晶体光学和电光器件

Crystal Optics and Electro-Optic Devices

（二）

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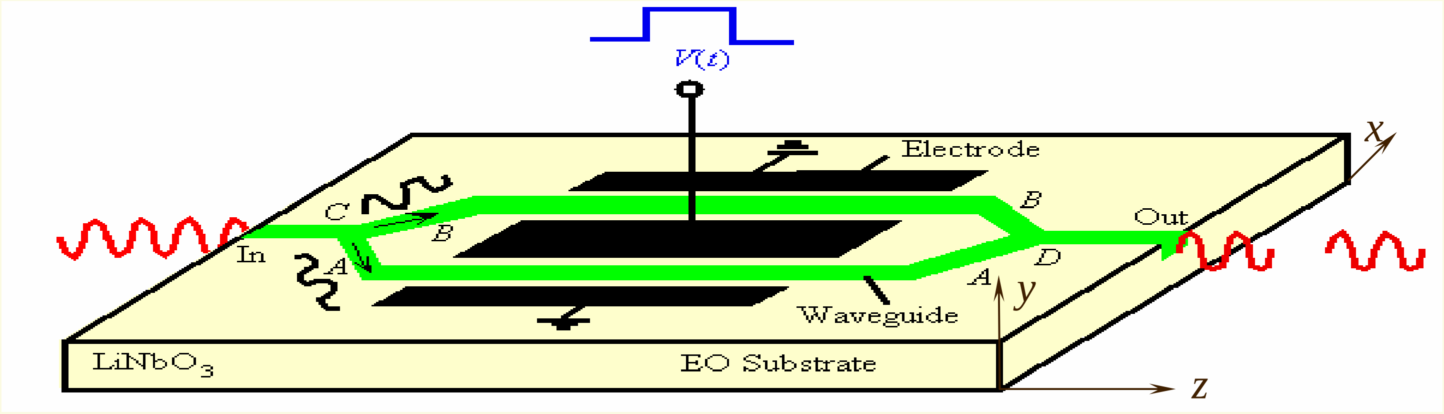
2007-4-23

Integrated Mach-Zehnder Optical Modulators

➢ Device Structure

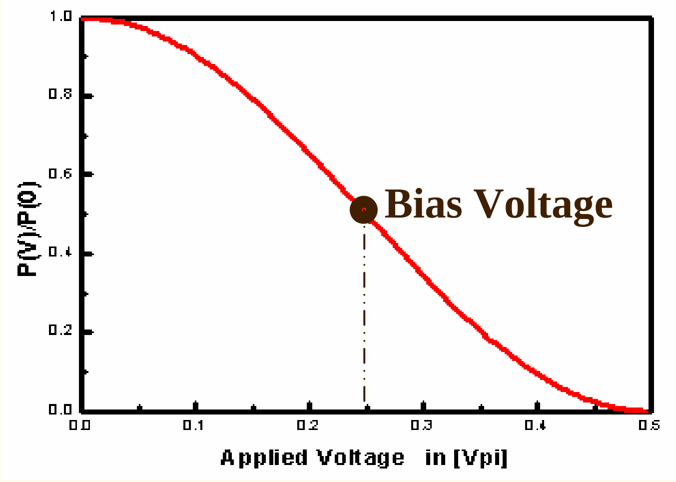


➢ Operating Principle

$$E_{out} \propto E_A \cos(\omega t + \Delta\phi) + E_B \cos(\omega t - \Delta\phi)$$
$$= 2E_A \cos(\Delta\phi)\cos(\omega t)$$

$$E_A \cong E_B \quad \text{and} \quad \Delta\phi = (\pi/V_\pi)V(t)$$

$$P_{out}[V(t)] = P_{out}(0)\cos^2[(\pi/V_\pi)V(t)]$$



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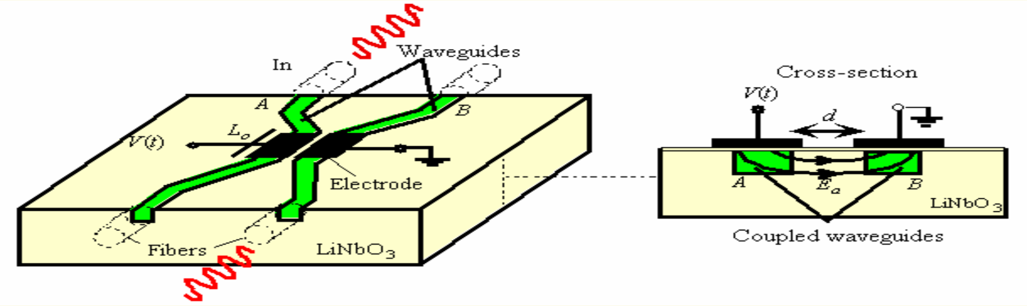
Electro-Optic Directional Couplers and Modulators

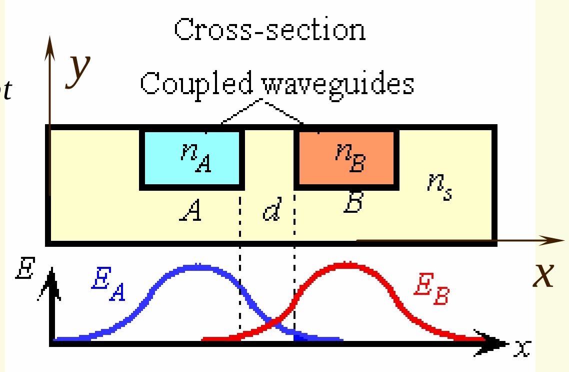
- **Device Structure**



- **Basic Principle**

Consider only the coupling between TE fields:

$$\mathbf{E} \cong \mathbf{e}_x \left[A(z) E_A(x,y) e^{-j\beta_A z} + B(z) E_B(x,y) e^{-j\beta_B z} \right] e^{j\omega t}$$



The general coupled wave Eqs. become:

$$dA/dz = -j\eta_A A - j\kappa_{AB} B e^{j(\beta_A - \beta_B)z}$$

$$dB/dz = -j\eta_B B - j\kappa_{BA} A e^{j(\beta_B - \beta_A)z}$$

If A and B are exactly the same, we have:

$\Delta\beta = \beta_A - \beta_B = 0$ for $V= 0$. Phase matching condition is perfectly satisfied and the two fields are strongly coupled. The power in A will be transferred to B.

For $V \neq 0$, $\Delta\beta \approx 2\Delta n(2\pi/\lambda)$. $\Delta n = (1/2)n_o \gamma_{22}(V/d)$. The phase mismatch will destroy the coupling between the two fields and the power transfer decays.

Solutions to The Coupled Wave Equations

- **Coupling Coefficiency**

$$\kappa_{AB}=\frac{k_0}{2n_A}\int_{\text{in B}}(n_B^2-n_s^2)E_A E_B^* dxdy \qquad \kappa_{BA}=\frac{k_0}{2n_B}\int_{\text{in A}}(n_A^2-n_s^2)E_B E_A^* dxdy$$

$$\delta_A=\frac{k_0}{2n_A}\int_{\text{in B}}(n_B^2-n_s^2)|E_A|^2 dxdy \qquad \delta_B=\frac{k_0}{2n_B}\int_{\text{in A}}(n_A^2-n_s^2)|E_B|^2 dxdy$$

- **Standard Form of The CWE**

$$A(z)=R(z)e^{-j[\eta_A+\delta]z} \qquad B(z)=S(z)e^{-j[\eta_B-\delta]z} \qquad (\delta=\Delta\beta/2)$$

$$\Longrightarrow R'-j\delta R=-j\kappa_{AB}S \qquad S'-j\delta S=-j\kappa_{BA}R \qquad (R(0)=1,\quad S(0)=0)$$

- **Solutions**

$$R(z)=\cos(\Omega z)+j\delta[\sin(\Omega z)/\Omega] \qquad S(z)=-j\kappa[\sin(\Omega z)/\Omega]$$

$$\kappa=|\kappa_{AB}|\approx|\kappa_{BA}| \qquad \Omega=\sqrt{\kappa^2+\delta^2}$$

For $\Delta\beta=0$, $P_A(z)=\cos^2(\kappa z)$ $P_B(z)=\sin^2(\kappa z)$ $\Longrightarrow$ $P_A(z)+P_B(z)=1$

Generally: $P_B(z)=P_{in}\left(\kappa^2/\Omega^2\right)\sin^2(\Omega z)$ $\quad P_A(z)=P_{in}-P_B(z)$

Summary

- **Length of the Coupler for Fully Power Transfer**

$$\kappa L = \pi/2$$

$\Rightarrow$ $P_A(0)=1\ P_A(L)=0$ and $P_B(0)=0\ P_B(L)=1$ for $\Delta\beta = 0$ ($V=0$)

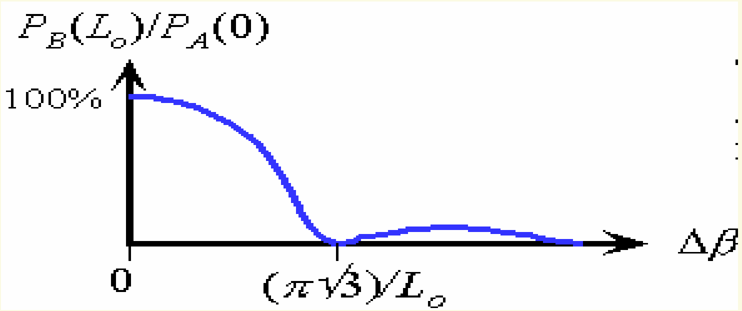
- **Switching Voltage**

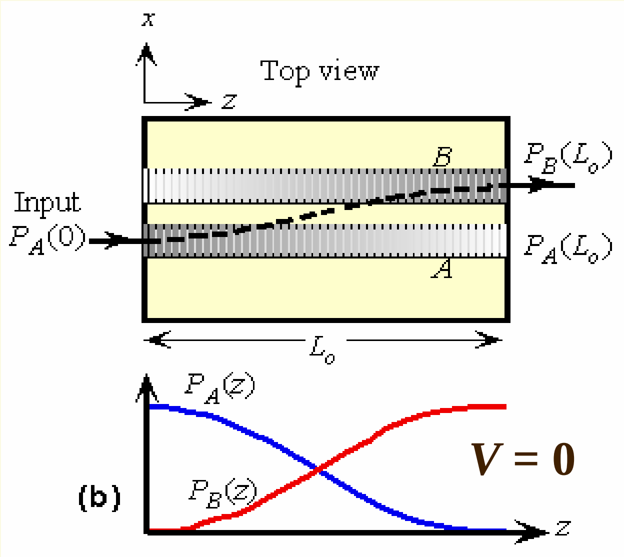
$$\delta L = \Delta\beta L/2 = n_o^3\gamma_{22}L(V/d)(\pi/\lambda) = \sqrt{3}\pi/2 \Rightarrow \Omega L = \pi$$

$\Rightarrow$ $P_A(0)=1\ P_A(L)=1$ and $P_B(0)=0\ P_B(L)=0$ **for** $V=\sqrt{3}\lambda d/2n_o^3\gamma_{22}L$

- **Coupling Efficiency**

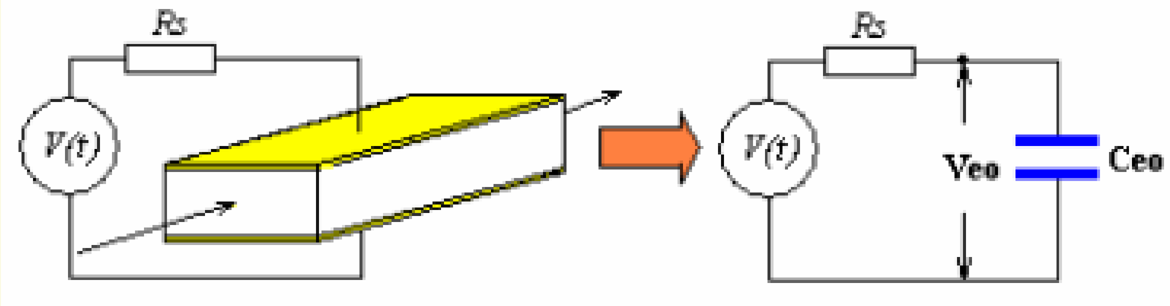
$$\frac{P_B(L)}{P_{in}} = \frac{\kappa^2}{\kappa^2+\delta^2}\sin^2\left(\sqrt{\kappa^2+\delta^2}L\right)$$





High Frequency Modulation: A Simple Case

- **Structure and Equivalent Circuit**



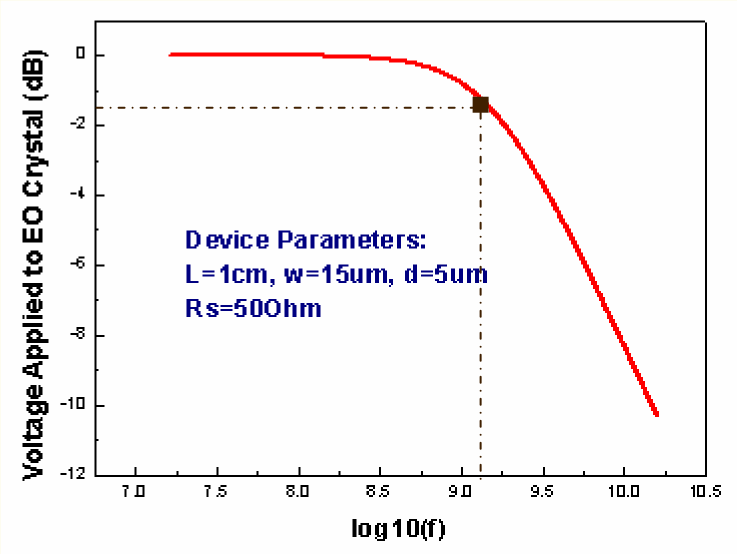
- **Effective Applied Voltage**

$$V_{eo}(\omega) \propto \frac{1}{\sqrt{1+\omega^2 R_s^2 C_{eo}^2}}$$

R_s is the internal resistance of the signal source; $C_{eo} = \varepsilon S/d$ is the capacitance of the modulator

$$f_{\max} = 1/\left(2\pi\sqrt{R_s C_{eo}}\right)$$

Reduce R_s and $C_{eo} = \varepsilon S/d$ to achieve high frequency modulation



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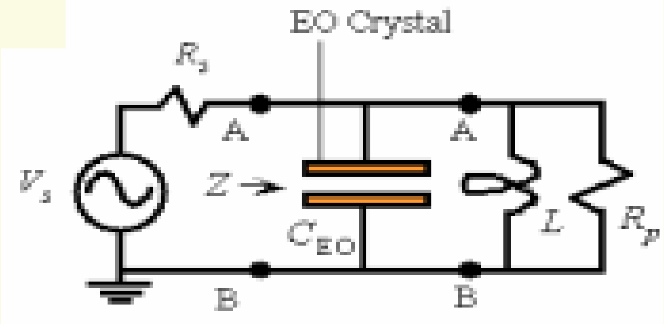
High Frequency Modulation: Improvement

- **Equivalent Circuit**



- **The Effective Applied Voltage**

$$V_{eo}(\omega) \propto \frac{1}{\sqrt{(\omega-\omega_0)^2+(1/2R_pC_{eo})^2}}$$

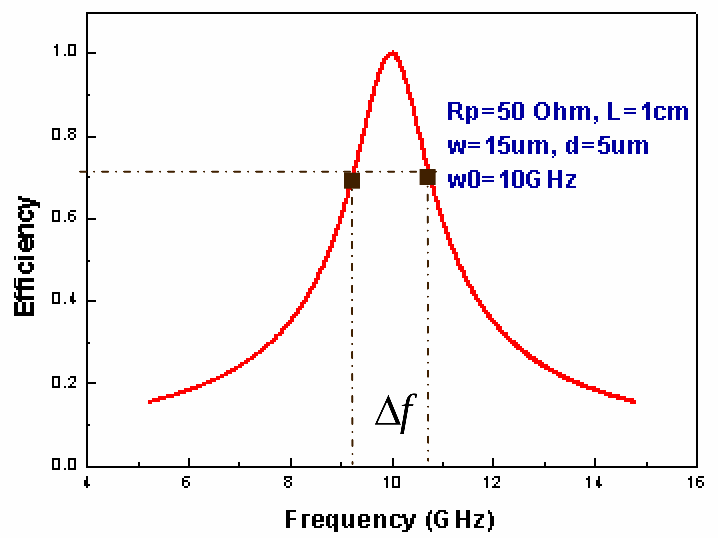
The efficiency is maximum at $\omega = \omega_0$

$$\omega_0 = 1/\sqrt{LC_{eo}}$$

is the center frequency of the modulator

- **Modulation Bandwidth**

$$\Delta f = 1/(2\pi R_p C_{eo})$$



- **Comments**

Reduce R_s , L, and C_{eo} to achieve high frequency modulation

R_p should be properly selected

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Transit-Time Limitations

➢ EO Retardation under High Speed Modulation

The EO phase retardation due to a low frequency field *E* can be written as:

$$\Delta\phi = \phi(E) - \phi(0) = aEL$$

For high speed modulation field *E*(*t*), the retardation should be:

$$\Delta\phi = a\int_0^L E(z)dz = \frac{ac}{n}\int_{t-\tau}^{t} E(t')dt'$$

$\tau = \frac{nL}{c}$ is the transit-time

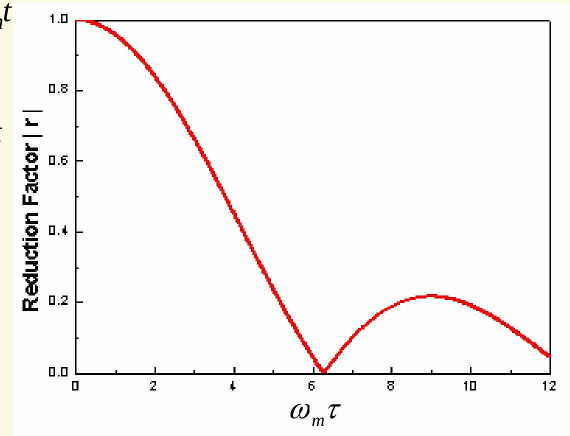
➢ Transit-Time Reduction Factor

Consider a sinusoid applied field: $E(t) = E_m e^{j\omega_m t}$

$$\Delta\phi = \frac{ac}{n}E_m\int_{t-\tau}^{t} e^{j\omega_m t'}dt' = aE_m L\left[\frac{1-e^{-j\omega_m\tau}}{j\omega_m\tau}\right]e^{j\omega_m t}$$

$r = (1-e^{-j\omega_m\tau})/(j\omega_m\tau)$ is the reduction factor

For $\omega_m\tau \ll 1$, $r = 1$, and $\Delta\phi = aE(t)L$



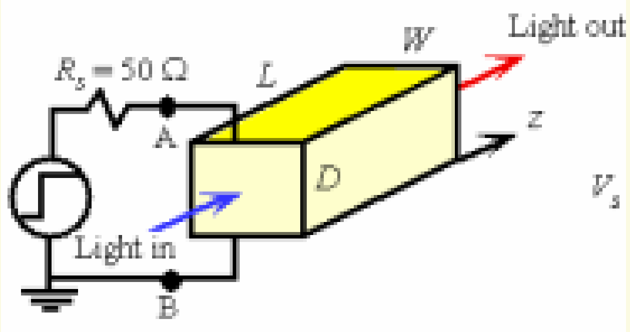
➢ The Highest Modulation Frequency

$r = 0.9$ for $\omega_m\tau = \pi/2$, $(f_m)_{max} = c/(4nL)$. For $n = 2.2$, $L = 1$cm, $(f_m)_{max} = 3.4$GHz

Traveling-Wave Modulators

- **The Traveling Modulation Field**

$$E(z,t)=E_m e^{j(\omega_m t-k_m z)}=E_m e^{j\omega_m(t-z/c_m)}$$



- **The Phase Retardation**

Consider the optical field entering at time t.

At some later time t', the field arrives at $z(t')=(c/n)(t'-t)$. At this point, the modulation field is:

$$E(z,t)=E_m e^{j\omega_m[t'-c(t'-t)/(nc_m)]}$$

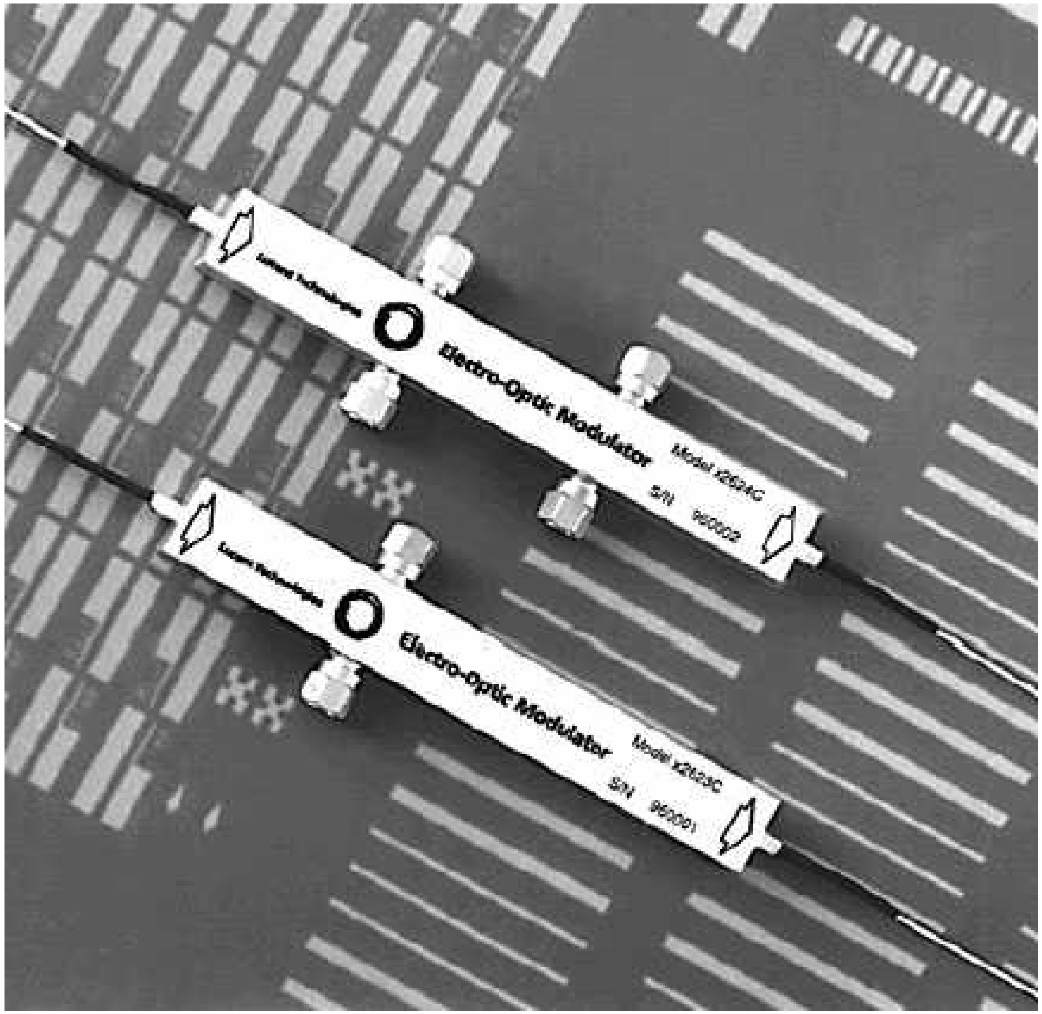
At time $t+\tau$ ($\tau = nL/c$), the optical field comes out from the crystal, and the total phase retardation obtained by the optical field is:

$$\Delta\phi = \frac{ac}{n}\int_t^{t+\tau} E(t', z(t'))dt' = aE_m L e^{j\omega_m t}\left[\frac{e^{j\omega_m\tau(1-c/nc_m)}-1}{j\omega_m\tau(1-c/nc_m)}\right]$$

- **The Reduction Factor and The Maximum Useful Frequency**

$$r = \frac{e^{j\omega_m\tau(1-c/nc_m)}-1}{j\omega_m\tau(1-c/nc_m)} \Longrightarrow (f_m)_{\max}=\frac{c}{4nL(1-c/nc_m)}$$

Ti Diffused $LiNbO_3$ MZ High Speed Modulators



Circular Birefringence Representation (CPR)

➢ The Basis Vectors in CPR

In CPR, the orthogonal and complete unit basis vector set are the CCW and CW rotating CP waves defined as $(1, 0)^T$ and $(0, 1)^T$. Arbitrary transverse field can be described as:

$$\mathbf{V} = V_+ \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + V_- \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} V_+ \\ V_- \end{Bmatrix}$$

In rectangular representation, the same field is described as:

$$\mathbf{V} = V_x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + V_y \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{Bmatrix} V_x \\ V_y \end{Bmatrix}$$

➢ The Transform Matrix

The two representation can be related by a transform matrix:

$$\begin{Bmatrix} V_+ \\ V_- \end{Bmatrix} = T \begin{pmatrix} V_x \\ V_y \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} V_x \\ V_y \end{pmatrix} = T^{-1} \begin{Bmatrix} V_+ \\ V_- \end{Bmatrix}$$

From $\{0, 1\}^T = T(1, j)^T$, $\{1, 0\}^T = T(1, -j)^T$, we get:

$$T = \frac{1}{2} \begin{pmatrix} 1 & j \\ 1 & -j \end{pmatrix} \quad \text{and} \quad T^{-1} = \begin{pmatrix} 1 & 1 \\ -j & j \end{pmatrix}$$

Wave Propagation in Circularly Birefringent Medium

➢ The Jones Matrix of a Circularly Birefringent Medium

In a CB medium, the indices of CCW and CW CP waves are denoted by n_+ and n_-. The Jones matrix of CB medium of length L in CPR is:

$$J = \begin{pmatrix} e^{-j\theta_+} & 0 \\ 0 & e^{-j\theta_-} \end{pmatrix} = e^{-j(\theta_+ + \theta_-)/2} \begin{pmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{pmatrix}$$

$$\begin{cases} \theta_+ = k_0 n_+ L, \ \theta_- = k_0 n_- L \\ \theta = (\theta_- - \theta_+)/2 \end{cases}$$

θ_+ and θ_- are the rotated angles of CCW and CW waves, respectively. Ignoring the common phase factor, the Jones matrix becomes:

$$J = \begin{pmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{pmatrix}$$

The matrix in rectangular representation is obtained by transformation matrix T:

$$J' = T^{-1} J\, T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = R(-\theta) \Rightarrow \begin{pmatrix} V_x(L) \\ V_y(L) \end{pmatrix} = J' \begin{pmatrix} V_x(0) \\ V_y(0) \end{pmatrix}$$

➢ Rotation of Polarization States

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

A linearly-polarized input field is rotated by an angle θ after propagation in the CB medium.

Magneto-Optic (Faraday) Effects

- **Faraday Rotation**

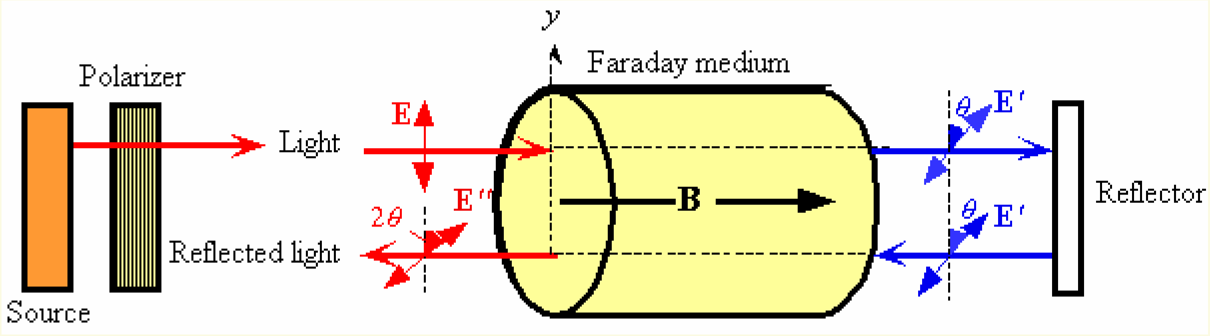
  Some materials exhibit circular birefringence when a magnetic field is applied along the wave propagation direction. The birefringence $\Delta n = n_+ - n_-$ is proportional to the applied magnetic field, so the rotation angle can be written as (Faraday effect):

  $$\theta = VBL$$ *V* is the Verdet constant of the material

  θ is the rotation angle about the direction of propagation.

- **Nonreciprocal Property of Faraday Rotation**

  For a wave traveling in the –B direction, the rotation is $-\theta$ about the new direction of propagation. So the Faraday rotation is *nonreciprocal*. For a given material, the rotation direction depends only on the direction of B.



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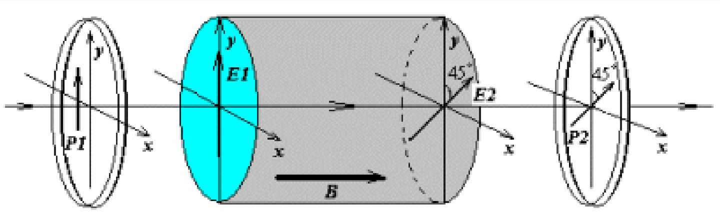
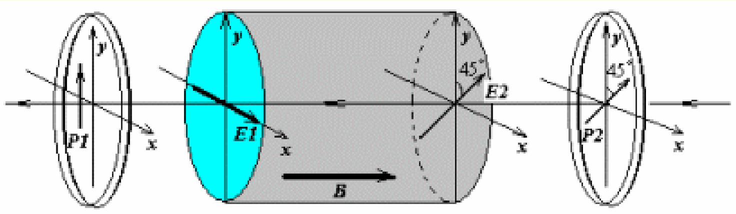
Optical Isolator

➢ Basic Principle



Forward Propagation Wave



Backward propagation Wave

➢ Improvement: Polarization Insensitivity