# USING THE WATER-BUDGET METHOD FOR THE DETERMINATION OF EVAPORATION FROM A LAKE

by

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#### Abstract

The water-budget method together with air humidity, air and water temperature and wind velocity was used for obtaining an equation for computing the evaporation from a lake. The period with witch this equation was obtained consisted of 21 rainless days. Then the evaporation equation was tested with an energy-budget calculation. This test made it clear, that the water-budget method may be useful, but for obtaining a more precise evaporation formula one must have more observation data.

#### Introduction

The program of the finnish IHD-activity in the representative basin Pääjärvi includes a study of evaporation from the central lake Pääjärvi of this basin. This study is carried out by a working group which is staffed by the Finnish Hydrological Office and the Meteorological Institute. The costs of the study have been paid by these institutes and the National Council for Sciences.

In this paper only the water-budget method for computing evaporation is used. The applicability of the method is restricted by the fact that the inflow into the lake should not be great as compared to the evaporation. A suitable period for this may be found in the year 1969 from July 26 to August 22 during which period no precipitation was measured at the lake. The observation instrumentation was being set up in that year only, so there are gaps in the observation data of discharge and in the operation of some meteorological instruments. For this reason a final evaporation formula cannot be obtained in this study but it may be investigated, if the instrumentation used is suitable for determination of evaporation.

# The Lake Pääjärvi

The lake Pääjärvi ( $\varphi=61^{\circ}3'N$ ,  $\lambda=25^{\circ}9'E$ ) and its situation can be seen in Fig. 1. The area of this lake is 13,6 km². Its mean depth is 14 m and the greatest depth nearly 80 m. The catchment area of the lake is 212 km² and the lake percentage 2. The amount of streamlets running to the lake is 25, the greatest of which has a catchment area 83 km². The lake is regulated for agricultural purposes and the regulation height is 80 cm.

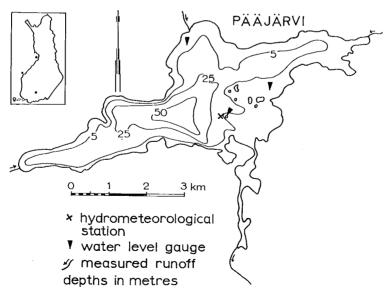


Fig. 1. Location map of the study area.

# Methods of measurements

The surface inflow to the lake was measured in three streamlets. These have a total catchment area of 138 km², or 65 % of the whole catchment area of the lake. Inflow from other streamlets and from land bordering the lake was estimated using the known amounts of inflow and size of the catchment areas. At the beginning of the period under study the total surface inflow was 3 mm/day and at the end of this period the inflow was 0,7 mm/day. The data of surface outflow from the lake is based on the discharge measurements carried out once a week, water level observations and measurements of the openings of the regulation dam. The surface outflow from the lake was 1,5 mm/day at the beginning of the period and 0,7 mm/day at the end of the period. Due to the regulation of the lake the outflow changed during three days. The greatest daily mean outflow was 4,4 mm/day.

Precipitation measurements were made at five precipitation stations situated around the lake and at one station situated on an islet in the lake. Because there was no precipitation during the period of investigation, it may be assumed that the precipitation data is without errors.

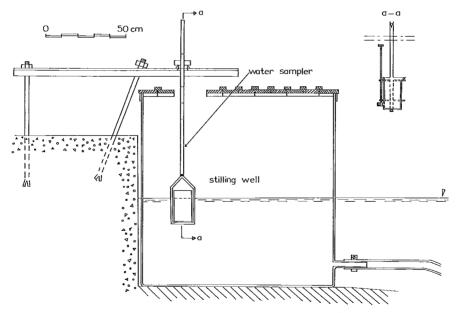


Fig. 2. Water level gauge. The water level height of the lake is obtained, when the amount of water in the water sampler is weighed.

The water level of the lake was measured with the instrument shown in Fig. 2. The purpose of the great tank is to damp the effect of the wind waves. Because the horizontal area of the water sampler is 100 cm<sup>2</sup>, the change of 1 mm of water level corresponds to an amount of 10 g of water. Three such units were placed in the lake (see Fig. 1). The observations were carried out every day at about 8 a.m. and p.m. It is difficult to estimate the real accuracy of this instrument. Perhaps at windless wheather we can obtain an accuracy close to 0,1 mm, but during heavy wind the result of observation is less accurate. It was noticed, that the observation made in the morning is better than that made in the evening. This may be due to the fact, that the effect of temperature on the instrument is less in the morning than in the evening.

Because the water-budget method alone is not sufficient for evaporation determinations, measurements of some meteorological quantities were also made. For determination of the energy-budget of the lake, measurements of the radiation balance were made at the hydrometeorological station in the lake (see Fig. 1) with the Middleton intrument. The measuring height was 1,5 m above the water level. The depth of the water was 2 m at this place. Measurements of total, reflected and diffused radiation were done with Kipp & Zonen solarimeters. For the determination of the heat energy of the lake thermal surveys were made once a week at 33 points in the lake.

Air temperature and humidity was measured with FRIEDRICHS  $3 \times 101$  Pt-100 electric aspirated psychrometers. These instruments were fixed to a mast placed in the lake near the hydrometeorological station (see Fig. 1). Measuring heights were 1 m, 2 m, 4 m and 6 m. At the same heights also the wind velocity was measured with LAMBRECHT cup anemometers. A common meteorological station with temperature and air moisture recorders was placed on the beach of the islet in the hydrometeorological station.

The temperature at the lake surface was recorded with two 1000-Pt resistors. These resistors were fixed under styrox plates which were floating in the lake. The measuring depth was 1—2 cm.

# Calculation of evaporation from the water-budget

The water-budget equation of a lake can be written in the form

$$P + I_s + I_g = O_s + O_g + E + \triangle W / \triangle t \tag{1}$$

where

P is precipitation (mm/day)

 $I_s$  surface inflow (mm/day)

 $I_g$  ground-water inflow (mm/day)

 $O_s$  surface outflow (mm/day)

 $O_g$  ground-water outflow (mm/day)

E evaporation (mm/day)

 $\triangle W$  change of water level (mm)

 $\triangle t$  time period (day)

The measured quantities in equation (1) are P,  $I_s$ ,  $O_s$  and  $\triangle W/\triangle t$ . The unknown quantities are E,  $I_g$  and  $O_g$ . Because the period studied was rainless and thus the water level of the lake is steadily decreasing, the bank storage may be thought to be constant and it is included in the quantity  $I_g$ .

In the following the same method is applied as is done in the United States for several lakes [3, 1, 7, 9]. As the ground-water flow is changing very slowly, the nett seepage  $O_g - I_g$  may be considered to be constant, or

$$O_{g} - I_{g} = c \tag{2}$$

In an earlier analysis [8] based on the same period as in this study, it was assumed, that the nett seepage is linearly dependent on time. However, the time-dependent term did not become significant. In that analysis the meteorological data was obtained from the common meteorological hut in the hydrometeorological station.

The most simple form for an evaporation formula is

$$E_i = au_i(e_0 - e_i) \tag{3}$$

where a is a constant, u is wind velocity,  $e_0$  is the saturated vapour pressure at the temperature of the lake surface and  $e_i$  is the vapour pressure in the air. The height of the measurement is denoted by the subscript i. Because the lake studied is rather deep, one can expect great differences between the lake surface and air temperatures affecting the vertical transport of water vapour. For this reason, in accordance with Lake Mead studies [4], an additional factor is added to the equation (3), or

$$E_{i} = au_{i} \left( 1 + \frac{b}{a} \left( T_{0} - T_{i} \right) \right) \left( e_{0} - e_{i} \right) \tag{4}$$

 $T_0$  is the temperature at the lake surface and  $T_i$  the temperature at the measuring height i. b is a constant.

Let us define a quantity  $E_w$  as

$$E_w = P + I_s - O_s - \triangle W / \triangle t \tag{5}$$

 $E_w$  can be computed from the measured data. With equations (1), (2), (4) and (5) the following formula is obtained for the mean of the  $E_w$ :

$$\bar{E}_w = \overline{au_i(e_0 - e_i)} + b\overline{u_i(e_0 - e_i)(T_0 - T_i)} + c \tag{6}$$

A bar above a symbol is denoting a twentyfour hours mean. In the following these means are computed by using day (8 a.m. to 8 p.m.) and night (8 p.m. to 8 a.m.) means of the measured quantities.

The constants a, b and c in equation (6) were determined with regression analysis from data covering the study period from 26 July to 28 August 1969. As there was malfunctioning of some instruments, all daily values cannot be used. The amount of observations in the regression analysis is 21.

The computation resulted in the following values:

i	a	b/a	$\boldsymbol{c}$	R	$s_0$
$\dot{\mathbf{m}}$	mm //m	$1/\mathrm{C}^{\circ}$	$\mathbf{m}\mathbf{m}$		$\mathbf{m}\mathbf{m}$
	$\frac{1}{\text{day}} / \left( \frac{1}{\text{s}} \text{ Hgmm} \right)$		$\overline{\mathrm{day}}$		$\overline{\mathrm{day}}$
1	0,184	0,032	-1,2	0,78	0,96
<b>2</b>	0,168	0,027	-1,3	0,79	0,92
4	0,133	0,050	-0,7	0,78	0,95

R is the total correlation coefficient and  $s_0$  the standard deviation of residuals.

From the values in the table one can see, that the correlation coefficient is rather small. From this it follows, that the values of the coefficients a, b and c are rather uncertain and thus the evaporation formula obtained ought to be tested by some independent method. In Fig. 3 the measured  $E_w$  is plotted against the computed evaporation  $E_2$ , which has been determined from the data measured at the height of 2 m.

The computed evaporation and energy-budget of the lake

By taking only the most significant terms of the energy budget, the energy equation of the lake can be written in the form

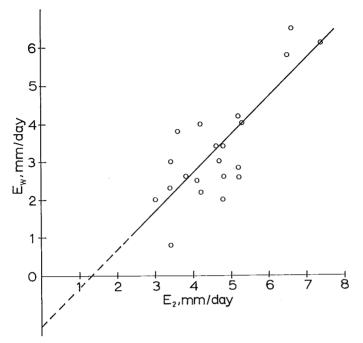


Fig. 3. Daily values of the quantity  $E_w$  plotted against the computed evaporation  $E_2 = 0.168 \; u_2 \; (1 + 0.027 (T_0 - T_2)) (e_0 - e_2) \; .$ 

$$Q_b - Q_s = Q_e + Q_a \tag{7}$$

where  $Q_b$  is radiation balance,  $Q_a$  energy utilized by evaporation,  $Q_a$  sensible heat and  $Q_s$  the change of the heat energy in the lake per time unit. Sensible heat  $Q_a$  is computed with the known  $Q_a$  and so called Bowen ratio from the equation

$$Q_a/Q_e = \gamma (T_0 - T_2)/(e_0 - e_2) \tag{8}$$

 $\gamma$  is a constant, the value of which is 0,49 Hgmm/C°.

Radiation balance  $Q_b$  was measured and  $Q_s$  was calculated with the thermal surveys carried out once a week.

In Fig. 4 the results are presented. The number indicates the ordinal of the period. The period from June 24 to July 2 is the first and the last, 18th is the period from October 21 to 30. Results have not been calculated for every period, as there has been some trouble with malfunctioning of some instruments. The ratio of the mean values of the

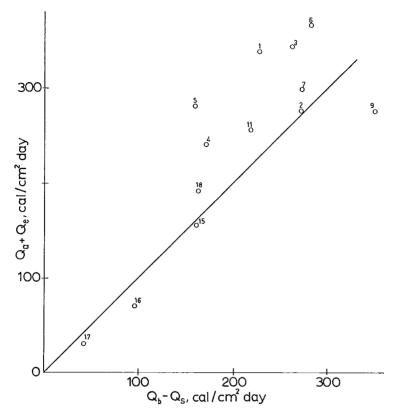


Fig. 4. Calculated sum  $Q_a + Q_e$  of energy utilized by evaporation and sensible heat plotted against the difference  $Q_b - Q_s$  between measured radiation balance and heat storage in the lake. Numbers refer to the ordinal of the period.

quantities  $Q_b - Q_s$  and  $Q_e + Q_a$  is 0,86. Because this ratio is deviating from unity there is some systematic error in the method. Probably this error has been caused by the error in the coefficient a in equation (4). Another source of error is associated with the calibration problems of the radiation balance meter. This calibration was done only for the short wave radiation. Also the use of equation (8) may carry an error. It is assumed in this equation, that the eddy conductivity for the heat energy and water vapour are equal. The value of the ratio of these coefficients has been noticed to vary between 0,5 and 1,7 [2]. If according to Halstead [5] the value 0,86 is used for this ratio, the value of the quantity  $(Q_b - Q_s)/(Q_e + Q_a)$  was computed to be 0,88.

The random deviations of points in Fig. 4 may partly result from the errors associated with the lake temperature data which is used for computing the quantity  $Q_s$ . In a deep lake as Pääjärvi the accuracy of 0,1 °C in temperature measurements made once a week is corresponding to a heat flow of 20 cal/cm² day. Also the fact that an equation like (4) does not in every situation give a true value of evaporation causes scattering of points in Fig. 4. Random errors associated with the measurements of air temperature and humidity may be considered to be small, as may be seen when comparing the measured values of evaporation obtained from different equations for  $E_i$ .

Also the terms of the advected energy ought to be taken into account in equation (7). However the amount of energy which was comming to the lake with inflow and precipitation and going away with the evaporated water and outflow was calculated to be small and was thus omitted.

### On the results

The basic assumption when using the above method is that the evaporation is stricly proportional to the quantity presented in equation (4) and that the quantities in this equation have been measured without such systematic errors which have influence on the constant c of equation (6). With the data presented in this study it is not possible to confirm the first assumption but reference is made to the studies carried out in the United States where the fluctuation of evaporation was greater. About the systematic errors of measurement perhaps the most significant one is associated with the measurements of temperature of the lake surface. Due to the long-wave radiation emitted by the body of water and evaporation there is formed a thin film at the surface the temperature of which is in common lower than the temperature below the surface. As the temperature measurements were carried out at the depth of 1 or 2 cm a systematic error may occur. In the Soviet Union [6] the difference between these two temperatures has been estimated. According to the calculation the difference may be as great as 1 C° but the monthly mean value would be about 0,1-0,2 C°. By differentiating the equation (4) one may state, that an error of 0,5 °C in the temperature  $T_0$  will correspond to an evaporation of about 0,4 mm/day at mean meteorological conditions. One part of this possible error will be included in the constant a and another part in the constant c.

Only the error entering into the constant  $\,c\,$  causes an error of the evaporation value.

As a conclusion from above one may state that in some cases the water-budget method may be used for determining the evaporation from a lake. The condition for this is that the observations of the water level must be accurate, measurements of precipitation must be good, and neither inflow nor outflow must be great. About the advantages of the method should be mentioned that the instruments are rather simple and there is no serious problem of calibrations.

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