Chapter 4 sampling of continous-time signals

4.1 periodic sampling

4.2 discrete-time processing of continuous-time signals

4.3 continuous-time processing of discrete-time signal

4.4 digital processing of analog signals

4.5 changing the sampling rate using discrete-time processing

4.1 periodic sampling

$x[n] = x_c(nT)$ $T : sample \quad period$,

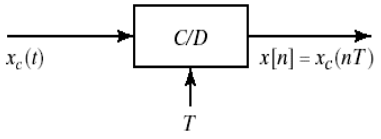
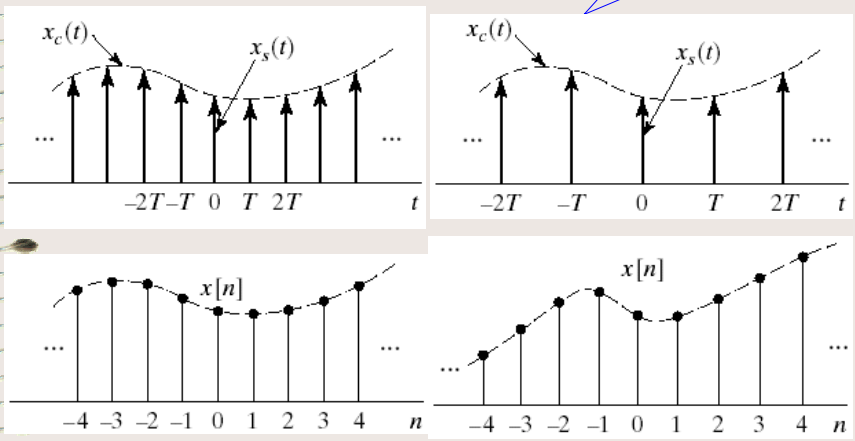


Figure 4.1 ideal continous-time-to-discrete-time(C/D)converter

T不同导致采样结果不同



3

理想采样的数学模型

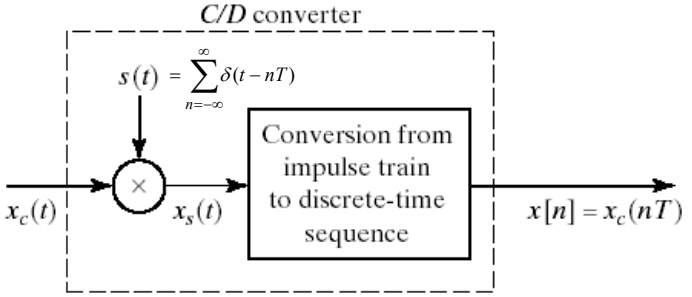
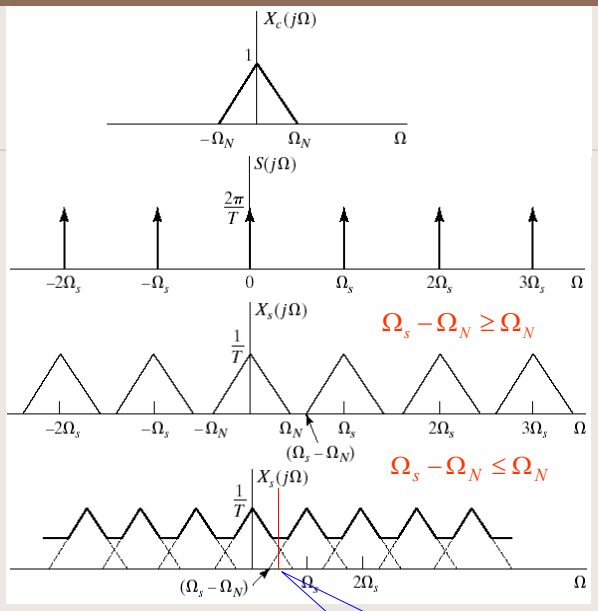


Figure 4.2(a)

4



理想采样频域变化，推导见课堂笔记

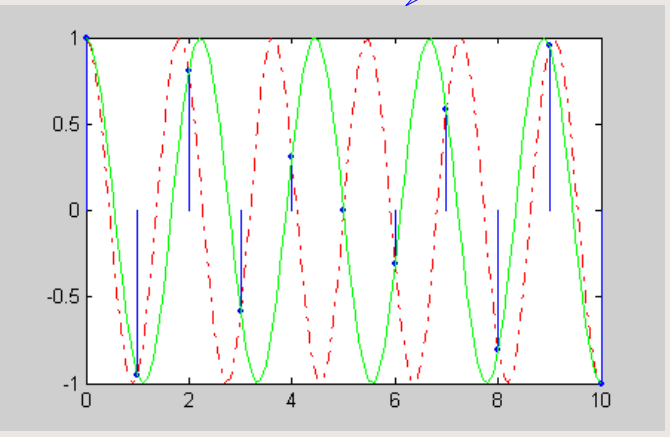
Figure 4.3

无混迭

有混迭

折叠频率

从时域理解高频折叠成低频



$\cos(1.1\pi n) = \cos(0.9\pi n)$

三角函数性质

重构的频域过程

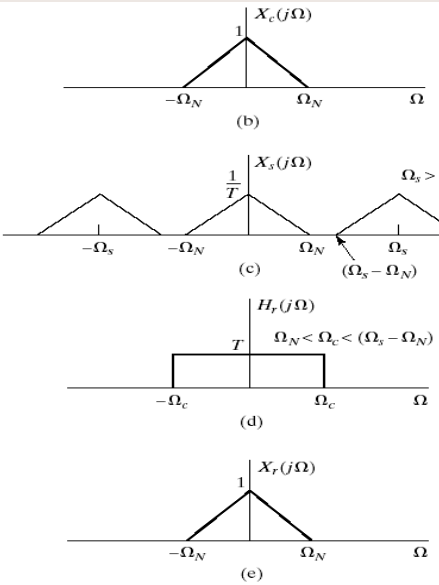


Figure 4.4

重构的数学模型

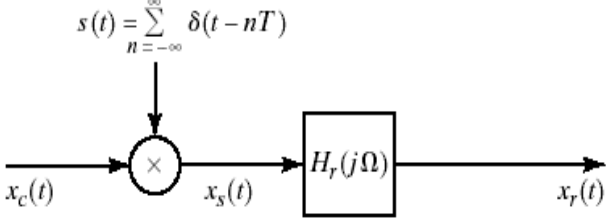
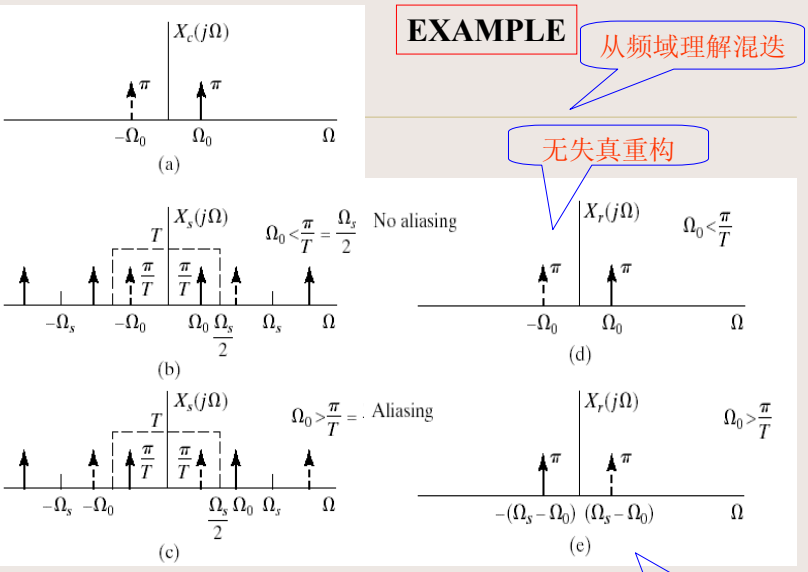


Figure 4.4(a)

EXAMPLE

从频域理解混迭

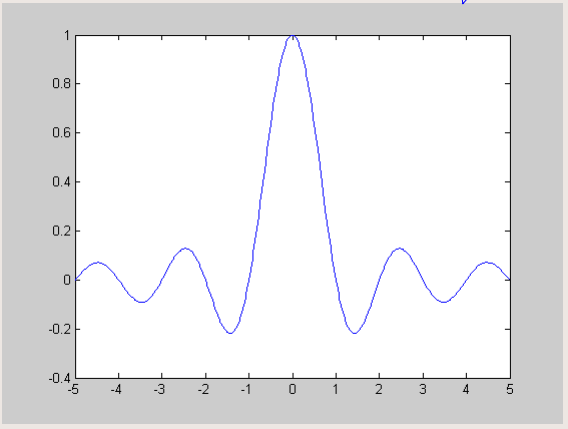


无失真重构

有失真重构

Figure 4.5

SINC（）函数



EXAMPLE

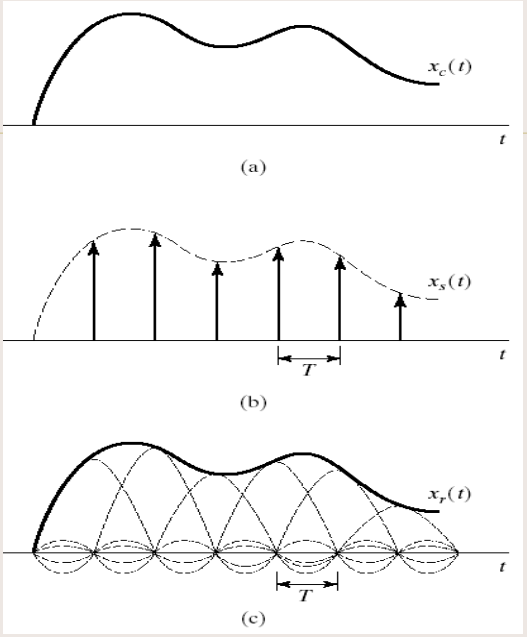
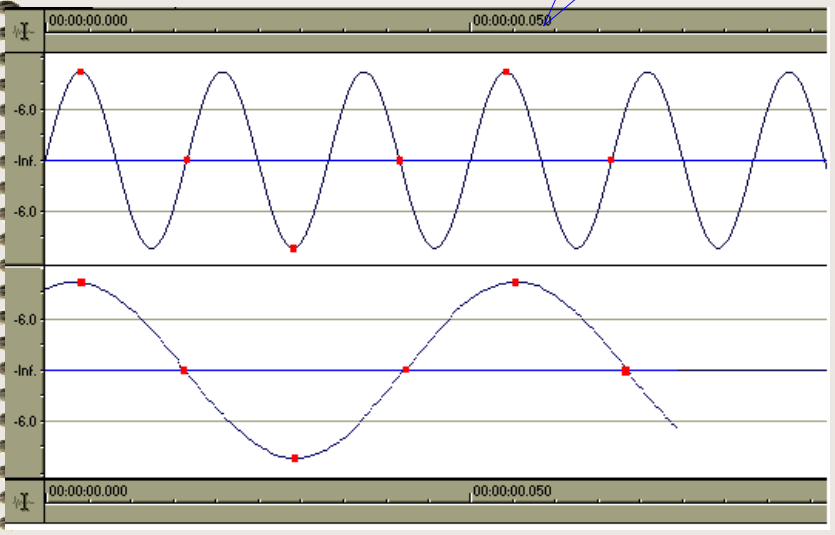


Figure 4.9

重构的时域内插

EXAMPLE

从时域理解混迭



EXAMPLE

MATLAB作图实现内插

$$x_a(t) = \cos(10\pi t), 0 \le t < 1, f = 5Hz$$

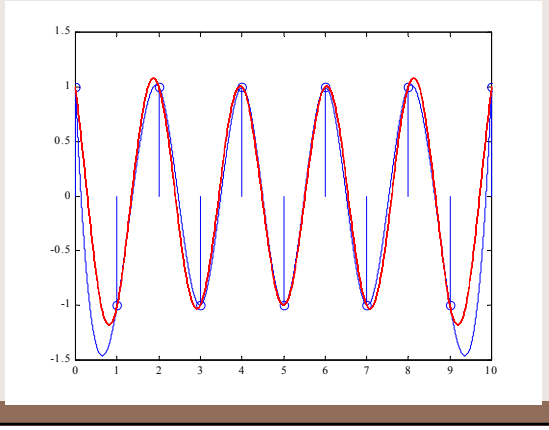
$$f_s = 10Hz(T = 0.1s)$$

$$x[n] = x_a(nT) = \cos(10\pi nT) = \cos(\pi n)$$

reconstruction signal :

$$y(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin[\pi(t - nT)/T]}{\pi(t - nT)/T}$$


```
T=0.1;
n=0:10;                x=cos(10*pi*n*T);     stem(n,x);
dt=0.001;                                    t=[0:dt:10];
y=spline(n,x,t);       hold on;              plot(t,y);
t=ones(11,1)* [0:dt:1];                      n=n'*ones(1,1/dt+1);
y=x*sinc((t-n*T)/T);                         plot(t/T,y,'r')
```

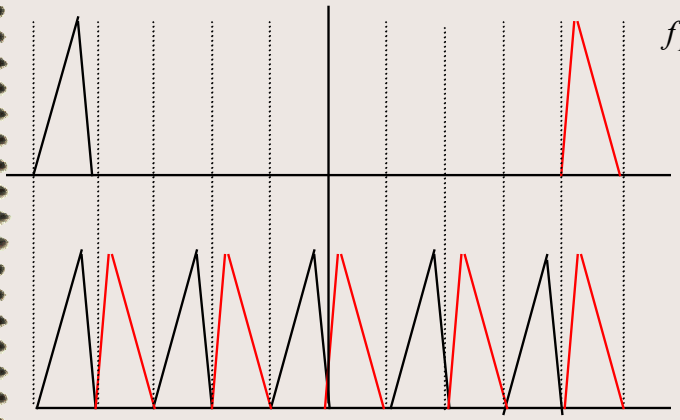


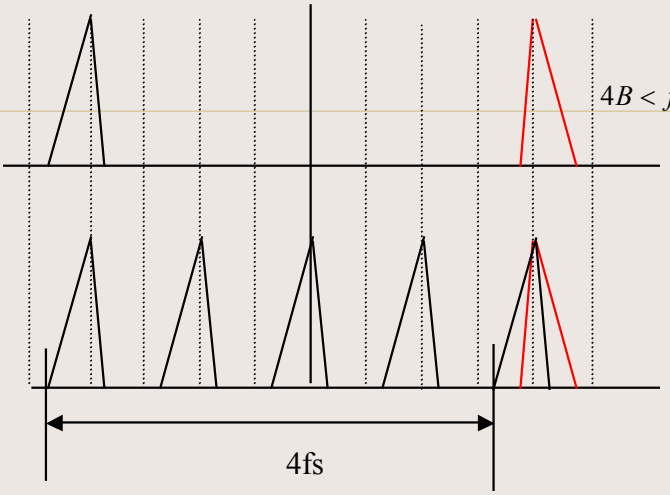
补充带通采样定理：

$$f_s = 2(f_H - f_L)(1 + M/N)$$

$$N = \text{int}\left(\frac{f_H}{f_H - f_L}\right)$$

$$M = \left(\frac{f_H}{f_H - f_L}\right) - N$$





$Nf_s = 2f_H$

$$f_s = 2f_H / N = 2B\frac{f_H / B}{N} = 2B\frac{(M+N)}{N} = 2B(1+M/N)$$

4.2 discrete-time processing of continuous-time signals

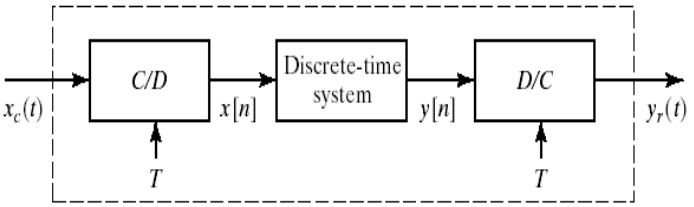


Figure 4.11

EXAMPLE

$$\therefore H_{eff}(j\Omega)=\begin{cases}H(e^{j\Omega T}) & |\Omega|<\pi/T\\ 0 & |\Omega|\geq\pi/T\end{cases}$$

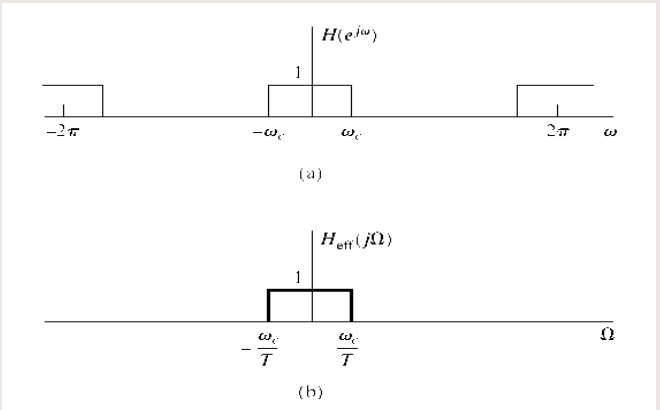
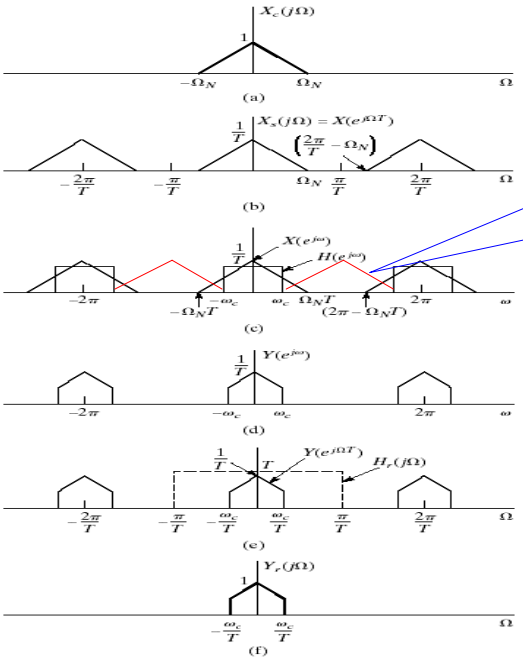


Figure 4.12



EXAMPLE

混迭发生在数字滤波器通带外则满足前述频响等效关系

Figure 4.13

4.3 continuous-time processing of discrete-time signal

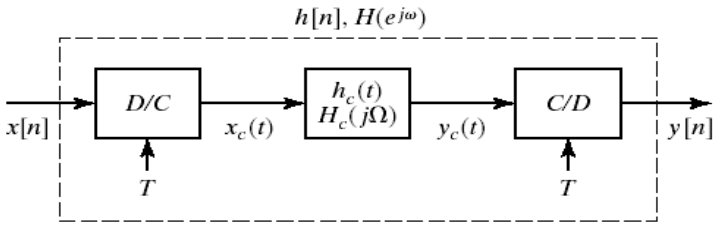


Figure 4.16

$$\therefore H(e^{j\omega}) = H_c(j\omega/T) \qquad , for\ |\omega| < \pi$$

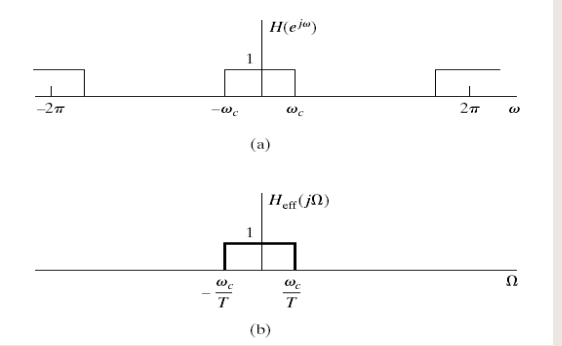
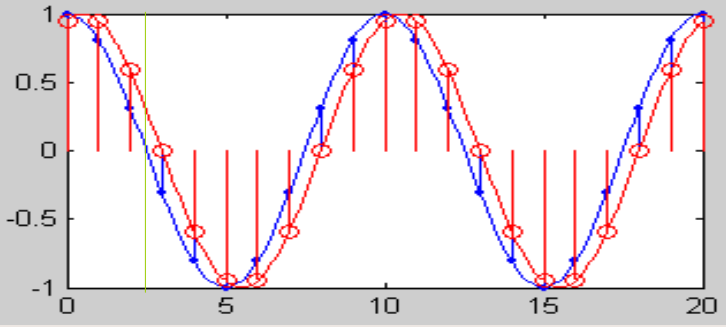


Figure 4.12

用前述系统解释非整数采样点延迟的数字系统

$$H(e^{j\omega}) = e^{-j\omega\Delta}$$



4.4 digital processing of analog signals

量化和编码

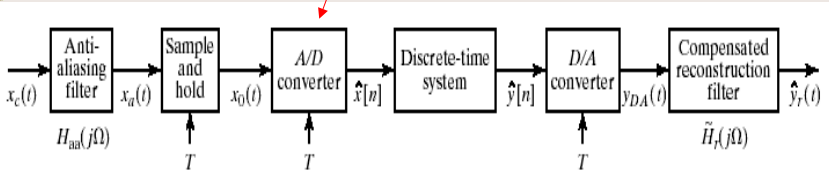


Figure 4.41

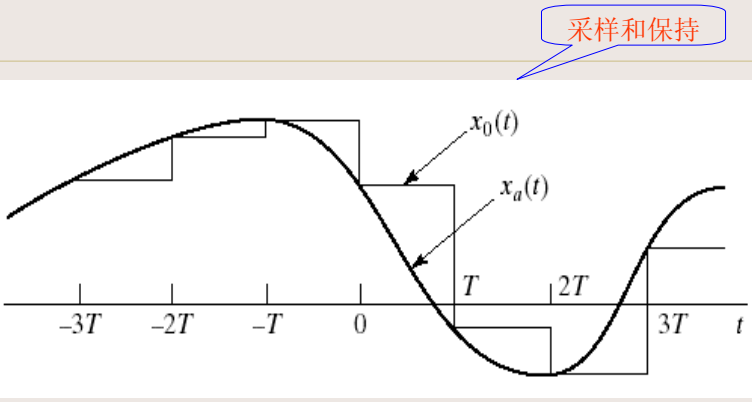


Figure 4.46(b)

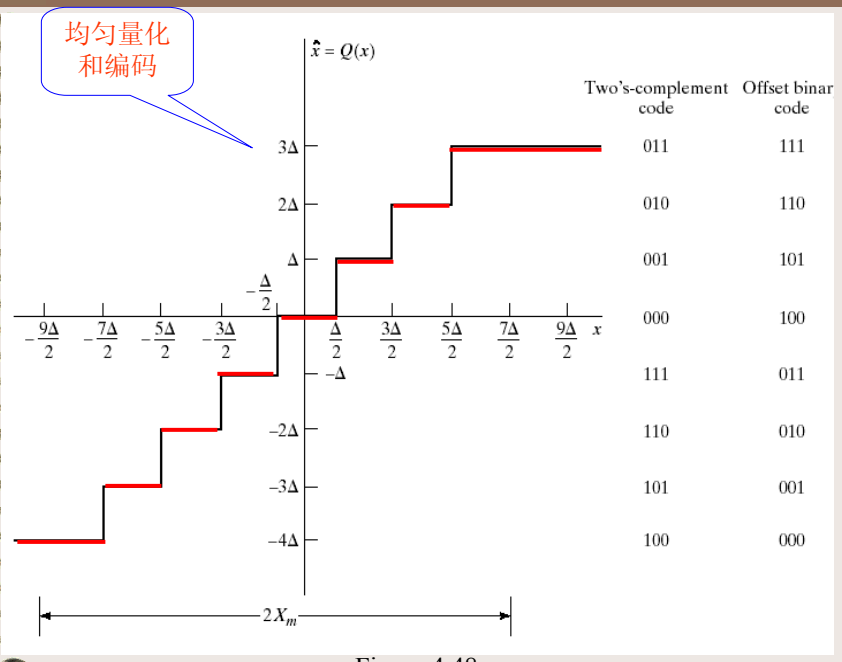


Figure 4.48

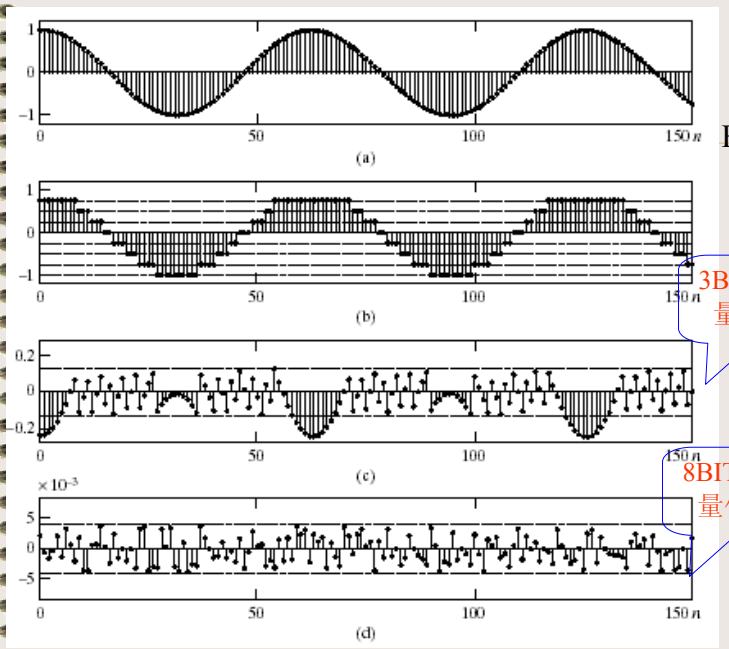
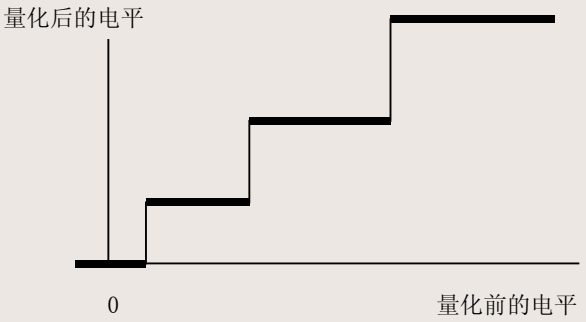


Figure 4.51

3BIT量化的量化误差

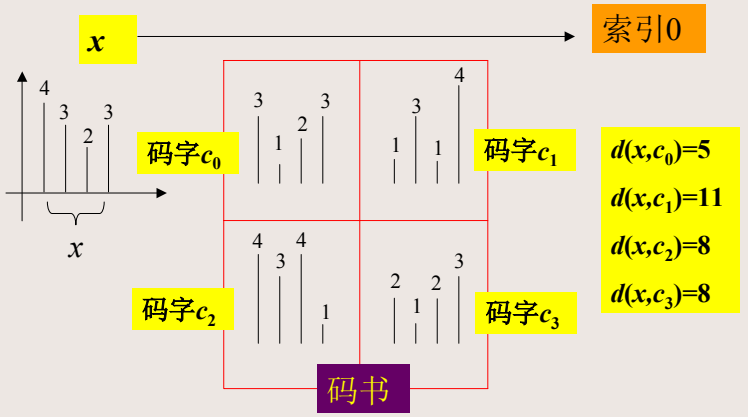
8BIT量化的量化误差

补充非均匀两化



补充矢量量化

✓一维信号例子:



补充矢量量化

✓ 图像编码例子:

原图象块（4灰度级，矢量维数 k=4×4=16）

码书 $C = \{y_0, y_1, y_2, y_3\}$

$d(x,y_0)=25$
$d(x,y_1)=5$
$d(x,y_2)=25$
$d(x,y_3)=46$

码字y_1最接近输入矢量图象块 $\boldsymbol{x}$，故用索引“01”编码

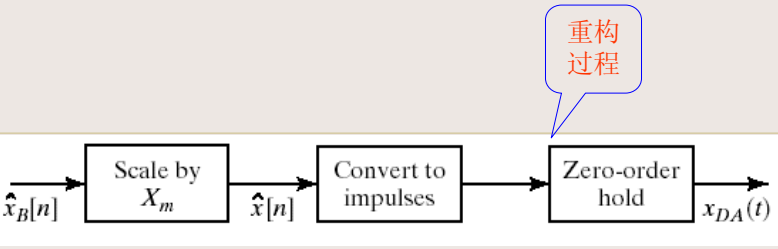
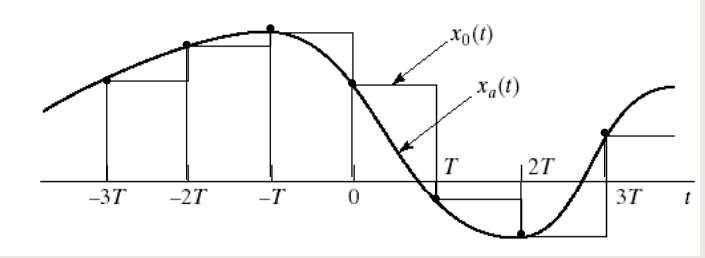
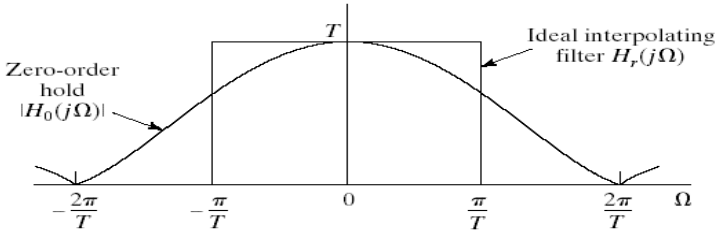


Figure 4.53



$$h(t) = R_N(t)$$



(a)

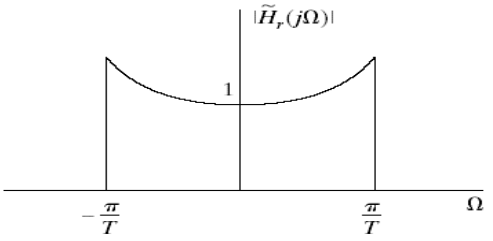
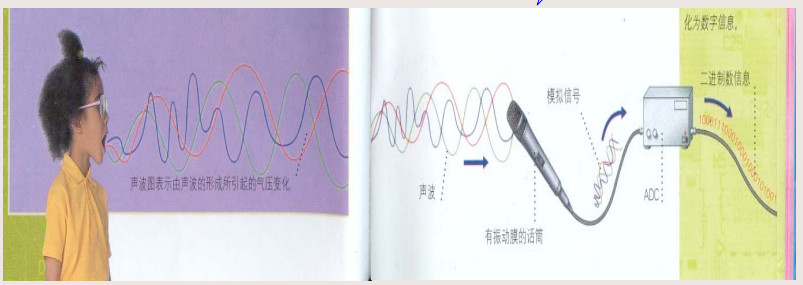
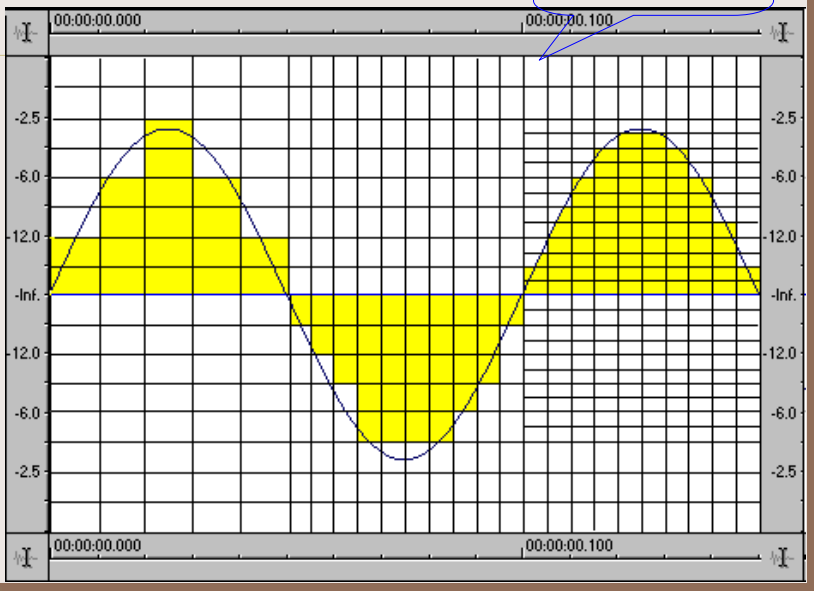


Figure 4.5

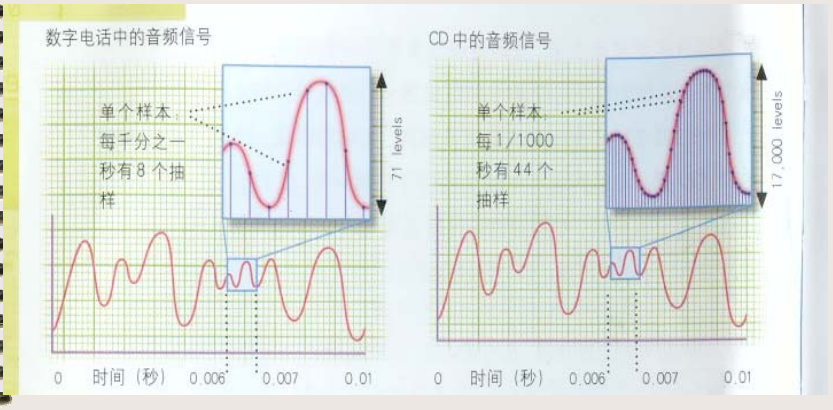
数字声音的录制过程



采样率和量化比特对音质的影响



不同的音质要求采用不同的采样率



4.1~4.4小结

1. 采样和重构的时域表示和频域变化情况，
   根据频域是否混迭得出采样定理
2. 用数字系统处理模拟信号或用模拟系统处理数字信号以解释某些数字系统，
   二者的频响在主周期线性关系
3. 模拟数字转换的实际步骤

要求和难点：

采样的时域和频域过程，频谱作图

采样定理的理解和应用

连续时间信号的离散化处理系统的频响关系

4.5 changing the sampling rate using discrete-time processing

4.5.1 sampling rate reduction by an integer factor (downsampling,decimation)

4.5.2 increasing the sampling rate by an integer factor (upsampling,interpolation)

4.5.3 changing the sampling rate by a noninteger fact

4.5.4 application of multirate signal processing

4.5.1 sampling rate reduction by an integer factor (downsampling,decimation)

$x_d[n] = x[nM]$

a sampling rate compressor:

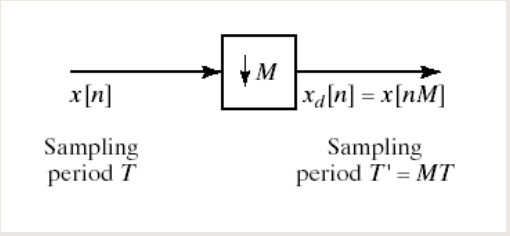
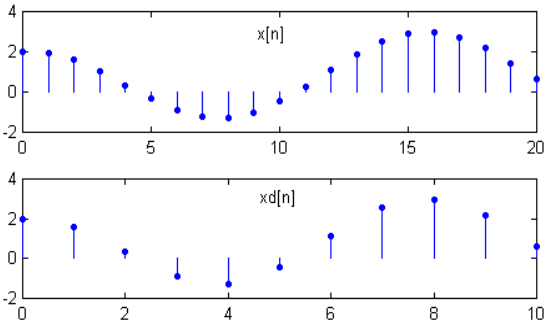


Figure 4.20



EXAMPLE

$$X_d(e^{j\omega}) = \frac{1}{2}\left[X\left(e^{j\omega/2}\right) + X\left(e^{j(\omega-2\pi)/2)}\right)\right]$$

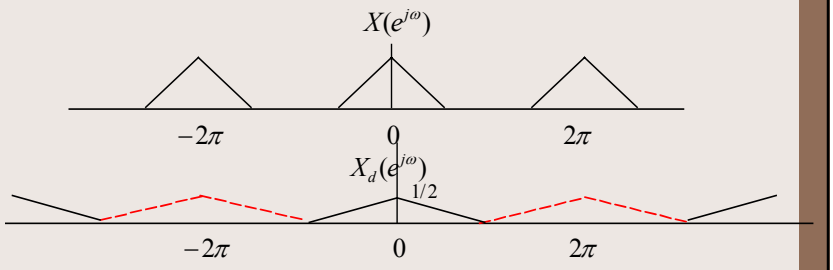
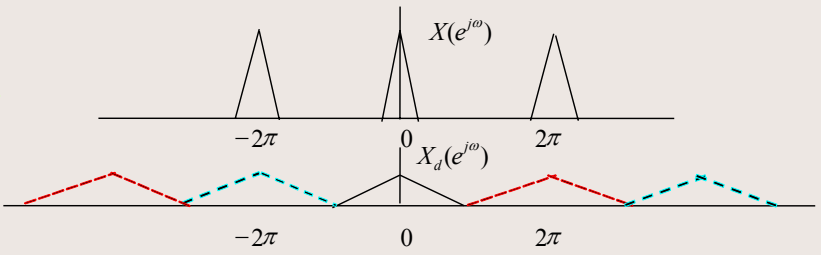


Figure 4.21(c)(d)

EXAMPLE

$$X_d(e^{j\omega}) = \frac{1}{3}\left[X\left(e^{j\omega/3}\right) + X\left(e^{j(\omega-2\pi)/3)}\right) + X\left(e^{j(\omega-4\pi)/3)}\right)\right]$$



EXAMPLE

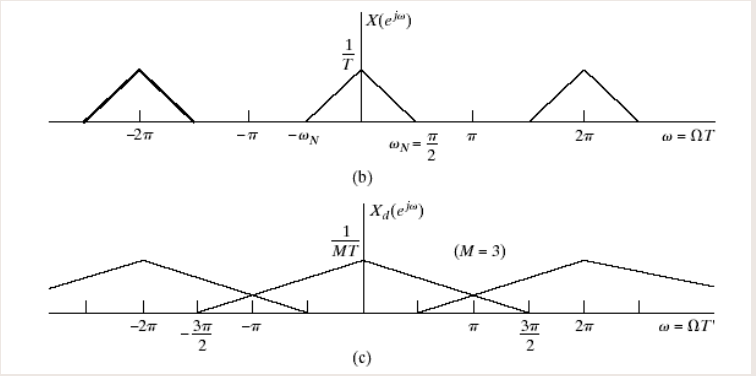


Figure 4.22(b)(c)

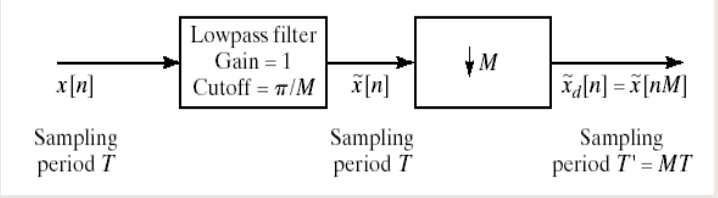


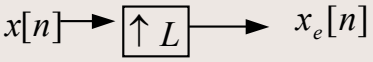
Figure 4.23

4.5.2 increasing the sampling rate by an integer factor (upsampling,interpolation)

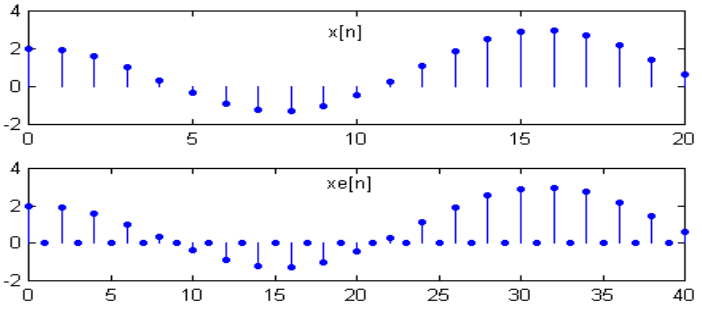
$$x_e[n]=\begin{cases} x[n/L] & n=0,\pm L,\pm 2L\cdots \\ 0 & other \end{cases}$$

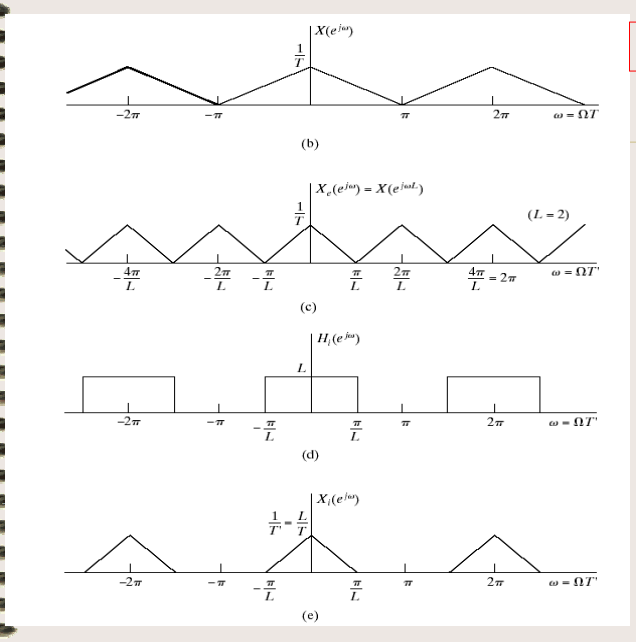
$$or, x_e[n]=\sum_{k=-\infty}^{\infty} x[k]\delta[n-kL]$$

a sampling rate exp*ander*:



L=2



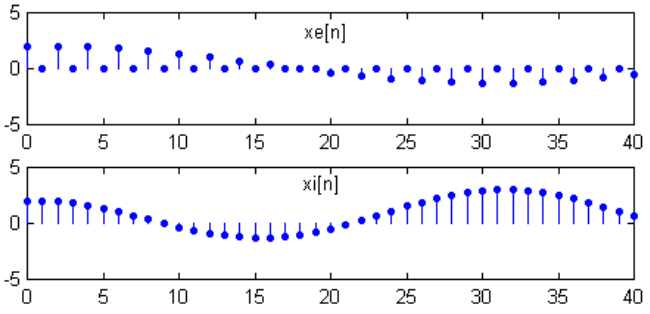


EXAMPLE

Figure 4.25

EXAMPLE

镜像滤波的时域过程



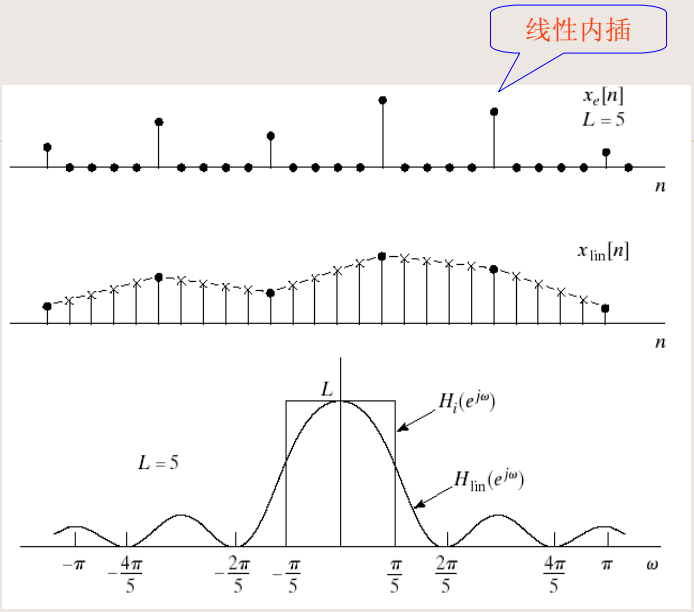


Figure 4.27

4.5.3 changing the sampling rate by a noninteger factor

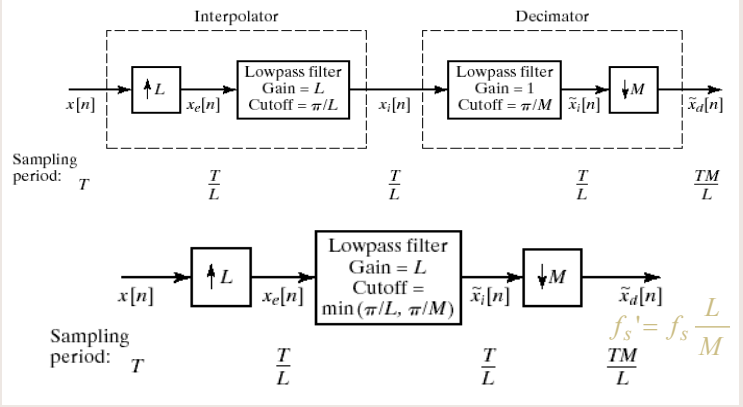
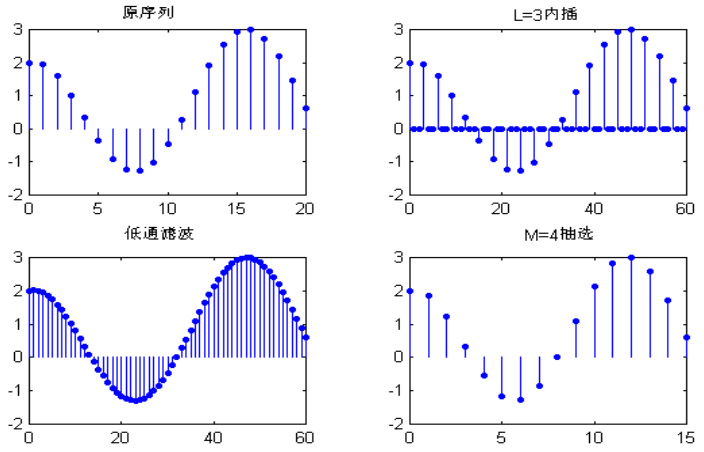


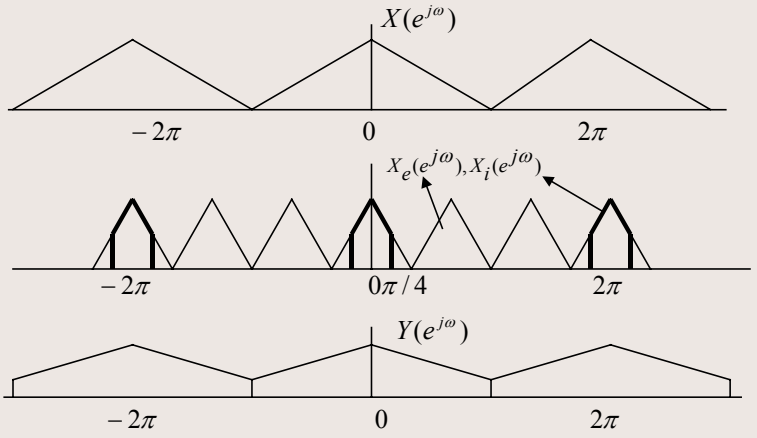
Figure 4.28

EXAMPLE

change $400Hz$*'s signal to* $300Hz$

$L=3, M=4$





4.5.4 application of multirate signal processing

采样系统

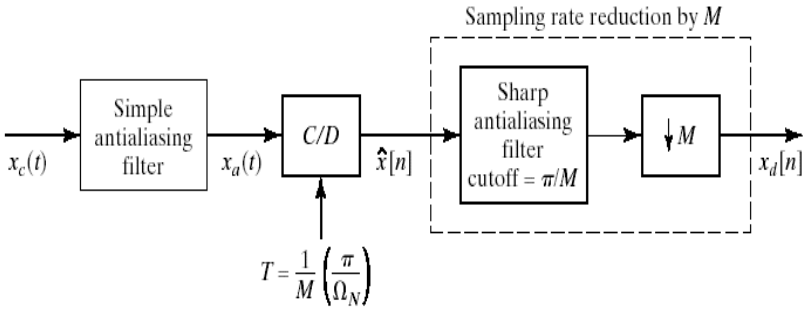


Figure 4.43

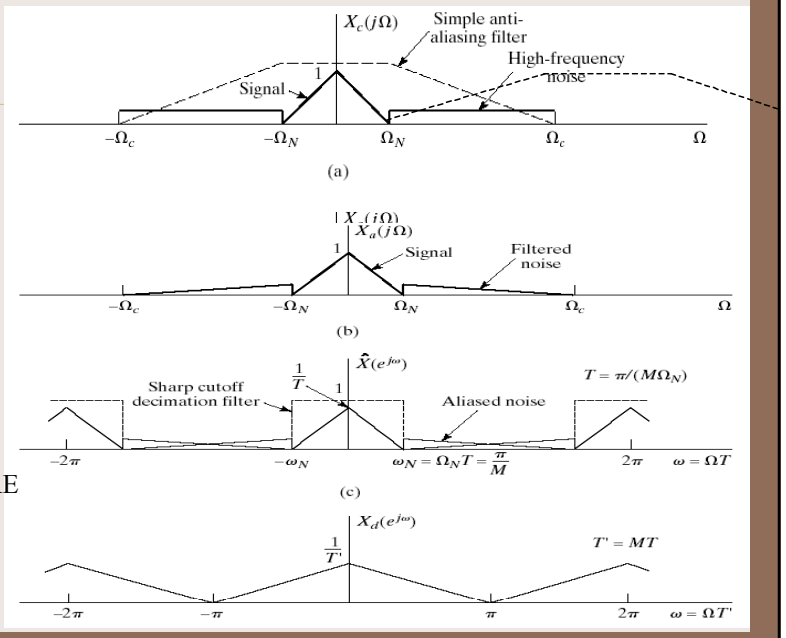
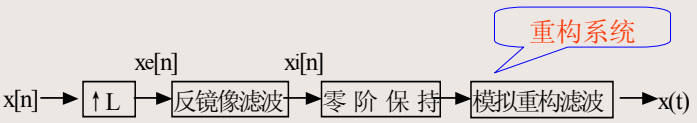
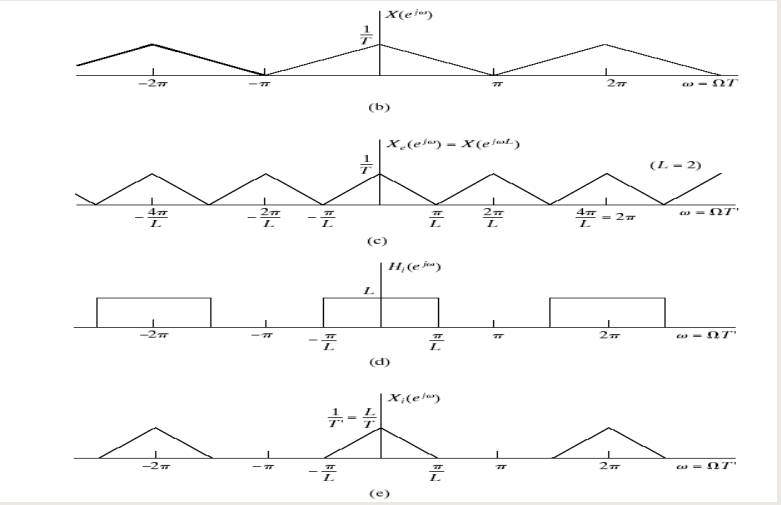


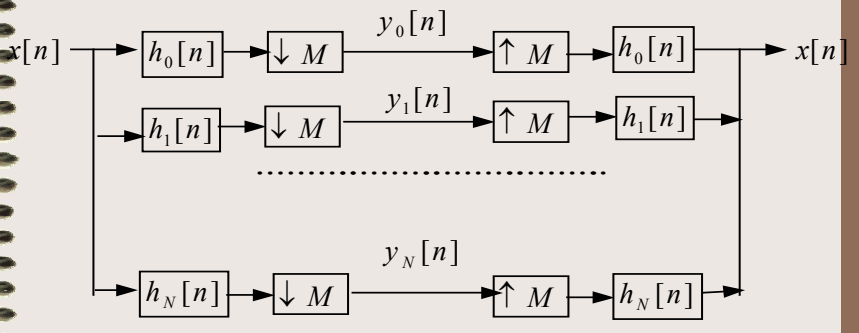
FIGURE 4.44

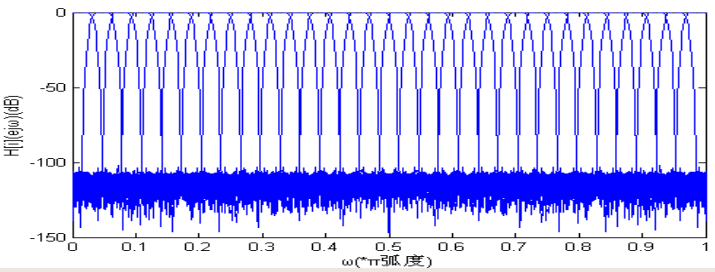
重构系统



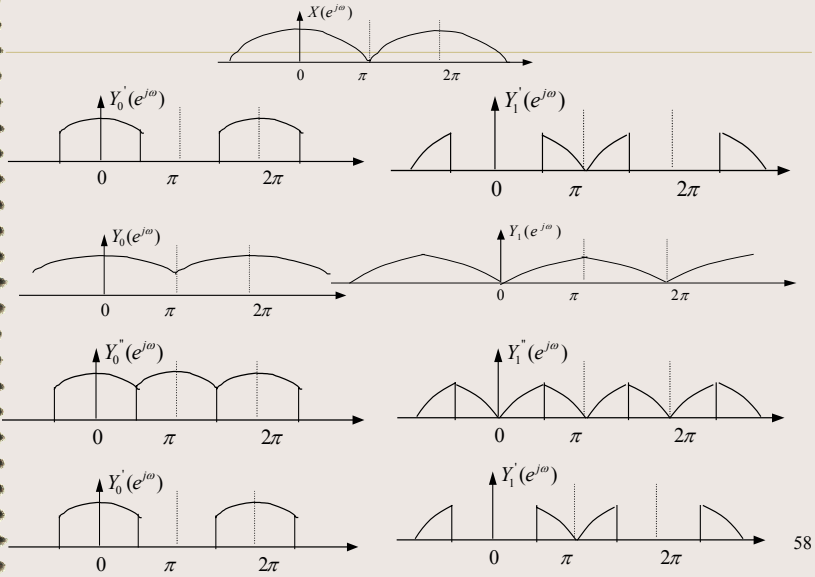


子带分析和综合




H(1)(e^jω)/(dB)
0
-50
-100
-150
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ω(*π弧度)

57


$X(e^{j\omega})$
$Y_0'(e^{j\omega})$
$Y_1'(e^{j\omega})$
$Y_0(e^{j\omega})$
$Y_1(e^{j\omega})$
$Y_0''(e^{j\omega})$
$Y_1''(e^{j\omega})$
$Y_0'(e^{j\omega})$
$Y_1'(e^{j\omega})$
0
π
2π

58

EXAMPLE

(1)*subband encode*:

$N = 2$

Bitrate: $16bit \cdot f_s \rightarrow 16bit \cdot fs/2 + 8bit \cdot fs/2 = 12bit \cdot fs$

(2)*encrypt* : *Dsubbands* · *Rsegment*

抽选和内插
实现变调






4.5小结

4.5.1 采样率降低整数倍
4.5.2 采样率升高整数倍
4.5.3 采样率改变有理数倍
4.5.4 多采样率信号处理的应用

要求：

抽选和内插的频域作图