[Technical Report]

Mathematical Model of One-dimensional Miscible Fluid Injection in Fractured Porous Media

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A mathematical model for miscible displacement in fractured porous media is developed. The model takes into account advective, gravitational and crossflow mechanisms of mass exchange between fracture and matrix. The model is normalized by using the dimensionless parameters which characterize the process, and the analytical solutions of the resulting system of equations are provided by utilizing the method of characteristics. The model developed has been compared with experimental results and with previous model, which includes only crossflow between fracture and matrix. There is very good agreement between experiments and this model prediction.

Keywords

Fractured porous media, Method of characteristics, Advection mass transfer, Miscible displacement, Mathematical modeling

1. Introduction

Fractured reservoirs (*e.g.* Asmari reservoirs in Iran) provide over 20% of the oil reserves in our nation. Iran is one of the world's leading energy producing countries with an estimated 9% of the world's remaining recoverable oil reserves and 17% of its natural gas reserves. Almost 90% of Iranian petroleum reservoirs are carbonated and these carbonated reservoirs are generally tight, and flow condition for oil in the matrix are poor. Therefore, the time needed to produce the oil will be longer than for high permeable sandstone reservoirs. Iranian carbonate reservoirs are fractured and consist of tight matrix blocks with fractures in between. The recovery factor for the Iranian fractured reservoirs is estimated to be in the range of 20 to 30%[1)]. In these reservoirs, the block heights are of the order of 3 to 15 m.

The declining oil production from Iranian fractured reservoirs after several decades of exploitation and the significant amount of oil still remaining in place are of great concern to the Iranian oil company and fully justify its interest in enhanced oil recovery (EOR) processes. One of the important mechanisms in EOR from fractured reservoirs is miscible fluid injection. Miscible fluid injection allows to recover substantial quantities of that oil trapped in the matrix.

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Miscible displacement within fractured porous media till recent time has not been sufficiently investigated. There are known only separate studies devoting to the interpretation of the field research data on intersoluble fluid displacement from heterogeneous porous media[2),3)]. Theoretical work on miscible displacement in fractured porous media is limited to a few paper published in the Russian literature. These Russian authors did not address the basic issues of miscible displacement in fractured porous media[4),5)]. A general model by, Bedrikovetsky *et al.* and Basniev and Bedrikovetsky includes the effect of gravity, diffusion, and convection. However, the assumptions which are required to proceed with the solution of the flow equations in a 3-D space are many and some unjustified. Zakirov *et al.*[6)] reported the results of experimental and theoretical investigations of miscible displacement process of compressible fluids within fractured reservoirs. On the basis of their studies the fluid miscible filtration mechanism within naturally fractured reservoirs was established. These researchers neglected crossflow between fracture and matrix. Thompson and Mungan[7)] reported the results of an experimental study on gravity drainage in fractured porous media under first contact miscible conditions. They used vertically mounted 2" diameter fractured and unfractured cores. Soltrol 130 was used as the oil and liquid normal butane as displacing fluid. Thompson and Mungan mainly studied the effect of displacement rate on recovery efficiency and didn't present any mathe-

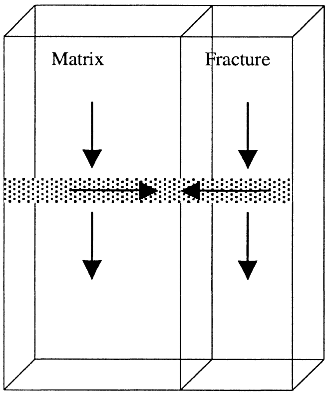

Fig. 1　Material Balance around Matrix and Fracture

matical model for their work. Tan and Firoozabadi[8] and Firoozabadi[9] used vertically blocks cores to study miscible displacement in fractured porous media. They concluded capillary pressure contrast of the fracture and the matrix is major parameter which causes low recovery efficiency of fractured reservoirs and reduction of reinfiltration (through elimination of capillary pressure) is one positive element of miscible displacement in fractured porous media. They neglected the advection mass transfer between matrix and fracture. They also presented a theory for case of no crossflow between matrix and fracture. They anticipated piston-like fronts propagating in both fracture and matrix. The prediction results base on their model had a very different trend than experimental data.

The purpose of this work is to provide a theoretical analysis of the miscible displacement in fractured porous media. Physical concepts related to the process will be emphasized in this work. In this study, a one-dimensional model will be used. As will be seen, the one-dimensional model can capture the main features of miscible displacement in fractured porous media.

2. Governing Equations

We assume that (a) all fluids are incompressible and first contact miscible, (b) each media has uniform properties, (c) viscous fingering is negligible, (d) the displacement is strictly one dimensional in two media, (e) there is no volume change of mixing, (f) diffusive mass transfer is negligible, and (g) viscosity is a linear function of solvent concentration.

Using the volume element in **Fig. 1** mass balance on a medium 1(fracture) yields:

Mass In − Mass Out = Mass Accumulated

$$\Delta t(C_1q_1|_x - C_1q_1|_{x+\Delta x} - \mathrm{SCR}\Delta x + m_1(C_2 - C_1)\Delta x) = \Delta x\varphi_1 h_1 w(C_1|_{t+\Delta t} - C_1|_t) \quad (1)$$

For the adjacent medium 2 (matrix), the solute cross-flowing out of medium 1 must necessarily end up on medium 2. Therefore, a similar mass balance yields

$$\Delta t(C_2q_2|_x - C_2q_2|_{x+\Delta x} + \mathrm{SCR}\Delta x - m_2(C_2 - C_1)\Delta x) = \Delta x\varphi_2 h_2 w(C_2|_{t+\Delta t} - C_2|_t) \quad (2)$$

In, Eqs. (1) and (2), SCR is solute crossflow term, which is determined as follows. The two sides of Eqs. (1) and (2) are divided to $\Delta t\Delta x$ and then both Δt and Δx are approached to zero. Thus,

$$-\frac{\partial(C_1q_1)}{\partial x} - \mathrm{SCR} + m_1(C_2 - C_1) = \varphi_1 A_1\frac{\partial C_1}{\partial t} \quad (3)$$

$$-\frac{\partial(C_2q_2)}{\partial x} + \mathrm{SCR} - m_2(C_2 - C_1) = \varphi_2 A_2\frac{\partial C_2}{\partial t} \quad (4)$$

The total mass balance (solute and solvent) in medium 1 yields:

Total crossflow = solute crossflow + solvent crossflow thus,

$$\text{Total crossflow} = -\frac{\partial q_1}{\partial x} \quad (5)$$

Therefore, solute crossflow will be:

$$\mathrm{SCR} = -C_1\frac{\partial q_1}{\partial x} \quad (6)$$

Hence, Eqs. (3) and (4) become:

$$\varphi_1 A_1\frac{\partial C_1}{\partial t} + q_1\frac{\partial C_1}{\partial x} - m_1(C_2 - C_1) = 0 \quad (7)$$

$$\varphi_2 A_2\frac{\partial C_2}{\partial t} + \frac{\partial(q_2C_2)}{\partial x} + C_1\frac{\partial q_1}{\partial x} + m_2(C_2 - C_1) = 0 \quad (8)$$

In addition to mass balance equations, we need to Darcy's law, density-concentration and viscosity-concentration relations:

$$\frac{\partial P_1}{\partial x} = -\frac{\mu_1 q_1}{k_1 A_1} + \rho_1 g \quad (9)$$

$$\frac{\partial P_2}{\partial x} = -\frac{\mu_2 q_2}{k_2 A_2} + \rho_2 g \quad (10)$$

$$\rho_1 = \rho_{\mathrm{solvent}} + C_1\Delta\rho \quad (11)$$

$$\rho_2 = \rho_{\mathrm{solvent}} + C_2\Delta\rho \quad (12)$$

$$\mu_1 = \mu_{\mathrm{solvent}} + C_1\Delta\mu \quad (13)$$

$$\mu_2 = \mu_{\mathrm{solvent}} + C_2\Delta\mu \quad (14)$$

$$\Delta\rho = \rho_{\mathrm{solute}} - \rho_{\mathrm{solvent}} \quad (15)$$

$$\Delta\mu = \mu_{\mathrm{solute}} - \mu_{\mathrm{solvent}} \quad (16)$$

In above equations, C_i is the solute concentration in medium i ($C_1 = 1$ for solute and $C_1 = 0$ for solvent). Subscripts 1, 2 refer to medium 1 (fracture) and medium 2 (matrix) and m_i is advection mass transfer coefficient. Grigorievich and Archer[10] and Bedrikovetsky and Evtjukhin[11] in their works showed that

$$m_1 = \frac{|q_2|A_1}{A_2 l_{\mathrm{w}}} \quad (17)$$

$$m_2 = \frac{|q_2|}{l_{\mathrm{w}}} \quad (18)$$

where l_{w} is average block size.

The above equations have to be solved with proper initial and boundary conditions. The initial and boundary conditions are

$C_1 = C_2 = 1$, $t = 0$ and $x = x$ (19)

Q_{t} = constant, $t = t$ and $x = x$ (20)

$C_1 = C_2 = 0$, $t = t$ and $x = 0$ (21)

Using dimensionless variables Eqs. (7) and (8) are transformed to

$$\alpha_1 \frac{\partial C_1}{\partial t_{-D}} + q_{-1D} \frac{\partial C_1}{\partial x_{-D}} + m_{1D}(C_1 - C_2) = 0 \qquad (22)$$

$$\alpha_2 \frac{\partial C_2}{\partial t_{-D}} + q_{-2D} \frac{\partial C_2}{\partial x_{-D}} + (C_1 - C_2)\frac{\partial q_{-1D}}{\partial x_{-D}} - m_{2D}(C_1 - C_2) = 0 \qquad (23)$$

For crossflow from matrix (medium 2) to fracture (medium 1) mass balance equations are

$$\alpha_1 \frac{\partial C_1}{\partial t_{-D}} + q_{-1D} \frac{\partial C_1}{\partial x_{-D}} + (C_1 - C_2)\frac{\partial q_{-1D}}{\partial x_{-D}} + m_{1D}(C_1 - C_2) = 0 \qquad (24)$$

$$\alpha_2 \frac{\partial C_2}{\partial t_{-D}} + q_{-2D} \frac{\partial C_2}{\partial x_{-D}} - m_{2D}(C_1 - C_2) = 0 \qquad (25)$$

where

$$t_{-D} = \frac{Q_t t}{\phi AL},\ x_{-D} = \frac{x}{L},\ q_{-1D} = \frac{q_1}{Q_t},\ q_{-2D} = \frac{q_2}{Q_t},$$
$$\alpha_1 = \frac{\varphi_1 A_1}{\phi A},\ \alpha_2 = \frac{\varphi_2 A_2}{\phi A}$$
$$\phi A = \varphi_1 A_1 + \varphi_2 A_2,\ m_{1D} = \frac{q_{2D} L A_1}{l_w A_2},\ m_{2D} = \frac{q_{2D} L}{l_w} \qquad (26)$$

The viscosity-concentration relations in the form of dimensionless variables become

$\mu_{-1D} = (1 + (v - 1)C_1)/v$ (27)

$\mu_{-2D} = (1 + (v - 1)C_2)/v$ (28)

where

$v = \mu_{solute}/\mu_{solvent}$ (29)

initial and boundary conditions with these dimensionless variables are

$C_1 = C_2 = 1$, $t_{-D} = 0$ and $x_{-D} = x_{-D}$ (30)

$Q_{-tD} = 1$, $t_{-D} = t_{-D}$ and $x_{-D} = x_{-D}$ (31)

$C_1 = C_2 = 0$, $t_{-D} = t_{-D}$ and $x_{-D} = 0$ (32)

Also, the total material balance is

$q_{-1D} + q_{-2D} = 1$ (33)

Let us assume the pressures of fluid in the fracture and in the matrix are equal. The assumption of the equality of pressures in fracture and matrix means the simultaneous hydraulic interaction. So the oil from matrix pushes the solvent in fracture up. The buoyancy force accelerates the flow in matrix and reduces the velocity of the flow in fracture. Thus,

$$\frac{\partial P_1}{\partial x} = \frac{\partial P_2}{\partial x} \qquad (34)$$

Combining Eqs. (9)-(16) and Eqs. (33) and (34), leads to

$$q_{-1D} = \frac{\mu_{-2D}\beta - \gamma\beta(C_2 - C_1)\left(\dfrac{A_2}{A_t}\right)}{\mu_{-1D} + \beta\mu_{-2D}} \qquad (35)$$

and

$$q_{-2D} = \frac{\mu_{-1D} + \gamma\beta(C_2 - C_1)\left(\dfrac{A_2}{A_t}\right)}{\mu_{-1D} + \beta\mu_{-2D}} \qquad (36)$$

where

$$\beta = \frac{k_1 A_1}{k_2 A_2} \qquad (37)$$

and

$$\gamma = \frac{\Delta\rho k_2 A_t}{\mu_{solute} Q_t} \qquad (38)$$

The formulation is now complete. Equations (22) through (25) must be solved simultaneously. Solution can be obtained from the method of characteristics. Utilizing method of characteristics, Eqs. (22) through (25) are formulated as an eigenvalue problem.

First, we must convert the Eqs. (22)-(25) as transport (traffic) equations[12]. This can be accomplished by substituting

$C_1 = U_1 \exp(-\xi_1)$ (39)

$C_2 = U_2 \exp(-\xi_2)$ (40)

where

$$\xi_1 = 0.5 m'_{1D}\left(\frac{t_{-D}}{\alpha_1} + \frac{x_{-D}}{q_{-1D}}\right) \qquad (41)$$

$$\xi_2 = 0.5 m'_{2D}\left(\frac{t_{-D}}{\alpha_2} + \frac{x_{-D}}{q_{-2D}}\right) \qquad (42)$$

$$m'_{1D} = m_{1D}\left(1 + \frac{\alpha_1 A_2}{\alpha_2 A_1}\right) \qquad (43)$$

$$m'_{2D} = m_{2D}\left(1 + \frac{\alpha_2 A_1}{\alpha_1 A_2}\right) \qquad (44)$$

Substitution of Eqs. (39) and (40) into Eqs. (22)-(25),

$$\alpha_1 \frac{\partial U_1}{\partial t_{-D}} + q_{-1D}\frac{\partial U_1}{\partial x_{-D}} = 0 \qquad (45)$$

$$\alpha_2 \frac{\partial U_2}{\partial t_{-D}} + q_{-2D}\frac{\partial U_2}{\partial x_{-D}} + (U_1 - U_2)\frac{\partial q_{-1D}}{\partial x_{-D}} = 0 \qquad (46)$$

for region I and

$$\alpha_1 \frac{\partial U_1}{\partial t_{-D}} + q_{-1D}\frac{\partial U_1}{\partial x_{-D}} + (U_1 - U_2)\frac{\partial q_{-1D}}{\partial x_{-D}} = 0 \qquad (47)$$

$$\alpha_2 \frac{\partial U_2}{\partial t_{-D}} + q_{-2D}\frac{\partial U_2}{\partial x_{-D}} = 0 \qquad (48)$$

for region II.

With these new variables, Eqs. (35) and (36) become

$$q_{-1D} = \frac{\mu_{-2D}\beta - \gamma\beta\big(U_2\exp(-\xi_2) - U_1\exp(-\xi_1)\big)\left(\dfrac{A_2}{A_t}\right)}{\mu_{-1D} + \beta\mu_{-2D}} \qquad (49)$$

$$q_{-2D} = \frac{\mu_{-1D} + \gamma\beta\big(U_2\exp(-\xi_2) - U_1\exp(-\xi_1)\big)\left(\dfrac{A_2}{A_t}\right)}{\mu_{-1D} + \beta\mu_{-2D}} \qquad (50)$$

The system of equations in matrix form is:

$\mathbf{A}\,\mathbf{U} = \eta\mathbf{U}$ (51)

where

$$\mathbf{A} = \begin{bmatrix} A_{11} \cdots\cdots A_{12} \\ A_{21} \cdots\cdots A_{22} \end{bmatrix} \qquad (52)$$

$$\mathbf{U} = \begin{bmatrix} \dfrac{\partial U_1}{\partial x_{-D}} \\ \dfrac{\partial U_2}{\partial x_{-D}} \end{bmatrix} \qquad (53)$$

The elements of matrix **A** for two regions (region I for crossflow from fracture to matrix and region II for crossflow from matrix to fracture) are defined as

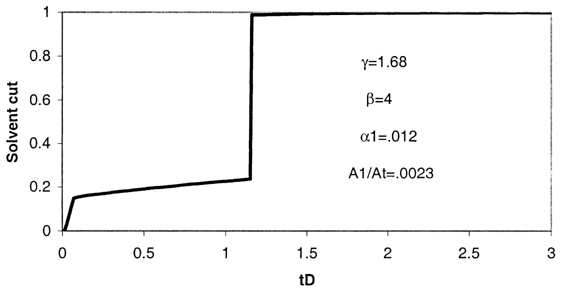

Fig. 2 Calculated Solvent Cut *vs.* Dimensionless Time

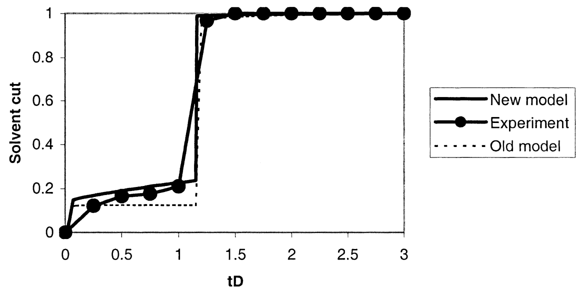

Fig. 3 Comparison between New Model and Old Model and Experiment Results (rate = 2.82×10^{-9} m^3/s)

$$A_{11} = \frac{q_{-1D}}{\alpha_1},\ A_{12} = 0,\ A_{21} = \frac{U_1 - U_2}{\alpha_2} \times \frac{\partial q_{-1D}}{\partial U_1}$$

$$A_{22} = \frac{q_{-2D}}{\alpha_2} + \frac{U_1 - U_2}{\alpha_2} \times \frac{\partial q_{-1D}}{\partial U_2} \quad (54)$$

for region I and

$$A_{11} = \frac{q_{-1D}}{\alpha_1} + \frac{U_1 - U_2}{\alpha_1} \times \frac{\partial q_{-1D}}{\partial U_1},\ A_{12} = \frac{U_1 - U_2}{\alpha_1} \times \frac{\partial q_{-1D}}{\partial U_2}$$

$$A_{21} = 0,\ A_{22} = \frac{q_{-2D}}{\alpha_2} \quad (55)$$

for region II.

Thus the eigenvalues (characteristic directions) for both regions respectively are

$$\eta^{\mathrm{I}} = A_{11} \text{ and } \eta^{\mathrm{II}} = A_{22} \quad (56)$$

Note that there is only one non-trivial characteristic direction.

The solution along the characteristic for each region is:

$$\frac{\mathrm{d}U_2}{\mathrm{d}U_1} = \frac{A_{21}}{A_{11} - A_{22}} \quad (57)$$

for region I and

$$\frac{\mathrm{d}U_1}{\mathrm{d}U_2} = \frac{A_{12}}{A_{22} - A_{11}} \quad (58)$$

for region II.

A convenient way to represent these solutions in graphical form is the time-distance diagram (characteristic directions) and the concentration path diagram (solution along the characteristic). The characteristic directions in their integrated form appear as an infinite number of straight lines fanning out from the common origin. The concentration path diagram and time-distance diagram provide all the information necessary to predict the solvent cut and oil recovery for two media miscible displacement.

3. Model Results

Figure 2 shows the computed solvent cut as a function of dimensionless time (t_{-D}) when there is crossflow and advection mass transfer between matrix and fracture.

4. Model and Experimental Results Comparisons

Jamshidnezhad *et al.* have provided data on miscible displacement tests for different fluids at various injection rates (2.82×10^{-9}, 4.527×10^{-9} and 1.13×10^{-8} m^3/s) in cylindrical Asmari fractured cores from Iranian reservoirs. Let us compare results of this new model with the previous model[8] and with the laboratory data. As shown in **Fig. 3** we observe that results from new model (solid line) which includes crossflow and advection mass exchange between fracture and matrix are closer to experimental results than previous model (dotted line)[8] which includes only crossflow between fracture and matrix and there is a very good agreement between experiments and new model predictions. Therefore, we can conclude that the one-dimensional model which takes into account advective, gravitational and crossflow mechanisms of mass exchange between fracture and matrix is adequate to apprehend essential features of miscible displacement in fractured porous media.

5. Conclusions

The development of a new mathematical model for miscible displacement in fractured-porous media and the laboratory data allow to make the following conclusions:

(1) A basic system of governing equations for the miscible displacement in fractured-porous media is derived. The model takes into account crossflow, gravitational and advective mechanisms of the fracture-matrix mass transfer.

(2) The model proposed contains five phenomenological parameters. In addition to normalized fracture capacity (α_1), productivity capacity ratio (β), viscosity ratio (ν) and gravity number (γ), for characterizing the miscible displacement process, the fifth parameter, advection mass transfer coefficients (m_{1D} and m_{2D}) are also introduced.

(3) Utilizing the method of characteristics provides the analytical solutions of the problem.

(4) The effect of advection mass transfer is investigated. We concluded advective mechanism improved the model predictions.
(5) Treatment of laboratory experiments by model derived shows that there is very good agreement between experiments and model predictions.

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Nomenclatures

C	: volumetric concentration per unit volume	[m^3/m^3]
m	: advection mass transfer coefficient	[m^2/s]
h	: height	[m]
H	: height	[m]
k	: permeability	[m^2]
L	: length	[m]
v	: viscosity ratio	[—]
P	: pressure	[Pa]
q	: volumetric flow rate	[m^3/s]
Q_t	: total volumetric flow rate	[m^3/s]
t	: time	[s]
W	: width	[m]
x	: distance	[m]
<Greeks>		
α	: define in Eq. (26)	[—]
β	: define in Eq. (37)	[—]
ρ	: density	[kg/m^3]
φ	: porosity	[fraction]
ϕ	: porosity	[fraction]
μ	: viscosity	[Pa・s]
γ	: define in Eq. (38)	[—]
<Subscripts>		
1, 2	: medium index (fracture = 1, matrix = 2)	
D	: dimensionless	
t	: total	

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要　旨

フラクチャー孔げき岩体におけるミシブル攻法の一次元数学モデル

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フラクチャー孔げき岩体内でのミシブル置換を表現する数学モデルを構築した。このモデルはフラクチャーとマトリックス間における物質交換に関して，移流，重力，それにクロスフローのメカニズムを考慮に入れている。モデルはプロセスを特徴づける無次元パラメーターによってノーマライズされており，結果としての数式システムの解析解は特性曲線法を応用することで得ることができた。構築したモデルは実験値と比較され，またフラクチャーとマトリックス間のクロスフローのみを考慮した以前のモデルとも比較された。そして実験値とモデル予測値との間には良好な一致のあることが確認できた。