CHAPTER 7 filter design techniques

7.0 introduction

7.1design of discrete-time IIR filters from continuous-time filters

7.1.1 filter design by impulse invariance

7.1.2 filter design by bilinear transform

7.2 design of FIR filters by windowing.

7.3 summary

7.0 introduction

1.ideal frequency-selective filter（理想选频滤波器）

Frequency-selective filter suggests a system that passes certain frequency components and totally rejects all others.

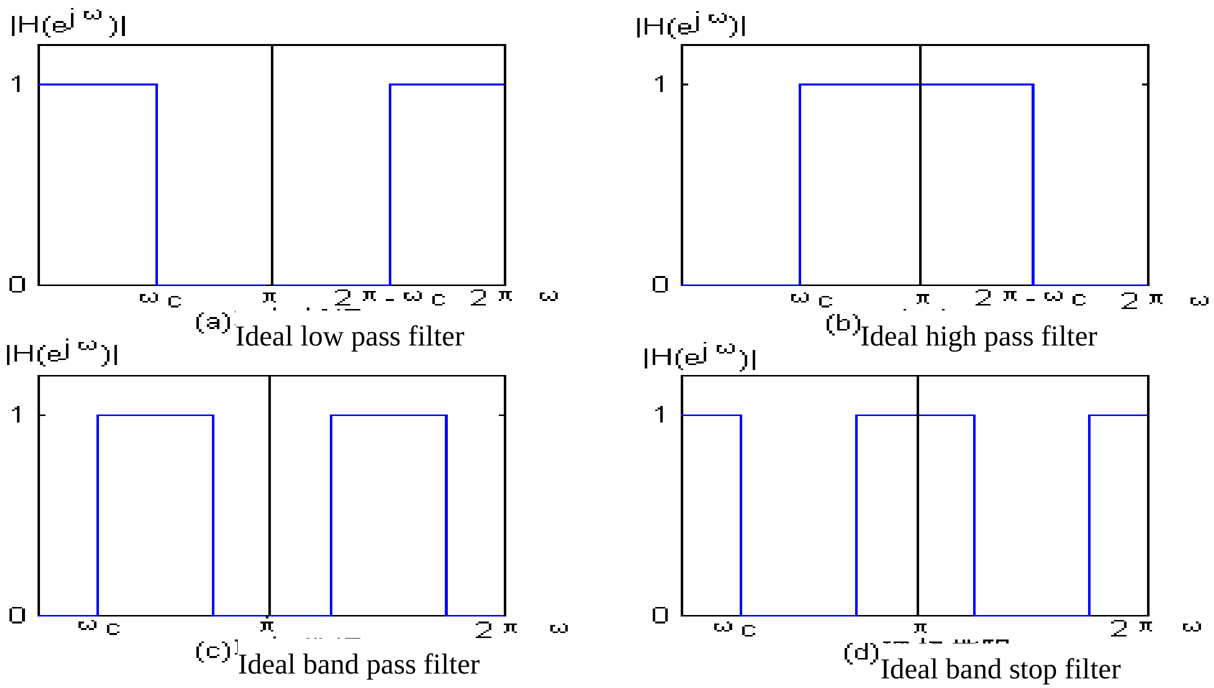


Figure 7.1

2.The design of filters involves the following stages:

1) **The specification（指标） of the desired properties of the system;**

2) **The approximation of the specifications using a causal discrete-time system;**

3) **The realization structure of the system.**

3.The specifications for the filter are typically given in the frequency domain.

Usually, a discrete-time filter is to be used for discrete-time processing of continuous-time signals as shown in Figure 7.2.

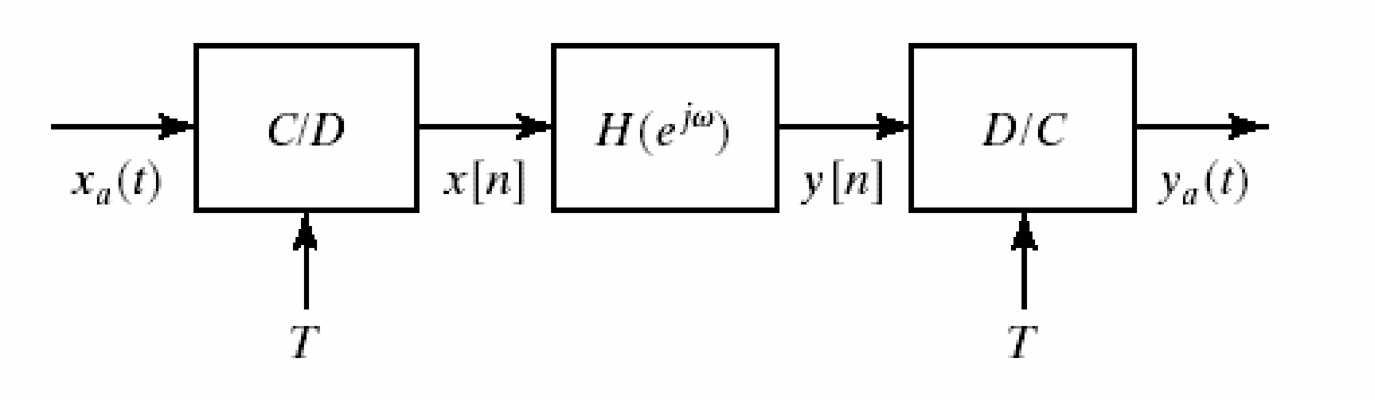
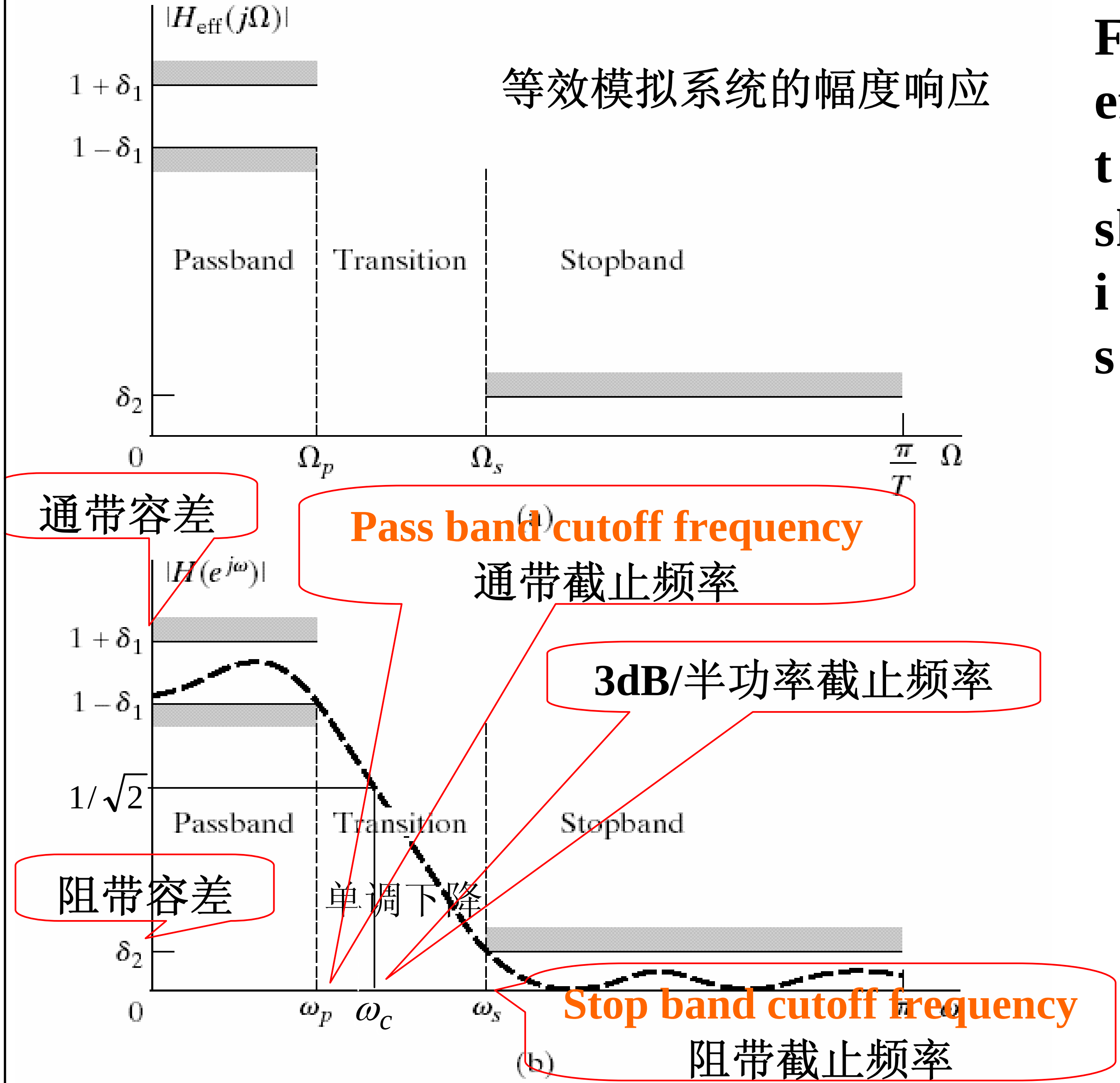


Figure 7.2

$$H_{eff}(j\Omega) = \begin{cases} H(e^{j\Omega T}) & |\Omega| < \pi / T \\ 0 & |\Omega| > \pi / T \end{cases}$$

So, the specifications in the frequency domain can be given for either the discrete-time filter or the effective continuous-time filter .



For specifications of effective continuous-time filter, we should change them into digital specifications:

$$\omega_p = \Omega_p T$$

$$\omega_s = \Omega_s T$$

Figure 7.2

It is common to express the maximum pass band and stop band gains in unit of decibels:

$$\alpha_p = 20 * \log_{10}(1-\delta_p)$$

$$\alpha_s = 20 * \log_{10}\delta_s$$

So, the maximum amplitude is scaled to 1, that is ,0 dB. For 3dB cutoff frequency :

$$|H(e^{j\omega_c})| = 1/\sqrt{2}$$

$$20\log_{10}|H(e^{j\omega_c})| = -3dB$$

7.1 design of discrete-time IIR filters from continuous-time filters

For designing a discrete-time high pass filter, we can realize it by three paths. We only concentrate on A/D transformation which transforms a continuous-time filter into a discrete-time filter.

$$ala\log y \quad low \quad pass \xrightarrow{A/D} digital \quad low \quad pass$$

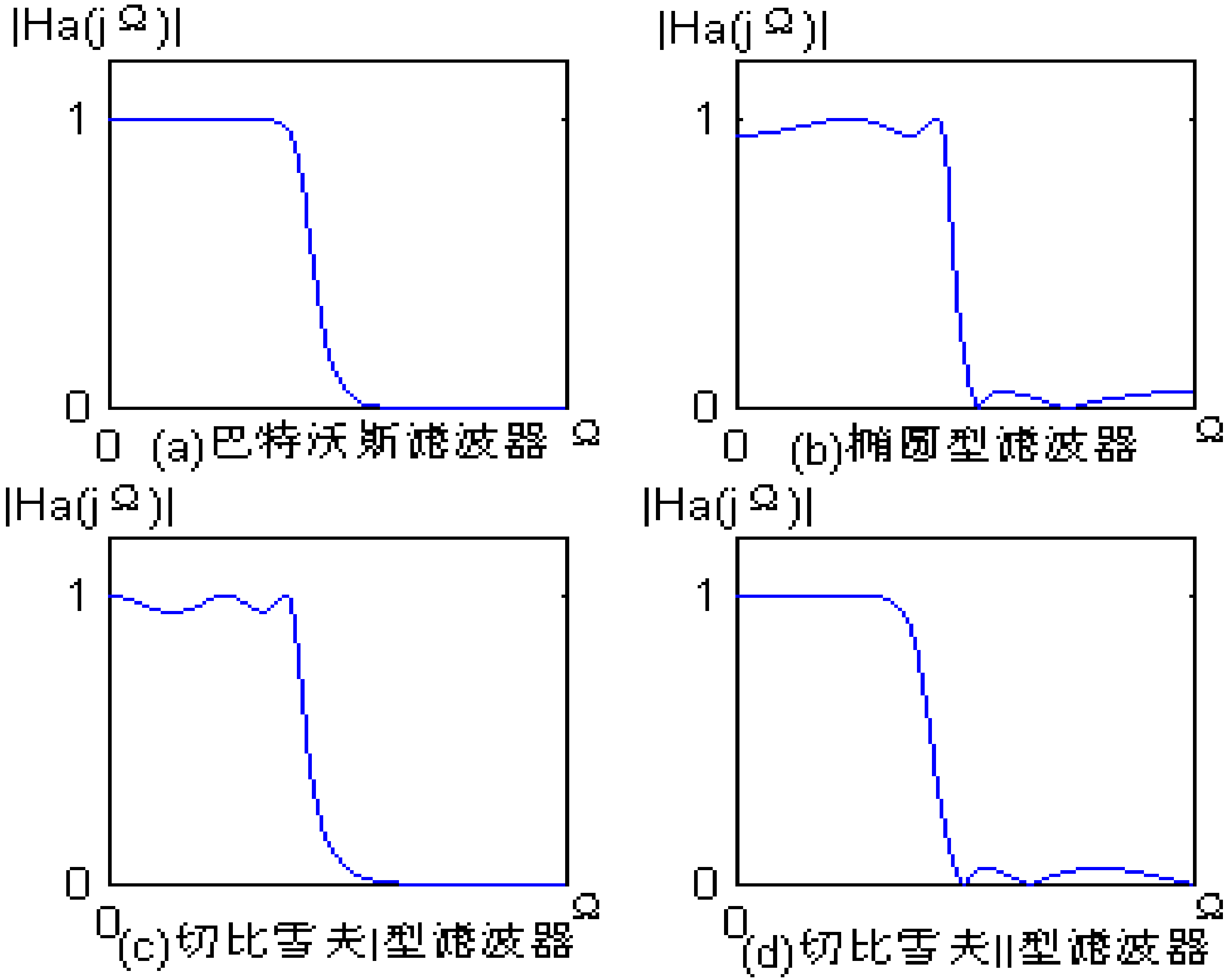
$$\downarrow \; frequency \quad transform \qquad \downarrow$$

$$ala\log y \quad high \quad pass \xrightarrow{A/D} digital \quad high \quad pass$$

7.1.0 introduction of analogy filter
7.1.1filter design by impulse invariance
7.1.2 filter design by bilinear transform

7.1.0 introduction of analogy filter

The magnitude function for three classes of low pass filters



比较：
（1）波动
（2）同阶，性能增
（3）设计复杂度增

We only give the formulas for Butterworth filter design :

Amplitude Function: $|H_c(j\Omega)|^2 = \dfrac{1}{1+(\Omega/\Omega_c)^{2N}}$

Then we have:

$$N = \left\lceil \log_{\Omega_p/\Omega_s}\left(\frac{10^{\alpha_p/10}-1}{10^{\alpha_s/10}-1}\right)^{1/2} \right\rceil$$

$$\Omega_c = \frac{\Omega_p}{\left(10^{\alpha_p/10}-1\right)^{\frac{1}{2N}}} \quad \text{OR} \quad \Omega_c = \frac{\Omega_s}{\left(10^{\alpha_s/10}-1\right)^{\frac{1}{2N}}}$$

The poles are: $s_k = \Omega_c e^{j\left(\frac{2k+1}{2N}+\frac{1}{2}\right)\pi}, k = 0 \cdots\cdots N-1$

Therefore we have the system function:

$$H_c(s) = \prod_{k=0}^{N-1} \frac{1}{1-\dfrac{s}{s_k}} = \frac{\Omega_c{}^N}{\prod_{k=0}^{N-1}(s-s_k)}$$

EXAMPLE

design a low pass analogy filter:

$$\Omega_p = 2\pi \cdot 1000\ rad/s, \Omega_s = 2\pi \cdot 2000\ rad/s, \alpha_p = 1dB, \alpha_s = 15dB$$

Solution 1:

```
[N,Wc]=buttord(2000*pi,4000*pi,1,15, ’s’)
[Bs,As]=butter(N,Wc, ’s’)
[H,W]=freqs(Bs,As);
plot(W/2/pi,20*(log10(abs(H))))
axis([1000,2000,-16,0])
grid on
```

OUTPUT:

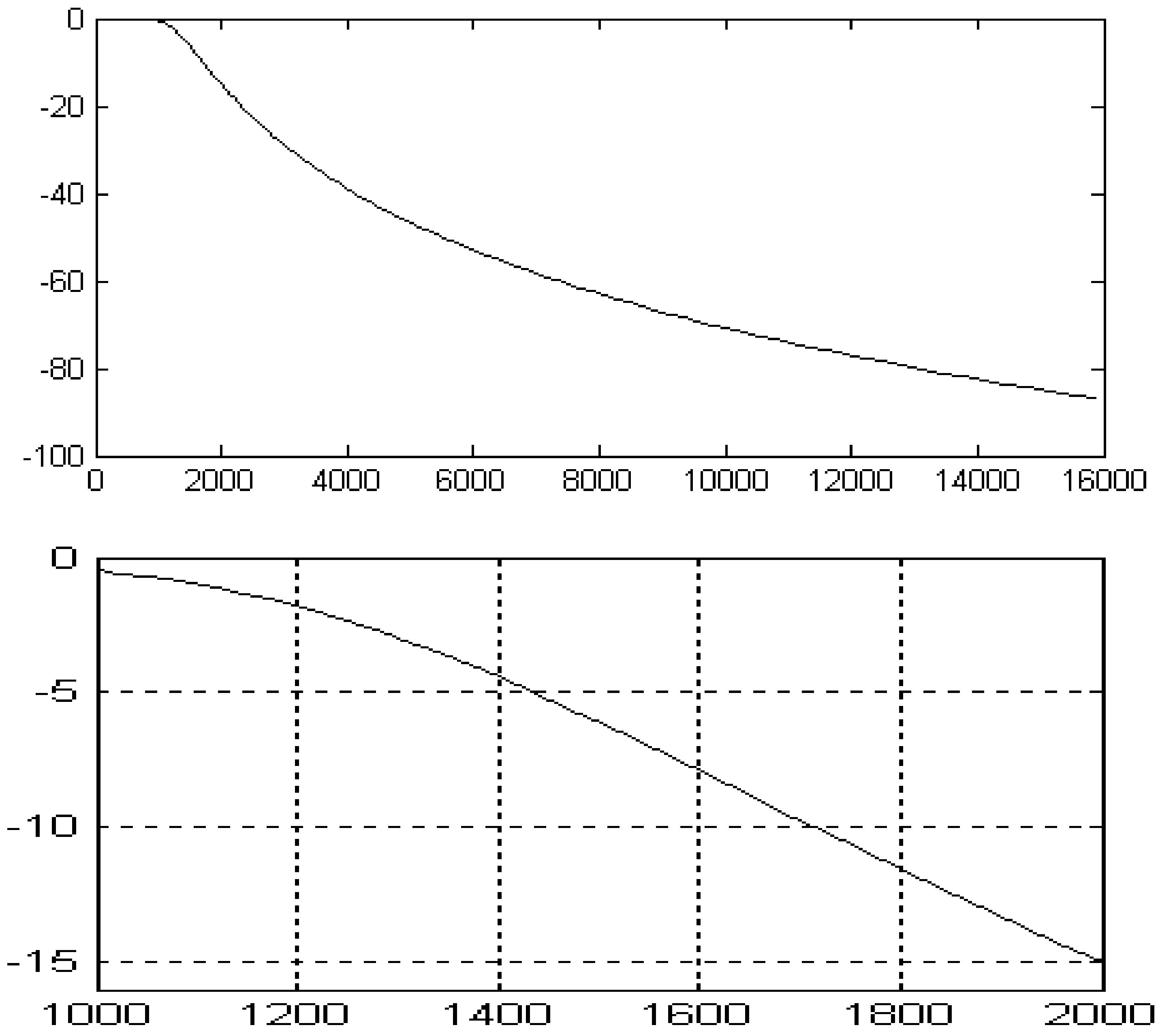
N = 4

Wc = 8.1932e+003

Bs = 1.0e+015 * 0 0 0 0 4.5063

As = 1.0e+015 * 0.0000 0.0000 0.0000 0.0014 4.5063

$$H_c(s) = \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4}$$


0
-20
-40
-60
-80
-100
0
2000
4000
6000
8000
10000
12000
14000
16000
0
-5
-10
-15
1000
1200
1400
1600
1800
2000

Solution 2:

[N,Wc]=buttord(2000*pi,4000*pi,1,15, ' s')

[z,p,k]=butter(N,Wc, ' s')

Output:

z = Empty matrix: 0-by-1

p = 1.0e+003 *

-7.5695 + 3.1354i

-7.5695 - 3.1354i

-3.1354 + 7.5695i

-3.1354 - 7.5695i

k = 4.5063e+015

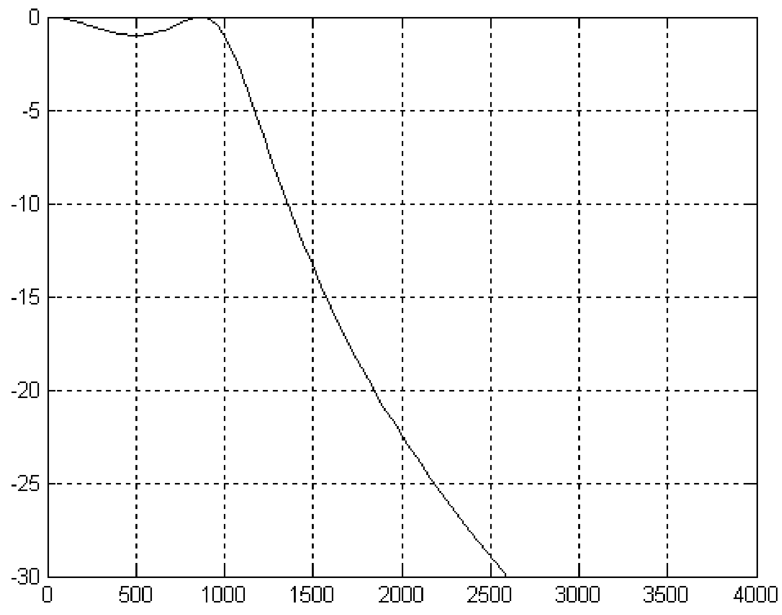
$$H_c(s) = \frac{k}{(s-p_0)(s-p_1)(s-p_2)(s-p_3)}$$

EXAMPLE

design a low pass chebyI analogy filter:

$$\Omega_p = 2\pi \cdot 1000\ rad/s, \Omega_s = 2\pi \cdot 2000\ rad/s, \alpha_p = 1dB, \alpha_s = 15\,dB$$

```
[N,Wc]=cheb1ord(2000*pi,4000*pi,1,15, ’s’)
[Bs,As]=cheby1(N,1,Wc, ’s’)
[H,W]=freqs(Bs,As);
plot(W/2/pi,20*(log10(abs(H))))
axis([0,4000,-30,0])
grid on
```

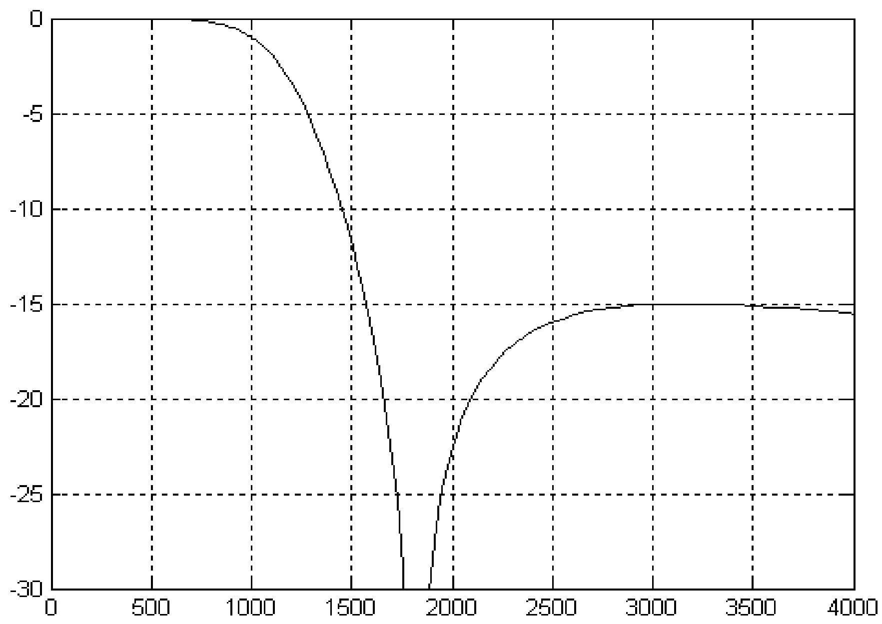


EXAMPLE

design a low pass chebyII analogy filter:

$$\Omega_p = 2\pi \cdot 1000\ rad/s, \Omega_s = 2\pi \cdot 2000\ rad/s, \alpha_p = 1dB, \alpha_s = 15dB$$

```
[N,Wc]=cheb2ord(2000*pi,4000*pi,1,15, ’ s’ )
[Bs,As]=cheby2(N,15,Wc, ’ s’ )
[H,W]=freqs(Bs,As);
plot(W/2/pi,20*(log10(abs(H))))
axis([0,4000,-30,0])
grid on
```

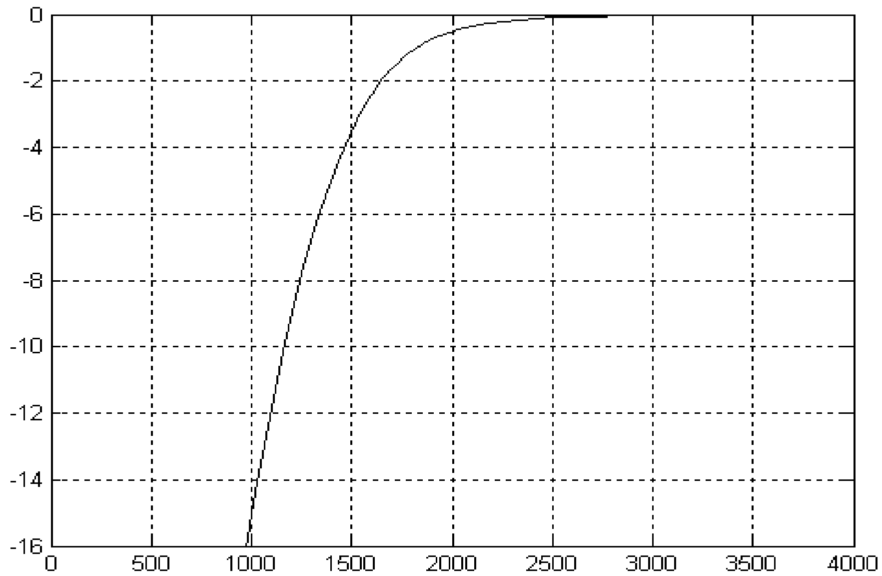


EXAMPLE

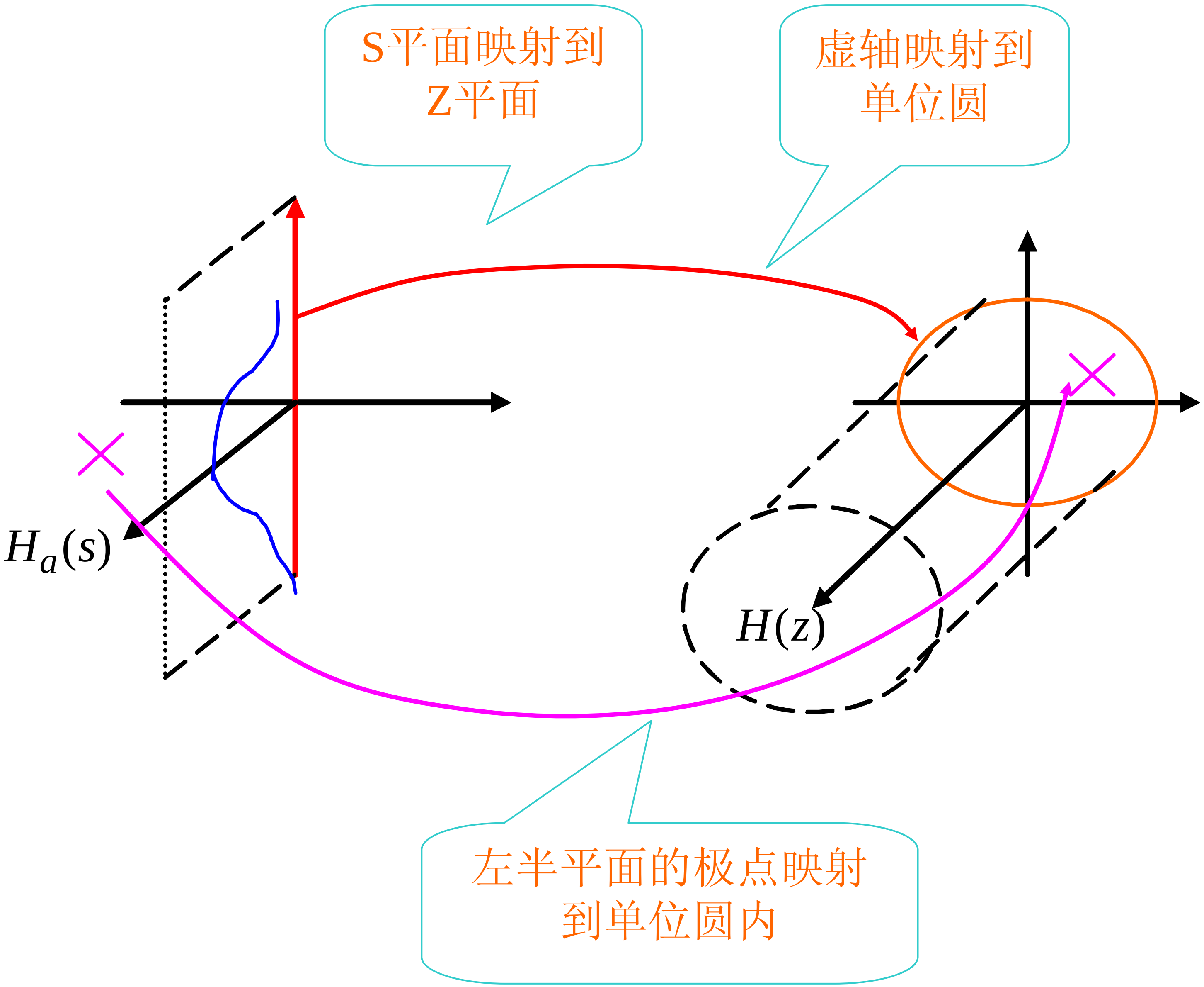
design a high pass analogy filter:

$$\Omega_p = 2\pi \cdot 2000\ rad/s, \Omega_s = 2\pi \cdot 1000\ rad/s, \alpha_p = 1dB, \alpha_s = 15\,dB$$

```
[N,Wc]=buttord(4000*pi,2000*pi,1,15,'s')
[Bs,As]=butter(N,Wc,'high','s')
[H,W]=freqs(Bs,As);
plot(W/2/pi,20*(log10(abs(H))))
axis([0,4000,-16,0])
grid on
```



In the transformation of a continuous-time filter into a discrete-time filter, we generally require that the essential properties of the continuous-time frequency response be preserved in the frequency response of the resulting discrete-time filter. Specifically, this implies that we want the imaginary axis of the s-plane to map onto the unit circle of the z-plane. A second conditions is that a stable continuous-time filter should be transformed to a stable discrete-time filter. This means that if the continuous-time system has poles only in the left half of the s-plane, then the discrete-time filter must have poles only inside the unit circle in the z-plane. These constraints are basic to all the techniques, including impulse invariance and bilinear transform which will be discussed in this section.


S平面映射到
Z平面
虚轴映射到
单位圆
$H_a(s)$
$H(z)$
左半平面的极点映射
到单位圆内

7.1.1 filter design by impulse invariance（冲击不变）

1.principle：

The impulse response of discrete-time filter is chosen proportional to equally spaced samples of the impulse response of continuous-time filter:

$$h[n] = T_d h_c[nT_d]$$

So we can get the system function of the discrete-time filter by the following steps:

2. Transforming Equation:

It is easy to carry out as a transformation on the system function. Let us consider the system function of the continuous-time filter expressed in terms of a partial fraction expansion:

$$H_c(s) = \sum_{k=0}^{N-1} \frac{A_k}{s - s_k}$$

The system function of the discrete-time filter can be expressed as:

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$$H(z) = H_c(s)\Big|_{\frac{1}{s-s_k} \to \frac{T_d}{1-e^{s_k T_d} z^{-1}}} = \sum_{k=0}^{N-1} \frac{T_d A_k}{1 - e^{s_k T_d} z^{-1}}$$

3. Relationship between the frequency response of the discrete-time filter and the continuous-time filter:

$$H(e^{i\omega}) = \sum_{k=-\infty}^{\infty} H_c\left(j\frac{\omega}{T_d} + j\frac{2\pi}{T_d}k\right)$$

If the continuous-time filter is bandlimited, so that

$$H_c(j\Omega) = 0 \qquad |\Omega| \geq \pi / T_d$$

then $$H(e^{j\omega}) = H_c\left(j\frac{\omega}{T_d}\right) \qquad |\omega| \leq \pi$$

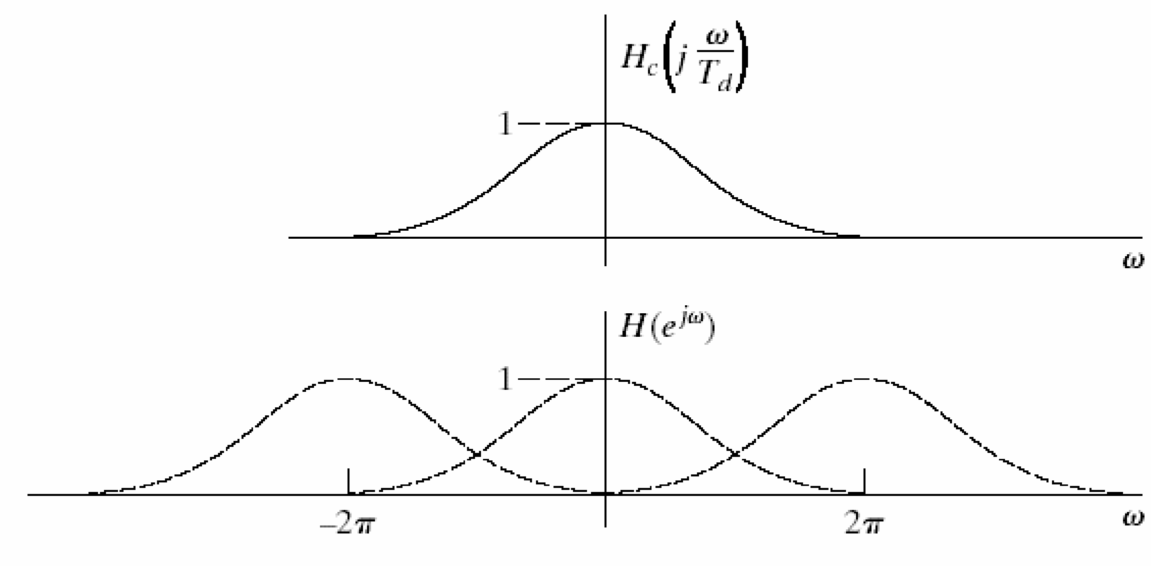


Figure 7.6

relation between frequencies of the discrete-time filter and the continuous-time filter:

$$\omega = \Omega T_d, -\pi < \omega < \pi, -\infty < \Omega < \infty$$

it's not one-to-one mapping. This effects the alias of the frequency response.

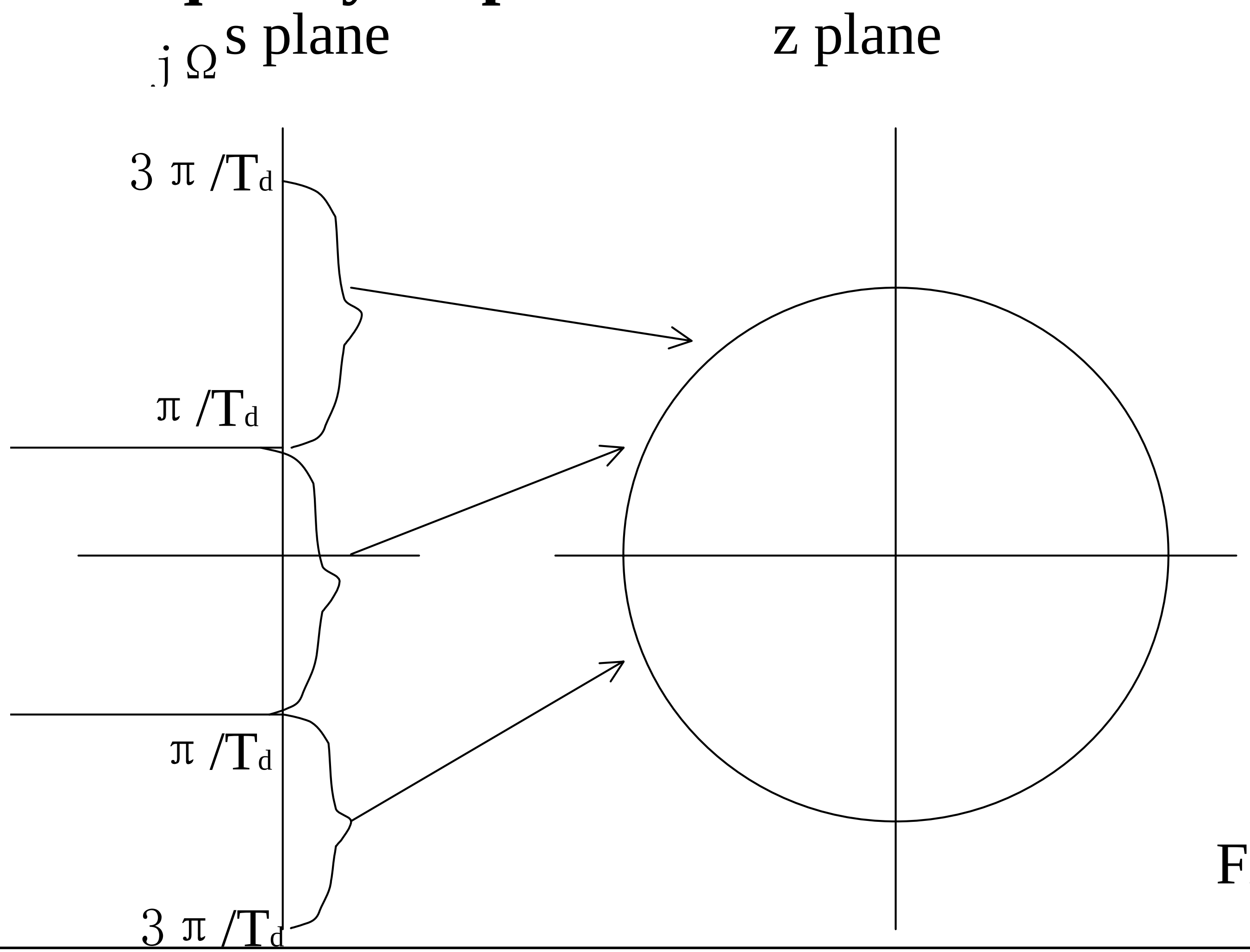


Figure 7.7

4.Stability (Relationship between poles):

$$s_k \rightarrow z_k = e^{s_k T_d}$$

$$if\ : \mathrm{Re}\{s_k\} < 0$$

$$then : |z_k| < 1$$

5.design steps：

(1)discrete-time specifications $\xrightarrow{\Omega_p=\omega_p/T_d,\ \Omega_S=\omega_S/T_d}$ **original continuous-time specifications**

(2) design $H_c(s)$

(3) $H(z)=H_c(s)\Big|_{\frac{1}{s-s_k}\to\frac{T_d}{1-e^{s_kT_d}z^{-1}}}$

6. About T_d :

1) **It has nothing to do with the sampling period T associated with the C/D and D/C conversion in figure 7.2;**
2) **Because we begin the design problem with the discrete-time filter specifications, the parameter T_d has no role in controlling aliasing and in the resulting discrete-time filter. we can choose arbitrarily the value of T_d , generally we choose 1 as a result. However, we should keep the same value of T_d in the system.**

7. Character:

Advantage: linear mapping of frequency;

Disadvantage: aliasing of frequency response ;

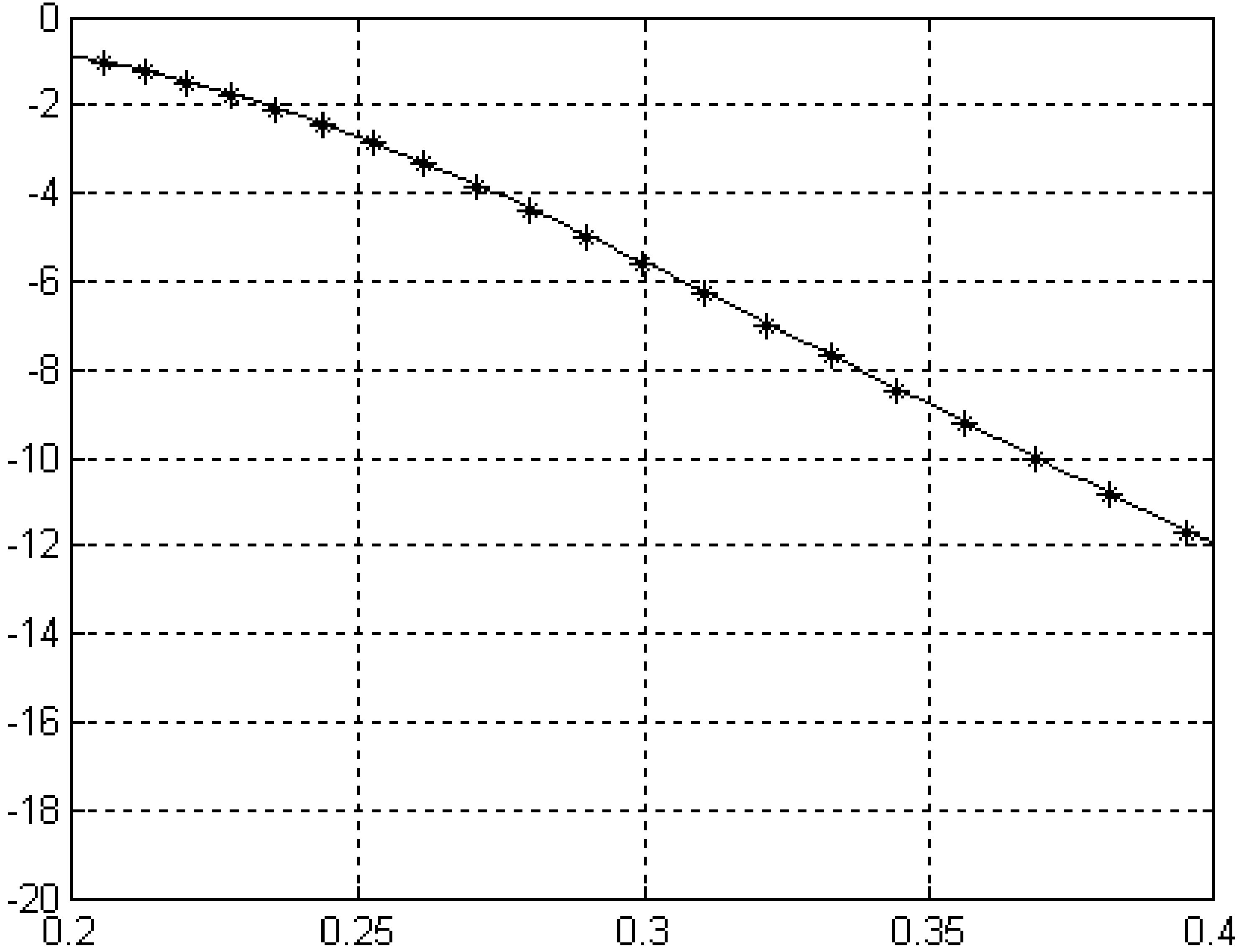
Restriction of application: can not be applies to high pass and band stop filters which are not band limited.

EXAMPLE

$$design \quad a \quad BW \quad digital \quad lowpasss \quad filter, need:$$

$$\omega_p = 0.2\pi, \omega_s = 0.4\pi, \alpha_p = 1dB, \alpha_s = 12dB,$$

```
wp=0.2*pi;                ws=0.4*pi
ap=1;                     as=12
Td=1;                     Wp=wp/Td;              Ws=ws/Td
[N,Wc]=buttord(Wp,Ws, ap , as, ’ s’ )
[Bs,As]=butter(N,Wc, ’ s’ )
[Bz,Az]=impinvar(Bs,As,1/Td)
[H,W]=freqs(Bs,As);
plot(W/pi,20*(log10(abs(H))), 'r*')
hold on
[H,w]=freqz(Bz,Az);
plot(w/pi,20*(log10(abs(H))))
axis([0.2,0.4,-20,0])
grid
```


0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
0.2
0.25
0.3
0.35
0.4

手算： $(1)\Omega_p = \dfrac{\omega_p}{T_d} = 0.2\pi rad/s, \Omega_s = \dfrac{\omega_s}{T_d} = 0.4\pi rad/s$

$$(2)N = \log_{\frac{\Omega_p}{\Omega_s}} \left[\frac{10^{\frac{\alpha_p}{10}} - 1}{10^{\frac{\alpha_s}{10}} - 1}\right]^{\frac{1}{2}} = 2.9208 \rightarrow 3$$

$$\Omega_c = \frac{\Omega_p}{\left(10^{\frac{\alpha_p}{10}} - 1\right)^{\frac{1}{2N}}} = 0.7870$$

$$s_k = \Omega_c e^{j\left(\frac{2k+1}{2N} + \frac{1}{2}\right)\pi}, k = 0,1,2 \qquad = -0.3935 \pm j0.6816 \qquad k = 0,2$$

$$= -0.7870 \qquad k = 1$$

$$H_c(s) = \prod_{k=0}^{N-1} \frac{1}{1 - \dfrac{s}{s_k}} = \frac{\Omega_c{}^N}{\prod_{k=0}^{N-1}(s - s_k)}$$

$$= \frac{0.7870^3}{(s^2 + 0.7870s + 0.6194)(s + 0.7870)} = \frac{-0.3934 - j0.2271}{s - (-0.3935 + j0.6816)} + \frac{0.7865}{s - (-0.7870)} + \frac{-0.3934 + j0.2271}{s - (-0.3935 - j0.6861)}$$

$$(3)H(z) = H_a(s)\Big|_{\frac{1}{s - s_k} -> \frac{1}{1 - e^{s_k T} z^{-1}}}$$

$$= \frac{-0.3934 - j0.2271}{1 - e^{0.3935 + j0.6861} z^{-1}} + \frac{0.7865}{1 - e^{-0.7870} z^{-1}} + \frac{-0.3934 + j0.2271}{1 - e^{-0.3935 - j0.6816} z^{-1}}$$

EXAMPLE

$$design \quad a \quad BW \quad digital \quad high-passs \quad filter, need:$$

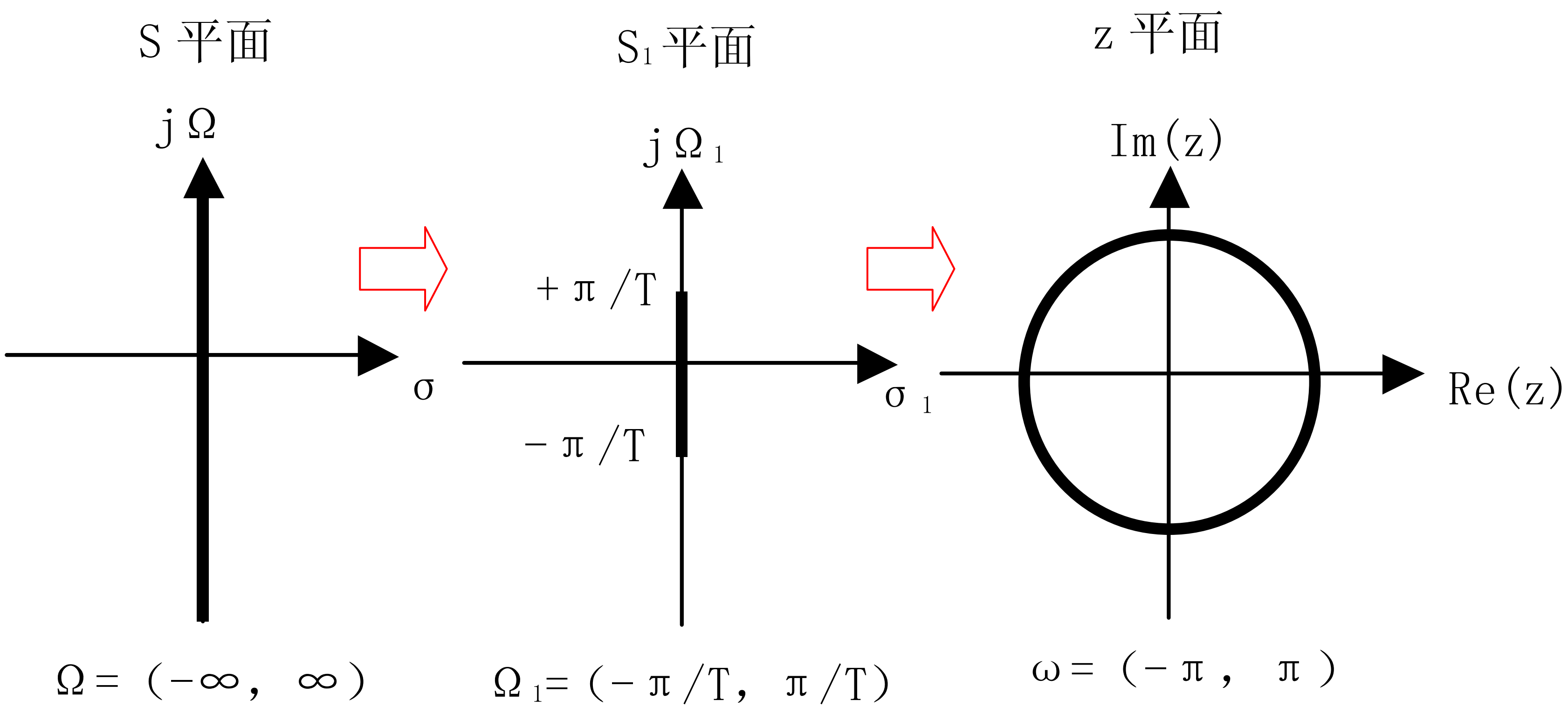
$$\omega_p = 0.6\pi, \omega_s = 0.5\pi, \alpha_p = 1dB, \alpha_s = 12dB,$$

```
[N,Wc]=buttord(0.6*pi,0.5*pi, 1 ,12, ’ s’ )
[Bs,As]=butter(N,Wc, 'high', ’ s’ )
[Bz,Az]=impinvar(Bs,As,1)
[H,W]=freqs(Bs,As);
plot(W/pi,20*(log10(abs(H))), 'r*')
figure
[H,w]=freqz(Bz,Az);
plot(w/pi,20*(log10(abs(H))))
```

7.1.2 filter design by bilinear transfor（双线性变换）

The technique discussed in this section avoids the problem of aliasing by using the bilinear transformation, an algebraic transformation between the variables s and z that maps the entire $j\Omega$ -axis in the s-plane to one revolution of the unit circle in the z-plane. The transformation between the continuous-time and discrete-time frequency variables must be nonlinear. Therefore, the use of this technique is restricted to situations in which the corresponding warping of the frequency axis is acceptable.

1. Principle



2. Transforming Equation:

With $H_c(s)$ denoting the continuous-time system function and H(z) the discrete-time system function, the bilinear transformation corresponds to replacing s by

$$s = \frac{2}{T_d}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

That is,

$$H(z) = H_c\left[\frac{2}{T_d}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)\right]$$

3.stable and causal:

we solve for z to obtain

$$z = \frac{1 + (T_d / 2)s}{1 - (T_d / 2)s}$$

and, substituting $s = \sigma + j\Omega$

$$z = \frac{1 + \sigma T_d / 2 + j\Omega T_d / 2}{1 - \sigma T_d / 2 - j\Omega T_d / 2}$$

If σ <0, then, it follows that |z|<1 for any value of Ω .

That is ,if a pole of $H_c(s)$ is in the left-half s-plane, its image in the z-plane will be inside the unit circle. Therefore, causal stable continuous-time filters map into causal stable discrete-time filters.

4. relation between frequencies:

If $s = j\Omega$ **then**

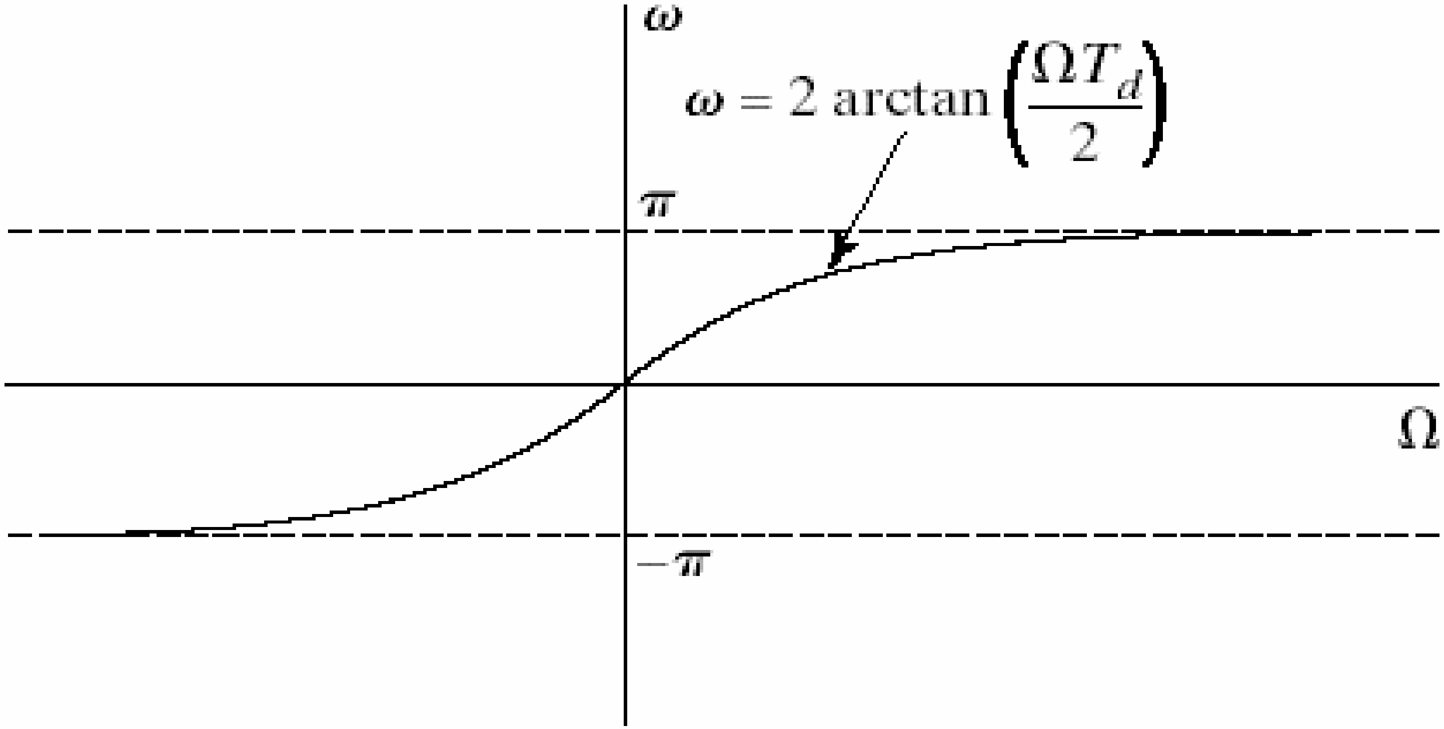
$$z = \frac{1 + j\Omega T_d / 2}{1 - j\Omega T_d / 2}$$

|z|=1. That is ,the $j\Omega$ **-axis maps onto the unit circle(one-to-one mapping).**

So, substitute $z = e^{i\omega}$ **, Get relation between frequencies**

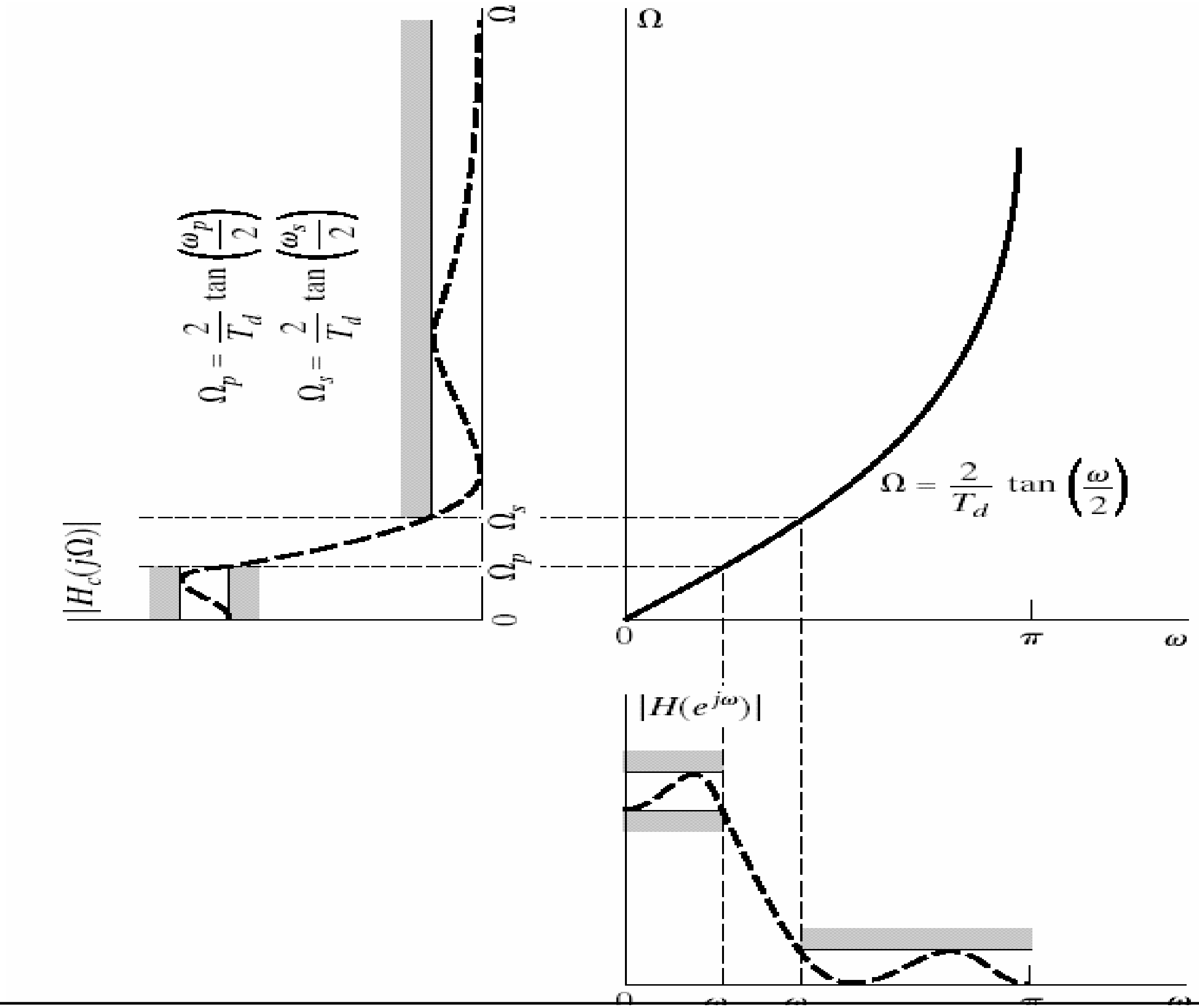
$$\omega = 2\arctan\left(\frac{T_d}{2}\Omega\right) \qquad \Omega = \frac{2}{T_d}\tan\left(\frac{\omega}{2}\right)$$

$$\omega = 2\arctan\left(\frac{T_d}{2}\Omega\right)$$



relation between frequency response:

$$H(e^{j\omega}) = H_c(j\Omega)\Big|_{\Omega=\frac{2}{T_d}\tan\left(\frac{\omega}{2}\right)}$$



So, the specification for the continuous-time filter should be calculated by pre-warp（预畸变）:

$$\begin{cases} \Omega_p = \dfrac{2}{T_d}\tan\left(\dfrac{\omega_p}{2}\right) \\ \Omega_s = \dfrac{2}{T_d}\tan\left(\dfrac{\omega_s}{2}\right) \end{cases}$$

5. design steps:

1)discrete-time specifications $\xrightarrow{\Omega_p = \frac{2}{T_d}\tan(\omega_p/2),\Omega_S = \frac{2}{T_d}\tan(\omega_s/2)}$

original continuous-time specifications

2) $H_c(s)$

3) $H(z) = H_c(s)|_{s=\frac{2}{T_d}\cdot\frac{1-z^{-1}}{1+z^{-1}}}$

6. About T_d

As with impulse invariance, the parameter T_d is of no consequence in the design procedure, since we assume that the design problem always begins with specifications on the discrete-time filter $H(e^{j\omega})$. When these specifications are mapped to continuous-time specifications and the continuous-time filter is then mapped back to a discrete-time filter, the effect of T_d will cancel.

7. Characters:

Advantage：avoids the problem of aliasing encountered with the use of impulse invariance, because it maps the entire imaginary axis of the s-plane onto the unit circle in the z-plane；

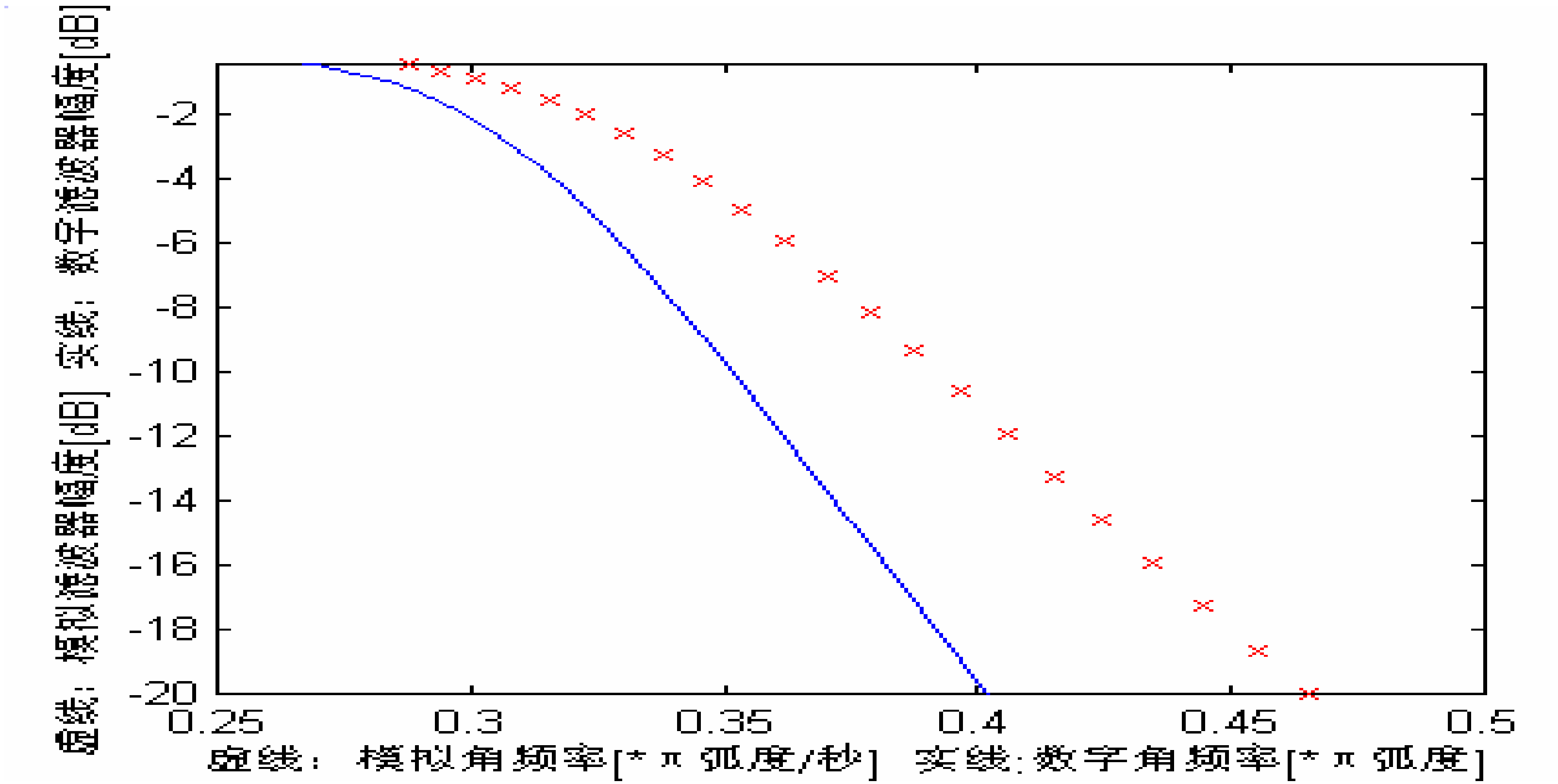
Disadvantage: the nonlinear compression of the frequency axis discussed before.

Application: the design of discrete-time filters using the bilinear transformation is useful only when this compression can be tolerated or compensated for.

EXAMPLE $design \quad a \quad lowpass \quad filter, need$

$$\omega_p = 0.2613\pi, \omega_s = 0.4018\pi, \alpha_p = 0.75dB, \alpha_s = 20dB,$$

```
wp=0.2613*pi;      ws=0.4018*pi;          ap=0.75;      as=20;
Td=1;              Ws=2/Td*tan(ws/2);  Wp=2/Td*tan(wp/2)
[N,Wc]=buttord(Wp,Ws,ap,as,’s’)
[Bs,As]=butter(N,Wc, ‘s’)
[Bz,Az]=bilinear(Bs,As,1/Td)
[H,W]=freqs(Bs,As);
plot(W/pi,20*(log10(abs(H))),’Rx‘)
hold on
[H,w]=freqz(Bz,Az);
plot(w/pi,20*(log10(abs(H))))
ylabel(‘虚线：模拟滤波器幅度[dB] 实线：数字滤波器幅度[dB]’)
xlabel(‘虚线：模拟角频率[*π弧度/秒] 实线:数字角频率[*π弧度]’)
axis([0.25,0.5,-20,-0.45])
grid
```



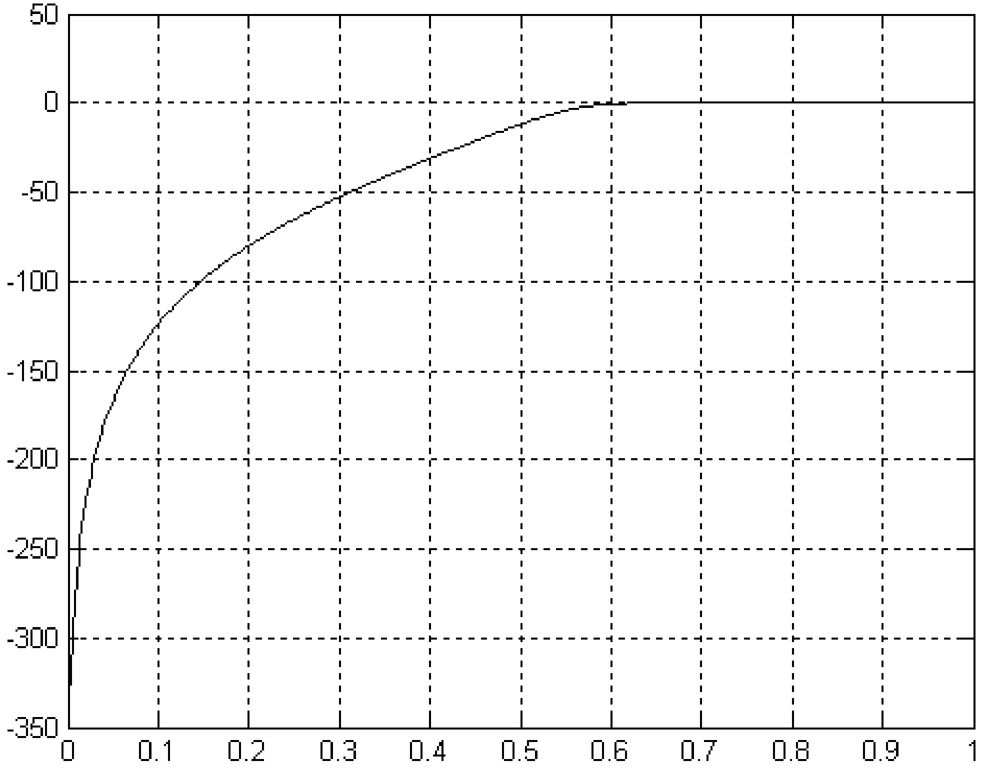
OR

```
wp=0.2613*pi;        ws=0.4018*pi;ap=0.75;        as=20
[N,wc]=buttord(wp/pi,ws/pi,ap,as)
[Bz,Az]=butter(N,wc)
[H,w]=freqz(Bz,Az);
plot(w/pi,20*(log10(abs(H))))
```

EXAMPLE *design a highpass filter, need*

$$\omega_p = 0.6\pi, \omega_s = 0.5\pi, \alpha_p = 1dB, \alpha_s = 12dB,$$

```
[N,Wc]=buttord(0.6,0.5,ap,as)
[Bz,Az]=butter(N,Wc,'high')
[H,w]=freqz(Bz,Az);
plot(w/pi,20*(log10(abs(H))))
grid
```



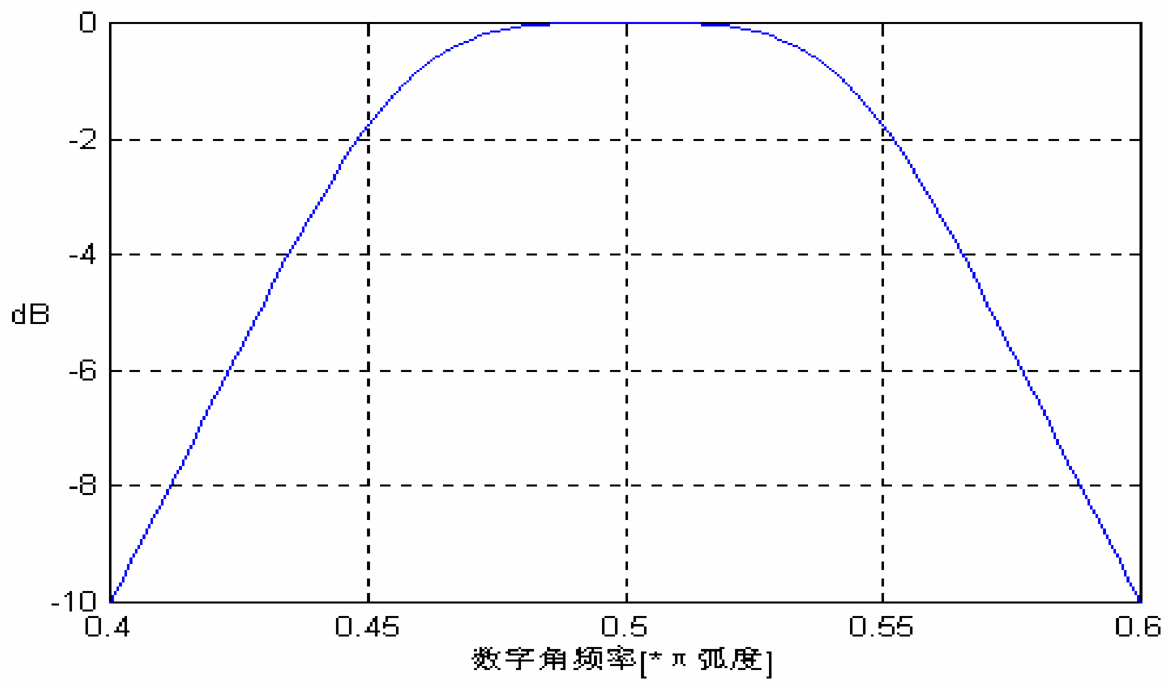
EXAMPLE

Design a band pass filter

$$\omega_0 = 0.5\pi rad,$$

$$\omega_{pl} = 0.45\pi rad, \omega_{pu} = 0.55\pi rad : 3dB$$

$$\omega_{sl} = 0.4\pi rad, \omega_{su} = 0.6\pi rad : 10dB$$



```
[N,wc]=buttord([0.45 0.55],[0.4 0.6],3,10) %双线性变换法
[B,A]=butter(N,wc)
[H,w]=freqz(B,A);
plot(w/pi,20*(log10(abs(H))))
ylabel('20log|H(ejω)| [dB]')
xlabel('数字角频率[*π 弧度]')
axis([0.4,0.6,-10,0]);                grid on
```

Output:

N = 2

wc = 0.4410 0.5590

B =	**0.0271**	**0**	**-0.0541**	**0**	**0.0271**
A =	**1.0000**	**0**	**1.4838**	**0**	**0.5920**

7.1.3 IIR summary

1.design steps

（1）等效的模拟系统指标 $\xrightarrow{\omega_p = \Omega'_p T, \omega_S = \Omega'_S T}$ 数字指标

$$\xrightarrow[\Omega_p = \frac{2}{T_d}\tan(\omega_p/2), \Omega_S = \frac{2}{T_d}\tan(\omega_s/2)]{\Omega_p = \omega_p/T_d, \Omega_S = \omega_S/T_d}$$ 原型模拟滤波器的指标

（2）$H_a(s)$

(3)$H_a(s) \xrightarrow[s = \frac{2}{T_d}\cdot\frac{1-z^{-1}}{1+z^{-1}}]{\frac{1}{s-s_k} -> \frac{T_d}{1-e^{s_k T_d} z^{-1}}} H$（$z$）

2.冲击响应不变法**:**

频率轴线性多对一映射**,**频响有混迭**,**不适用于高通等

双线性变换法**:**

频率轴有畸变一对一映射**,**频响无混迭**,**不适用于微分器

7.1.4 频率变换

模拟域频率变换：

低通$\rightarrow$低通：$s \rightarrow s/\Omega_c$

低通$\rightarrow$高通：$s \rightarrow \Omega_c / s$

低通$\rightarrow$带通：$s \rightarrow (s^2 + \Omega_{c1}\Omega_{c2})/[s(\Omega_{c2} - \Omega_{c1})]$

低通$\rightarrow$带阻：$s \rightarrow [s(\Omega_{c2} - \Omega_{c1})]/(s^2 + \Omega_{c1}\Omega_{c2})$

Ω_{c2}:上截止频率，Ω_{c1}：下截止频率

数字域频率变换：

低通→低通：$z^{-1} \to \dfrac{z^{-1}-\alpha}{1-z^{-1}\alpha}$

低通→高通：$z^{-1} \to -\dfrac{z^{-1}+\alpha}{1+z^{-1}\alpha}$

低通→带通：$z^{-1} \to -\dfrac{z^{-2}-\dfrac{2\alpha k}{k+1}z^{-1}+\dfrac{k-1}{k+1}}{\dfrac{k-1}{k+1}z^{-2}-\dfrac{2\alpha k}{k+1}z^{-1}+1}$

低通→带阻：$z^{-1} \to \dfrac{z^{-2}-\dfrac{2\alpha k}{k+1}z^{-1}+\dfrac{k-1}{k+1}}{-\dfrac{k-1}{k+1}z^{-2}-\dfrac{2\alpha k}{k+1}z^{-1}+1}$

α, k与转换前后滤波器的截止频率有关

7.2 design of FIR filters by windowing

7.2.1 design ideas
7.2.2 properties of commonly used windows
7.2.3 effect to frequency response
7.2.4 design step

7.2.1 design ideas

The window method generally begins with an ideal desired frequency response, it's inverse transform is

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

Many idealized systems are defined by piecewise-constant or piecewise-functional frequency responses with discontinuities at the boundaries between bands. As a results, these systems have impulse responses that are non-causal and infinitely long.

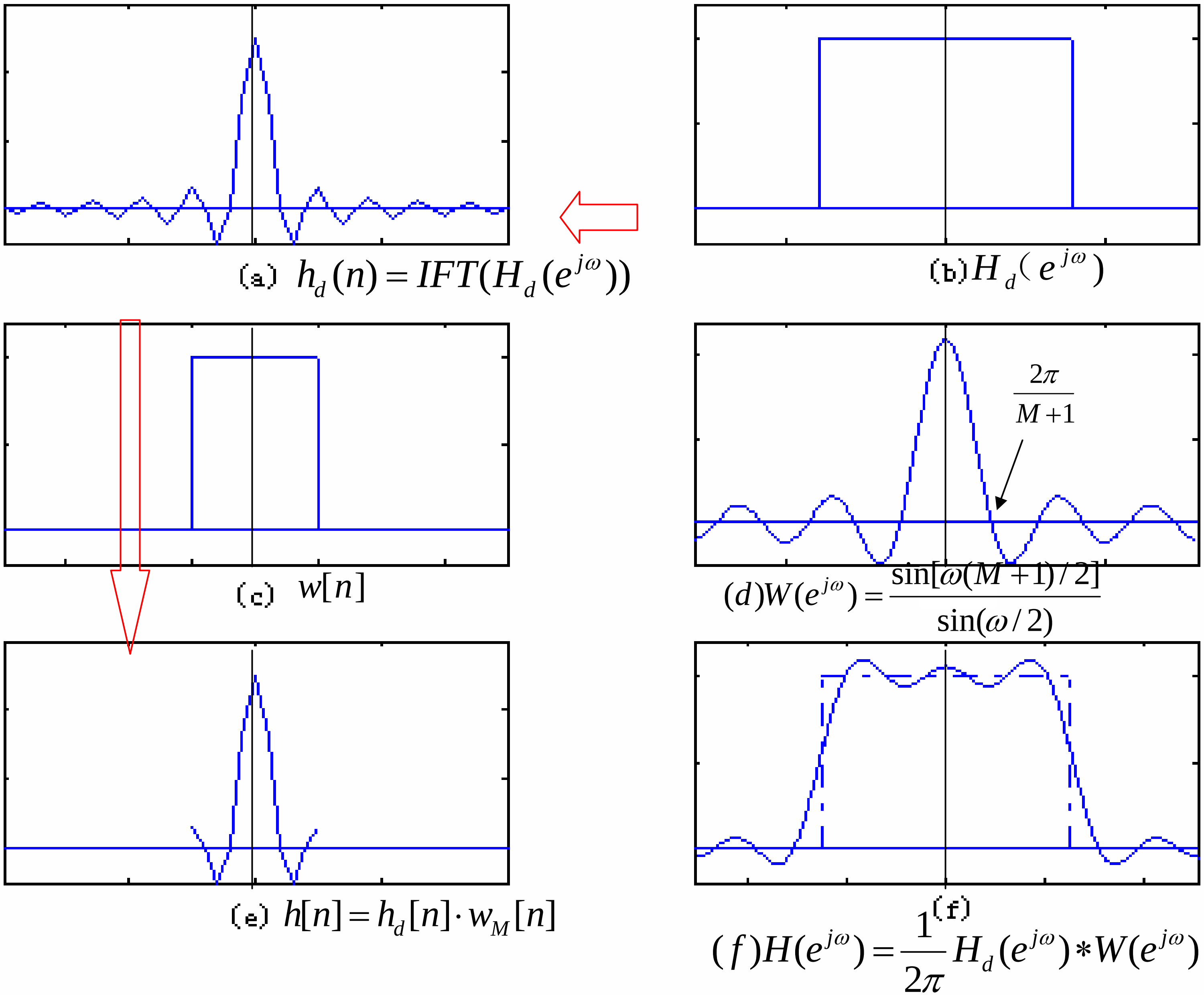
The most straightforward approach to obtaining a causal FIR approximation to such systems is to truncate the ideal response:

$$h[n] = h_d[n]w[n]$$

where, the window w[n] is the rectangular window.

The corresponding effect in the frequency domain is

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta})W(e^{j(\omega-\theta)})d\theta$$



Gibbs phenomenon

(1)The width of the resulting transition band is determined by the width of the main lobe of the Fourier transform of the window.

(2)The pass band and stop band ripples are determined by the side lobes of the Fourier transform of the window. The ripples in the pass band and the stop band are approximately the same, and are not dependent on M and can be changed only by changing the shape of the window.

滤波器的过渡带宽←窗频谱的主瓣宽←窗长和窗形状
滤波器的阻带衰减←窗频谱的旁瓣相对幅度←窗形状

7.2.2 properties of commonly used windows

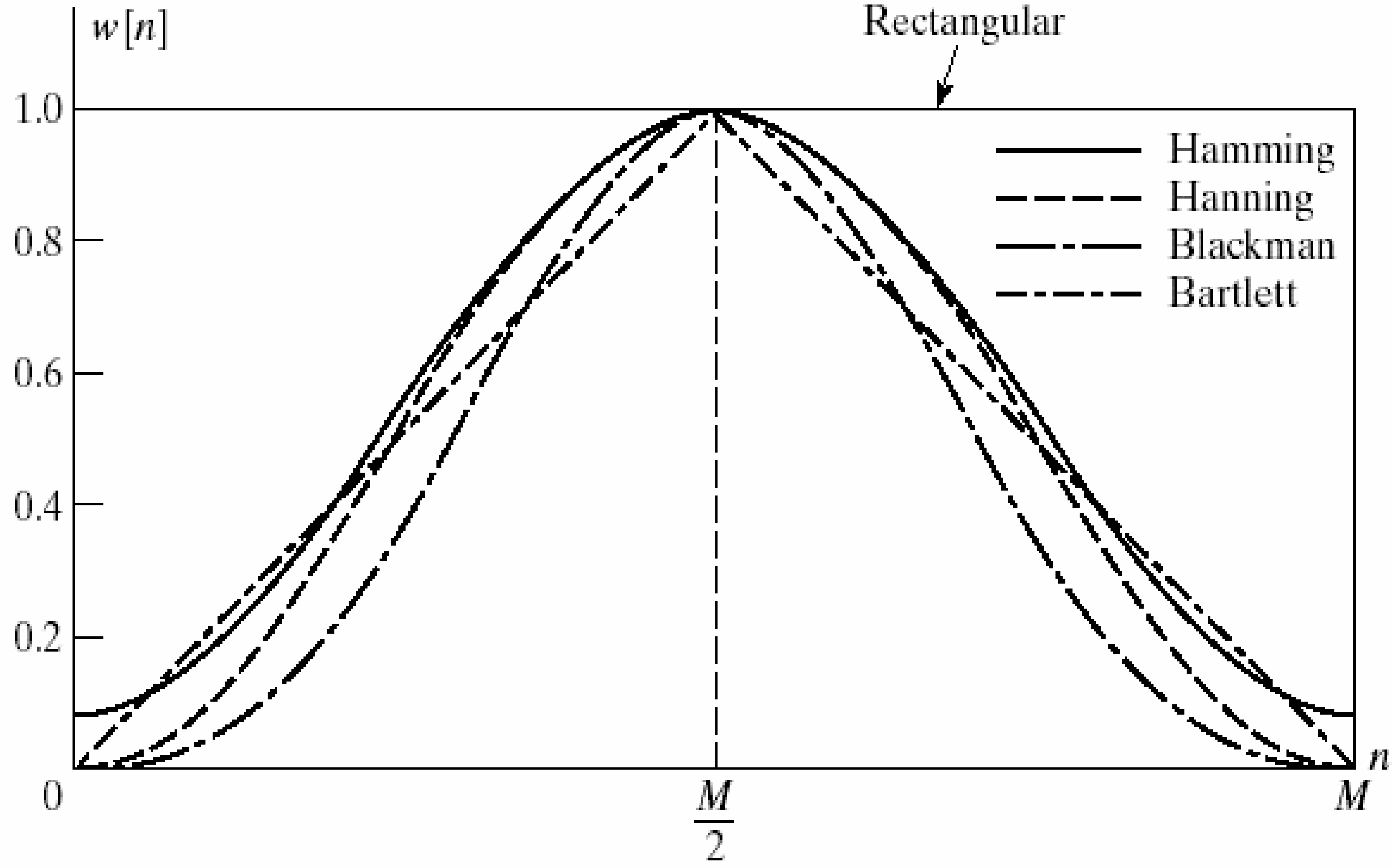
$$(1)\ rectangular: w[n]=\begin{cases}1, & 0\le n\le M\\ 0, & other\end{cases}$$

$$(2)\ bartlett\ \ (triangular): w[n]=\begin{cases}\dfrac{2n}{M}, & 0\le n\le \dfrac{M}{2}\\ 2-\dfrac{2n}{M}, & \dfrac{M}{2}\le n\le M\\ 0, & other\end{cases}$$

$$(3)\ hanning: w[n]=\begin{cases}0.5-0.5\cos\left(\dfrac{2\pi n}{M}\right), & 0\le n\le M\\ 0 & other\end{cases}$$

$$(4)\ hamming: w[n]=\begin{cases}0.54-0.46\cos\left(\dfrac{2\pi n}{M}\right), & 0\le n\le M\\ 0 & other\end{cases}$$

$$(5)\ blackman: w[n]=\begin{cases}0.42-0.5\cos\left(\dfrac{2\pi n}{M}\right)+0.08\cos\left(\dfrac{4\pi n}{M}\right), & 0\le n\le M\\ 0 & other\end{cases}$$



w[n]
Rectangular
1.0
0.8
0.6
0.4
0.2
0
Hamming
Hanning
Blackman
Bartlett
n
M/2
M

Figure 7.21

$$(6)kaiser : w[n] = \begin{cases} \dfrac{I_0\left(\beta(1-[(n-\frac{M}{2})/\left(\frac{M}{2}\right)]^2)^{\frac{1}{2}}\right)}{I_0(\beta)} & 0 \le n \le M \\ 0 & other \end{cases}$$

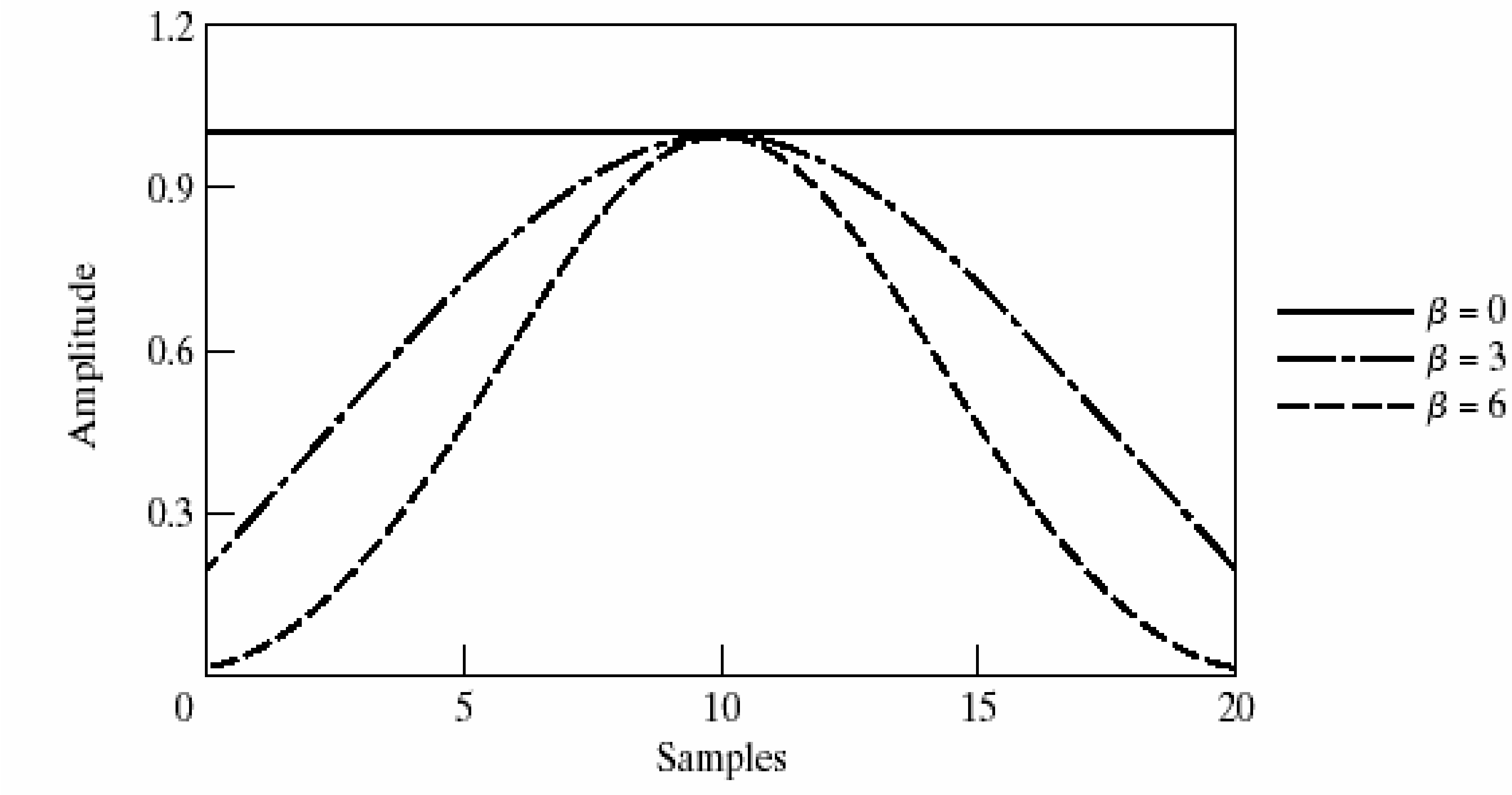


Figure 7.24(a)

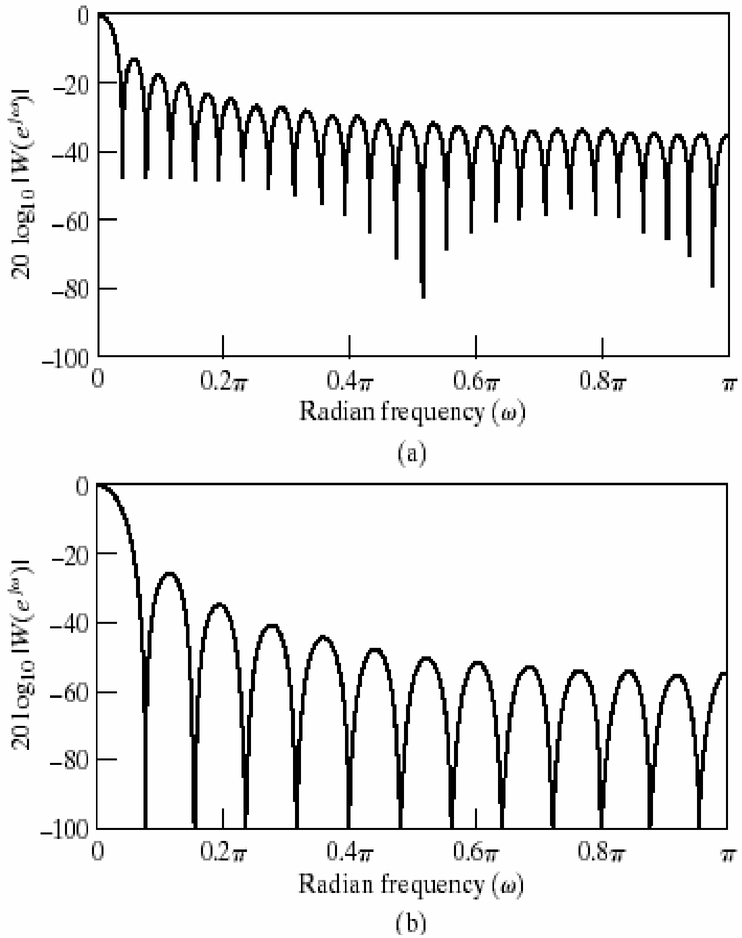
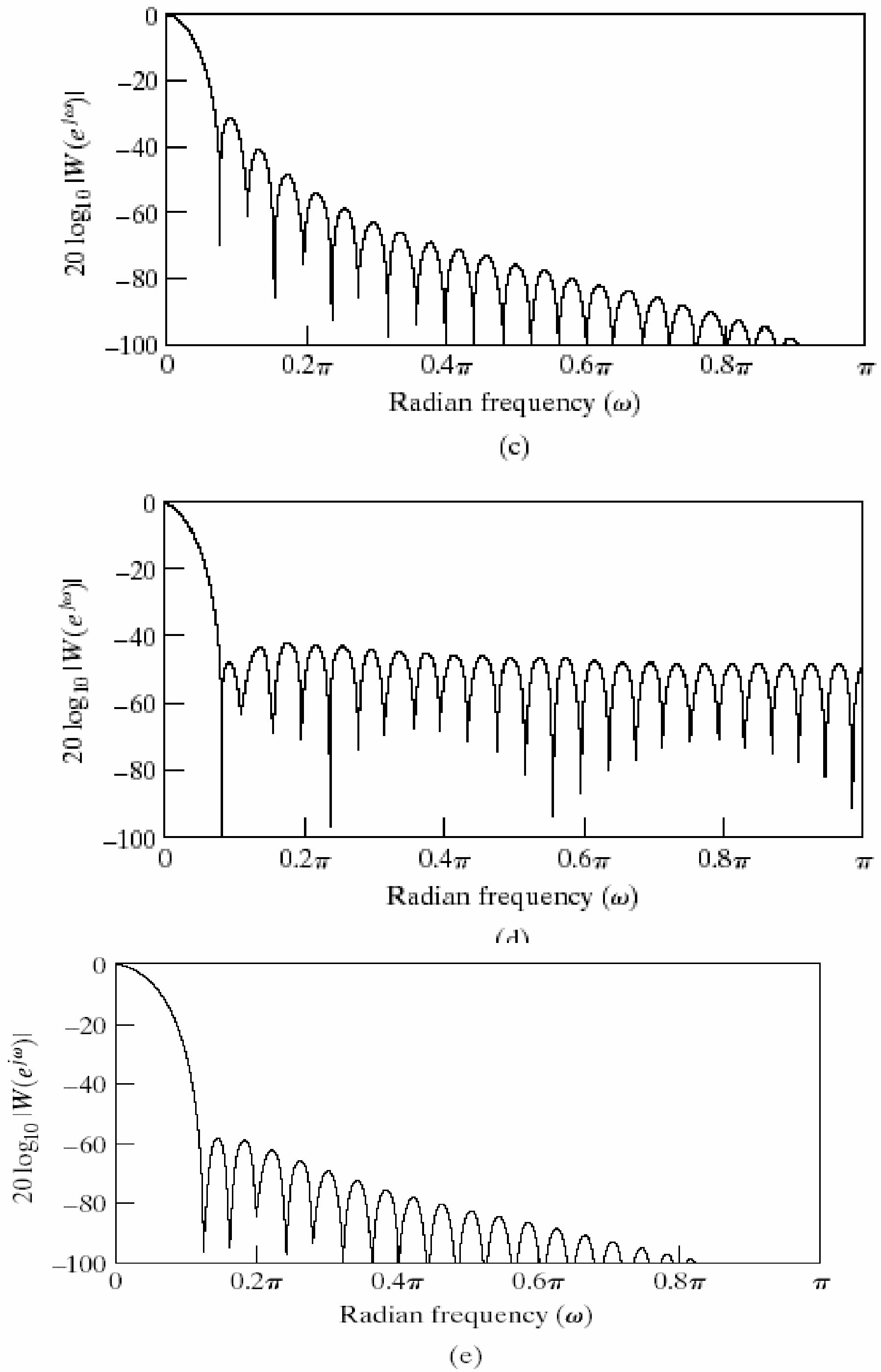
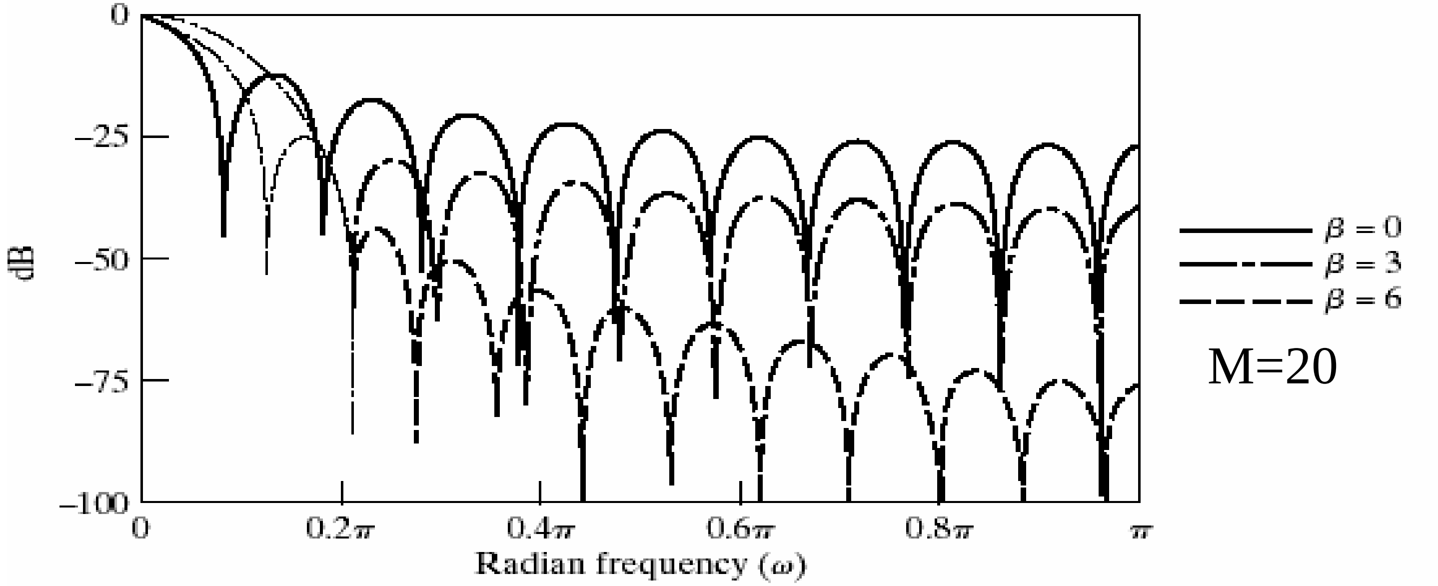


Figure 7.22 Fourier transforms (log magnitude) of windows of Figure 7.21. with $M = 50$. (a) Rectangular. (b) Bartlett. (c) Hanning. (d) Hamming. (e) Blackman.



Blackman family：

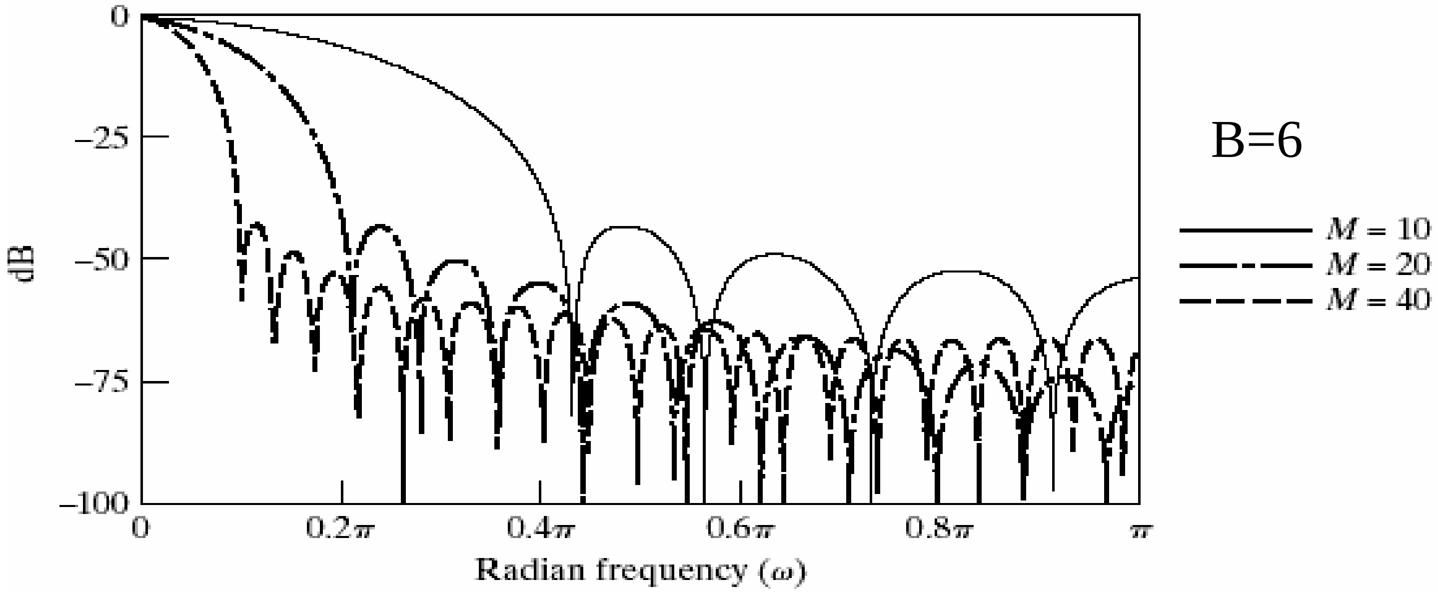
(a)-(e)旁瓣衰减增，主瓣宽增



(b)

M=20

随着**B**增：
旁瓣衰减增，
主瓣宽增



(c)

B=6

随着**N**增：
旁瓣衰减不变
主瓣宽减

Figure 7.24 Fourier transform of Kaiser family

	窗频谱的旁瓣衰减	窗频谱的主瓣宽	滤波器的阻带衰减		滤波器的过度带宽
Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)	Equivalent Kaiser Window, β	Transition Width of Equivalent Kaiser Window
Rectangular	−13	$4\pi/(M+1)$	−21	0	$1.81\pi/M$
Bartlett	−25	$8\pi/M$	−25	1.33	$2.37\pi/M$
Hanning	−31	$8\pi/M$	−44	3.86	$5.01\pi/M$
Hamming	−41	$8\pi/M$	−53	4.86	$6.27\pi/M$
Blackman	−57	$12\pi/M$	−74	7.04	$9.19\pi/M$

Table 7.1

滤波器的过度带宽略小于主瓣宽

同样阻带衰减的凯窗

7.2.3 effect to frequency response

布窗查表**7.1**

凯窗公式计算

$$\beta = \begin{cases} 0.1102(A-8.7) & A > 50 \\ 0.584(A-21)^{0.4} + 0.07886(A-21) & 21 \le A \le 50 \\ 0 & A < 21 \end{cases}$$

$$\Delta\omega = \frac{A-8}{2.285M}$$

Δ ω：**transition width,** $A = -20\log_{10}\delta$

7.2.4 design step

1.obtain the ideal impulse response

$$\omega_c = (\omega_p + \omega_s)/2$$

$$H_d(e^{j\omega}) = H_d(\omega)e^{j\Phi(\omega)} = \begin{cases} e^{-j\frac{M}{2}\omega} & pass \quad band \\ 0 & stop \quad band \end{cases}$$

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

2.Determine the window shape according to pass band a stop band ripples:

$$\delta = \min(\delta_p, \delta_s) \qquad A = -20\log_{10}\delta$$

（1）refer to tables 7.1 if Blackman window;

（2）refer to the following formulation if Kaiser window:

$$\beta = \begin{cases} 0.1102(A-8.7) & A > 50 \\ 0.584(A-21)^{0.4} + 0.07886(A-21) & 21 \le A \le 50 \\ 0 & A < 21 \end{cases}$$

3.Determine the window length according the transition width (M is even for high-pass and band-stop filter）

$$\Delta\omega = |\omega_s - \omega_p|$$

(1) refer to tables 7.1 if Blackman window: $M = \dfrac{D\pi}{2\Delta\omega}$

(2) refer to the formulation if Kaiser window: $M = \dfrac{A-8}{2.285\Delta\omega}$

4.truncate the ideal impulse response

$$h[n] = h_d[n] \cdot w[n]$$

5.verify $H(e^{j\omega})$

and adjust ω_c, M, and β **until** $H(e^{j\omega})$ **satisfy the specifications**. (MATLAB)

Example 1

Blackman window

$$design \quad a \quad FIR \quad high \quad pass \quad filter\ , need\ :$$

$$\omega_p = 0.7\pi, \omega_s = 0.54\pi, \alpha_p = 3dB, \alpha_s = 40\,dB$$

(1) $\omega_c = (\omega_p + \omega_s)/2 = 0.62\pi$

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{\sin[\pi(n-M/2)]}{\pi(n-M/2)} - \frac{\sin[0.62\pi(n-M/2)]}{0.62\pi(n-M/2)}, n = -\infty......\infty$$

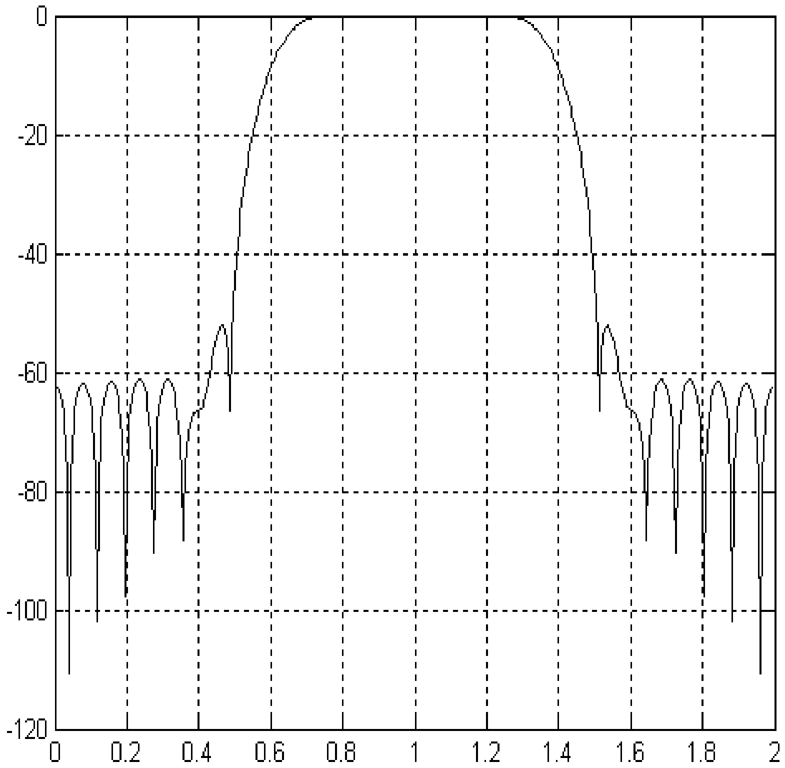
(2) $\delta = \min(\delta_p, \delta_s) = \min(1-10^{-\alpha_p/20}, 10^{-\alpha_s/20}) = 0.01, A = 40dB$ **hamming**

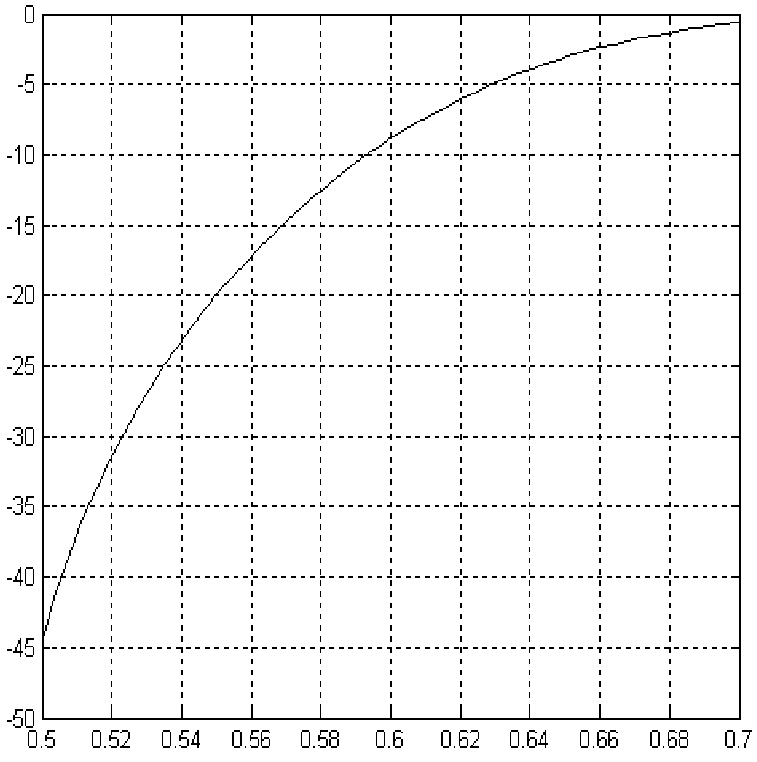
(3) $|\omega_p - \omega_s| = \frac{8\pi}{M}/2 = (0.7-0.54)\pi, \therefore M = 25 \to 26$

(4) $h[n] = \left\{\frac{\sin[\pi(n-13)]}{\pi(n-13)} - \frac{\sin[0.62\pi(n-13)]}{\pi(n-13)}\right\} \cdot \left[0.54 - 0.46\cos(\frac{2\pi n}{26})\right] \cdot R_{27}[n]$

（5）MATLAB verify

```
h=fir1(26,0.62,'high',hamming(27))
H=fft(h,512);
plot([0:511]/256,20*log10(abs(H)))
axis([0.5,0.7,-50,0]);                    grid on
```

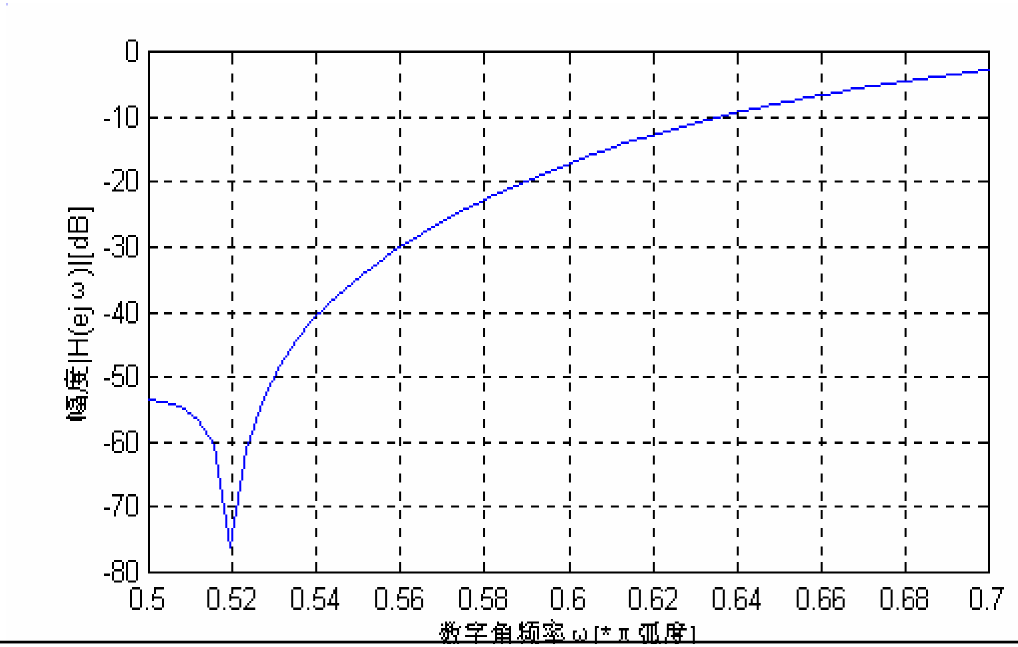




（6）adjust

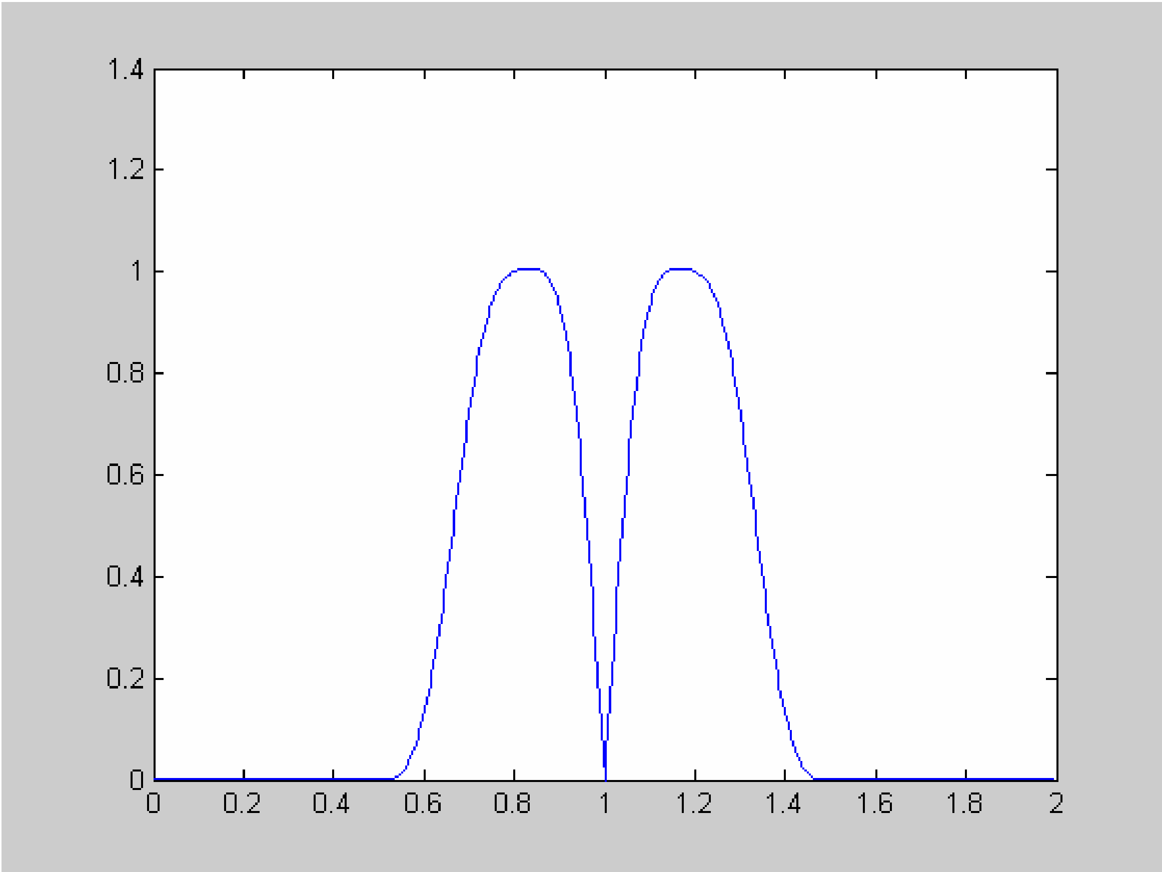
```
h=fir1(24,0.665,’high’,hamming(25))
H=fft(h,512);
plot([0:511]/256,20*log10(abs(H)))
axis([0.5,0.7,-80,0]);
grid on
```

h= 0.0001 0.0023 -0.0040 0.0004 0.0104 -0.0170 0.0009 0.0358
-0.0538 0.0014 0.1288 -0.2727 0.3357 -0.2727 0.1288 0.0014
-0.0538 0.0358 0.0009 -0.0170 0.0104
0.0004 -0.0040 0.0023 0.0001



M=25

```
n=0:25;
h=(sinc(n-12.5)-sinc((n-12.5)*0.665)*0.665).*hamming(26)';
H=fft(h,512);
plot([0:511]/256,abs(H))
```



Example 2 **Kaiser window**

design a FIR high pass filter, need :

$$\omega_p = 0.7\pi, \omega_s = 0.54\pi, \alpha_p = 3dB, \alpha_s = 40\,dB$$

Solution1:

(1) $\omega_c = (\omega_p + \omega_s)/2 = 0.62\pi$

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega = \frac{\sin[\pi(n-M/2)]}{\pi(n-M/2)} - \frac{\sin[0.62\pi(n-M/2)]}{0.62\pi(n-M/2)}, n=-\infty.....\infty$$

(2) $\delta = \min(\delta_p, \delta_s) = \min(1-10^{-\alpha_p/20}, 10^{-\alpha_s/20}) = 0.01, A = 40dB$

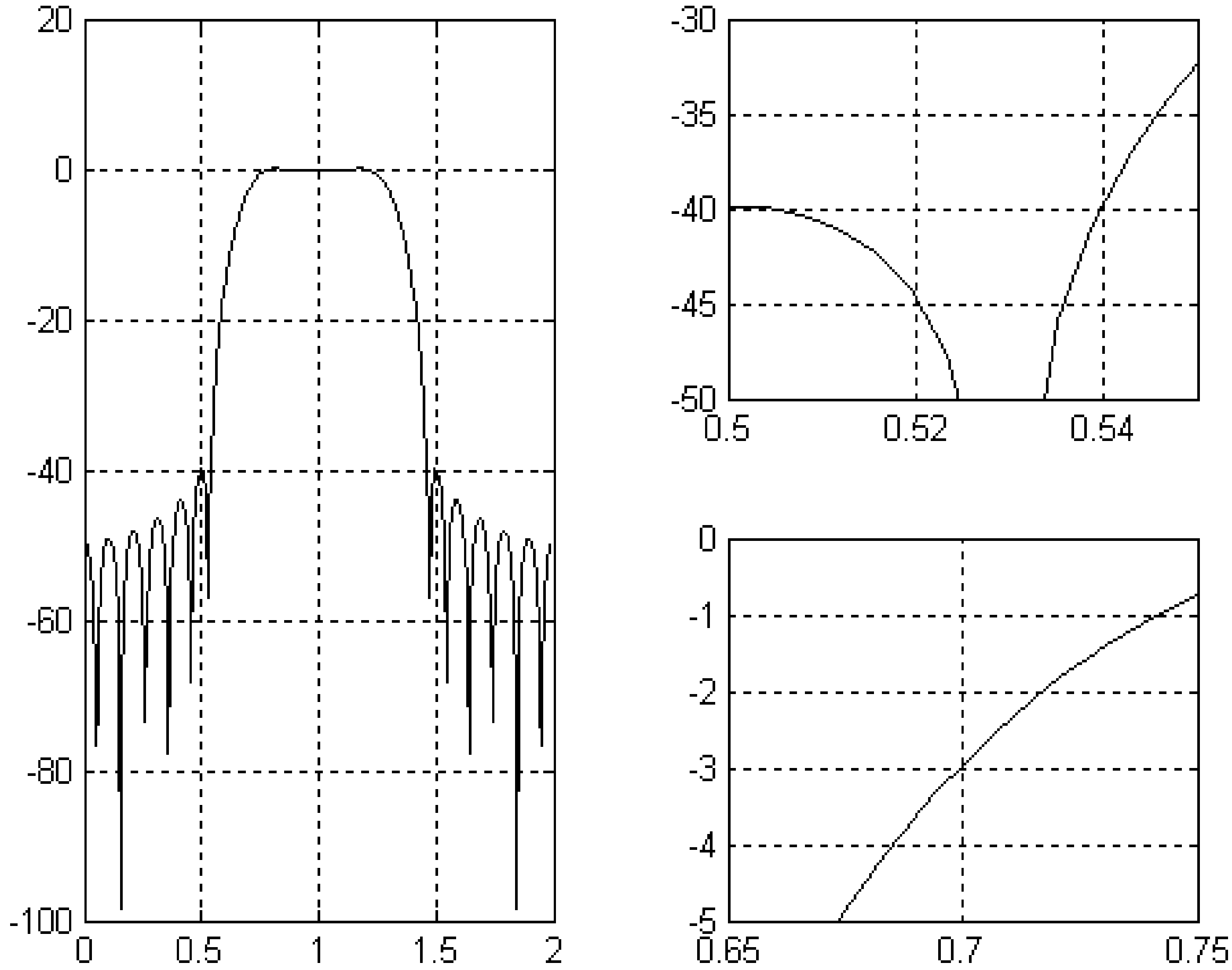
$$\beta = 0.5842(40-21)^{0.4} + 0.07886(40-21) = 3.3953$$

(3) $M = \dfrac{40-8}{2.285(0.7\pi - 0.54\pi)} = 27.86 \rightarrow 28$

(4) $h[n] = \dfrac{\sin[\pi(n-14)] - \sin[0.62\pi(n-14)]}{\pi(n-14)} \cdot w[n]$

（5）MATLAB verify and adjust

```
h=fir1(18,0.662, 'high', kaiser (19,3.2953));          H=fft(h,512);
k=0:511;
subplot(1,2,1);              plot(k/256,20*log10(abs(H)));  grid on;
subplot(2,2,2);              plot(k/256,20*log10(abs(H)));
axis([0.5,0.55,-50,-0]);     grid on
subplot(2,2,4);              plot(k/256,20*log10(abs(H)));
axis([0.65,0.75,-5,0]);      grid on
```



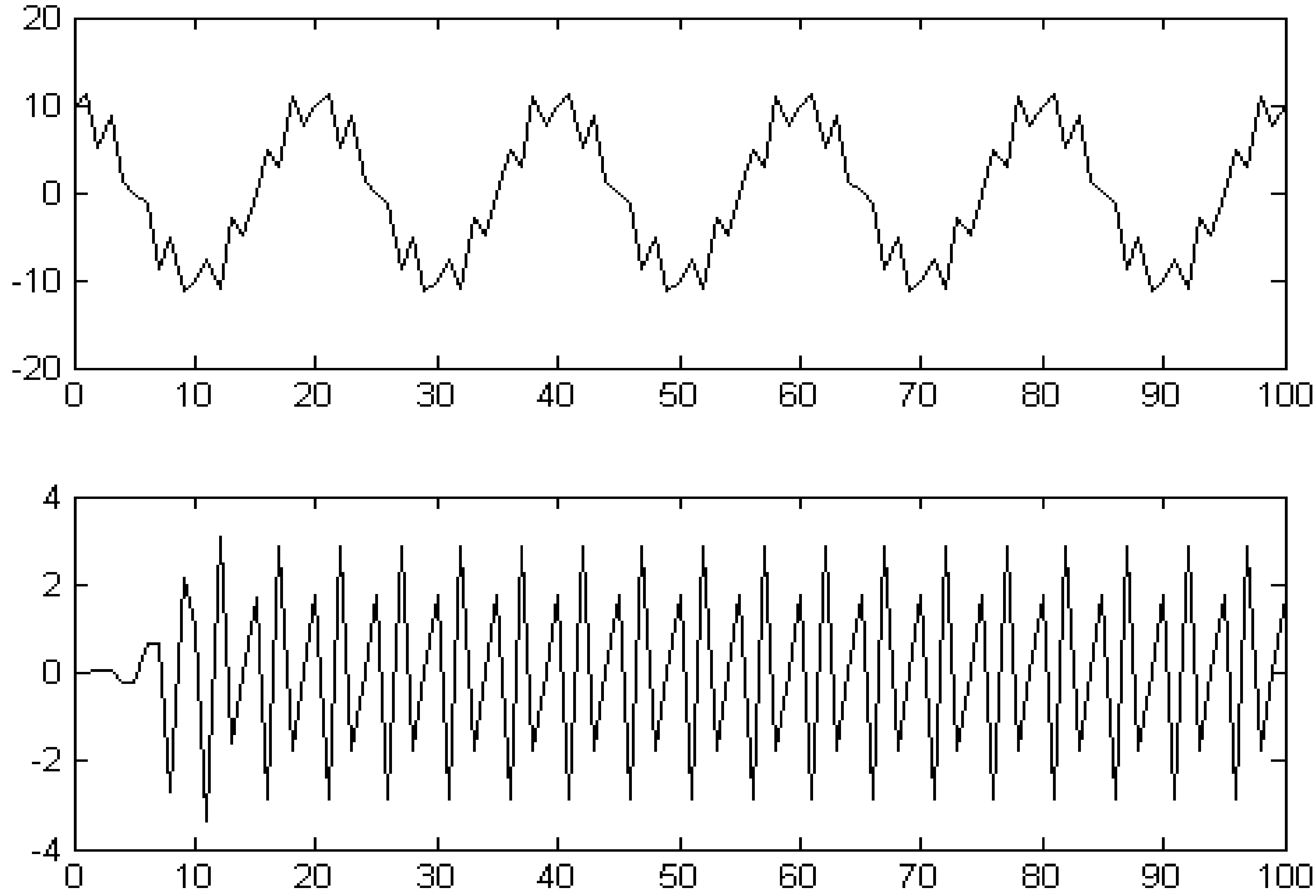
20
0
-20
-40
-60
-80
-100
0
0.5
1
1.5
2
-30
-35
-40
-45
-50
0.5
0.52
0.54
0
-1
-2
-3
-4
-5
0.65
0.7
0.75

Example 3

Perform high pass filtering to signal

$$x[n] = 10\cos(0.1\pi) + 3\sin(0.8\pi)R_{20}[n]$$

```
h=fir1(18,0.5, 'high',kaiser(19,6));
n=[0:100];      x=10*cos(0.1*pi*n)+3*sin(0.8*pi*n) ;
subplot(2,1,1);          plot(n,x)
y=filter(h,1,x) ;          subplot(2,1,2);          plot(n,y)
```


20
10
0
-10
-20
0 10 20 30 40 50 60 70 80 90 100
4
2
0
-2
-4
0 10 20 30 40 50 60 70 80 90 100

7.2.5 FIR summary

$w[n]$	$W(e^{j\omega})$	$H(e^{j\omega})$
窗形状	窗频谱的旁瓣衰减（表第一列）	滤波器的频响的阻带衰减（表第三列）
窗长	窗的主瓣宽度（表第二列）	滤波器的频响的过度带宽（表第二列/**2**）

7.3 滤波器的其他matlab设计简介

1. **IIR的频域最小均方误差法：invfreqz( )**
2. **FIR的频率取样法：fir2( )**
3. **FIR的最小均方误差法：firls( )**
4. **FIR的最优等波纹逼近法的Parks-McCellan/Remez算法：remez( )**

7.4 summary

FIR数字滤波器	**IIR**数字滤波器
单位取样响应有限长	单位取样响应无限长
总是稳定	要考虑稳定性
采用卷积或递归实现	不能用卷积实现，采用递归
极易实现线性相位	不能实现真正的线性相位
要达到同样的性能指标 阶数比**IIR**数字滤波器高很多	由于递归的采用，能以较低 阶数逼近指标
可以采用**FFT**快速实现	无快速算法

7.1 **IIR**设计（模拟滤波器法）

7.1.1冲击响应不变法

7.1.2双线性变换法

7.2 **FIR**设计（窗函数法）

7.2.1设计思想

7.2.2常用窗的性质

7.2.3加窗对频响的影响

7.2.4设计步骤

7.3 比较**IIR**和**FIR**

要求：

理解冲击响应不变法和双线性变换法的原理
二者的映射特点，应用场合
理解窗函数法的设计思路
用**MATLAB**设计各种滤波器

难点：

原型模拟滤波器、数字滤波器、等效模拟滤波器间的关系；
窗函数法中窗形状和窗长对系统特性的影响；
冲击不变法和**II**类**FIR**不能用于高通的原因。

Exercise and experiment

7.16 7.18 7.22(a)(b) 7.23

第二次实验

50
52
56
57
59