Entanglement Diversion of Entangled Squeezed Vacuum State*

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Abstract: By analyzing the action of a beam splitter on squeezed vacuum states (SVSs), it is found that, if the two input light beams of the beam splitter have the same squeezing amplitude and phase, the output state is a two-mode SVS. If the two input light beams of the beam splitter have the same squeezing amplitude but a phase difference of π, the output state is a direct product state of two single-mode SVSs. In the two-mode SVS, each mode has the same photon number with both even and odd photons, while the single-mode SVS which contains only even-number photon in the photon number-state representation. Based on these results, a scheme to divert the entanglement of entangled SVS from its initial locations to another location is suggested. In this scheme, the two SVSs are utilized as the quantum channel. The process of the entanglement diversion is achieved by using the 50/50 symmetric beam splitters and the photon detectors with the help of classical information.

Key words: Entanglement diversion; Entangled SVS; Beam-splitter; Detection of photon
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0 Introduction

Quantum entanglement is a key ingredient of quantum mechanics. In recent years, entanglement has generated many interests in the quantum information processing, such as quantum teleportation[1], superdense coding[2], quantum key distribution[3], and telecoloning[4]. In particular, quantum entanglement has been viewed as an essential resource for quantum information processing[5-6]. Thus some methods of controlling entanglement have been suggested[7-8]. The original study for quantum information processing focused on the systems with a finite-dimensional (discrete variable) state space, such as the polarizations of a photon or the discrete levels of an atom. Since the experimental demonstration of quantum teleportation of coherent states[9], many interests have arisen in continuous variable quantum information processing[10]. Thereupon, the entangled states with continuous variable receive much attention.

The squeezed radiation field has wide applications in many different arenas of quantum information processing, such as it can be used to encode quantum information on continuous variables. In quantum information processing, the entangled squeezed state is normally categorized into a two modes continuous variable state. However, there is a suggestion to implement a logical qubit encoding through treating a coherent superposition state, which can make a single mode continuous variable state as a qubit in two dimensional Hilbert space[11]. Thereupon, many schemes have been proposed for realizing quantum information processing by using the entangled squeezed vacuum states.

In the previous papers, we have researched using the entangled squeezed vacuum states to realize quantum information processing such as preparation of entangled states[12-13], entanglement concentration and purification[14], quantum teleportation[15-16]. In the present work, the action of a beam splitter on squeezed vacuum states is analyzed and studied in detail, and a scheme for realizing entanglement diversion of entangled squeezed vacuum state is proposed.

1 The function of a beam splitter

We first briefly review the function of a beam splitter. The lossless symmetric 50/50 beam splitter is described by

$$\hat{B}_{12} = \exp\left[\mathrm{i}\,\frac{\pi}{4}(\hat{a}_1^{+}\hat{a}_2 + \hat{a}_2^{+}\hat{a}_1)\right] \quad (1)$$

where $\hat{a}_i$ and $\hat{a}_i^{+}$ (i denote 1, 2) is the bosonic annihilation and creation operators of system i, respectively. It transforms the coherent state $|\alpha\rangle_1 |\beta\rangle_2$ as

$$\hat{B}_{12}\,|\alpha\rangle_1\,|\beta\rangle_2 = |(\alpha+\mathrm{i}\beta)/\sqrt{2}\rangle_1\,|(\beta+\mathrm{i}\alpha)/\sqrt{2}\rangle_2 \quad (2)$$

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This transformation plays an important role in the realization of quantum information processing involving coherent states. It is more intricate to transform SVSs using beam splitters. Before presenting our entanglement diversion scheme, we first analyze the action of the beam splitter on SVSs. Let $\hat{a}_1$ and $\hat{a}_2$ denote the annihilation operators for the two light beams entering the two input ports of the beam-splitter. Let $\hat{A}_1$ and $\hat{A}_2$ denote the annihilation operators for the two light beams leaving the two output ports of the 50/50 beam-splitter. The boundary conditions at the surface of the beam-splitter lead to the well-known input-output relations

$$\hat{A}_1=\frac{1}{\sqrt{2}}(\hat{a}_1+\mathrm{i}\hat{a}_2),\hat{A}_2=\frac{1}{\sqrt{2}}(\hat{a}_2+\mathrm{i}\hat{a}_1) \tag{3}$$

We assume that the two input light beams of the beam-splitter are in SVSs

$$|\psi\rangle_{\mathrm{in}}=\hat{S}_1(\xi_1)\hat{S}_2(\xi_2)|0,0\rangle=|\xi\rangle_1|\xi\rangle_2 \tag{4}$$

where $\xi=re^{\mathrm{i}\varphi}$ is any complex number with squeezing amplitude r and squeezing angle φ and

$$\hat{S}_i(\xi_i)=\exp\left(-\frac{\xi_i}{2}\hat{a}_i^{+2}+\frac{\xi_i^*}{2}\hat{a}_i^2\right) \tag{5}$$

are the single-mode squeezing operators for mode i ($i=1,2$). An expansion of single-mode SVS in terms of Fork states is

$$|\xi\rangle=\sqrt{\sec hr}\sum_{n=0}^{\infty}\frac{\sqrt{(2n)!}}{n!}\left(-\frac{1}{2}e^{\mathrm{i}\varphi}\tan hr\right)^n|2n\rangle \tag{6}$$

After interacting with the beam-splitter, the state vector of the output light beams becomes

$$|\hat{\psi}\rangle_{\mathrm{out}}=\exp\left[-\frac{1}{4}(\xi_1-\xi_2)\hat{A}_1^{+2}+\frac{1}{4}(\xi_1^*-\xi_2^*)A_1^2+\frac{1}{4}(\xi_1-\xi_2)\hat{A}_2^{+2}-\frac{1}{4}(\xi_1^*-\xi_2^*)\hat{A}_2^2-\frac{1}{2}(\xi_1+\xi_2)\hat{A}_1^+\hat{A}_2^+-\frac{1}{2}(\xi_1^*+\xi_2^*)\hat{A}_1\hat{A}_2\right]|0,0\rangle \tag{7}$$

The following two cases play a key role in our entanglement diversion scheme.

Case 1 The two input light beams have the same squeezing amplitude and phase, i. e., $r_1=r_2=r$ and $\varphi_1=\varphi_2=\varphi$. In this case, from Eq. (7) it is easy to see that the state of output light beams is simply a two-mode SVS

$$|\psi\rangle_{\mathrm{out}}=\exp\left[-re^{\mathrm{i}(\varphi+\pi/2)}\hat{A}_1^+\hat{A}_2^++re^{-\mathrm{i}(\varphi+\pi/2)}\hat{A}_1\hat{A}_2\right]|0,0\rangle \tag{8}$$

An expansion of two-mode SVS in terms of Fork states is

$$|\xi\rangle_{12}=\sec hr\sum_{n=0}^{\infty}(-e^{\mathrm{i}\varphi}\tan hr)^n|n,n\rangle_{12} \tag{9}$$

Case 2 The two input light beams have the same squeezing amplitude but a phase difference of π, i. e., $r_1=r_2=r$ and $\varphi_2-\varphi_1=\pi$. In this case, the output state of the beam-splitter becomes a direct product of two single-mode SVSs

$$|\psi\rangle_{\mathrm{out}}=\exp\left(-\frac{1}{2}re^{\mathrm{i}\varphi}\hat{A}_1^{+2}+\frac{1}{2}re^{-\mathrm{i}\varphi}\hat{A}_1^2\right)\cdot\exp\left(-\frac{1}{2}re^{\mathrm{i}(\varphi+\pi)}\hat{A}_2^{+2}+\frac{1}{2}re^{-\mathrm{i}(\varphi+\pi)}\hat{A}_2^2\right)|0,0\rangle \tag{10}$$

2 Entanglement diversion

The method to transform SVSs using beam splitters has been discussed. Next, we present our scheme of entanglement diversion.

Assume there are three remote partners (Alice, Bob, Clara) who want to establish a connection. First, Alice shared the quantum channel $|\psi\rangle_{13}$ with Bob and quantum channel $|\psi\rangle_{24}$ with Clara, where

$$|\psi\rangle_{13}=\frac{1}{\sqrt{N_-}}(|\xi\rangle_1|\xi\rangle_3-|-\xi\rangle_1|-\xi\rangle_3)$$

$$|\psi\rangle_{24}=\frac{1}{\sqrt{N_-}}(|\xi\rangle_2|\xi\rangle_4-|-\xi\rangle_2|-\xi\rangle_4) \tag{11}$$

are entangled SVS and $N_-=2[1-\mathrm{sec}h(2r)]$ is the normalization factor. We have proven that they are the maximally entangled states; their amount of entanglement is exactly one ebit and the entanglement is independent of the parameters involved[12]. The methods for preparing the entangled SVSs has been discussed in Ref. [11-12]. Then the initial state of the whole system consisting of system 1, 2, 3 and 4 is given by

$$|\psi\rangle_{1234}=|\psi\rangle_{13}\otimes|\psi\rangle_{24}=\frac{1}{N_-}(|\xi\rangle_1|\xi\rangle_2|\xi\rangle_3|\xi\rangle_4-|\xi\rangle_1|-\xi\rangle_2|\xi\rangle_3|-\xi\rangle_4-|-\xi\rangle_1|\xi\rangle_2|-\xi\rangle_3|\xi\rangle_4+|-\xi\rangle_1|-\xi\rangle_2|-\xi\rangle_3|-\xi\rangle_4) \tag{12}$$

Note that at this time the modes 1 and 2 are located at the Alice's location, mode 3 is at Bob's location and mode 4 is at Clara's location.

Now Bob wants to establish a direct connection with Clara for realizing quantum communication between her and Clara. Then Bob sends a request through a classical channel to Alice. Alice, here as an operator, accepts this request and lets her modes 1 and 2 enter the input ports of the beam-splitter BS_2. After interacting with the beam-splitter, we can see from Eq. (7) that the output state of the whole system becomes

$$|\psi'\rangle_{1234}=\frac{1}{N_-}(\hat{S}_{12}(\xi)|0,0\rangle_{12}|\xi\rangle_3|\xi\rangle_4-|\xi\rangle_1|-\xi\rangle_2|\xi\rangle_3|-\xi\rangle_4-|-\xi\rangle_1|\xi\rangle_2|-\xi\rangle_3|\xi\rangle_4+\hat{S}_{12}(-\xi)|0,0\rangle_{12}|-\xi\rangle_3|-\xi\rangle_4) \tag{13}$$

where the two-mode squeezed operator is given by

$$\hat{S}_{12}(\xi)=\exp(-\xi\hat{A}_1^+\hat{A}_2^++\xi^*\hat{A}_2\hat{A}_1) \tag{14}$$

Subsequently Alice makes the two-mode

photon number measurement on modes 1 and 2 at her side and sends the classical information to Bob and Clara to finish the entanglement diversion process. From Eq. (6, 9 and 13) it can be seen that for systems 1 and 2, the first and fourth terms on the right-hand side are the two-mode SVS in which each mode has the same photon number with both even and odd photons in the photon number-state representation, while the second and third terms are the single-mode SVS which contains only even-number photon in their number-state representation. When the results of measurements are the odd number of photon, from Eq. (13) it can be seen that modes 3 and 4 collapse into

$$|\psi'\rangle_{34}=\frac{1}{\sqrt{N_-}}(|\xi\rangle_3|\xi\rangle_4-|-\xi\rangle_3|-\xi\rangle_4) \quad (15)$$

where we have considered normalization factor. Then a quantum channel between Bob and Clara has been established.

3 Summary

In summary, we have presented a simple scheme to divert the entanglement among Alice, Bob and Clara in bipartite entangled SVS. In this scheme, we exploited the important features of the beam splitter transforming SVSs, which are that if the two input light beams of the beam splitter have the same squeezing amplitude and phase, the output state is simply a two-mode SVS; and that if the two input light beams of the beam splitter have the same squeezing amplitude but a phase difference of π, the output state is a direct product state of two single-mode SVSs. The key factor in our entanglement division scheme is photon number measurement which must be sensitive enough to measure the number of photons and determine whether the number is even or odd. In practice this is difficult, especially for large numbers of photons, but in principle this can be done. In Ref. [17] Xiao-Guang Wang described one method to distinguish even and odd n by coupling the field to a two-level atom through dispersive interaction, which can enhance the possibility to realize this scheme.

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纠缠压缩真空态的纠缠转移

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摘　要:通过分析光学分束器对压缩真空态光场的作用,发现如果分束器的输入光是两束具有同样振幅和相位的单模压缩真空态光场,则输出光为双模压缩真空态光场;若分束器的输入光是两束具有同样振幅但有 π 相位差的单模压缩真空态光场,则输出光仍为两束单模压缩真空态光场.对于双模压缩真空态光场,每个模中容纳的光子数可以是基数或偶数.而对于单模压缩真空态光场,每个模中只能包含偶数个光子.根据这些结果,提出了一个纠缠转移的方案.在这个方案中,两个纠缠压缩真空态光场被用作量子信道,通过利用光学分束器作用和光子数探测的方法,并在经典通讯的帮助下,实现了三个通讯伙伴之间的纠缠转移.
关键词:纠缠转移;纠缠压缩真空态;分束器;光子探测

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