

# 非线性光纤光学

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# 主要参考书

- Govind P.Agrawal, “Nonlinear Fiber Optics”.
- 钱梅珍, “激光物理”
- 经典综述性文献

# 主要内容

- 电磁理论基础
- 光波导的一般理论
- 自相位调制 (SPM)
- 交叉相位调制 (XPM)
- 四波混频 (FWM)
- 量子光学初步
- 受激Raman散射 (SRS)
- 受激Brillouin散射 (SBS)

# 经典电磁理论基础

The background is a gradient of blue and black. A prominent curved line starts from the left edge and curves downwards towards the bottom right, separating the dark blue upper section from the lighter blue lower section. The title text is positioned in the upper dark blue section.

# Maxwell电磁场方程

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho_f$$

$$\nabla \cdot \mathbf{B} = 0$$

电磁场是一个统一的物质体系  
电场和磁场并不是两个独立的  
矢量场，二者紧密相关。

**E:** 电场强度[V/m]

**H:** 磁场强度[A/m]

**D:** 电位移矢量[C/m<sup>2</sup>]

**B:** 磁感应强度[T=V·s/m<sup>2</sup>]

$\rho_f$ : 自由电荷密度[C/m<sup>3</sup>]

$\mathbf{J}_f$ : 自由电流体密度[A/m<sup>2</sup>]

在绝缘介质内部， $\rho_f = 0$ ； $\mathbf{J}_f =$

坐标系为  
右手系统

# 介质的时空特性

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}$$

$\mu_0$ : 真空磁导率[H/m]

$\varepsilon_0$ : 真空介电系数[F/m]

**M**: 外场在介质内感应出的宏观磁偶极矩分布。对一般非铁磁介质**M=0**

**P**: 外场在介质内感应出的宏观电偶极矩分布。对一般线性介质:

$$\mathbf{P}(\mathbf{r}, t) = \varepsilon_0 \int_{-\infty}^{+\infty} \chi(\mathbf{r}, t - \tau) \mathbf{E}(\mathbf{r}, \tau) d\tau$$

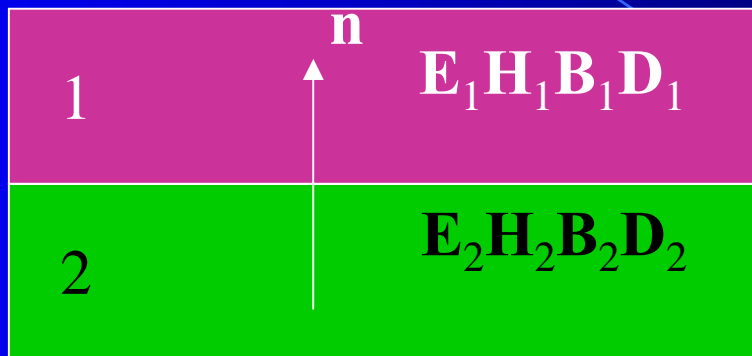
变换到频域为:

$$\mathbf{P}(\omega) = \varepsilon_0 \chi(\omega) \mathbf{E}(\omega) \Rightarrow \mathbf{D}(\omega) = \varepsilon_0 \varepsilon_r(\omega) \mathbf{E}(\omega)$$

$$\varepsilon(\omega) = \varepsilon_0 \varepsilon_r(\omega), \quad \varepsilon_r(\omega) = 1 + \chi(\omega)$$

对于一般各向同性介质,  $\varepsilon_r$  为一标量

# 电磁场边值关系



$$\mathbf{n} \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$$

$$\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_{sf}$$

$$\mathbf{n} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_{sf}$$

$$\mathbf{n} \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$$

在绝缘介质边界上，电磁场切向分量连续

# 电荷守恒定律

$$\nabla \cdot \mathbf{J}_f + \frac{\partial \rho_f}{\partial t} = 0$$

单位时间从空间某点单位体积内流出的电荷量等于该体积内电荷的减少

# 欧姆定律

$$\mathbf{J}_f = \sigma \mathbf{E}$$

$\sigma$ 为介质的电导率 $[\Omega^{-1} \cdot \text{m}^{-1}]$

# 磁导率

一般光学介质对磁场的响应与真空相同：

$$\mu = \mu_0$$

铁磁性介质的形式较为复杂



# 相对介电常数和折射率

在最一般情况下， $\chi$ ， $\epsilon_r$ 为复二阶张量，  
且与众多物理量有关：

$$\epsilon_r = \epsilon_r(\mathbf{r}, t, \mathbf{E}, \mathbf{H}, \mathbf{S}, \mathbf{T}, \dots)$$

$$P_i(\omega) = \epsilon_0 \chi_{ij}(\omega) E_j(\omega)$$

$$D_i(\omega) = \epsilon_0 \epsilon_{ij}(\omega) E_j(\omega)$$

通过适当选择坐标系， $\epsilon_r$ 的形式可以得到简化

对于双轴晶体：

$$\epsilon_r = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix}$$

对于单轴晶体：

$$\epsilon_r = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix}$$

定义介质的折射率： $n_i = \sqrt{\epsilon_{ii}} \quad i=1,2,3$

# Maxwell方程的频域形式

$$\nabla \times \mathbf{E} = -i\omega\mathbf{B}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + i\omega\mathbf{D}$$

$$\nabla \cdot \mathbf{D} = \rho_f$$

$$\nabla \cdot \mathbf{B} = 0$$

## 介质方程

$$\mathbf{D}(\mathbf{r}, \omega) = \varepsilon(\mathbf{r}, \omega)\mathbf{E}(\mathbf{r}, \omega)$$

$$\mathbf{B}(\mathbf{r}, \omega) = \mu_0\mathbf{H}(\mathbf{r}, \omega)$$

$$\varepsilon(\omega) = \varepsilon_0\varepsilon_r(\omega) = \varepsilon_0[1 + \chi(\omega)]$$

# 电磁场波动方程

在通常的光学介质内  $\rho_f = 0$ ,  $\mathbf{J}_f = 0$ ; 由Maxwell方程得到:

$$\nabla \times \nabla \times \mathbf{E} = \omega^2 \varepsilon \mu_0 \mathbf{E}$$

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\longrightarrow \nabla^2 \mathbf{E} + k^2 \mathbf{E} = -\nabla[\nabla(\ln \varepsilon) \cdot \mathbf{E}]$$

弱导近似下:  $\nabla \varepsilon \approx 0$

$$\longrightarrow \nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0$$

$$\nabla^2 \mathbf{H} + k^2 \mathbf{H} = -\nabla(\ln \varepsilon) \times (\nabla \times \mathbf{H})$$


弱导近似下:  $\nabla \varepsilon \approx 0$


$$\longrightarrow \nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0$$

$$k = \omega \sqrt{\varepsilon \mu_0} = \omega \sqrt{\varepsilon_r} \sqrt{\varepsilon_0 \mu_0} = \frac{\omega}{c} \sqrt{\varepsilon_r} = k_0 \sqrt{\varepsilon_r} = k_0 n$$


# 横场和纵场

$$\mathbf{E} = \mathbf{e}_x E_x + \mathbf{e}_y E_y + \mathbf{e}_z E_z, \quad \mathbf{H} = \mathbf{e}_x H_x + \mathbf{e}_y H_y + \mathbf{e}_z H_z$$


$$\mathbf{E} = \mathbf{E}_t + \mathbf{e}_z E_z$$


$$\mathbf{H} = \mathbf{H}_t + \mathbf{e}_z H_z$$

$$\nabla = \mathbf{e}_x \frac{\partial}{\partial x} + \mathbf{e}_y \frac{\partial}{\partial y} + \mathbf{e}_z \frac{\partial}{\partial z}$$


$$\nabla = \nabla_t + \mathbf{e}_z \frac{\partial}{\partial z}$$

$$H_z = \frac{1}{j\omega\mu_0} \nabla_t \cdot (\mathbf{e}_z \times \mathbf{E}_t)$$

$$E_z = -\frac{1}{j\omega\epsilon} \nabla_t \cdot (\mathbf{e}_z \times \mathbf{H}_t)$$

$$\mathbf{e}_z \times \frac{\partial \mathbf{E}_t}{\partial z} = -j\omega\mu_0 \mathbf{H}_t - \frac{1}{j\omega} \mathbf{e}_z \times \nabla_t \left( \frac{1}{\epsilon} \nabla_t \cdot (\mathbf{e}_z \times \mathbf{H}_t) \right)$$

$$\mathbf{e}_z \times \frac{\partial \mathbf{H}_t}{\partial z} = j\omega\epsilon_0 \mathbf{E}_t + \frac{1}{j\omega\mu_0} \mathbf{e}_z \times \nabla_t (\nabla_t \cdot (\mathbf{e}_z \times \mathbf{E}_t))$$

# 自由空间中的单色平面波

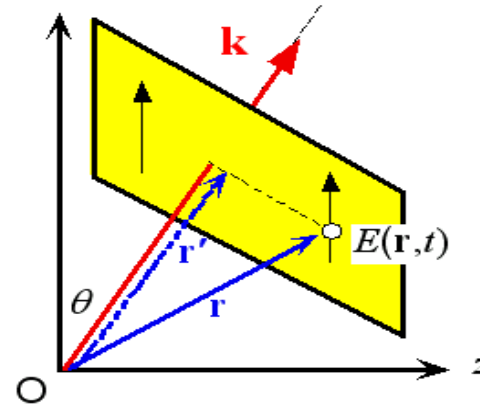
$$\mathbf{E}(\mathbf{r}) = E_0 \exp(-jk \cdot \mathbf{r})$$

$$\mathbf{H}(\mathbf{r}) = H_0 \exp(-jk \cdot \mathbf{r})$$

$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y + k_z \mathbf{e}_z$$

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

Direction of propagation



A travelling plane EM wave along a direction  $\mathbf{k}$

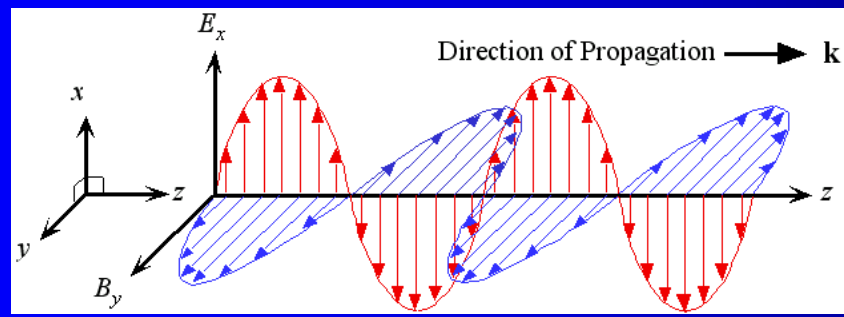
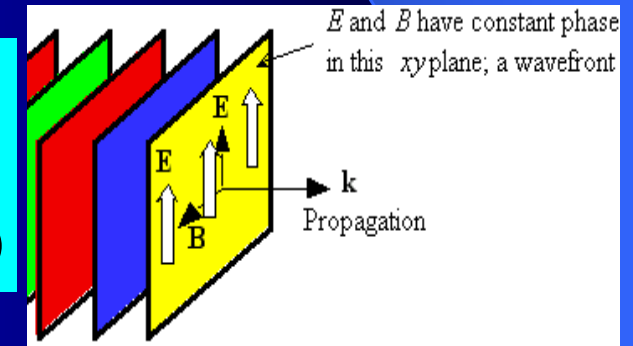
$\nabla \Rightarrow -jk$  Maxwell Eq.

$$\mathbf{k} \times \mathbf{E}_0 = \omega \mu_0 \mathbf{H}_0$$

$$\mathbf{k} \times \mathbf{H}_0 = -\omega \epsilon \mathbf{E}_0$$

$$\mathbf{k} \cdot \mathbf{E}_0 = \mathbf{k} \cdot \mathbf{H}_0 = 0$$

$\mathbf{E}$ 、 $\mathbf{H}$ 、 $\mathbf{k}$ 相互正交  
TEM波



$$E_x = E_0 \exp^{j(\omega t - kz)}, E_y = E_z = 0$$

$$H_y = H_0 \exp^{j(\omega t - kz)}, H_x = H_z = 0$$

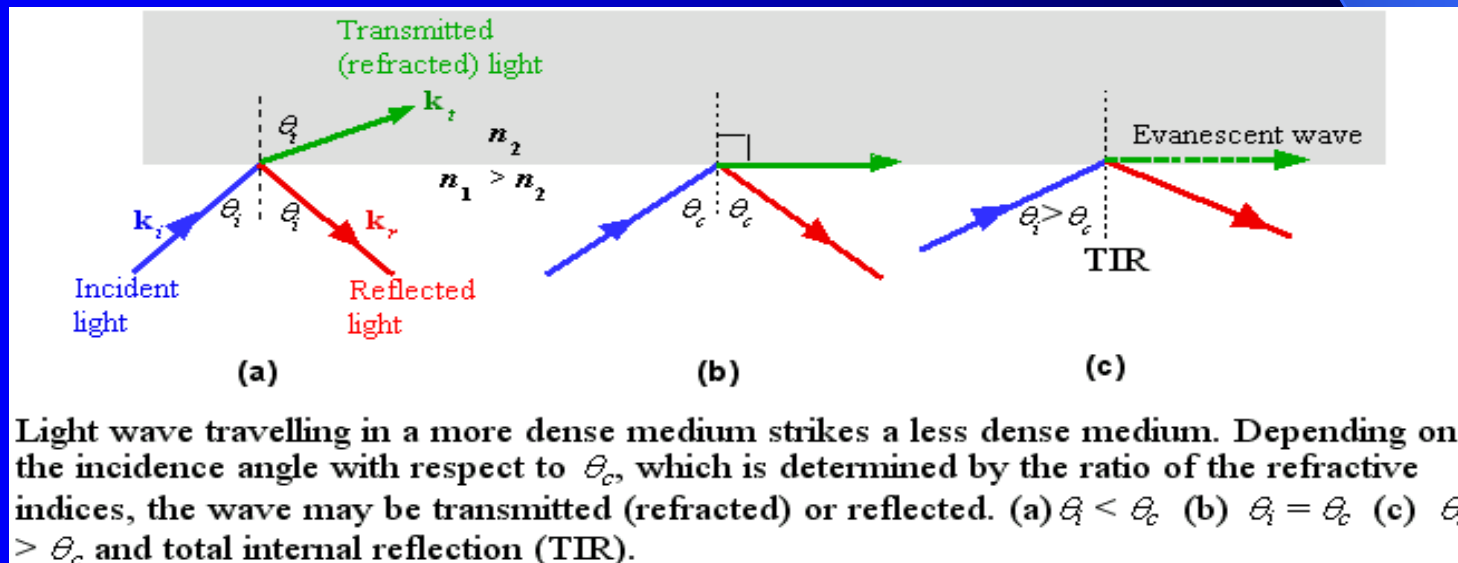
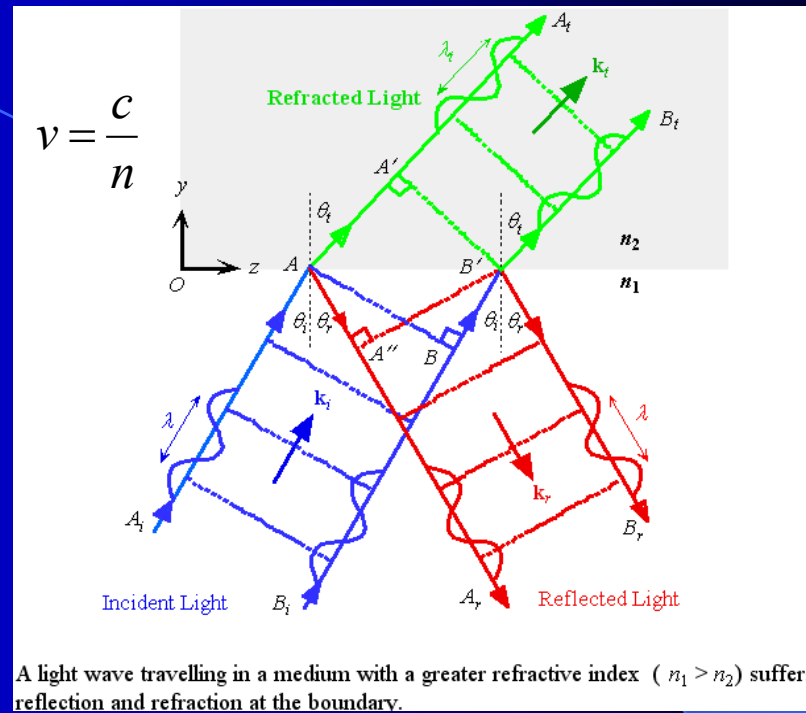
# 反射和折射

Snell定律:

$$n_1 \sin \theta_i = n_2 \sin \theta_t, \theta_i = \theta_r$$

全内反射和临界角

$$\theta_i > \theta_c = \sin^{-1} \frac{n_2}{n_1}$$



# Snell定律

$$\mathbf{E}_i = \mathbf{E}_{i0} e^{-jk_i \cdot \mathbf{r}} \quad \mathbf{E}_r = \mathbf{E}_{r0} e^{-jk_r \cdot \mathbf{r}} \quad \mathbf{E}_t = \mathbf{E}_{t0} e^{-jk_t \cdot \mathbf{r}}$$

选择坐标系使 $k_{ix}=0$ :  $\longrightarrow$   $\mathbf{k}_i = \mathbf{e}_y k_{iy} + \mathbf{e}_z k_{iz}$

$$\mathbf{n} \times \left( \mathbf{E}_{i0} e^{-i(\mathbf{k}_i \cdot \mathbf{r} - \omega t)} + \mathbf{E}_{r0} e^{-i(\mathbf{k}_r \cdot \mathbf{r} - \omega t)} \right) = \mathbf{n} \times \mathbf{E}_{t0} e^{-i(\mathbf{k}_t \cdot \mathbf{r} - \omega t)} \quad (y=0)$$

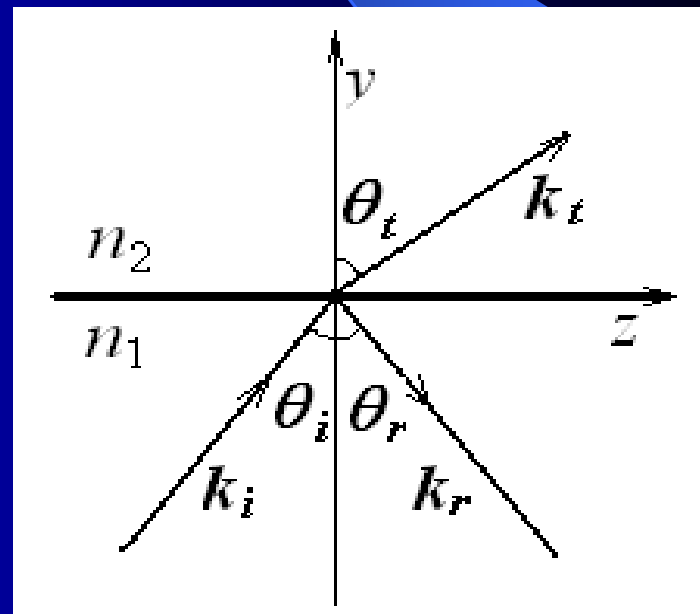
$$\longrightarrow \mathbf{k}_i \cdot \mathbf{r} = \mathbf{k}_r \cdot \mathbf{r} = \mathbf{k}_t \cdot \mathbf{r} \quad (y=0)$$

$$\longrightarrow zk_{iz} = xk_{rx} + zk_{rz} = xk_{tx} + zk_{tz}$$

$$\longrightarrow k_{iz} = k_{rz} = k_{tz}, \quad k_{rx} = k_{tx} = 0$$

$$\longrightarrow k_i \sin \theta_i = k_r \sin \theta_r = k_t \sin \theta_t$$

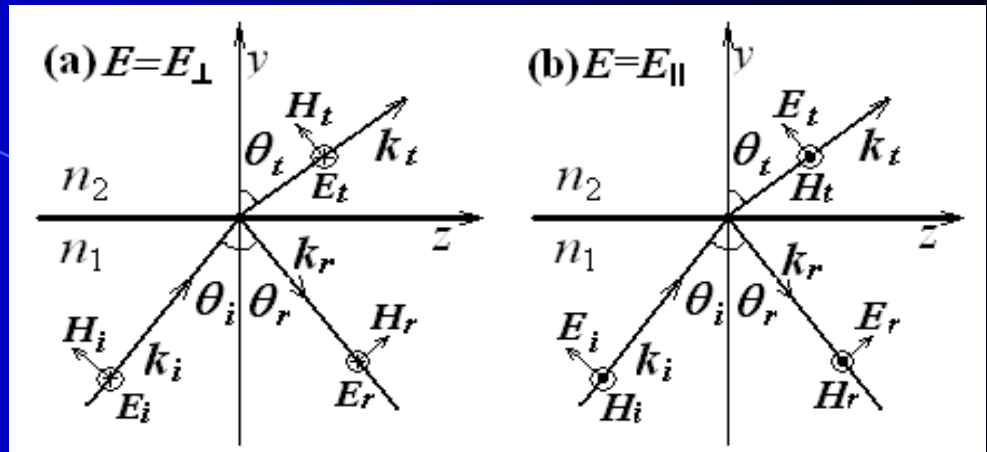
$$\longrightarrow \theta_i = \theta_r, \quad n_1 \sin \theta_i = n_2 \sin \theta_t$$



# Fresnel公式

$$\mathbf{E} = \mathbf{E}_{//} + \mathbf{E}_{\perp}$$

分别考虑平行和垂直情形



(1)  $\mathbf{E}_i = \mathbf{E}_{\perp}$

在  $y = 0$  的分界面上，电磁场切向分量连续

$$E_{i0} + E_{r0} = E_{t0}$$

$$-H_{i0} \cos \theta_i + H_{r0} \cos \theta_r = -H_{t0} \cos \theta_t \Rightarrow n_1(E_{i0} - E_{r0}) \cos \theta_i = n_2 E_{t0} \cos \theta_t$$

$$\Rightarrow r_{\perp} = \frac{E_{r0}}{E_{i0}} = \frac{\cos \theta_i - [n^2 - \sin^2 \theta_i]^{1/2}}{\cos \theta_i + [n^2 - \sin^2 \theta_i]^{1/2}}, \quad t_{\perp} = \frac{E_{t0}}{E_{i0}} = \frac{2 \cos \theta_i}{\cos \theta_i + [n^2 - \sin^2 \theta_i]^{1/2}}$$

(2)  $\mathbf{E}_i = \mathbf{E}_{//}$

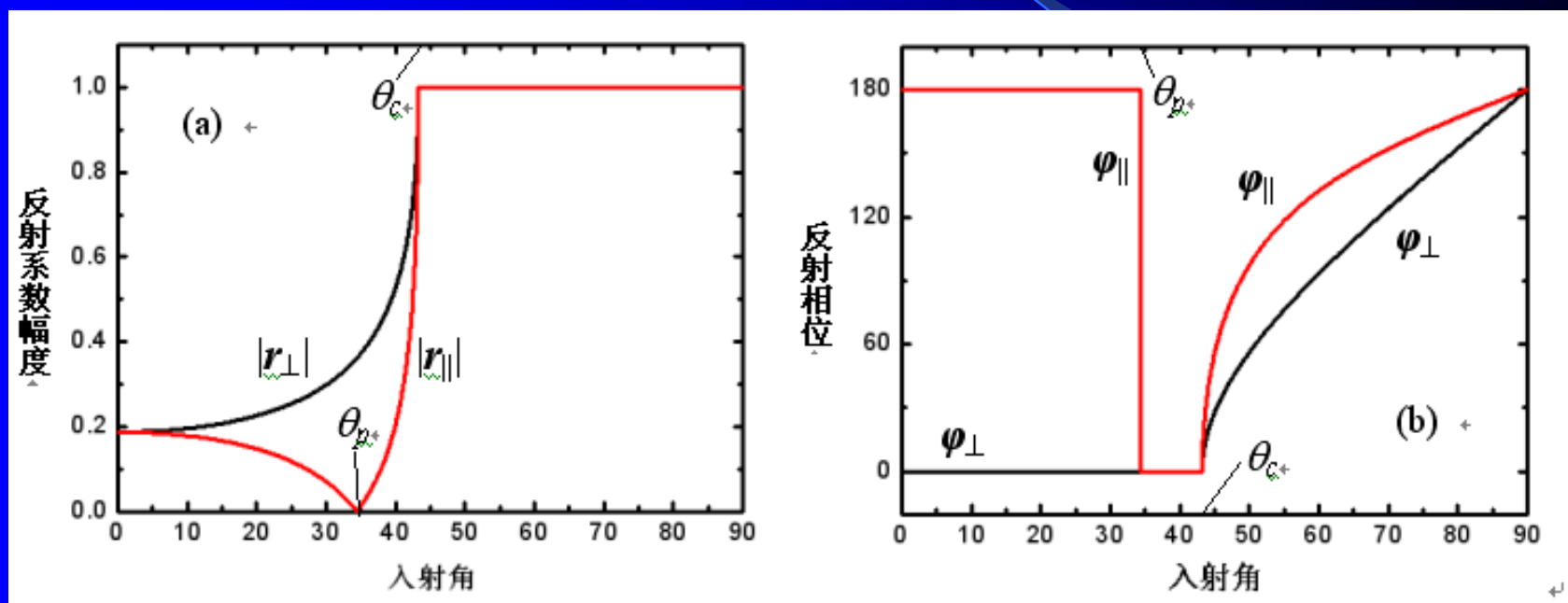
$$r_{//} = \frac{[n^2 - \sin^2 \theta_i]^{1/2} - n^2 \cos \theta_i}{[n^2 - \sin^2 \theta_i]^{1/2} + n^2 \cos \theta_i}, \quad t_{//} = \frac{2n \cos \theta_i}{n^2 \cos \theta_i + [n^2 - \sin^2 \theta_i]^{1/2}}$$

$$n = \frac{n_2}{n_1}$$



# 一个例子

$$n_1=1.46, n_2=1$$



Brewster角:

$$\theta_p = \tan^{-1} \frac{n_2}{n_1}$$

# 相速度、色散与群速度

## (1) 相速度 (波阵面推进的速度)

考虑沿  $z$  方向传输的单色波平面波:

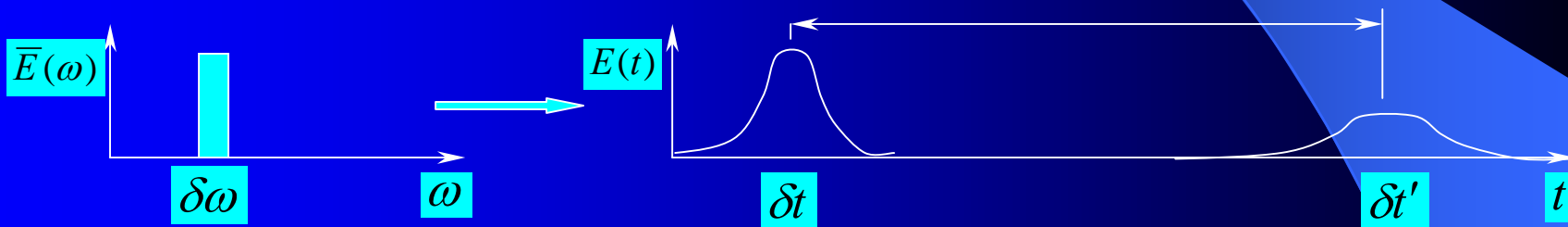
$$\mathbf{E} = \mathbf{E}_0 e^{j(\omega t - kz)}$$

其等相位面  $\omega t - kz = \text{constant}$  推进的速度为:  $v = dz/dt = \omega/k = c/n$

## (2) 色散 (波速随频率的变化)

$$\varepsilon_r(\omega) \Rightarrow n(\omega) \Rightarrow v(\omega) \Rightarrow k(\omega)$$

## (3) 群速度 (波群或波包的运动速度)



$$E(t) = \int_{-\infty}^{+\infty} E(\omega) e^{j(\omega t - kz)} d\omega = e^{j(\omega_0 t - k_0 z)} \int_{-\infty}^{+\infty} E(\Omega) e^{j\Omega(t - \frac{dk}{d\omega} z)} d\Omega = E\left(t - \frac{dk}{d\omega} z\right) e^{j(\omega_0 t - k_0 z)}$$

波包的运动速度为:  $v_g = dz/dt = d\omega/dk$

群折射率

$$k = \frac{\omega n(\omega)}{c} \Rightarrow \frac{dk}{d\omega} = \frac{1}{c} \left[ n + \omega \frac{dn}{d\omega} \right] = \frac{n_g}{c} \Rightarrow v_g = \frac{c}{n_g} \Rightarrow n_g = n + \omega \frac{dn}{d\omega}$$

# 电磁场的能量和能流

## (1) 电磁场能量守恒定律

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\frac{\partial}{\partial t} \left( \frac{1}{2} \mu_0 \mathbf{H}^2 + \frac{1}{2} \varepsilon \mathbf{E}^2 \right)$$

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H}) = -\mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} = \frac{\partial}{\partial t} \left( \frac{1}{2} \mu_0 \mathbf{H}^2 \right), \quad \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} = \frac{\partial}{\partial t} \left( \frac{1}{2} \varepsilon \mathbf{E}^2 \right) \quad \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}, \quad \mathbf{B} = \mu_0 \mathbf{H}$$

(2) Poyting矢量 (能流密度矢量)  $\mathbf{P} = \mathbf{E} \times \mathbf{H}$ , 复数形式:  $\mathbf{P} = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*)$

单色平面波:  $\mathbf{P} = \frac{|\mathbf{E}_0|^2}{2\omega\mu_0} \mathbf{k} = \frac{|\mathbf{H}_0|^2}{2\omega\varepsilon_0} \mathbf{k} \quad [\text{W} \cdot \text{m}^{-2}]$

(3) 电磁场能量密度  $w = \frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu_0 H^2 \quad [\text{J} \cdot \text{m}^{-3}]$

# 光的粒子性

$$E = \hbar\omega, \quad \mathbf{p} = \hbar\mathbf{k}$$

单位体积内的光子数（光子密度）：

$$N = \frac{w}{\hbar\omega}$$

$$w = \frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu_0 H^2$$

光子流密度：

$$\mathbf{S} = \frac{1}{\hbar\omega} \mathbf{P}$$

$$\mathbf{P} = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*)$$

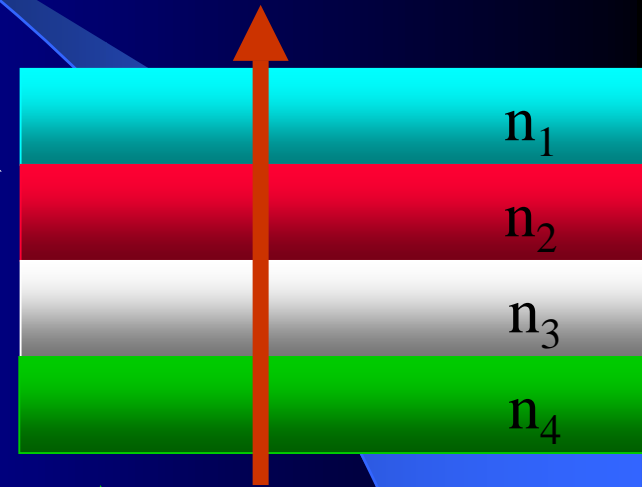
# 一维平面光波导

## 平面光波导

半导体光电子器件、LiNbO<sub>3</sub>波导器件、平面光波回路PLC

## 一维平面光波导的基本结构

由多层平板介质构成的波导结构，折射率在垂直于介质分界面的方向上发生变化。

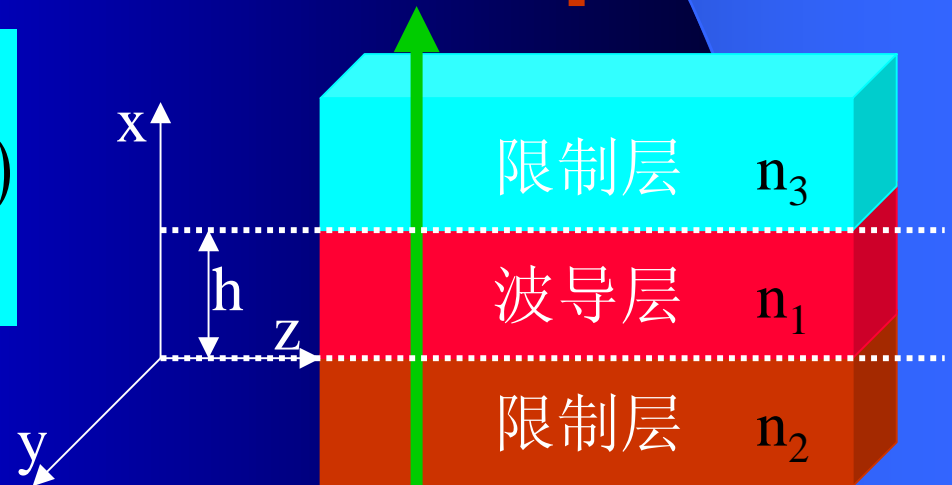


## 三层均匀一维平面光波导

$$n(x) = \begin{cases} n_1, & 0 \leq x \leq h \\ n_2, & x \leq 0 \\ n_3, & x \geq h \end{cases} \quad (n_1 > n_2 \geq n_3)$$

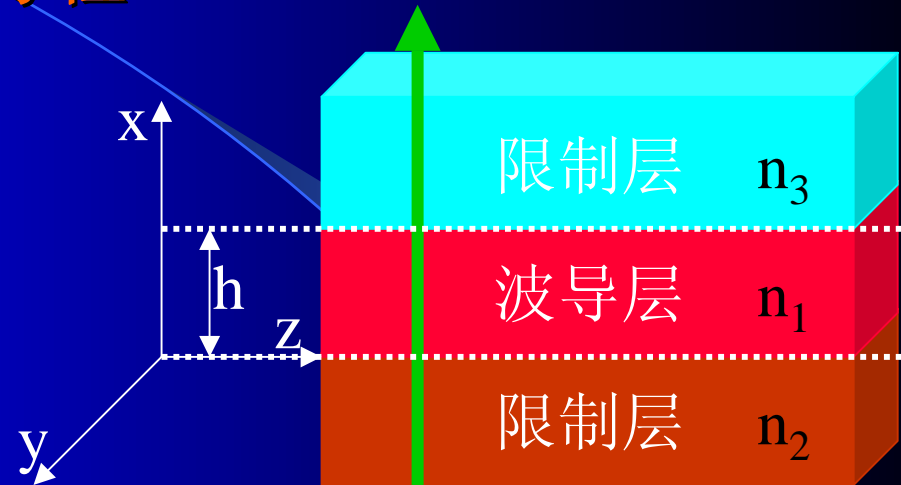
对称结构:  $n_2 = n_3$

非对称结构:  $n_2 \neq n_3$



# 一维平面光波导中的电磁场方程

光波导将空间分为三个均匀的区域，各区域内电磁场分量的切向分量在介质分界面上满足连续性条件。三个区域内的电磁场的各个直角分量均满足下述波动方程：



$$\nabla^2 \Psi + k^2 \Psi = 0$$

$$k = k_0 n, k_0 = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$$

$$\psi = \psi(x) \exp(-j\beta z)$$

$$\nabla^2 = \nabla_t^2 + \frac{\partial^2}{\partial z^2}$$

$$\frac{d^2 \psi}{dx^2} + (k_0^2 n_j^2 - \beta^2) \psi = 0, j = 1, 2, 3$$

$$TE : \psi = E_y$$

$$TM : \psi = H_y$$

## 场方程

$$\frac{d^2\psi}{dx^2} + (k_0^2 n_j^2 - \beta^2)\psi = 0, j = 1, 2, 3$$

$$TE: \psi = E_y$$

$$TM: \psi = H_y$$

$$\text{导模条件: } k_0 n_2 < \beta < k_0 n_1$$

## 解

$$\psi(x) = \begin{cases} A \cos(\delta x) + B \sin(\delta x) & 0 \leq x \leq h \\ A \exp(\gamma_2 x) & , \quad x < 0 \\ [A \cos \delta h + B \sin(\delta h)] \exp[-\gamma_3(x-h)] & x > h \end{cases}$$

$$\delta^2 = k_1^2 - \beta^2, \gamma_2^2 = \beta^2 - k_2^2, \gamma_3^2 = \beta^2 - k_3^2, k_j = k_0 n_j, (j = 1, 2, 3)$$

$\gamma$ : 衰减系数

切向分量连续

$\Psi(E_y, H_y), (E_z, H_z)?$

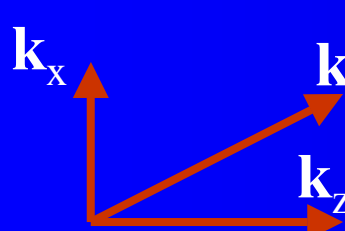
# 特征方程

TE模

$H_z$ 连续

$$\tan(\delta h) = \frac{\delta(\gamma_2 + \gamma_3)}{\delta^2 - \gamma_2\gamma_3}$$

$$H_z = \frac{j}{\omega\mu_0} \frac{dE_y}{dx}$$



$$\delta^2 = k_1^2 - \beta^2, \gamma_2^2 = \beta^2 - k_2^2, \gamma_3^2 = \beta^2 - k_3^2, k_j = k_0 n_j, (j = 1, 2, 3)$$

$$\delta^2 = k_1^2 - \beta^2 = k_x^2$$

$$2k_x h + \varphi_2 + \varphi_3 = 2m\pi$$

$$k_x h = \arctan\left[\frac{\delta(\gamma_2 + \gamma_3)}{\delta^2 - \gamma_2\gamma_3}\right] + m\pi$$



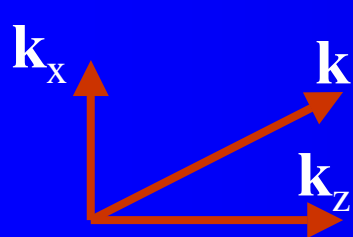
# 特征方程

TM模

$E_z$ 连续

$$\tan(\delta h) = \frac{\delta k_1^2 (k_3^2 \gamma_2 + k_2^2 \gamma_3)}{k_2^2 k_3^2 \delta^2 - k_1^4 \gamma_2 \gamma_3}$$

$$E_z = -\frac{j}{\omega \epsilon} \frac{dH_y}{dx}$$



$$\delta^2 = k_1^2 - \beta^2, \gamma_2^2 = \beta^2 - k_2^2, \gamma_3^2 = \beta^2 - k_3^2, k_j = k_0 n_j, (j = 1, 2, 3)$$

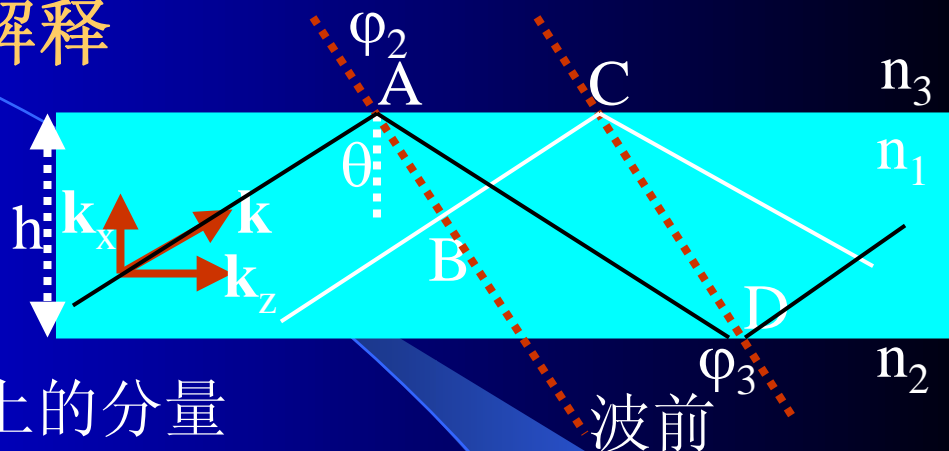
$$\delta^2 = k_1^2 - \beta^2 = k_x^2$$

# 光波导中模式的几何光学解释

全反射条件

$$\theta > \theta_{c12} > \theta_{c13}$$

$$\sin \theta > n_2/n_1$$



传输常数：波矢量在传输方向上的分量

$$\beta = k_z = k_0 n_1 \sin \theta$$

$$\mathbf{k} = \mathbf{k}_0 n_1$$

$$k_0 n_2 < \beta < k_0 n_1$$

相干加强条件：同一波阵面上相位相同或相差 $2\pi$ 整数倍

满足全反射条件时，只有某些以特定角度入射的光线才能在波导内传导，每一种可以传导的电磁波称为波导的一种**模式**。

$$\left(\overline{AD} - \overline{BC}\right) \frac{2\pi n_1}{\lambda} + \varphi_2 + \varphi_3 = 2m\pi$$

特征方程

$$AD - BC = 2h \cos \theta$$

$$2k_0 n_1 h \cos \theta + \varphi_2 + \varphi_3 = 2m\pi$$

## 小结

- 波动光学获得TE模式和TM模式的特征方程与几何光学分析获得的横向谐振条件类似，但是表述起来更为严格。
- 求解特征方程，可以获得传输常数的一系列解，每一个解对应一个模式。
- 由传输常数可以获得其他的分布参数，进一步得出电磁场的各个场分量。

$$\delta^2 = k_1^2 - \beta^2, \gamma_2^2 = \beta^2 - k_2^2, \gamma_3^2 = \beta^2 - k_3^2, k_j = k_0 n_j, (j=1,2,3)$$

## 模式的基本性质

### 波动光学

限制层内衰减

$$k_0 n_2 < \beta < k_0 n_1$$

衰减速度

$$n_1 > n_2 \geq n_3$$

$$\gamma_3 > \gamma_2$$

近截止

$$\beta_m \approx k_2, \gamma_{2m} \approx 0, \delta_m \approx \sqrt{k_1^2 - k_2^2}$$

场分布进入 $n_2$ 介质内很深

远离截止

$$\beta_m \approx k_1, \delta_m \approx 0, \gamma_{2m} \approx \sqrt{k_1^2 - k_2^2} = \frac{2\pi}{\lambda} \sqrt{n_1^2 - n_2^2}$$

场分布很好地限制在波导层内

### 几何光学

全反射条件

分布深度

# 光波导的一般理论