《摄影测量学》（上）第三章

单片空间后方交会

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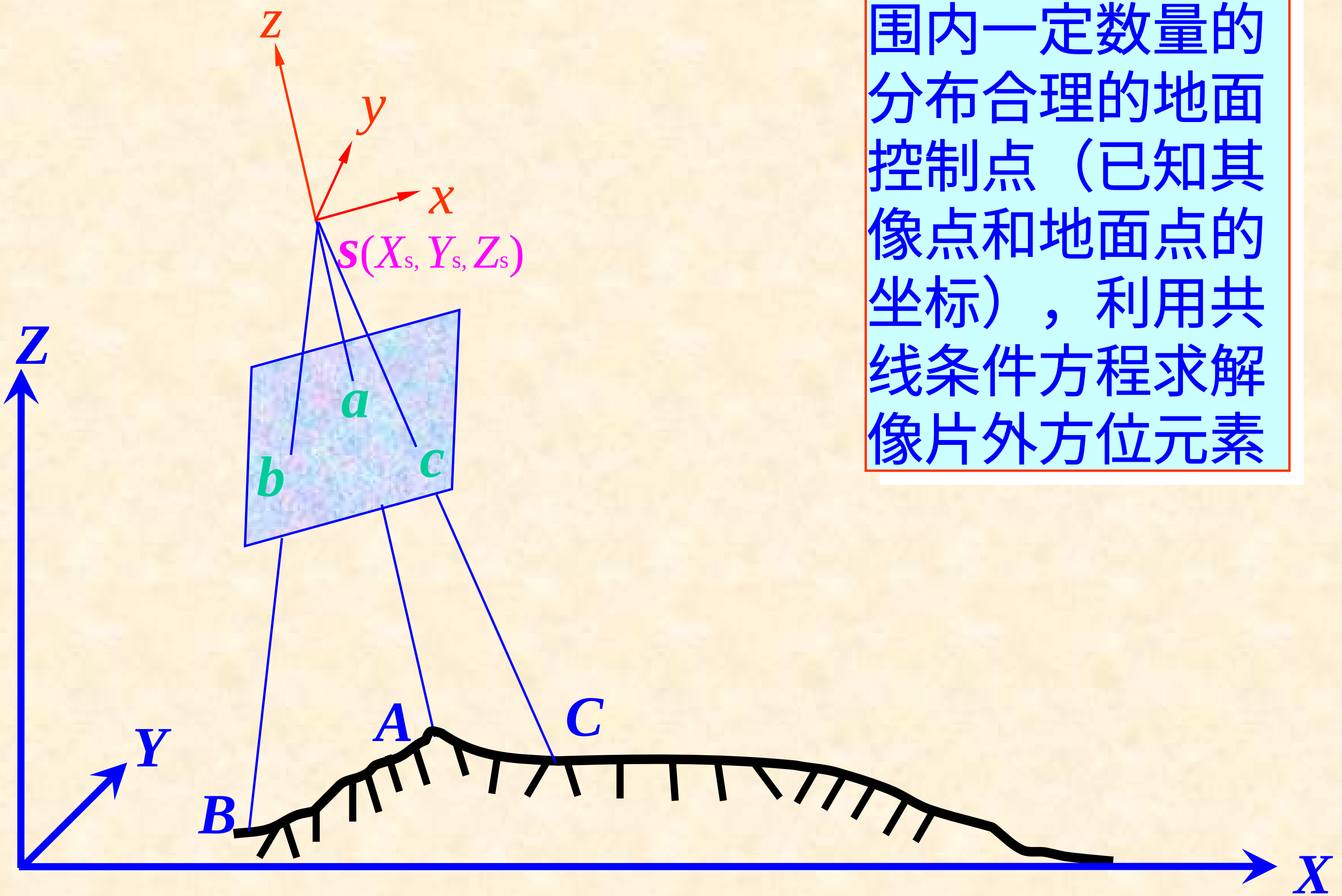
主要内容

一、定义

二、误差方程和法方程

三、计算过程

一、定义



根据影像覆盖范围内一定数量的分布合理的地面控制点（已知其像点和地面点的坐标），利用共线条件方程求解像片外方位元素

二、误差方程

- 已知值 $x_0, y_0, f, m, \mathbf{X}, \mathbf{Y}, \mathbf{Z}$
- 观测值 x, y
- 未知数 $X_S, Y_S, Z_S, \varphi, \omega, \kappa$
- 泰勒级数展开

$$v_x = \frac{\partial x}{\partial \varphi}\Delta\varphi + \frac{\partial x}{\partial \omega}\Delta\omega + \frac{\partial x}{\partial \kappa}\Delta\kappa + \frac{\partial x}{\partial X_s}\Delta X_s + \frac{\partial x}{\partial Y_s}\Delta Y_s + \frac{\partial x}{\partial Z_s}\Delta Z_s + x^0 - x$$

$$v_y = \frac{\partial y}{\partial \varphi}\Delta\varphi + \frac{\partial y}{\partial \omega}\Delta\omega + \frac{\partial y}{\partial \kappa}\Delta\kappa + \frac{\partial y}{\partial X_s}\Delta X_s + \frac{\partial y}{\partial Y_s}\Delta Y_s + \frac{\partial y}{\partial Z_s}\Delta Z_s + y^0 - y$$

共线条件方程

$$x - x_0 = -f\frac{a_1(X - X_s) + b_1(Y - Y_s) + c_1(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} = -f\frac{\overline{X}}{\overline{Z}}$$

$$y - y_0 = -f\frac{a_2(X - X_s) + b_2(Y - Y_s) + c_2(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} = -f\frac{\overline{Y}}{\overline{Z}}$$

$$\begin{bmatrix} \overline{X} \\ \overline{Y} \\ \overline{Z} \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} X - X_s \\ Y - Y_s \\ Z - Z_s \end{bmatrix} = \boldsymbol{R}^{-1} \begin{bmatrix} X - X_s \\ Y - Y_s \\ Z - Z_s \end{bmatrix}$$

偏导数 1

$$\frac{\partial x}{\partial X_s} = -\frac{f}{\bar{Z}^2}\left(\frac{\partial \bar{X}}{\partial X_s}\bar{Z} - \frac{\partial \bar{Z}}{\partial X_s}\bar{X}\right)$$

$$= -\frac{f}{\bar{Z}^2}(-a_1\bar{Z} + a_3\bar{X})$$

$$= \frac{1}{\bar{Z}}\left(a_1 f - f\frac{\bar{X}}{\bar{Z}}a_3\right)$$

$$= \frac{1}{\bar{Z}}\left[a_1 f + a_3(x - x_0)\right]$$

偏导数 2

$$\frac{\partial x}{\partial \varphi} = -\frac{f}{\bar{Z}^2}\left(\frac{\partial \bar{X}}{\partial \varphi}\bar{Z} - \frac{\partial \bar{Z}}{\partial \varphi}\bar{X}\right)$$

$$\frac{\partial x}{\partial \varphi} = -\frac{f}{\bar{Z}}\left(\frac{\partial \bar{X}}{\partial \varphi} - \frac{\bar{X}}{\bar{Z}}\frac{\partial \bar{Z}}{\partial \varphi}\right)$$

$$= -\frac{f}{\bar{Z}}\left\{b_2\bar{Z} - b_3\bar{Y} - \frac{\bar{X}}{\bar{Z}}(b_1\bar{Y} - b_2\bar{X})\right\}$$

$$= -b_2 f + b_3 f\frac{\bar{Y}}{\bar{Z}} + f\frac{\bar{X}}{\bar{Z}}\left(b_1\frac{\bar{Y}}{\bar{Z}} - b_2\frac{\bar{X}}{\bar{Z}}\right)$$

$$\frac{\partial x}{\partial \varphi} = -f\cos\omega\cos\kappa + (y-y_0)\sin\omega - (x-x_0)\left[-\frac{(y-y_0)}{f}\cos\omega\sin\kappa + \frac{(x-x_0)}{f}\cos\omega\cos\kappa\right]$$

$$= (y-y_0)\sin\omega - \left\{\frac{(x-x_0)}{f}\left[(x-x_0)\cos\kappa - (y-y_0)\sin\kappa\right] + f\cos\kappa\right\}\cos\omega$$

偏导数 2-1

$$\frac{\partial \boldsymbol{R}^{-1}}{\partial \varphi}=\frac{\partial(\boldsymbol{R}_\kappa^{-1}\boldsymbol{R}_\omega^{-1}\boldsymbol{R}_\varphi^{-1})}{\partial \varphi}=\boldsymbol{R}_\kappa^{-1}\boldsymbol{R}_\omega^{-1}\frac{\partial \boldsymbol{R}_\varphi^{-1}}{\partial \varphi}=\boldsymbol{R}_\kappa^{-1}\boldsymbol{R}_\omega^{-1}\boldsymbol{R}_\varphi^{-1}\boldsymbol{R}_\varphi\frac{\partial \boldsymbol{R}_\varphi^{-1}}{\partial \varphi}=\boldsymbol{R}^{-1}\boldsymbol{R}_\varphi\frac{\partial \boldsymbol{R}_\varphi^{-1}}{\partial \varphi}$$

$$\boldsymbol{R}_\varphi\frac{\partial \boldsymbol{R}_\varphi^{-1}}{\partial \varphi}=\begin{bmatrix}\cos\varphi & 0 & -\sin\varphi\\ 0 & 1 & 0\\ \sin\varphi & 0 & \cos\varphi\end{bmatrix}\begin{bmatrix}-\sin\varphi & 0 & \cos\varphi\\ 0 & 0 & 0\\ -\cos\varphi & 0 & -\sin\varphi\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}$$

$$\frac{\partial\begin{bmatrix}\bar{X}\\ \bar{Y}\\ \bar{Z}\end{bmatrix}}{\partial \varphi}=\boldsymbol{R}^{-1}\boldsymbol{R}_\varphi\frac{\partial \boldsymbol{R}_\varphi^{-1}}{\partial \varphi}\begin{bmatrix}X-X_s\\ Y-Y_s\\ Z-Z_s\end{bmatrix}=\boldsymbol{R}^{-1}\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}\begin{bmatrix}X-X_s\\ Y-Y_s\\ Z-Z_s\end{bmatrix}$$

偏导数 2-2

$$\frac{\partial\begin{bmatrix}\bar{X}\\ \bar{Y}\\ \bar{Z}\end{bmatrix}}{\partial\varphi}=\boldsymbol{R}^{-1}\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}\boldsymbol{R}\begin{bmatrix}\bar{X}\\ \bar{Y}\\ \bar{Z}\end{bmatrix}$$

$$=\begin{bmatrix}-c_1 & 0 & a_1\\ -c_2 & 0 & a_2\\ -c_3 & 0 & a_3\end{bmatrix}\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}\begin{bmatrix}\bar{X}\\ \bar{Y}\\ \bar{Z}\end{bmatrix}$$

$$=\begin{bmatrix}0 & a_1c_2-a_2c_1 & a_1c_3-a_3c_1\\ a_2c_1-a_1c_2 & 0 & a_2c_3-a_3c_2\\ a_3c_1-a_1c_3 & a_3c_2-a_2c_3 & 0\end{bmatrix}\begin{bmatrix}\bar{X}\\ \bar{Y}\\ \bar{Z}\end{bmatrix}$$

$$=\begin{bmatrix}0 & -b_3 & b_2\\ b_3 & 0 & -b_1\\ -b_2 & b_1 & 0\end{bmatrix}\begin{bmatrix}\bar{X}\\ \bar{Y}\\ \bar{Z}\end{bmatrix}$$

$$=\begin{bmatrix}b_2\bar{Z}-b_3\bar{Y}\\ b_3\bar{X}-b_1\bar{Z}\\ b_1\bar{Y}-b_2\bar{X}\end{bmatrix}$$

$$\begin{bmatrix}\bar{X}\\ \bar{Y}\\ \bar{Z}\end{bmatrix}=\boldsymbol{R}^{-1}\begin{bmatrix}X-X_s\\ Y-Y_s\\ Z-Z_s\end{bmatrix}$$

正交矩阵的每一个元素等于它的代数余子式

误差方程

$$v_x = a_{11}\Delta X_s + a_{12}\Delta Y_s + a_{13}\Delta Z_s + a_{14}\Delta\varphi + a_{15}\Delta\omega + a_{16}\Delta\kappa + x^0 - x$$

$$v_y = a_{21}\Delta X_s + a_{22}\Delta Y_s + a_{23}\Delta Z_s + a_{24}\Delta\varphi + a_{25}\Delta\omega + a_{26}\Delta\kappa + y^0 - y$$

垂直摄影情况下，可取$\varphi=\omega=0$，保留κ，则

$$a_{11} = -\frac{f}{H}\cos\kappa$$

$$a_{12} = -\frac{f}{H}\sin\kappa$$

$$a_{13} = -\frac{x-x_0}{H}$$

$$a_{14} = -(f + \frac{(x-x_0)^2}{f})\cos\kappa + \frac{(x-x_0)(y-y_0)}{f}\sin\kappa$$

$$a_{15} = -\frac{(x-x_0)(y-y_0)}{f}\cos\kappa - (f + \frac{(x-x_0)^2}{f})\sin\kappa$$

$$a_{16} = +(y-y_0)$$

$$a_{21} = +\frac{f}{H}\sin\kappa$$

$$a_{22} = -\frac{f}{H}\cos\kappa$$

$$a_{23} = -\frac{y-y_0}{H}$$

$$a_{24} = -\frac{(x-x_0)(y-y_0)}{f}\cos\kappa + (f + \frac{(y-y_0)^2}{f})\sin\kappa$$

$$a_{25} = -(f + \frac{(y-y_0)^2}{f})\cos\kappa - \frac{(x-x_0)(y-y_0)}{f}\sin\kappa$$

$$a_{26} = -(x-x_0)$$

外方位元素的计算

当一张像片上至少有三个控制点时，误差方程矩阵形式

$$\boldsymbol{V} = \boldsymbol{A}\boldsymbol{x} - \boldsymbol{l}$$

$$\boldsymbol{x} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{l})$$

$$\sigma_0 = \sqrt{\frac{\boldsymbol{V}^{\mathrm{T}}\boldsymbol{V}}{2n-6}}$$

$$\boldsymbol{V} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} \varDelta X_s \\ \varDelta Y_s \\ \varDelta Z_s \\ \varDelta \varphi \\ \varDelta \omega \\ \varDelta \kappa \end{bmatrix}, \quad \boldsymbol{l} = \begin{bmatrix} x^0 - x \\ y^0 - y \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \end{bmatrix}$$

$$\boldsymbol{Q}_{xx} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}$$

$$m_i = \sigma_0 \sqrt{\boldsymbol{Q}_{xx}^{ii}}$$

三、计算过程

- 获取已知数据 $m, x_0, y_0, f, \boldsymbol{X}_{tp}, \boldsymbol{Y}_{tp}, \boldsymbol{Z}_{tp}$
- 量测控制点像点坐标 x,y
- 确定未知数初值 $\boldsymbol{X}_{s0}, \boldsymbol{Y}_{s0}, \boldsymbol{Z}_{s0}, \varphi_0, \omega_0, \kappa_0$
- 组成误差方程式并法化
- 解求外方位元素改正数
- 检查迭代是否收敛

本讲参考资料

教材

作业：

PP.39，第7题；a_{15}

张剑清，潘励，王树根 编著，《摄影测量学》，武汉大学出版社

参考书

1、李德仁，周月琴 等编，《摄影测量与遥感概论》，测绘出版社

2、李德仁，郑肇葆 编著，《解析摄影测量学》，测绘出版社

课间实习

内容

用 C 或 C++ 语言编写单片空间后方交会程序

时间

4小时（先写出程序源代码，后上机调试）

要求

1、提交实习报告：程序框图、程序源代码、计算结果

2、计算结果：像点坐标、地面坐标、单位权中误差、外方位元素及其精度

3、数据： **PP.39，第9题**