

Ota-C Based Proportional-Integral-Derivative (PID) Controller and Calculating Optimum Parameter Tolerances

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Abstract

Proportional-integral-derivative (PID) controllers are one of the most important control elements used in the process control industry. In practice, operational amplifiers are generally used in analog controllers. On the other hand, the operational transconductance amplifiers recently developed, which have some positive properties compared to operational amplifiers, are not used in analog controller design at present. Furthermore, the optimum parameter tolerances for the proposed PID circuit by the use of parameter sensitivities are determined. These tolerances keep the relative error at the output of the controller due to parameter variations in the tolerance region.

Key Words: *PID controller, operational transconductance amplifier, optimum parameter tolerances.*

1. Introduction

In practice, operational amplifiers are generally used in analog controllers. On the other hand, the operational transconductance amplifiers (OTAs) recently developed have some positive properties such as a broader frequency band, smaller dissipation factors, suitability for integration, electronic tunability of the transconductance gain and better stability characteristics compared to op amps [1,2].

But OTAs have not been used in design of analog controllers so far. The purpose of this study is to present a synthesis procedure for the realization of an analog proportional-integral-derivative (PID) controller by the use of OTAs together with grounded capacitors. This procedure is based on the signal-flow graph, which is a powerful tool in active circuit design. The circuit which will be obtained requires eight OTAs and two grounded capacitors for the most general PID controller.

The simulation of the proposed circuit is performed and the results are discussed. Furthermore, the optimum parameter tolerances by the use of parameter sensitivities are determined. These tolerances keep the relative error of the output of the controller due to parameter variations in the tolerance region and they can also be used to improve and to control the sensitivity performance of the proposed PID controller.

2. Operational Transconductance Amplifier (OTA)

An OTA device is ideally defined with the following terminal equations and shown such as in Fig. 1 [1,2],

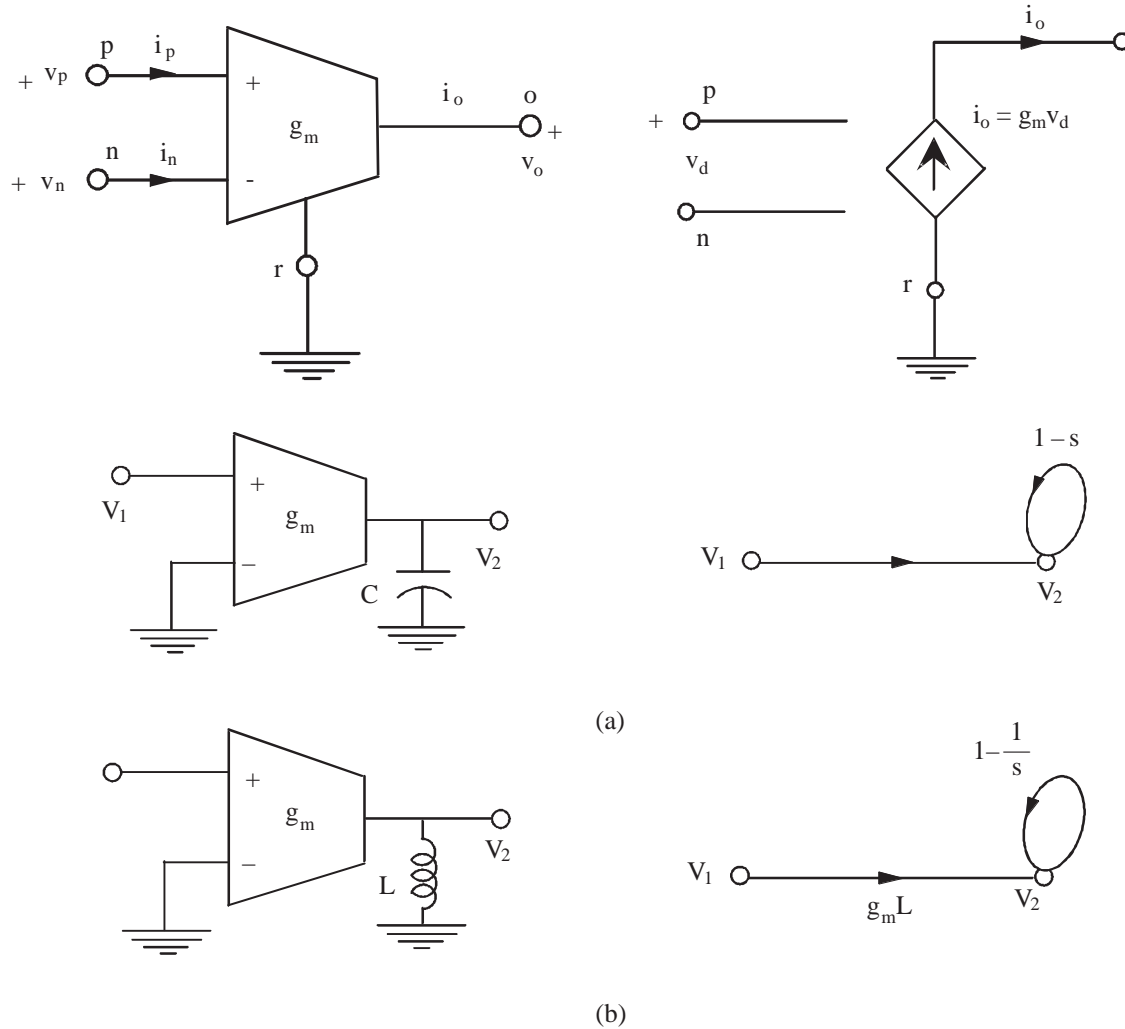


Figure 1. (a) OTA element, (b) Equivalent circuit

where $v_d = v_p - v_n$ stands for differential input voltage, p and n are non-inverting input and inverting input respectively g_m is the transconductance transferring differential input voltage and the output current.

Depending on the configuration of OTA, g_m can be adjusted to the desired value with a control voltage applied from outside. This property makes OTA very convenient for the design and implementation of any circuit. Transconductance, g_m , is proportional to the control voltage in OTAs realized with CMOS technology as follows:

$$g_m = hV_{GG} \quad (1)$$

where h is a constant. In a CMOS-OTA supplied with symmetrical supply voltages of $\pm 5V$, the control voltage, V_{GG} , will be held between $-4V$ and $-2V$. In this interval of control voltage transconductance, g_m will be between 0.25 mA/V and 2 mA/V .

Of course there are some differences between a real OTA and the ideal OTA characteristics given above, namely input capacitance C_i between inputs and reference points and differential input capacitance C_d between p and n inputs. Also it will be suitable to add output resistance r_o , and the output capacitance C_o between the output and the reference point. If it is desired to describe the output dynamics of the circuit, output saturation current I_S and output saturation voltage V_S must be taken into consideration [2].

As seen, the main difference between OTA and OP-AMP is that OTA has a current output. By using OTA, an amplifier, an integrator, a derivative circuit, and a summing circuit can be implemented easily [2]. These implementations are examined in part 3.

3. Synthesis Procedure

Consider the operational transconductance circuits shown in Fig. 2. The signal-flow graph of these circuits can be drawn by using the defining equation of the active element [2].

In Fig. 2(a), an integration circuit and its signal-flow graph are shown. The integration time constant of this circuit is C/g_m as can be easily seen from the signal graph. In Fig. 2(b), an OTA-C based derivative circuit and its signal flow graph are shown. The problem in the circuit is to realize the required ideal inductance element. Because of the difficulty of the realization and the direct implementation, the inductance element has been simulated synthetically. For this purpose, the OTA-C based gyrator circuit, as shown in Fig. 2(c), can be used, terminated by a capacitor. Routine analysis shows that the input function of this gyrator can be calculated as follows:

$$Z_i = \frac{1}{g_{m1}g_{m2}Z_0} = s \frac{C}{g_{m1}g_{m2}} \quad (2)$$

Since Z_i is proportional by s , this circuit behaves as an inductor.

The proposed three mathematical functions, i.e., multiplication, integration and derivation, are transmitted to the output by the OTA based summing circuit shown in Fig. 2(d).

If a given transfer function is represented by a signal-flow graph composed of subgraphs shown in Figure 1, then the circuit corresponding to the signal-flow graph can be realized by interconnecting the building blocks of Fig. 2.

The transfer function of a general analog PID controller can be written as follows [3]:

$$T(s) = \frac{V_o(s)}{V_i(s)} = K_P + \frac{K_I}{s} + sK_D \quad (3)$$

where $v_o(t)$ is the output voltage, $v_i(t)$ is the input voltage, K_P is the proportional gain, K_I is the integral gain, and K_D is the derivative gain.

A signal-flow graph of this transfer function of an analog PID controller can be drawn such as in Fig. 3.

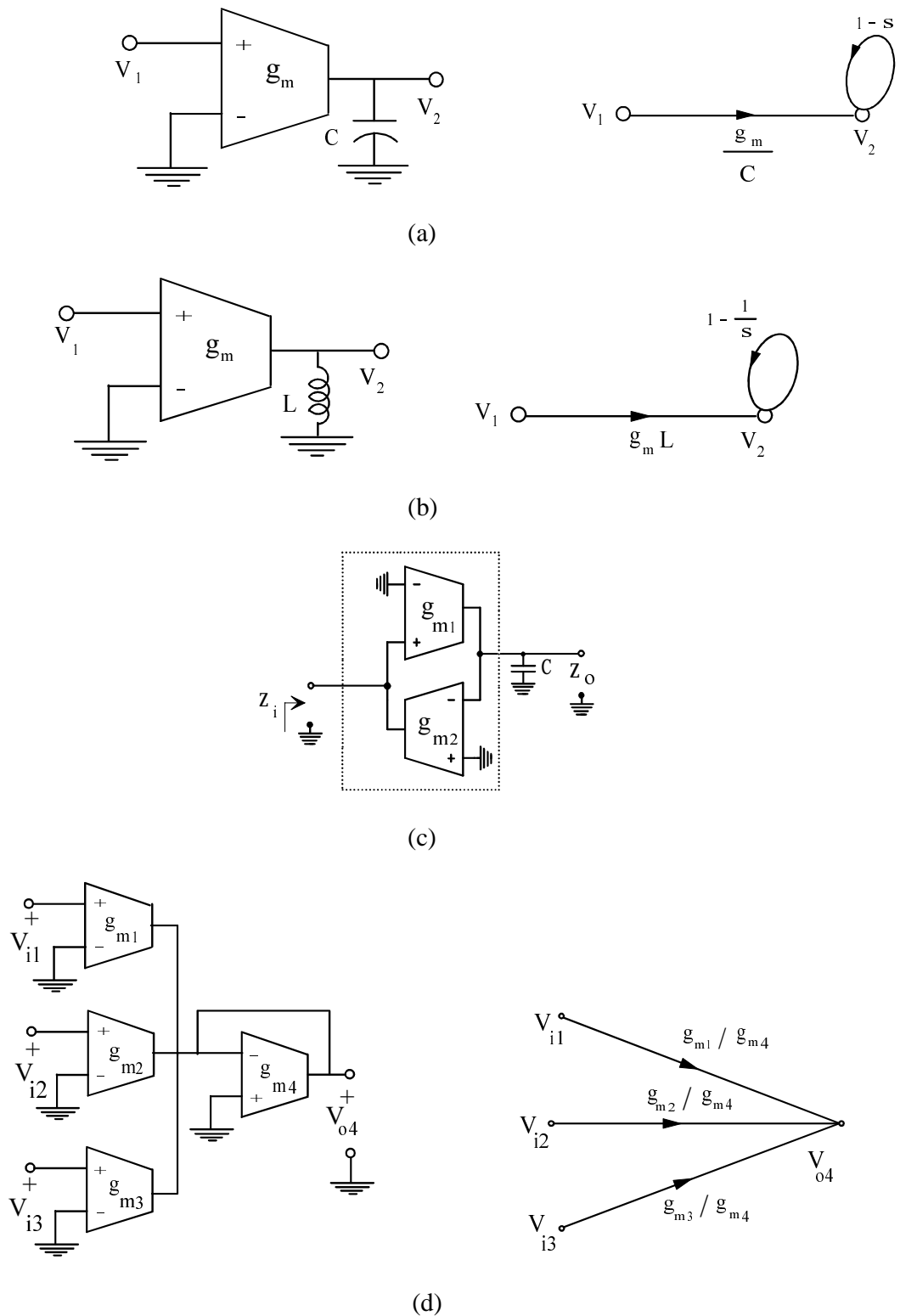


Figure 2. Basic building blocks using OTAs and grounded capacitors together with corresponding signal flow graphs (a) Integrator circuit (b) Derivative circuit (c) Inductance element (d) Summing circuit

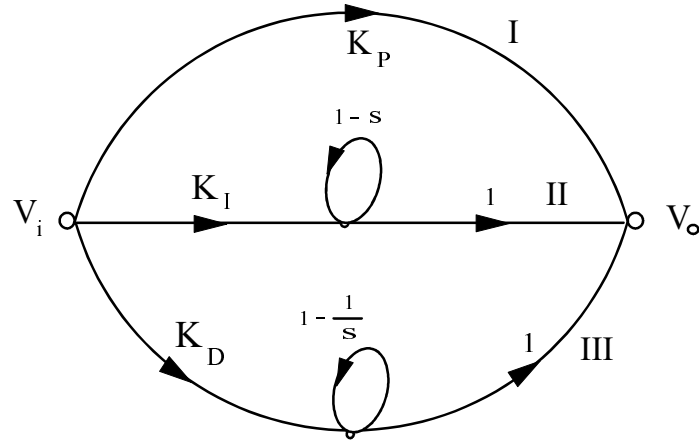


Figure 3. Signal flow graph corresponding to the transfer function of the proportional-integral-derivative (PID) controller

The realization of the analog OTA-C based PID controller circuit corresponding to the signal-flow graph in Fig. 3, which is realized by the subcircuits in Fig. 2, is shown in Fig. 4 [4].

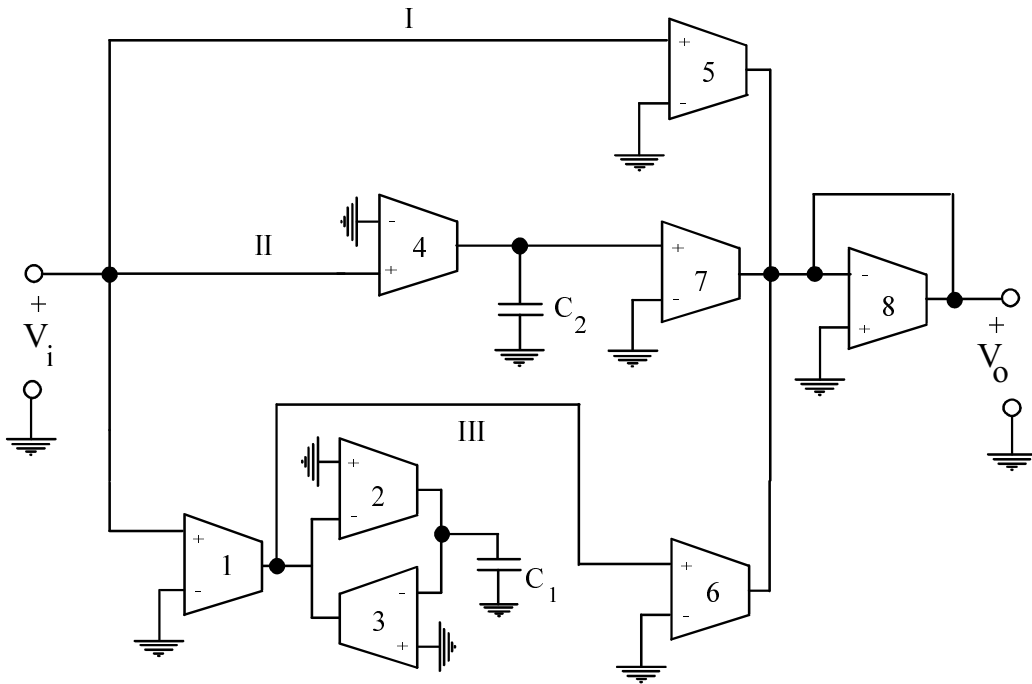


Figure 4. An OTA-C based proportional-integral-derivative (PID) controller realization corresponding to the signal flow graph in Fig. 3.

In Fig. 4, the OTAs numbered 2 and 3, together with the capacitor C_1 , make gyration and this gyration simulates an inductance with the value of $L = C_1/g_{m2}g_{m3}$. This inductance and the OTA numbered 1 configure the derivative circuit. The capacitor C_2 , together with the OTA numbered 4, forms the integration circuit. The OTAs numbered 5, 6, 7 and 8 form the weighted summing circuit. And the OTAs numbered 5 and 8 describe the proportional gain.

In Fig. 4, an OTA-C based PI controller circuit and an OTA-C based PD controller circuit can be obtained by removing path III and path II, respectively, between input and output.

If the circuit in Fig. 4 is analyzed, the controller gains K_P , K_I , and K_D will be obtained as follows:

$$K_P = \frac{g_{m5}}{g_{m8}} \quad (4a)$$

$$K_I = \frac{g_{m4} g_{m7}}{C_2 g_{m8}} \quad (4b)$$

$$K_D = \frac{g_{m1} C_1 g_{m6}}{g_{m2} g_{m3} g_{m8}} \quad (4c)$$

The controller gains depend mostly on the ratios of transconductances of the OTAs. Hence the effects of the deviations of g_{m_s} due to environmental conditions on the controller gains are expected to be reduced. On the other hand, OTAs' transconductances, $g_{m_i}, i = 1, \dots, 8$, can be adjusted to the desired values by DC control voltages, so the controller gains can be assigned to the prescribed values.

4. The Results of Simulation

In order to confirm the theoretical results, the PID circuit given in Fig. 4 is simulated in the SPICE program for the step input voltage with 0.2 Volt amplitude and 0.1 s pulse width. In this simulation, the symmetrical circuit structure given in Fig. 5 is used for OTAs.

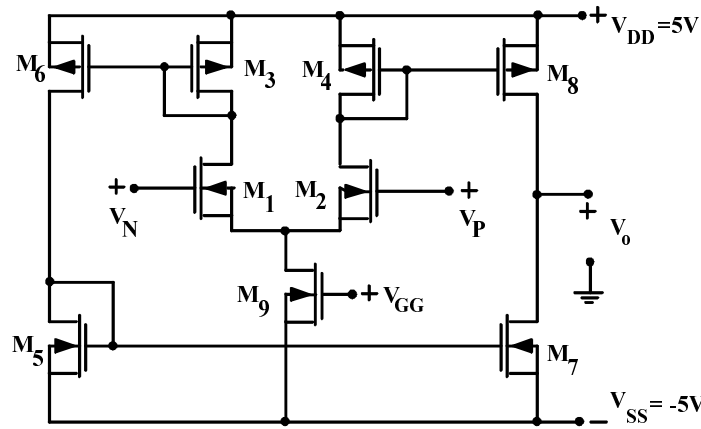


Figure 5. Symmetrical CMOS OTA structure used in SPICE simulation

In the simulations for the proposed controller in Fig.4, 0.8 μm AMS (Austria Mikro Systeme Int. A.G.) model parameters given in Table 1 were taken. The transistor dimensions, which are used in SPICE simulations, are also shown in Table 2 [5].

In this circuit, for the symmetrical supply voltages of $\pm 5V$, a control voltage $V_{GG} = -3.24V$ is applied to the circuit to get a transconductance of 0.532 mA/V. The values of the capacitors, C_1 and C_2 , in the circuit are varied in the range 0.5 μF to 2 μF with 0.5 μF increments. The capacitance values are selected so as to have a better illustration. Note that it is necessary to use an OTA based generalized impedance converter circuit (GIC) for capacitance scaling when realizing the PID circuit.

Table 1. AMS(Austria Mikro Systeme Int. A.G.) 0.8 μm Level 2 model parameters

AMS (Austria Mikro Systeme Int. A.G.) 0.8 μm Level 2 model parameters:
.MODEL nmos NMOS (LEVEL=2 LD=0.414747E-6 TOX=505.0E-10 + NSUB=1.35634E16 VTO=0.864893 KP=44.9E-6 GAMMA=0.981 + PHI=0.6 UEXP=0.211012 UCRIT=107603 DELTA=3.53172E-5 + VMAX=100000 XJ=0.4E-6 LAMBDA=0.0107351 NFS=1E11 + NEFF=1.001 NSS=1E12 TPG=1 RSH=9.925 CGDO=2.83588E-10 + CGSO=2.83588E-10 CGBO=7.968E-10 CJ=0.0003924 + MJ=0.456300 CJSW=5.284E-10 MJSW=0.3199 PB=0.7 + XQC=1 UO=656)
.MODEL pmos PMOS (LEVEL=2 LD=0.580687E-6 TOX=432.0E-10 + NSUB=1E16 VTO=-0.944048 KP=18.5E-6 GAMMA=0.435 + PHI=0.6 UEXP=0.242315 UCRIT=20581.4 DELTA=4.32096E-5 + VMAX=33274.4 XJ=0.4E-6 LAMBDA=0.0620118 NFS=1E11 + NEFF=1.001 NSS=1E12 TPG=-1 RSH=10.25 CGDO=4.83117E-10 + CGSO=4.83117E-10 CGBO=1.293E-9 CJ=0.0001307 MJ=0.4247 + CJSW=4.613E-10 MJSW=0.2185 PB=0.75 XQC=1 UO=271)

Table 2. Dimensions of MOS devices used in CMOS OTA for SPICE simulation

MOS Tranzistor	W(μm)	L(μm)
M1	30	3
M2	30	3
M3	12	3
M4	12	3
M5	12	3
M6	36	3
M7	12	3
M8	36	3
M9	45	3

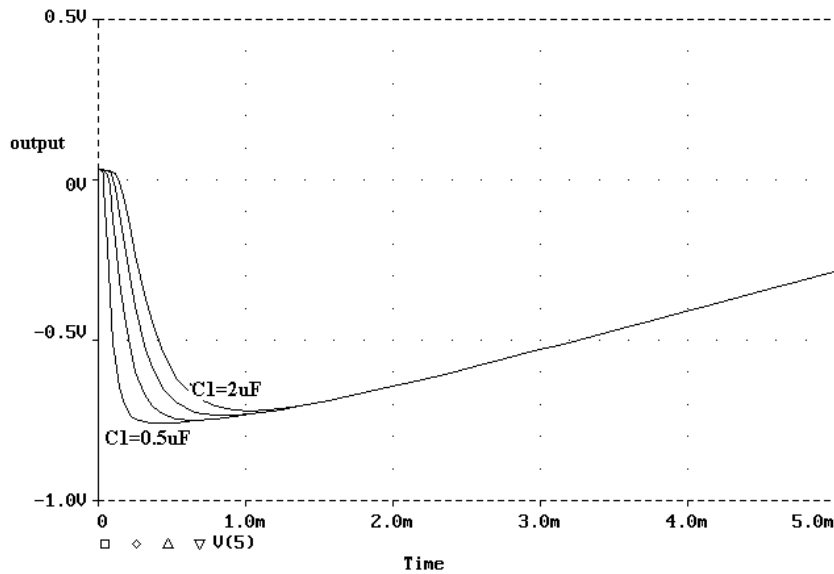


Figure 6. Result of the simulation of the OTA-C based PID controller (For $C_1 = 0.5\mu\text{F}, 1\mu\text{F}, 1.5\mu\text{F}, 2\mu\text{F}$)

The simulation results of the output of the OTA-C based PID controller for $C_2 = 1 \mu F$ is given in Fig. 6 for various values of the derivation capacitor C_1 . Similarly, the simulation results for the output of the OTA-C based PID controller for $C_1 = 1 \mu F$ are also given for various values of the integration capacitor C_2 in Fig. 7. In both situations, the proportional gains are taken to be constant and equal to each other for better illustrations.

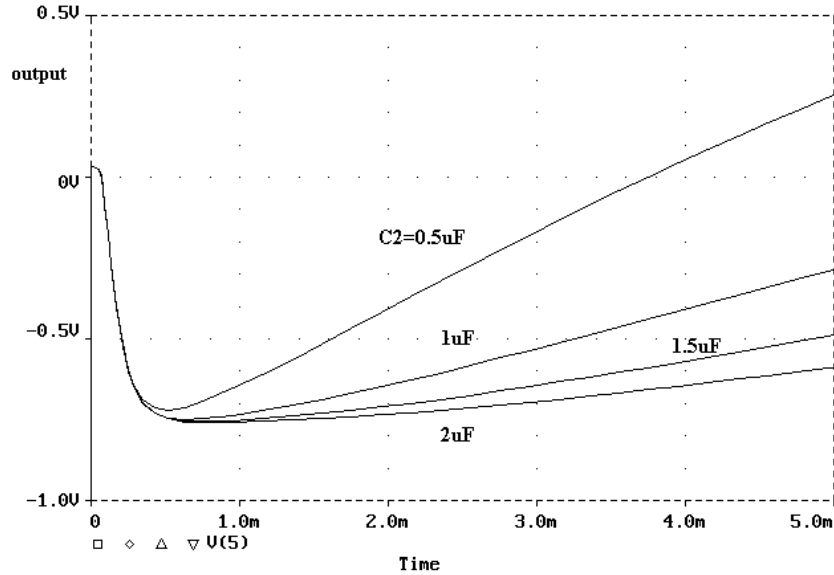


Figure 7. Result of the simulation of the OTA-C based PID controller (For $C_2 = 0.5\mu F, 1\mu F, 1.5\mu F, 2\mu F$)

From Figs. 6 and 7 it is clear that the results are in good agreement with the theoretical expectations.

Bipolar OTAs with a larger transconductance can be used to realize a PID controller as well. In this case, the finite input resistance must be taken into account when the non-idealities of the circuit are considered.

5. Calculating Optimum Parameter Tolerances

The optimum parameter tolerances are defined as the tolerances whose contributions to the upper bound of the relative error, $|\Delta V_0/V_0|$ at the output of the controller given in Fig. 4 are equal to each other. This type of definition of optimum tolerances is quite reasonable since the designer expects the contribution of each parameter variation on output deviation to be equal. In general, the designer does not know in advance how much each parameter contributes to the output error. Using the above definition of the optimum tolerances, we are sure that, considering the upper bound of the error, all parameter deviations contribute equally to the output deviation. Moreover, optimum tolerances are generally not equal to each other. Formulation of these tolerances was given by Erdal [6,7]. Considering this fact, we can define the optimum parameter tolerances as

$$t_{x_i} = t_0/n|S_{x_i}^T(\omega_i)|, \quad i = 1, \dots, 10 \quad (5)$$

where t_{x_i} is the i th parameter tolerance; t_0 is the output tolerance of the controller, n is the parameter number, i.e., $n = 10$ for the given configuration, and ω_i is the angular frequency at which $|S_{x_i}^T(\omega)|$ takes its maximum value, i.e.,

$$\left| S_{x_i}^T(\omega_i) \right|_{\max} = \max \left\{ \left| S_{x_i}^T(\omega) \right| \right\}, \quad \omega \in [\omega_1, \omega_2] \quad (6)$$

where $\omega \in [\omega_1, \omega_2]$ describes the designer's specified frequency band. Hence $|S_{x_i}(\omega)| \leq |S_{x_i}^T(\omega_i)|$, $\omega \in [\omega_1, \omega_2]$. It should be noted that ω_i belongs to the interval $\omega \in [\omega_1, \omega_2]$, and $|S_{x_i}^T(\omega)|$ has its maximum value at this frequency. The designer can easily determine ω_i by plotting $|S_{x_i}^T(\omega)|$ at this interval or by using already existing mathematical programs like Matlab.

For example, let us assume that the proportional gain, $K_P = 10$, the integral gain, $K_I = 2 \text{ s}^{-1}$, and the derivative gain, $K_D = 5 \text{ s}$, are given. Then the parameter values can be selected in Fig. 4 as follows:

$$g_{m1} = g_{m5} = g_{m6} = 2 \text{ mA/V}; \quad (7a)$$

$$g_{m2} = g_{m3} = g_{m4} = g_{m7} = g_{m8} = 0.2 \text{ mA/V}; \quad (7b)$$

$$C_1 = 10\mu F; \quad C_2 = 100\mu F. \quad (7c)$$

Assuming that the designer wants $|\Delta V_o/V_o|$ to be less than or equal to 0.01, then the parameter tolerances are to be chosen as follows:

$$t_{g_{mk}} = t_{C_1} = t_{C_2} = 0.1\%, \quad k = 1, \dots, 8 \quad (8)$$

Although optimum tolerances generally are not equal to each other, for this particular example they all turned out to be equal to each other. Choosing the parameter tolerances such as above, the designer can guarantee that the maximum deviation due to the parameter variations because of the environmental effects, at the output of the controller, will be less than or equal to 0.01.

If the designer wants $|\Delta V_o/V_o|$ to be less than or equal to 0.1, then the parameter tolerances must be chosen ten times larger than the ones in Eq. (8) and so forth.

6. Conclusions

In this study, an OTA-C based PID design procedure is given first and a PID circuit is proposed. This proposed circuit, which consists of only eight OTAs and two grounded capacitors, is very convenient to integrate in a chip. This circuit is also very suitable in situations when the stable control is required since OTAs have a broader band than operational amplifiers. Besides the controller gains, K_P , K_I and K_D depend only on the control voltages of OTAs, so they can easily be adjusted to any desired value and this is another advantage of the circuit.

Furthermore, the optimum parameter tolerances are determined. These tolerances keep the relative error of the output of the OTA-C based PID controller circuit due to parameter variations in the prescribed tolerance region.

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