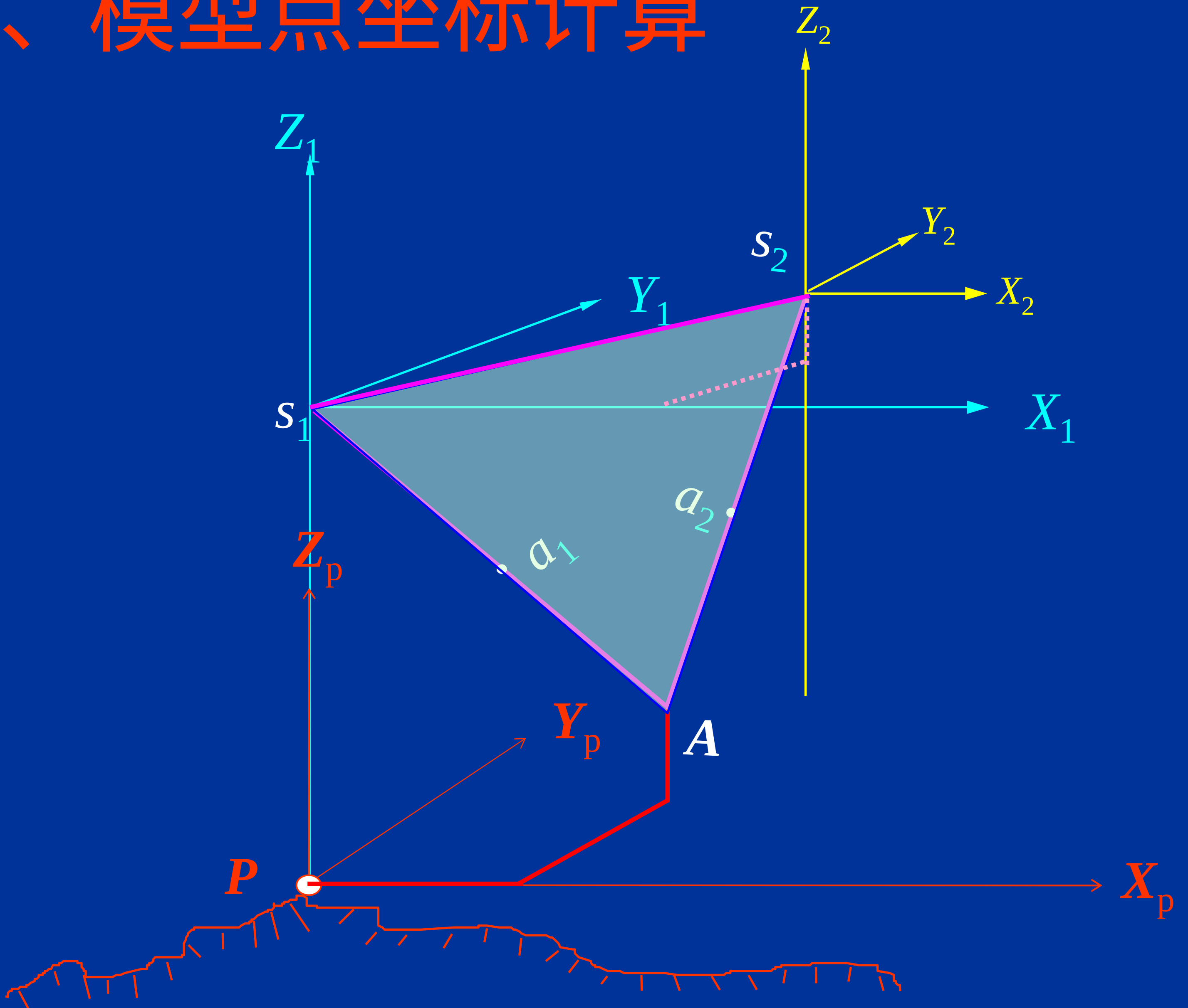
$$\boldsymbol{Q}_{xx} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PA})^{-1}$$

$$m_i = \sigma_0\sqrt{\boldsymbol{Q}_{xx}^{ii}}$$

三、相对定向元素计算

- 获取已知数据 x_0, y_0, f， x_1, y_1, x_2, y_2
- 假定摄影基线 $B_x = x_1 - x_2$
- 设定相对定向元素的初值 $\mu = \nu = \varphi = \omega = \kappa = 0$
- 由相对定向元素计算像空间辅助坐标 $X_1, Y_1, Z_1, X_2, Y_2, Z_2$
- 逐点计算误差方程式的系数和常数项并法化
- 解法方程，求相对定向元素改正数
- 求相对定向元素的新值
- 判断迭代是否收敛（限差0.01′=3×10^{-5}）

四、模型点坐标计算



模型点坐标

$$X_A = X_{s1} + mN_1X_1 = mN_1X_1$$

$$Y_A = \frac{1}{2}m(Y_{s1} + N_1Y_1 + Y_{s2} + N_2Y_2) = \frac{1}{2}m(N_1Y_1 + N_2Y_2 + B_y)$$

$$Z_A = Z_{s1} + mN_1Z_1 = mf + mN_1Z_1$$

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \boldsymbol{R}_1 \begin{bmatrix} x_1 \\ y_1 \\ -f \end{bmatrix}, \quad \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \boldsymbol{R}_2 \begin{bmatrix} x_2 \\ y_2 \\ -f \end{bmatrix}$$

$$N_1 = \frac{B_X Z_2 - B_Z X_2}{X_1 Z_2 - X_2 Z_1}$$

$$N_2 = \frac{B_X Z_1 - B_Z X_1}{X_1 Z_2 - X_2 Z_1}$$

本讲参考资料

教材

作业：

PP.40，第13、14题

张剑清，潘励，王树根 编著，《摄影测量学》，武汉大学出版社

参考书

1、李德仁，周月琴 等编，《摄影测量与遥感概论》，测绘出版社

2、李德仁，郑肇葆 编著，《解析摄影测量学》，测绘出版社