The Consistent Histories Interpretation of Quantum Mechanics

Abstract

The consistent histories (CH) reformulation of quantum mechanics (QM) was developed by Robert Griffiths, given a formal logical systematization by Roland Omnès, and under the label 'decoherent histories', was independently developed by Murray Gell-Mann and James Hartle and extended to quantum cosmology. Criticisms of CH involve issues of meaning, truth, objectivity, and coherence, a mixture of philosophy and physics. We will briefly consider the original formulation of CH and some basic objections. The reply to these objections, like the objections themselves, involves a mixture of physics and philosophy. These replies support an evaluation of the CH formulation as a replacement

1 The Consistent Histories Formulation

The Griffiths formulation of Consistent Histories broke with the orthodox interpretation by treating closed systems, by not assigning measurement a foundational role, and by insisting that quantum mechanics supply an account of all basic processes including measurements[1].

There are three basic features. First, there is the specification of a closed system at particular times by a series of events. An event is the specification of the properties of a system through a projection operator for the Hilbert subspace representing the property. Second,the time development is stochastic, involving many histories. Though Griffiths relied on Schrödinger dynamics, he treated it as going from event to event, rather than as a foundation for unitary evolution of a system prior to measurement and collapse. The events could be stages in a uniform evolution, measurements, interaction with the environment, or a virtual interaction. At this stage there is no distinction between real and virtual processes. A history is a time- ordered sequence of events. It is represented by projectors on a tensor product of the Hilbert spaces of the events. Third, a consistency condition is imposed on histories, or families of histories. Only those that may be assigned probabilities are given a physical interpretation.

The developers of the CH interpretation are insistent on presenting this as a replacement for 'the measurement interpretation'. Why does this need replacement? For Griffiths (2002a, Preface.) and Omnès (1994, chap. 2, 1999, p. 80) the basic reason is that a measurement-based interpretation does not accord with what a fundamental theory should be. It subordinates the mathematical formalism to the language of experimental physics. A fundamental theory should supply a basis for interpreting experiments. In evaluating this we will stress the idea that formulations and interpretations involve different criteria of evaluation.

A comparison with classical physics clarifies the status accorded quantum histories. In classical physics, stochastic dynamics is generally introduced because of ignorance of precise values. Consider a fair flip of a coin n times. The 2^n possible outcomes represents a sample space with histories of the form, HHTHT... For a closer parallel, consider classical statistical mechanics,where the state of a system is represented by a point in phase space and the evolution of the system, or its history, by the trajectory of this point. The phase space may be coarse-grained by dividing it into a set of cells of arbitrary size that are mutually exclusive and jointly exhaustive. A cell will be assigned a value 1 if the point representing the system is in the cell, and has the value 0 otherwise.. We introduce a variable, B_i for these 0 and 1 values, where the subscript, i, indexes the cells. These variables satisfy

$$\sum_i B_i = 1 \qquad\qquad B_i B_j = \delta_{ij} B_j$$

This assignment of 0 and 1 values supports a Boolean algebra. To represent a history, construct a Cartesian product of copies of the phase space and let them represent the system at times $t_0, t_1, \ldots, t_n$. Then the product of the variables, , B_i, for these time slices represents a history. The relation to classical probabilities can be given an intuitive expression. The tensor product of the successive phase spaces has a volume with an a priori probability of 1. Each history is like a hole dug by a phase-space worm through this volume. Its a priori probability is the ratio of the volume of the worm hole to the total volume. The probability of two histories is additive provided the worm holes don't overlap. In the limit the total volume is the sum of a set of worm holes that are mutually exclusive and jointly exhaustive.

Quantum mechanics uses Hilbert space, rather than phase space and represents properties by sub-spaces. The correlate to dividing phase space into cells is a decomposition of the identity, dividing Hilbert space into mutually exclusive and jointly exhaustive subspaces whose projectors satisfy:

$$\sum_i B_i = 1 \qquad\qquad B_i^\dagger = B_i \qquad\qquad B_i B_j = \delta_{ij} B_j \tag{1}$$

Each history generates a subspace wormhole through the tensor product of Hilbert spaces. The a priori probability of a particular history is the ratio of the volume of its wormhole to the total volume. A history might have incompatible quantities at different stages, e.g. of σ_x at t_1 and σ_y at t_2, but has only projectors for compatible properties at each time slice. Corresponding to the intuitive idea of a wormhole volume the *weight* for a history is

$$K(Y) = E_1 T(t_1, t_2) E_2 T(t_2, t_3) \cdots T(t_{n-1}, t_n) E_n, \tag{2}$$

where E stands for an event or its orthogonal projection operator, $T(t_1, t_2)$ is the operator for the evolution of the system from t_l to t_2. Eq. (2) can be simplified by using the Heisenberg projection operators

$$\hat{E}_j = T(t_r, t_j) E_j T(t_j, t_r), \tag{3}$$

where t_r is a reference time independent of the value of t_j leading to

$$\hat{K}(Y) = \hat{E}_1 \hat{E}_2 \cdots \hat{E}_n. \tag{4}$$

Then the weight of a history may be defined in terms of a product

$$W(Y) = \langle K(Y), K(Y') \rangle = \langle \hat{K}, \hat{K}' \rangle \rangle. \tag{5}$$

The significance of this equation, defined on the space of operators, may be seen by the phase-space comparison used earlier. Classical weights used to assign probabilities are additive functions on the sample space. If E and F are two disjoint collections of phase-space histories, then $W(E \bigcup F) = W(E) + W(F)$. Quantum weights should also satisfy this requirement, since they yield classical probabilities and must be non-negative. As Griffiths (2002a, 121-124) shows, Eq. (5) achieves this. Quantum histories behave like classical histories to the degree that mutual interference is negligible. This is the key idea behind the varying formulations of a consistency condition. If two histories are sufficiently orthogonal, $\langle K(Y), K(Y') \rangle \approx 0$, then their weights are additive and can be interpreted as relative probabilities. This idea of mutual compatibility may be extended to a *family* of histories. A family is a sample space of compatible histories. Such a family is represented by a consistent Boolean algebra of history projectors. This may be extended from a family of projectors, $\mathfrak{F}$ to a refinement, $\mathfrak{G}$, that contains every projector in $\mathfrak{F}$.

Consistency considerations lead to the basic unit for interpretative consistency, a *framework*, a single Boolean algebra of commuting projectors based upon a particular decomposition of the identity[2]. A framework supplies the basis for quantum reasoning in CH. Almost all the objections to the CH interpretation are countered by showing they violate the single framework rule, or by a straightforward extension, the single family rule. Quantum claims that are meaningful in a particular framework may be meaningless in a different framework. This notion, accordingly, requires critical analysis.

There are two aspects to consider: the relation between a framework and quantum reasoning, and whether the framework rule is an *ad hoc* imposition. The first point is developed in different ways by Omnès and Griffiths. Omnès develops what he calls consistent (or sensible) logics. In the standard philosophical application of logic to theories, one first develops a logic system, or syntax, and then applies it. The content to which it is applied does not alter the logic. Omnès (1994, sect. 5.2) uses 'logic' for an interpreted set of propositions. This terminology does not imply a non-standard logic.

Griffiths focuses on frameworks. He develops the logic of frameworks by considering simple examples and using them as a springboard to general rules The distinctive features of this reasoning confined to a framework can be seen by contrast with more familiar reasoning. Consider a system that may be characterized by two or more complete sets of compatible properties. The Hilbert space representing the system may be decomposed into different sets of subspaces corresponding to the different sets of compatible properties. To simplify the issue take σ_x^+ and σ_z^+ as the properties. Can one attach a significance or assign a probability to 'σ_x^+ AND σ_z^+'? In CH propositions are represented by projectors of Hilbert subspaces. The representation of σ_x requires a two-dimensional subspace with states $\mid X^+\rangle$ and $\mid X^-\rangle$, projectors $X^\pm = \mid X^\pm\rangle\langle X^\pm \mid$, and the identity, $I = X^+ + X^-$. One cannot represent 'σ_x^+ AND σ_z^+' in any of the allowed subspaces. Accordingly it is dismissed as ‘meaningless’.

The distinctive features and associated difficulties of this framework reasoning are illustrated by Griffiths’s reworking of Wheeler’s (1983) delayed choice experiment. Both Wheeler and Griffiths (1998) consider a highly idealized Mach-Zehender interferometer.

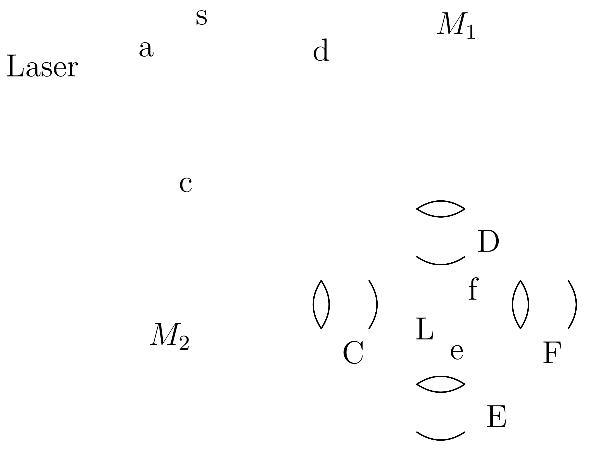

Figure 1: A Mach-Zehender Interferometer

The classical description in terms of the interference of light waves may be extended to an idealized situation where the intensity of the laser is reduced so low that only one photon goes through at a time. Here S and L are beam-splitters, M_1 and M_2 are perfect mirrors, and C, D, E, and F are detectors. If D registers, one infers path d; if C registers, then the path is c. If C and D are removed, then the detectors E and F can be used to determine whether the photon is in a superposition of states. Wheeler’s delayed choice was based on the idealization that detectors C and D could be removed after the photon had passed through S. It is now possible to implement such delayed choice experiments, though not in the simplistic fashion depicted.

To see the resulting paradox assume that detectors C and D are removed and that the first beam splitter leads to the superposition, which can be symbolized in abbreviated notation as

$$\mid a\rangle \mapsto \mid s\rangle \;=\; (\mid c\rangle + \mid d\rangle)/\sqrt{2}, \tag{6}$$

where $\mid a\rangle$, $\mid c\rangle$, and $\mid d\rangle$ are wave packets at the entrance and in the indicated arms. Assume that the second beam splitter L leads to a unitary transformation

$$\mid c\rangle \mapsto \mid u\rangle \;=\; (\mid e\rangle + \mid f\rangle)/\sqrt{2}, \quad \mid d\rangle \mapsto \mid v\rangle \;=\; (- \mid e\rangle + \mid f\rangle)/\sqrt{2}, \tag{7}$$

with the net result that

$$\mid a\rangle \mapsto \mid s\rangle \mapsto \mid f\rangle. \tag{8}$$

Equations (6) and (8) bring out the paradox. If the detectors, C and D were in place, then the photon would have been detected by either C or D. If it is detected by C, then it must have been in the c arm. If the detectors are removed and the F detector registers, then it is reasonable to assume that the photon passed through the interferometer in the superposition of states given by eq. (6). The detectors were removed while the photon was already in the interferometer. It may seem reasonable to ask what state the photon was in before the detectors were removed. Here, however, intuition is a misleading guide to the proper formulation of questions in a quantum context.

Griffiths treats this paradox by considering different families of possible histories. Using C and D for the ready state of detectors, considered as quantum systems, and C* and D* for triggered states then one consistent family for the combined photon-detector system is

$$|a\rangle|CD\rangle \longrightarrow \begin{pmatrix} |c\rangle|CD\rangle \longrightarrow |C^*D\rangle \\ |d\rangle|CD\rangle \longrightarrow |CD^*\rangle \end{pmatrix} \tag{9}$$

Here $|a\rangle|CD\rangle$ represents a tensor product of the Hilbert spaces of the photon and the detector. Eq. (9) represents a situation in which the photon enters the interferometer and then proceeds either along the c arm, triggering C^* or along the d arm, triggering D^*. These paths and outcomes are mutually exclusive.

For the superposition alternative, treated in eqs. (6)–(8), there is a different consistent family of histories,

$$|a\rangle|EF\rangle \longrightarrow |s\rangle|EF\rangle \longrightarrow \begin{pmatrix} |e\rangle|EF\rangle \longrightarrow |E^*F\rangle \\ |f\rangle|EF\rangle \longrightarrow |EF^*\rangle \end{pmatrix} \tag{10}$$

Eq. (10) represents superposition inside the interferometer and exclusive alternatives after the photon leaves the interferometer. In accord with eq. (8) the upper history in eq. (10) has a probability of 0 and F^* is triggered.

Suppose that we replace the situation represented in eq. (10) by one in which the photon is in either the c or d arms. There is no superposition within the interferometer, but there is when the photon leaves the interferometer. This can be represented by another consistent family of histories,

$$|a\rangle|EF\rangle \longrightarrow \begin{pmatrix} |c\rangle|EF\rangle \longrightarrow |u\rangle|EF\rangle \longrightarrow |U\rangle \\ |d\rangle|EF\rangle \longrightarrow |v\rangle|EF\rangle \longrightarrow |V\rangle \end{pmatrix}, \tag{11}$$

where

$$|U\rangle \;=\; (|E^*F\rangle \;+\; |EF^*\rangle)/\sqrt{2},$$

$$|V\rangle \;=\; (-|E^*F\rangle \,+\, |EF^*\rangle)/\sqrt{2}.$$

Both $|U\rangle$ and $|F\rangle$ are Macroscopic Quantum States (MQS), or Schrödinger cat states. The formalism allows for such states. However, they are not observed and do not represent measurement outcomes. This delayed choice example represents the way traditional quantum paradoxes are dissolved in CH. Reasoning is confined to a framework. Truth is framework-relative. The framework is selected by the questions the physicist imposes on nature. If a measurement has an outcome, then one must choose a framework that includes the outcome. Within a particular framework, there is no contradiction. One is dealing with consistent histories. The traditional paradoxes all involve combining elements drawn from incompatible histories.

Measurement is a catchall term for a grab bag of problems. For present purposes we consider three aspects. The first is the traditional theory of measurement stemming from von Neumann (1955, chap. 6) and Wigner[3]. The object to be measured and the measuring apparatus together can be represented by a state function, whose evolution is given by the Schrödinger equation. This is linear dynamics leading from a superposition of states only to further superpositions. Von Neumann‘s projection postulate, and similar collapse postulates, were introduced to explain how a superposition becomes a mixture in a measurement situation. Omnès‘s treatment of this will be discussed later. Revisionary interpretations of QM generally reject collapse postulates as *ad hoc* principles.

By ‘measurement situation’ we refer to a laboratory situation of an experimenter conducting an experiment, or in the now fashionable jargon performing a measurement. Here the maxim is: Properly performed measurements yield results. A measurement interpretation of QM has been treated elsewhere (MacKinnon 2007). This differs from the von Neumann approach in taking the distinctive results of quantum measurements as its point of departure for developing the formalism. Griffiths's development also tailors the formalism of QM to fit experimental measurements situations. The Schrödinger equation is treated as one method of path development, not as an overall governing principle. This leads to two general principles: 1) *A quantum mechanical description of a measurement with particular outcomes must employ a framework in which these outcomes are represented.* 2) *The framework used to describe the measuring process must include the measured properties at a time before the measurement took place.* This embodies the experimental practice of interpreting a pointer reading in the apparatus after the measurement as recording a property value characterizing a system before the measurement.

1.1 Extending the Formalism

Gell-Mann and Hartle independently developed a consistent history formalism as a transformation of Feynman's sum-over-histories formulation[4]. Quantum cosmology, their concern, requires a quantum mechanical treatment of closed systems. The universe does not admit of an outside observer. The universe is the ultimate closed system. Now it is characterized by formidable complexity, of which we have only a very fragmentary knowledge. The assumptions behind the big bang hypothesis confer plausibility on the further assumption that in the instant of its origin the universe was a simple unified quantum system. If we sidestep the problem of a state function and boundary conditions characterizing the earliest stages[5], we may skip to stages later than the Planck era, where space-time was effectively decoupled. Then the problem of quantum gravity may be avoided. The universe branched into subsystems. Even when the background perspective recedes over the horizon, a methodological residue remains, the treatment of closed, rather than open systems. To present the basic idea in the simplest form, consider a closed system characterized by a single scalar field, $\phi(x)$. The dynamic evolution of the system through a sequence of spacelike surfaces is generated by a Hamiltonian labeled by the time at each surface. This Hamiltonian is a function of $\phi(\mathbf{x}, t)$ and the conjugate momentum, $\pi(\mathbf{x}, t)$. On a spacelike surface these obey the commutation relations, $[\phi(\mathbf{x}, t), \pi(\mathbf{x}', t)] = \imath\delta(\mathbf{x}, \mathbf{x}')$ (with $\hbar, c = 1$). Various field quantities (aka observables) can be generated by ϕ and π. To simplify we consider only non-fuzzy 'yes-no' observables. These can be represented by projection operators, $P(t)$. In the Heisenberg representation, $P(t) = e^{\imath Ht}\, P(t_0)\, e^{-\imath Ht}$.

The novel factor introduced here is a coarse graining of histories. Coarse graining begins by selecting only certain times and by collecting chains into classes. The decoherence functional is defined as

$$D(\alpha', \alpha) \;=\; \mathrm{Tr}[C'_{\alpha}\, \rho\, C^{\dagger}_{\alpha}], \tag{12}$$

where ρ is the density matrix representing the initial conditions. In this context 'decoherence' has a special meaning. It refers to a complex functional defined over pairs of chains of historical projectors. The basic idea is the one we have already seen. Two coarse grained histories decohere if there is negligible interference between them. Only decoherent histories can be assigned probabilities. Different decoherence conditions can be set. We will consider two[6].

$$Weak: \quad Re\, Tr[C'_{\alpha}\, \rho\, C^{\dagger}_{\alpha}] \;= \delta(\alpha'\alpha)P(\alpha) \tag{13}$$

$$Medium: \quad Tr[C'_{\alpha}\, \rho\, C^{\dagger}_{\alpha}] \;= \delta(\alpha'\alpha)P(\alpha) \tag{14}$$

Weak decoherence is the necessary condition for assigning probabilities to histories. When it obtains the probability of a history, abbreviated as α is $P(\alpha) \;=\; D(\alpha\alpha)$. Medium decoherence relates to the possibility of generalized records. Here is the gist of the argument. Consider a pure initial state, $|\psi\rangle$ with $\rho \;=\; |\psi\rangle\langle\psi|$. Alternative histories obeying exact medium decoherence can be resolved into branches that are orthogonal, $|\psi\rangle \;=\; \sum_{\alpha} C_{\alpha}|\psi_{\alpha}\rangle$. If the projectors did not form a complete set, as in weak decoherence, then the past is not fixed. Other decompositions are possible. This relates to the more familiar notion of records when the wave function is split into two parts, one representing a system and the other representing the environment, $R_{\alpha}(t)$.

These could not count as environmental records of the state of a system if the past could be changed by selecting a different decomposition. Thus, medium decoherence, or a stricter condition such as strong decoherence, is a necessary condition for the emergence of a quasiclassical order.

It is far from a sufficient condition. The order represented in classical physics presupposes deterministic laws obtaining over vast stretches of time and space. The GH program must show that it has the resources required to produce a quasiclassical order in which there are very high approximations to such large scale deterministic laws. At the present time the operative issue is the possibility of deducing such quasi-deterministic laws. The deduction of detailed laws from first principles is much too complex. Zurek, Feynman and Vernon, Caldeira and Leggett, and others initiated the process by considering simplified linear models. The GH program puts these efforts into a cosmological framework and develops methods for going beyond linear models. The standard implementation of a linear model represents the environment, or a thermal bath, by a collection of simple harmonic oscillators. In an appropriate model the action can be split into two parts: a distinguished observable, q^i, and the other variables, Q_i, the ignored variables that are summed over.

The G-H program extends this to non-linear models, at least in a programmatic way. I will indicate the methods and the conclusions. As a first step we introduce new variables for the average and difference of the arguments used in the decoherence function:

$$\begin{aligned} X(t) &= 1/2(x'(t) + x(t)) \\ \xi(t) &= x'(t) - x(t) \\ D(\alpha', \alpha) &= f(X, \xi), \end{aligned} \qquad (15)$$

where $x'(t)$ and $x(t)$ refer to events. The rhs of eq. (15) is small except when $\xi(t) \approx 0$. This means that the histories with the largest probabilities are those whose average values are correlated with classical equations of motion. Classical behavior requires sufficient coarse graining and interaction for decoherence, but sufficient inertia to resist the deviations from predictability that the coarse graining and interactions provide. This is effectively handled by an analog of the classical equation of motion. In the simple linear models, and in the first step beyond these, it is possible to separate a distinguished variable, and the other variables that are summed over. In such cases, the analog of the equation of motion has a term corresponding to the classical equation of motion, and a further series of terms corresponding to interference, noise and dissipation. The factors that produce decoherence also produce noise and dissipation. This is handled, in the case of particular models, by tradeoffs between these conflicting requirements. The goal is to produce an optimum characteristic scale for the emergence of classical action. In more realistic cases, where this isolation of a distinguished variable is not possible, they develop a coarse graining with respect to hydrodynamic variables, such as average values of energy, momentum, and other conserved, or approximately conserved, quantities. A considerable amount of coarse graining is needed to approximate classical deterministic laws. Further complications, such as the branching of a system into subsystems,e.g. galaxies,

systems, planets, present problems not yet explored in a detailed way. Nevertheless the authors argue that they could be handled by further extensions of the methods just outlined.

Omnès has recently offered a speculative extension the CH formulation that addresses the measurement problem[7]. If QM is the basic science of reality, then it should somehow explain the fact that properly performed quantum measurements yield unique results. In this context a quantum measurement can be thought of as a two stage process. The first stage is the transformation of a superposition of states to a mixture, the traditional measurement problem. When this is treated as a pure theoretical problem, then it has no solution within the framework of QM applied to an isolated system. Omnès accepts the now common assumption that decoherence reduces a superposition to a mixture FAPP. A mixture of states assigns different probabilities to different components. The actual measurement selects one of these possibilities, effectively reducing all the other probabilities to 0. This reduction also leads to distinctively classical patterns, e.g., ionization, bubbles, tracks. Standard treatments of QM do not attempt to explain how this reduction happens. They rely on the fact that QM is intrinsically probabilistic. These probabilities are considered objective, rather than the subjective probabilities associated with guessing whether a tossed coin is heads or tails.

Omnès's attempt to explain measurement relies on a particular assumption about the way the probabilities in a mixture evolve. One evolves to a value of 1, while all the others evolve to a value of 0. This does not follow from the Schrödinger equation. Others have tried to deduce this reduction by modifying the Schrödinger equation (see Pearle 2007). Omnès effectively reverses the procedure. What follows from the assumption that the probabilities do evolve in this way and evolve very quickly? The key conclusion he draws is that $\mathrm{Tr}(\rho^2) \approx 1$, where ρ is the density matrix of the measuring system. Standard physics leads to the conclusion that $\mathrm{Tr}(\rho^2) << 1$. Omnès's conclusion entails that the measuring system is in an almost pure state. This, he argues, would obtain if the universe were in a pure state. Then reduction is interpreted as the breaking and regeneration of classicality.

Omnès presents a possible mechanism to explain this. "Reduction is a universal process and therefore its explanation must be universal." If this be so, then the development of any particular case should illustrate the universal process. There is a pure state of the universe that controls **everything**. One should use this, rather than phenomenological physics as a basis. The Hawking-Hartle cosmology assumes a pure state function for the universe. However, the formalism does not relate to particular measurement processes. I do not find this argument convincing. On a phenomenological level, reduction is ubiquitous, but involves different mechanisms like friction or approach to equilibrium. These need not have a common solution. Regardless of whether one finds the proposed solution plausible, the new challenge remains. If QM is the basic science of reality, then one should attempt to explain everything physical on the basis of a QM formulation that is not parasitic on classical physics.

2 Criticisms of Consistent Histories

The objections brought against the CH interpretation cluster around the border separating physics from philosophy. The technical physical objections have been answered largely by showing that confining quantum reasoning to a framework eliminates contradictions (See Griffiths 1997, 1998, and 2002a, chaps. 20-25). Here we will focus on the more philosophical aspects and group them under three headings: Meaning, Truth, and Arbitrariness. The first two share a core objection. The CH interpretation makes meaning and truth framework relative. Critics take this as an *ad hoc* restriction that violates accepted norms concerning truth and meaning. The issue of arbitrariness concerns the selection of histories. The formalism allows in principle a very large number of histories The CH interpretation selects a few privileged histories. Critics object that the formalism supplies no basis for the selection. The G-H project specifies the conditions for the emergence of quasiclassicality. The formalism allows an indefinitely large number of extensions of the quasiclassical framework. Only a minute fraction of them preserve quasiclassicality. Again, critics object that the formalism supplies no basis for selecting only the members of this minute fraction.

Adrian Kent has brought the issue of meaning to the forefront[8]. Consider two histories with the same initial and final states and intermediate states σ_x and σ_z, respectively. In each history one can infer the intermediate state with probability 1. A simple conjunction of two true propositions yields '$\sigma_x \,\mathrm{AND}\, \sigma_z$'. Griffiths and Hartle contend, and Kent concedes, that there is no formal contradiction since the intermediate states are in separate histories. Kent finds this defense arbitrary and counter-intuitive. Our concepts of logical contradiction and inference are established prior to and independent of their application of quantum histories. If each intermediate state can be inferred, then their conjunction is meaningful.

The issue of truth comes to the forefront when one considers the ontological significance of assigning quantitative values to properties. In classical physics assigning a value to a property means that the property possesses the value. Copenhagen quantum physics fudges this issue. The CH interpretation exacerbates the difficulty. A realistic interpretation of projectors take them as representing the properties a system possesses at a time. This does not fit the Griffiths treatment of the delayed choice experiment when one asks what position the photon *really* had at time t_2. Thus, d'Espagnat (1995, chap. 11) argues that the CH interpretation involves inconsistent property assignments. In a similar vein Bub (1997, p. 236) expressed the objection that if there are two quasiclassical histories of Schrödinger's cat, then one does not really know whether the cat is alive or dead. Bassi and Ghiradi(1999) make the issue of truth explicit. The attribution of properties to a system is true if and only if (iff) the system actually possesses the properties. They find Griffiths's reasoning "shifty and weak", implying the coexistence of physically senseless decoherent families. This criticism extends to probabilities. From an ontological perspective probabilities of properties must refer to objective and intrinsic properties of physical systems. There is, they claim, no other reasonable alternative. If they referred to the possibilities of measurement results, then this would be a measurement interpretation, not a replacement for it. Goldstein (1998) argues that the CH interpretation cannot be true, since it contradicts established no-go theorems.

To treat the framework relevance of truth we should distinguish 'truth' and 'true'. In philosophical contexts 'truth' inevitably conjures up theories of truth: correspondence theories, coherence theories, pragmatic theories, assertive-redundancy theories, and others. The most pertinent, the correspondence theory of truth, generates controversies concerning Aristotle's original doctrine, Tarski's specification of 'true' for a formal language, and puzzles concerning the way a proposition corresponds to a state of affairs. The criticisms brought against the CH interpretation seem to presuppose only a minimal sense:

“The cat is on the mat” is true iff the cat is on the mat.

This looks unproblematic in the context of someone who sees the cat and understands the claim. It becomes highly problematic when one argues from the acceptance of a theory as true to what the world must be like to make it true. Thus Hughes (1989, p. 82) asks Feynman’s forbidden question: What must the world be like if quantum mechanics is true of it?

In forbidding such questions Feynman was following the normal practice of physicists. Claims presented as true do not depend on a philosophical theory of truth, but on the normal use of language in physics. This will be treated in much greater detail elsewhere (MacKinnon forthcoming). Here we will simply exploit Donald Davidson’s truth semantics to indicate how ‘true’ can be interpreted as a semantic primitive whose use is not dependent on theories of truth. Davidson’s gradual abandonment of an extensional theory of ‘true’ led to a critical rethinking of the interrelation of truth, language, interpretation, and ontology. I will summarize the overview presented in his (2001, Essay 14). Philosophers have been traditionally concerned with three different types of knowledge: of my own mind; of the world; and of other minds. The varied attempts to reduce some of these forms to the one taken as basic have all proved abortive. Davidson’s method of interrelating them hinges on his notion of radical interpretation. My attempt to interpret the speech of another person relies on the functional assumption that she has a basic coherence in her intentions, beliefs, and utterances. Interpreting her speech on the most basic level involves assuming that she holds an utterance true and intends to be understood. The source of the concept 'true' is interpersonal communication. Without a shared language there is no way to distinguish what is the case from what is thought to be the case. I also assume that by and large she responds to the same features of the world that I do. Without this sharing in common stimuli thought and speech have no real content. The three different types of knowledge are related by triangulation. I can draw a baseline between my mind and another mind only if we can both line up the same aspects of reality. Knowledge of other minds and knowledge of the world are mutually dependent. "Communication, and the knowledge of other minds that it presupposes, is the basis of our concept of objectivity, our recognition of a distinction between false and true beliefs". (*Ibid.*, p. 217).

Our ordinary language picture of reality is not a theory. It is a shared vehicle of communication involving a representation of ourselves as agents in the world and members of a community of agents, and of tools and terms for identifying objects, events, and properties. Extensions and applications may be erroneous. There can be factual mistakes, false beliefs, incorrect usages, and various inconsistencies. But, the designation of some practice as anomalous is only meaningful against a background of established practices that set the norms. Our description of reality and reality as described are interrelated, not in a vicious circle, but in a developing spiral. The acceptance of any particular claim as true implicitly presupposes the acceptance of a vast but amorphous collection of presuppositions. (See Davidson 1984, esp. chap. 14.) These come into focal awareness only in specialized contexts, such as translating material from a primitive or ancient culture with quite different presuppositions or programming a robot to cope with a particular environment.

The acceptance as true of a scientific claim, whether an experimental report of a theoretical deduction, implicitly presupposes the acceptance of a vast, but not so amorphous, collection of claims as true, e.g. the reliability and calibration of instruments, established theories, basic physical facts, the validity of a deduction, the honesty of an experimental report, etc. Any particular claim may be called into question when there are grounds for doubting its truth or pertinence. However, it is not possible to call all the presuppositions into question and continue the practice of science. In the mid 1920s the normal function of implicit presuppositions began to cause serious difficulties in quantum contexts. The most striking example was the way the experimenters, Davisson and Germer (1927, 1928) backed into the acceptance of truth claims as framework relative. Their experimental research of scattering slowly moving electrons off a nickel surface were interrupted when the vacuum tube containing the nickel target burst. They heated the nickel target to remove impurities and then slowly cooled it. When they resumed their scattering experiments they were amazed to find that the earlier random scattering was replaced by a regular pattern very similar to wave reflection. The explanation that gradually emerged was that the heating and slow cooling of the target led to the formation of relatively large nickel crystals. They reluctantly accepted the then novel contention that electrons scattered off crystals behave like waves. The previous presupposition that electrons travel in trajectories is only true in particular experimental situations. Born's probabilistic interpretation of the ψ-function implicitly accommodated this framework relevance by according $int\psi\psi^*$ a value of 1 only when the integration is carried out in the proper environment. In the CH formulation 'true' is interpreted as having a probability of 1 relative to a framework. This is in accord with the normal usage of 'true' in quantum physics. It does not invoke any version of the correspondence theory of truth and does not support a context-independent attribution of possessed properties.

Truth is related to implication. In formal logic a contradiction implies anything. This relation was recognized informally long before the development of formal systems. The medieval adage was: "*Ex falso sequitur quodlibet*". If anything follows then no implications are reliable. As Kent noted, the CH formulation should accord with the normal relation between implication and contradiction. Here, it is important to recognize the way this was modified in the normal language of quantum physics. The mid 1920s difficulties just noted led to a situation where normal reliance on implicit presuppositions led to fundamental contradictions in the context of quantum experiments.

1a. **Electromagnetic radiation is continuously distributed in space.** The high precision optical instruments used in measurements depend on interference, which depends on the physical reality of wavelengths.

1b. **Electromagnetic radiation is not continuously distributed in space.** This is most clearly shown in the analysis of X-rays as needle radiation and in Compton's interpretation of his eponymous effect as a localized collision between a photon and an electron.

2a. **Electromagnetic radiation propagates in wave fronts.** This is an immediate consequence of Maxwell's equations.

2b. **Electromagnetic radiation travels in trajectories.** Again, theory and observation support his. The theory is Einstein's account of directed radiation. The observations concern X-rays traveling from a point source to a point target.

3a. **Photons function as discrete individual units.** The key assumption used to explain the three effects treated in Einstein's original paper is that an individual photon is either absorbed as a unit or not absorbed at all. Subsequent experiments supported this.

3b. **Photons cannot be counted as discrete units** Physicists backed into this by fudging Boltzmann statistics. It became explicit in Bose- Einstein statistics.

These and further contradictions concerning electronic orbits, were not contradictions derived from a theory. The Bohr-Somerfeld atomic theory had become a thing of rags and patches. These contradictions were encountered in attempts to give a coherent framework for interpreting different experimental results.

We will distinguish the language of phenomena from the language of theories. Bohr's resolution of these problems included a reformation of the language of phenomena. In resolving this crisis, Bohr introduced something of a Gestalt shift, from analyzing the apparently contradictory *properties* attributed to objects and systems to analyzing the *concepts* used. As Bohr saw it, the difficulties were rooted in "... an essential failure of the pictures in space and time on which the description of natural phenomena has hitherto been based."[9]. Bohr reinterpreted the role of the language used to give space-time descriptions of sub-microscopic objects and properties.

The description of experiments and the reporting of results must meet the conditions of unambiguous communication of information. This requires ordinary language supplemented by the terms and usages developed through the progress of physics. Thus, the meanings of the crucial terms 'particle' and 'wave' were set by their use in classical physics. Each of these terms is at the center of a cluster of concepts that play an inferential role in the interpretation of experiments. From tracks on photographic plates experimenters infer that a particle originated at a point, traveled in a trajectory, collided with another particle, penetrated an atom, and displaced an inner electron. Waves do not travel in trajectories. They propagate in wave fronts, interfere with each other, are diffracted or absorbed. A straightforward extension of both concepts to different contexts generated contradictions.

Bohr's new guidelines, centering on complementarity, resolved these contradiction by restricting the meaningful use of classical concepts to contexts where these concepts could be related to real or ideal measurements. Concepts proper to one measurement context could not be meaningfully extended to a complementary measurement context. Bohr treated the mathematical formalism as a tool and regarded these analyses of idealized experiments as the chief means of establishing the consistency of the language of quantum physics[10]. This explains the chiaroscuro nature of his analyses featuring detailed representations of grossly unrealistic experiments: diaphragms rigidly clamped to heavy wooden tables, clocks with the primitive mechanism showing, a scale supported by a dime-store spring. These are belligerently classical tools used to illustrate the limits of applicability of classical concepts in atomic and particle experiments. Bohr thought he achieved an overall consistency only after 1937. Subsequently he introduced an idiosyncratic use of 'phenomenon' as a unit of explanation. The object studied, together with the apparatus needed to study it constitute a phenomenon, an epistemologically irreducible unit. Wheeler's analysis of the delayed choice experiment draws on Bohr's terminology: "No elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon."Idealized thought experiments supplied the basic tool for testing consistency.

After these modifications were assimilated into normal linguistic usage in the quantum community the linguistic crisis that precipitated the Gestalt shift receded from collective memory. This forgetfulness allowed critics to couple normal physical language with incompatible extensions of a correspondence theory of truth. The CH formulation is in strict accord with the Bohrian semantics just summarized. We can make this explicit for the issues of meaning, implication, frameworks, and truth. CH relates to experiments through its analysis of measurement situations, i.e., the normal practice of experimenters. Thus, in the delayed choice experiments analyzed earlier if the C or D detectors detected a particle one could infer the trajectory of the photon. If the C and D detectors are removed and the F detector is triggered then one can infer that the photon was in a superposition of states. Bohr's use of 'phenomenon' treats each experimental situation as an epistemologically irreducible unit. Within a particular experimental analysis one can rely on classical logic and normal experimental inferences. These inferences cannot be extended to a complementary experimental analysis.lness allowed critics to couple normal physical language with incompatible extensions of a correspondence theory of truth. The CH formulation is in strict accord with the Bohrian semantics just summarized. We can make this explicit for the issues of meaning, implication, frameworks, and truth. CH relates to experiments through its analysis of measurement situations, i.e., the normal practice of experimenters. Thus, in the delayed choice experiments analyzed earlier if the C or D detectors detected a particle one could infer the trajectory of the photon. If the C and D detectors are removed and the F detector is triggered then one can infer that

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Griffiths's use of 'framework' corresponds to Bohr's use of 'phenomenon'. Within a framework one uses Boolean logic and relies on normal experimental inferences. However, one cannot juxtapose incompatible frameworks or detach inferences from the framework in which they function. These limitations on allowed inferences were introduced to avoid generating contradictions. Thus, in disallowing the meaningfulness of such juxtapositions as 'σ_x^+ AND σ_z^+', where these are intermediate states in different histories, the CH interpretation is in strict accord with the prior rules governing contradiction and implication in quantum contexts. Asserting that the photon traveled through the c arm is equivalent to

"The photon traveled through the c arm" is true (t).

Physicists do not invoke an assertive-redundancy account of truth. They do rely on the normal linguistic practice of assertion encapsulated in (t). When one switches from this normal reliance on 'true' to 'truth', based on some kind of correspondence theory, then it seems to make sense to ask where the photon really was before the detection. This use of 'really' and its ontological implications are not allowed in either Bohrian semantics of the CH formulation.

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Dowker and Kent (1995, 1996) criticized the CH interpretation as arbitrary and incomplete. We will separate this criticism from the problems related to quasiclassicality. Consider a system whose initial density matrix, ρ_i is given along with the normal complement of Hilbert-space observables. Events are specified by sets, σ_j of orthogonal Hermitian projectors, $P^{(i)}$, characterizing projective decompositions of the identity at definite times. Thus,

$$\sigma_j(t_i) \;=\; \{{P_I}^{(i)} : i \;=\; 1, 2, \ldots, n_j\}_{t_j}$$

defines a set of projectors obeying eq. (1) at time t_i. Consider a list of sets and time sequences. The histories given by choosing one projection from each set in all possible ways are an exhaustive and exclusive set of alternatives, $\mathcal{S}$. Dowker and Kent impose the Gell-Mann–Hartle medium decoherent consistency conditions, restrict their considerations to exactly countable sets, consider consistent extensions of $\mathcal{S}$, $\mathcal{S}'$, and then ask how many consistent sets a finite Hilbert space supports. The answer is a very large number. This prompts two interrelated questions. How is one set picked out as the physically relevant set? What sort of reality can be attributed to the collection of sets?

Griffiths (1998) countered that these extended sets are meaningless. Their construction leads to histories that could not be assigned probabilities. To make the difficulty more concrete consider the simplest idealized realization of the Dowker-Kent *Ansatz*, a silver atom passing through a Stern-Gerlach (SG) magnet. We will use the simplified notation, X, Y, and Z, for spin in these directions. At t_1 there are three families:

$$X_+(t_1), X_-(t_1) \qquad Y_+(t_1), Y_-(t_1) \qquad Z_+(t_1) Z_-(t_1)$$

The passage from t_1 to t_n allows of 6^{2n} possible histories. For the simple point we wish to make we consider 6 of the 36 possible histories leading form t_1 to t_2

$$(a) X_+(t_1) X_+(t_2) \quad (c) X_+(t_1) Y_+(t_2) \quad (e) X_+(t_1) Z_+(t_2)$$

$$(b) X_+(t_1) X_-(t_2) \quad (d) X_+(t_1) Y_-(t_2) \quad (f) X_+(t_1) Z_-(t_2)$$

The formalism does not assign probabilities to these histories. Here the appropriate experimental context would be successive SG magnets with various orientations. Suppose that the atom passes through an SG magnet with a X orientation at t_1 and one with a Z orientation at t_2, then only (e) and (f) can have non-zero probabilities. The selection of histories as meaningful is determined by the questions put to nature in the form of actual or idealized experimental setups. The fact that the formalism does not make the selection is not a shortcoming.

The final objection we will consider is the Dowker-Kent claim that the GH program cannot demonstrate the preservation of a quasiclassical order. Here again, it is misleading to expect the formalism to supply a selection principle. The GH program was set up more like a problem in reverse engineering, than as the interpretation of a formalism.

> In a universe governed at a fundamental level by quantum-mechanical laws, characterized by indeterminacy and distributed probabilities, what is the origin of the phenomenological, deterministic laws that approximately govern the quasiclassical domain of everyday experience? What features of classical laws can be traced to their underlying quantum-mechanical origin?[11]

The G-H project was never presented as a deductive theory. The goal was to see whether the acceptance of QM as the fundamental science of physical reality allowed for an explanation of the large-scale deterministic laws characterizing classical physics, a reverse engineering project that might eventually lead to a more formal theory.

Consider a hacker trying to reverse engineer a computer game of shooting down alien invaders and assume that he has developed a machine language formulation that accommodates the distinctive features of the alien game at a certain stage of the action. Any such machine language formulation admits of an indefinitely large number of extensions, only a minute fraction of which would preserve ‘quasialienality’. This is not an impediment. The hacker is guided by a goal, reproducing a functioning game, rather than by the unlimited possibilities of extending machine-language code. The GH program has shown the possibility of programmatically reproducing basic features of the deterministic laws of classical physics. To achieve this goal the program relies on decoherence and various approximations. It is misleading to treat the result as if it were an exact solution capable of indefinite extension.

When the consistent histories formulation and the Gell-Mann–Hartle project utilizing this formulation are put in the proper interpretative perspective, then they can adequately meet both the philosophical and the physical objections brought against them. Should the CH formulation be accepted as a replacement for the Copenhagen interpretation? My answer to this begins with the Landau-Lifshitz sense of ‘quasiclassical’. The CH analysis of actual and idealized experiments relies on quasiclassical state functions like $|C^{*}D\rangle$, indicating that the C detector has been triggered and the D detector was not. These are place holders for equivalence classes of state functions, that will never be specified in purely quantum terms. In an actual measurement one does not rely on $|C^{*}D\rangle$, but on a description of a measurement situation in the standard language of physics. This put us back in the realm where the Copenhagen interpretation has a well established success. The CH formulation/interpretation is not a stand alone interpretation in this practical sense. In the laboratory one carries on with physics as usual. Because of the way it is constructed the CH formulation parallels the Copenhagen interpretation with a projection postulate, or the measurement interpretation, as explained elsewhere (MacKinnon 2008b).

However, it does serve as a replacement for the Copenhagen interpretation in certain theoretical contexts. We have effectively considered two such contexts in the present article. The first is the acceptance of quantum mechanics as the basic science of reality. It cannot be understood as a basic independent science while its formulation has a parasitic relation with classical physics. Some other proposed replacements for Copenhagen exclude the orthodox treatment of measurements, thus generating a measurement problem. The CH formulation does not have this difficulty. Accordingly, it supplies a consistent *formulation* of QM as a foundational science. This does not imply that it can stand alone in the normal practice of physics.The second context is an application of QM that excludes the possibility of outside observers. Cosmology is the prime example. Here again, one needs a formulation that is independent of, but compatible with, the orthodox interpretation. The CH formulation meets this requirement. We leave open the issue of whether it is superior to other proposed replacements, such as some version of the many-worlds interpretation.

Notes

[1] This is based on Griffiths 1984, 1996, 1997, 2002a, 2002b; on Griffiths and Hartle 1997 and on Griffiths's helpful comments on earlier drafts of this material.

[2] This idea of a distinctive form of quantum reasoning was developed in Omnès, R.: (1994), Chaps. 9, 12, and in Griffiths 1999, and 2002a chap. 10.

[3] In a conversation with Abner Shimony Wigner claimed "I have learned much about quantum theory from Johnny, but the material in his Chapter Six Johnny learned all from me, cited from Aczel, p. 102.

[4] Gell-Mann, M. and Hartle, J. 1990; 1993. The differences between this program and older forms of reductionism is discussed in MacKinnon 2008a

[5] This is treated in Hartle, 2002a, 2002b

[6] Gell-Mann and Hartle 1994b

[7] Omnès, 2008 I am grateful to Professor Omnès for an advance copy of this article.

[8] Kent 1996 was answered by Griffiths and Hartle 1997, which was answered by Kent, 1998.

[9] This was from a talk given in August 1925 before Bohr was familiar with

Heisenberg's new formulation of QM. It is reproduced in Bohr 1934, p. 34. Bohr never intended or presented an interpretation of QM as a theory. See Gomatam 2007

[10] "The physical content of quantum mechanics is exhausted by its power to formulate statistical laws governing observations obtained under conditions specified in plain language". (Bohr 1958, p. 12)

[11] Gell- Mann and Hartle 1993, p. 3345

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