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COLLISIONAL PLANETARY RINGS: THE ROLE OF THE WIDTH AND THE OPTICAL DEPTH

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RESUMEN

Se demuestra analíticamente que la torca ejercida por un satélite sobre un anillo planetario (no-colisional) centrado en una resonancia de primer orden, depende exclusivamente del producto Wt, donde W es el ancho físico del anillo y t el tiempo desde que se "crea" el satélite. En $t < 0$ las partículas están en órbitas keplerianas no perturbadas alrededor del planeta. El trabajo analítico se desarrolla al orden más bajo posible en las variables perturbadas. Se presentan también resultados numéricos de la integración de las ecuaciones completas de movimiento de las partículas que forman los anillos (con y sin colisiones). Los resultados numéricos para el caso sin colisiones muestran que: *i*) la torca es función del producto tW para $t < t_r$ donde t_r es el tiempo en que el problema se vuelve nolineal. *ii*) En presencia de colisiones la torca es función de tW para $t < t_{coll} < t_r$, donde t_{coll} es el tiempo medio entre colisiones. Para $t > t_{coll}$ la torca es constante o comienza a disminuir dependiendo del valor de t_{coll} y de la profundidad del "gap" que se abre.

ABSTRACT

We show analytically that the torque exerted by a satellite over a (collisionless) planetary ring centered at a first order resonance, is a function only of the product Wt, where W is the width of the ring and t is the time since the satellite is "created". For $t < 0$ particles are in unperturbed keplerian orbits around the planet. The analytical work is made to lowest possible order in the perturbed variables. We also show the results of numerical integrations of the full Newtonian equations of motion for collisionless and collisional rings and show that: i) For collisionless rings, the value of the mean torque is a function of tW for $t < t_r$, where t_r represents the time when non-linearities are no longer negligible. ii) For collisional rings, the mean torque is a function only of tW as long as $t < t_{coll} < t_r$, where t_{coll} is the mean time between consecutive collisions among the particles. For $t > t_{coll}$, the mean torque stays constant or starts diminishing depending on the value of the optical depth and the depth of the gap that opens at the resonance.

Key Words: **ORBITAL RESONANCES — PLANETARY RINGS — SATELLITE TORQUES**

Most of the structure observed in planetary rings is located near resonances with the satellites that orbit around (or within) them. This work has two parts. The first one is analytical and the second is numerical. In the analytical approach we calculate the initial response of an annulus made of individual particles (constant surface density) centered at a first order resonance with a small satellite. Equations are solved to lowest possible order in the perturbed variables and at $t = 0$ the satellite is created. For $t < 0$ particles are in unperturbed

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keplerian orbits around the planet. The change in angular momentum per unit semimajor axis is integrated over semimajor axis. When the limits in semimajor axis $\rightarrow \infty$, one retrieves the constant linear standard torque, $\mathcal{T}_{GT}$, first derived by Goldreich & Tremaine (1978). When one uses finite limits, it is shown that the torque normalized to $\mathcal{T}_{GT}$ exclusively depends on the product tW, where W is the physical width of the annulus and t is the time.

In the numerical simulations the full Newtonian equations of motion for each test particle in the annulus are integrated individually in the frame of the restricted three body problem. Initially, particles are uniformly distributed in semimajor axis on both sides of the resonant semimajor axis (a 2:1 Inner Lindblad Resonance). As the integration of each particle's orbit proceeds, values of angular momentum are saved for a predetermined set of time instants. At the end the code adds up the angular momentum of all of the particles in the annulus at each instant of time obtaining the time evolution of the total angular momentum of the annulus. In the first experiments no collisions are introduced. Simulations were perfomed for annuli of different widths. An initial transient (negative) torque is observed for all of the annuli. The absolute value of this initial torque increases as the width of the annulus is increased. It is also shown that the value of the torque during the transient is not constant in time, it is a function that depends exclusively on the product tW, as predicted in the analytical work; the transient lasts up to, $t = t_r$ where t_r represents the time when non-linearities are no longer negligible. The growth of non-linearities causes the initial transient (negative) torque to reverse in sign and eventually, as the particles get out of phase in their periodic exchange of angular momentum with the satellite, the torque becomes null as expected in a dissipationless system (see Espresate & Lissauer 1999). From this simulations it is also seen that there is minimum width, W_m, such that if $W \geq W_m$, the initial transient torque significantly approaches 95% of $\mathcal{T}_{GT}$ *before* the problem becomes non-linear at $t = t_r$ and it can be seen that if $W \rightarrow \infty$ the initial transient torque asymptotically approaches $\mathcal{T}_{GT}$. For $W < W_m$ the torque does not get close to the linear standard torque before the problem becomes nonlinear.

For the collisional simulations, the same procedure is followed as in the collisionless case except that collisions are introduced artificially during the numerical integration of individual particles in a pseudo-random way (see Espresate 1997). The mean time between collisions goes as the inverse of the optical depth, τ. There is a critical value of the optical depth τ_c such that, if $\tau < \tau_c$ and $W > W_m$, it is not possible to obtain a secular torque close to $\mathcal{T}_{GT}$ (after the initial transient). That is, for $t > t_r$ the torque decreases in absolute value and stays practically constant for the rest of the integration interval. For $\tau > \tau_c$, the torque critically depends on the width. The larger is the optical depth, the larger is the minimum width required to approach $\mathcal{T}_{GT}$. The reason behind this result is that the region over which most of the torque is exerted, shrinks as $1/t$. The mean time between collisions, t_{coll} goes as the inverse of the optical depth. One can imagine that when $t \sim t_{coll}$ most of the particles have suffered at least one collision and the whole system has lost memory of where it was and where to go, therefore, the system reaches an equilibrium and its temporal evolution is halted. The shorter is t_{coll} (larger optical depth), the region over which most of the torque is exerted is larger or the larger is t_{coll}, the more concentrated the torque is around the resonance and the width required to obtain a torque very close to $\mathcal{T}_{GT}$ is decreased. This relation between width and optical depth is only valid for $\tau > \tau_c$. If $\tau < \tau_c$, when the system halts its time evolution, non-linearities are somewhat (or fully) developed and analytical predictions are not expected to be valid. Finally if $\tau > \tau_c$, the value of the torque at $t = t_{coll}$ when the system halts its time evolution exclusively depends on the product Wt_{coll}.

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