

Control of beam halo-chaos using real Morlet wavelet function in a periodic-focusing channel^{*}

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Abstract: The Kapchinsky-Vladimirsky (K-V) beam through an axisymmetric periodic-focusing magnetic field is studied using the particle-core model. The beam halo-chaos is found, and a real Morlet wavelet function controller is proposed based on the mechanism of halo formation and the strategy of controlling halo-chaos. The method is applied to the multi-particle simulation to control the halo. The numerical results show that the halo-chaos and its regeneration can be eliminated effectively by using the real Morlet wavelet function control method. At the same time, the radial particle density is uniform at the center of the beam as long as the control method and appropriate parameter are chosen.

Key words: High-current particle beam; Halo-chaos; Periodic-focusing magnetic field; Real Morlet wavelet function

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In recent years, the high intensity particle beam has been utilized widely due to its attractive features in possible breakthrough applications, such as clean nuclear power systems, nuclear science, the production of radioactive isotopes for medical purpose^[1-18]. However, the halo of a particle beam not only reduces the accelerator efficiency, but also damages the body and environment. It is necessary to control the halo-chaos^[5-18] of the beam. Although controlling the beam halo-chaos through a periodic-focusing magnetic field has been studied most of the time, the total number of the particles is only 5×10^4 in Ref. [5-13]. In this paper, we have proposed a real Morlet wavelet function for suppressing the halo in the periodic-focusing channel using more particles. The real Morlet wavelet function has strong nonlinearity and stability. The halo-chaos is a spatio-temporal chaos which is sensitive and unstable, but we have found that the beam halo-chaos can be controlled effectively by the real Morlet wavelet function control method. Then, we have studied the characteristics of the particle beam when the halo is not suppressed or is suppressed. The obtained results indicate that we can control not only the halo-chaos using this method, but also the beam distribution in some sense.

1 Numerical method and the controller

At present, the particle-core model is used to investigate the beam halo-chaos widely. This model assumes that the Kapchinsky-Vladimirsky(K-V) beam is round and continuous, thus the dimensionless nonlinear equation of the beam envelope and the transverse equations of motion for a single particle in a periodic-focusing system are^[2-3]

$$d^2 r_b / ds^2 + \kappa_z(s) r_b - K / r_b - 1 / r_b^3 = 0 \quad (1)$$

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$$\frac{d^2x}{ds^2} + \kappa_z(s)x + \frac{q}{\gamma_b^3\beta_b^2mc^2} \frac{\partial\Phi_s(x,y,s)}{\partial x} = 0 \quad (2)$$

$$\frac{d^2y}{ds^2} + \kappa_z(s)y + \frac{q}{\gamma_b^3\beta_b^2mc^2} \frac{\partial\Phi_s(x,y,s)}{\partial y} = 0 \quad (3)$$

where r_b is the beam radius and $s=vt$ is the axial displacement, in which v is the axial velocity of the beam particles, K is a measure of the beam self-field, c is the speed of light; $\beta_b = v/c$, $\gamma_b = 1/\sqrt{1-\beta_b^2}$, is relativistic mass factor; q and m are the particle charge and rest mass, respectively; $\Phi_s(x,y,s)$ is the self-potential. The periodic function $\kappa_z(s)$ characterizes the strength of the periodic-focusing exterior magnetic field as shown in Fig. 1^[5-13], $\kappa_z(s) = \kappa_z(s+S)$ (S is a period), and Γ is the filling factor of magnetic field.

In the particle-core-interaction model, the self-field force acting on a particle is given by

$$F_r = -q\nabla\Phi_s(x,y,s) \quad (4)$$

The nonlinear feedback control method is proposed based on the general strategy of controlling chaos in Ref [5], which applies a nonlinear feedback controller G to the right-hand side of Eq. (4), that is

$$F_r = -q\nabla\Phi_s(x,y,s) + G \quad (5)$$

The force acting on an particle can be changed by controller G , at the same time the transverse velocity and energy of particles are reduced or translated, so the tendency of the particles escaping from the core is restrained. The loose particles in beam are compressed to the core field, and the particles become concentrative after the halo-chaos is controlled, thus the halo-chaos is suppressed^[17-18].

It is critical to select an effective controller G for decreasing the halo-chaos. In our work, the real Morlet wavelet function is used, which has a strong nonlinearity, stability, and excellent localization property as shown in Fig. 2, namely

$$f(x) = \cos(x) \cdot \exp(-x^2) \quad (6)$$

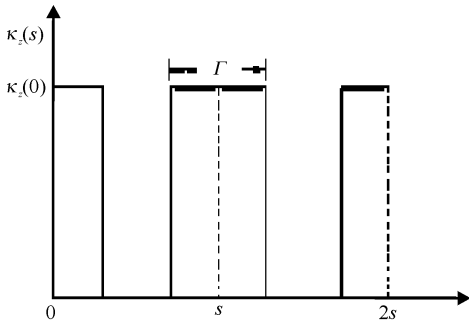


Fig. 1 Periodic function and filling factor

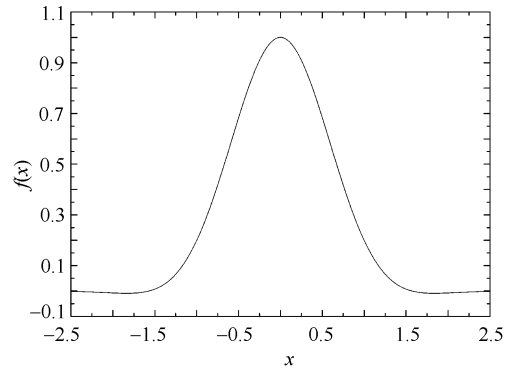


Fig. 2 Real Morlet wavelet function

Thus, the real Morlet wavelet function controller is put forward for suppressing the halo using the real Morlet wavelet function as follows:

$$G = g[f(r_{\text{rms}}) - f(a_m)] \quad (7)$$

where r_{rms} is the average root-mean-square radius, a_m is the matching radius. So we apply a variational electric field $E(r_{\text{rms}})$ that is radial around the periodic-focusing channel^[18], namely

$$E(r_{\text{rms}}) = \frac{g}{q}[f(r_{\text{rms}}) - f(a_m)] \quad (8)$$

The controller G is designed to be

$$G = E(r_{\text{rms}})q = 10[f(r_{\text{rms}}) - f(a_m)] \quad (9)$$

When $r_{\text{rms}} \rightarrow a_m \rightarrow 0$, G vanishes automatically.

2 Numerical results and discussion

The particle-in-cell(PIC) simulation is used for studying the beam under the periodic-focusing channel. The main parameters used are as follows: the total number of particles is 5×10^5 , the vacuum phase advance

$\sigma_0=115^\circ$, filling factor $\Gamma=0.80$, tune-depression $\eta=0.80$, $\eta=1/a_m^2\sigma_0$, which characterizes the strength of space-charge effect. Mismatch factor $M=1.5$, where the mismatch parameter gives the ratio of the initial beam radius to the matched radius. The calculated parameters are as follows; matched radius $a_m=0.7891642$ and perveance $K=0.9032079$. The evolution periodic steps are 1800.

2.1 Comparison of particle distributions in plane

Figure 3 shows the cross section of the beam before and after controlling the halo-chaos. Before the halo-chaos is suppressed, the radial distribution range of beam particles is large and noncompact in the vicinity of the beam core as shown in Fig. 3(a). Because space-charges cause nonlinear effects and energy exchange takes place between particles and the core, some particles can escape from the core and form a halo surrounding the core. After controlling the halo-chaos by the real Morlet wavelet function controller, the halo is eliminated, and the radial particle density becomes uniform. At the same time, the ratio of the cross section of the beam after suppressing the halo to that before controlling is 1/12 or so.

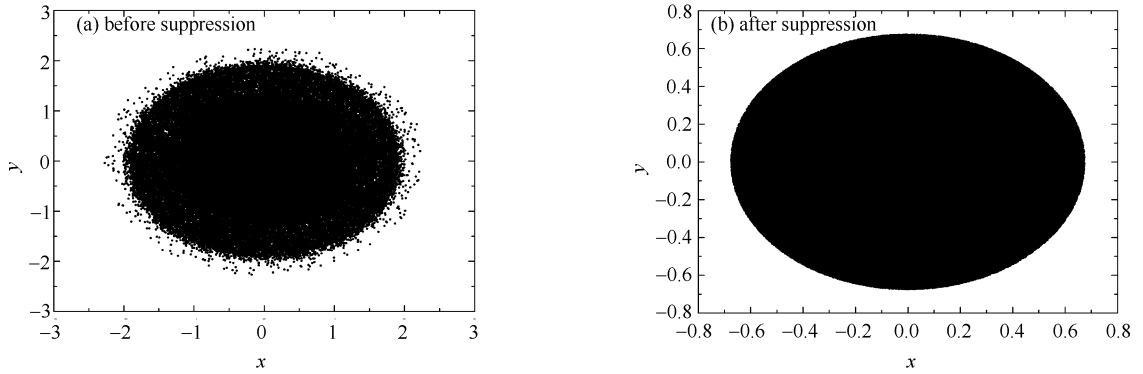


Fig. 3 Particle distribution at periodic section

2.2 Comparison of statistical variables of beams

In the simulation, a halo-chaos strength factor H is defined, which is the ratio of the particles outside $1.75a_m$ to the total particles. H is a measure of the halo control. In Fig. 4(a), H is not zero, which means that the halo exists all the time before suppression. After suppressing the halo, H is zero, which means that the halo is removed by the real Morlet wavelet function controller(as shown in Fig. 4(b)).

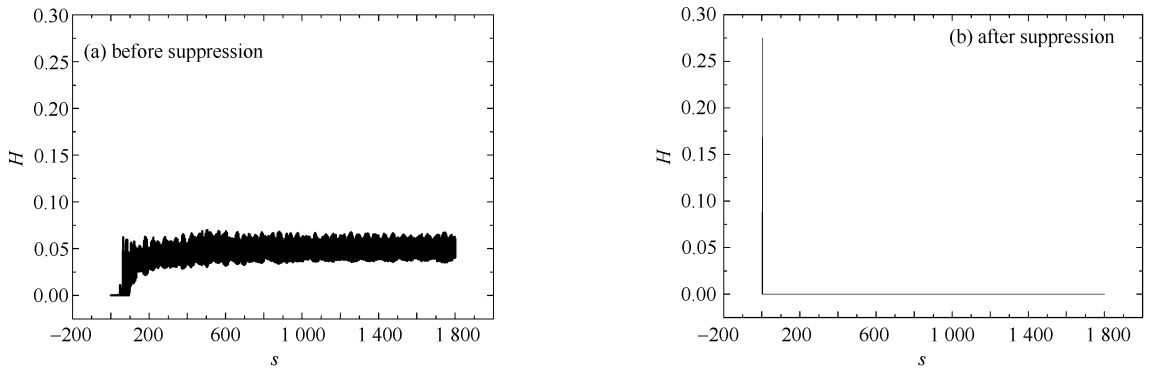


Fig. 4 Halo-chaos strength factor vs axial displacement

The evolution of root-mean-square radius r_{rms} is smart and irregular before suppressing the halo as shown in Fig. 5(a). In Fig. 5(b), the evolution of r_{rms} is regular and its amplitude becomes small after the halo is suppressed. This indicates that the tendency of the particles escaping form the core is under effective control.

2.3 Analysis of radial particle density in high-intensity particle beam

The initial distribution of the K-V beam is uniform as shown in Fig. 6. When the beam is not controlled, the simulation result indicates that the uniform distribution is destroyed rapidly, and the radial particle density of beam varies acutely as shown in Fig. 7(a). From the particle-core-interaction model, space-charge force is linear in the beam core, but nonlinear outside the core. At the same time, particles of mismatched beam

are affected by the external magnetic field. The complexity of the force acting on particles may cause that more and more particles escape from the core and form the halo, and result in such a radial particle density evolution curve of beam.

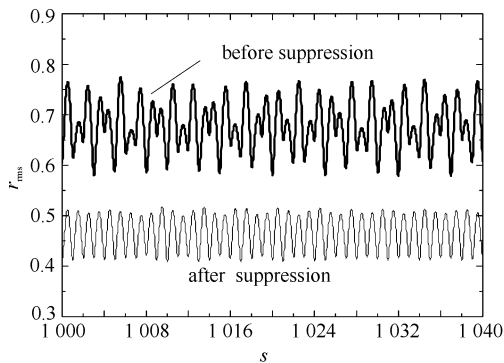


Fig. 5 Evolution of r_{rms} vs axial displacement

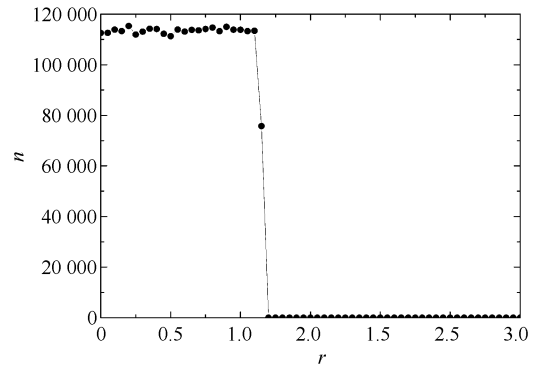


Fig. 6 Initial density distribution of K-V beam

When the beam halo-chaos is suppressed effectively by the real Morlet wavelet function controller, change in radial particle density occurs. In Fig. 7(b), uniform density appears at the beam's center compared with the density distribution before the beam-halo is prevented, and the area of particle distribution becomes smaller and more uniform than the area of the initial distribution. This indicates that the particle density might change its spatial distribution after the halo-chaos is suppressed.

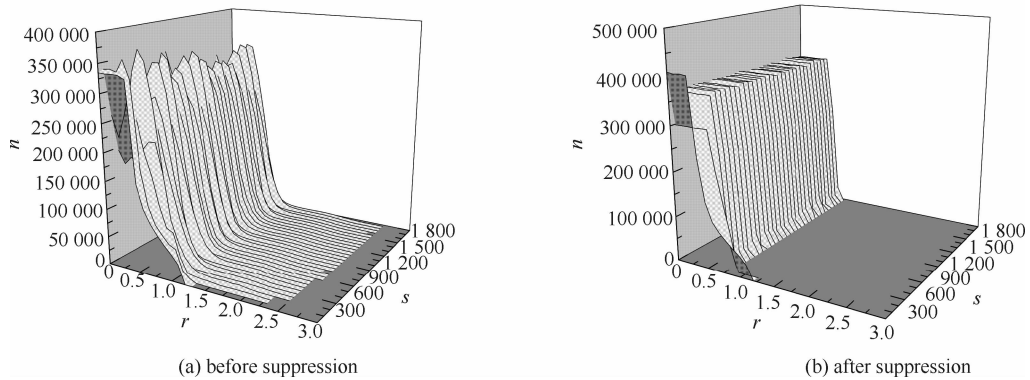


Fig. 7 Evolution of radial particle density of beam

3 Summary

The simulative results of controlling the beam halo-chaos and radial particle density show that, as long as appropriate system parameters are chosen, the halo can be suppressed by the real Morlet wavelet function controller, and the density uniformity of beam can be obtained. Using real Morlet wavelet function to control beam halo-chaos is a valuable try for the particle beam applications in the future.

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周期聚焦磁场中束晕-混沌的实 Morlet 小波函数控制

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摘 要: 基于束晕-混沌的非线性控制策略, 对周期性聚焦磁场中初始分布满足 K-V 分布的粒子束进行模拟研究, 提出了控制其束晕-混沌的实 Morlet 小波函数控制器, 并给出具体的实施方案。数值模拟研究表明, 在适当的参数条件下, 运用这种方法不仅可以消除束晕及其再生现象, 达到对束晕-混沌的有效控制, 而且可以控制束流到均匀分布。

关键词: 强流粒子束; 束晕-混沌; 周期聚焦磁场; 实 Morlet 小波函数