J. Astrophys. Astr. (2003) **24,** 3–10

Is the Age Problem Resolved?

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Received 2001 October 8; accepted 2003 January 16

Abstract. The cosmological, astrophysical, and nucleocosmochronological methods for estimating the age of the universe and the corresponding uncertainties are comparatively studied in the present paper. We are led to the conclusion that the new measurements of cosmological parameters, and the recent estimates of the age of globular clusters have led to the gradual disappearance of the age problem from the arena of modern cosmology.

Key words. Cosmology: age of the universe, cosmological parameters.

1. Introduction

Three main streams of research seem to have converged to consistent estimates of the age of the universe within the corresponding uncertainties. First, recent observational measurements of matter density parameter give (Coles & Ellis 1994):

$$\Omega_m = 0.1 - 0.4. \qquad (1)$$

This range epitomizes the result of a variety of methods for calculating Ω_m, as reviewed by Coles & Ellis (1994). These include gravitational lensing, galaxy clustering observations, measurements of the peculiar velocities of galaxies, and dipole analysis of cosmic background radiation. Secondly, the recent Boomerang experiment provides us with (deBernandis *et al.* 2000):

$$0.88 < \Omega_o < 1.12, \qquad (2)$$

where $\Omega_o = \Omega_m + \Omega_\Lambda$, and Ω_Λ is the vacuum energy density parameter related to the cosmological constant, Λ , by $\Omega_\lambda = \frac{\Lambda c^2}{3H_o^2}$. Since $\Omega_o = 1$ corresponds to a flat universe, the Boomerang experiment shows the flatness of the universe to a fairly good degree of accuracy. Thirdly, the Hipparcos parallax catalog, together with other techniques (Chaboyer *et al.* 1998), indicates that globular clusters are farther away than previously believed, implying a reduction in the age estimates. Moreover, studies of high redshift supernovae have led to a non-vanishing cosmological constant, which affects the cosmological age estimates. In the following sections we discuss these issues in more detail.

2. Cosmological estimates of the age of the universe

In the context of current cosmological theory, which is based on the validity of the cosmological principle, the cosmic time elapsed since the big bang singularity is approximately given by (Riazi 1991):

$$\tau_c = \frac{1}{H_o} \int_o^1 \frac{\sqrt{r} dr}{\sqrt{\Omega_m + (1 - \Omega_m - \Omega_\Lambda) r + \Omega_\Lambda r^3}}. \tag{3}$$

This equation involves an approximation, because it is the result of integration of the Friedmann's equations after recombination (i.e., by neglecting the quantum and radiation dominated era). The approximation holds because the value which the integral assumes by the insertion of customary values of H_o, Ω_m, and Ω_Λ in the above integral is considerably larger than the Planck time (10^{-43}s) and the age of the early universe before recombination ($\simeq 10^5$ yr). Setting aside the logical and physical controversies underlying the conception of a beginning for the universe, and ignoring our lack of knowledge as to what happened to the universe during the Planck era, when a quantum theory of gravity predominated, modern cosmologists have come to call τ_c "the age of the universe". According to equations (1) and (2), the energy density of vacuum has had a paramount role in the evolution of the universe. This leads, according to equation (3), to a considerable increase of the age of the universe as compared to the age of a flat universe without a cosmological constant (which is 8.6 Gyr for $H_o = 75\,\mathrm{kms^{-1}Mpc^{-1}}$). The matter density parameter can be decomposed according to

$$\Omega_m = \Omega_b + \Omega_{cdm} + \Omega_{hdm}, \tag{4}$$

where Ω_b, Ω_{cdm}, and Ω_{hdm} denote the density parameter of baryonic matter, cold dark matter, and hot dark matter, respectively. Assuming the existence of hot dark matter and a flat universe for which $\Omega_o = 1$, Novosyadlyj and his coworkers (2000) have recently embarked on a thoroughgoing statistical analysis of cosmological parameters on the basis of a wide set of observational data including the Abell-ACO cluster power spectrum and mass function, peculiar velocities of galaxies, the distribution of $Ly - \alpha$ clouds, and CMB temperature fluctuations. Using a χ^2 minimization method, Novosyadlyj *et al.* (2000) argue that the cosmological parameters which match observational data best are

$$h_o = H_o/100 = 0.70 \pm 0.12 \text{ km s}^{-1} \text{ Mpc}^{-1},$$
$$\Omega_m = 0.41 \pm 0.11 \;\; (\Omega_\Lambda = 0.59 \pm 0.11),$$
$$\Omega_{cdm} = 0.31 \pm 0.15, \;\; \Omega_{hdm} = 0.059 \pm 0.028,$$

and

$$\Omega_b = 0.039 \pm 0.014.$$

In each of these equalities, the first number indicates the most probable value and the second number is the corresponding standard error. For $\Omega_o = 1$, the cosmological age integral reduces to

$$\tau_c = \frac{2}{3 H_o \Omega_\Lambda^{1/2}} \sinh^{-1} \left(\frac{\Omega_\Lambda}{\Omega_m} \right)^{1/2}, \tag{5}$$

which by insertion of the data obtained by Novosyadlyj *et al.* leads to

$$\tau_c = 12.3\,\text{Gyr}. \tag{6}$$

According to a well known theorem in the theory of errors (Barford 1995), the standard error of $z = f(x, y, ...)$ is related to the standard errors of $x, y, ...$ according to

$$S(z) = \left[\left(\frac{\partial f}{\partial x}\right)^2_{x=x_o, y=y_o,...} S^2(x) + \left(\frac{\partial f}{\partial y}\right)^2_{x=x_o, y=y_o,...} S^2(y) + ... \right]^{1/2}. \tag{7}$$

Hence, if we rewrite (5) by using $\Omega_o = 1$ as

$$\tau_c = \frac{2}{3H_o\Omega_\Lambda^{1/2}} \sinh^{-1}\left(\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right)^{1/2}, \tag{8}$$

then, based on Novosyadlyj's data,

$$\frac{\partial \tau_c}{\partial H_o} = -\frac{2}{3H_o^2\Omega_\Lambda^{1/2}} \sinh^{-1}\left(\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right)^{1/2} = -172.8\ \text{Gyr}^2, \tag{9}$$

and

$$\frac{\partial \tau_c}{\partial \Omega_\Lambda} = -\frac{1}{3H_o\Omega_\Lambda}\left[\frac{1}{(1-\Omega_\Lambda)^{1/2}(2\Omega_\Lambda^2 - 2\Omega_\lambda + 1)^{1/2}} - \frac{1}{\Omega_\Lambda^{1/2}} \sinh^{-1}\left(\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right)^{1/2}\right] = 6.73\ \text{Gyr}. \tag{10}$$

Thus, finally,

$$\tau_c = 12.3 \pm 2.2\ \text{Gyr}. \tag{11}$$

There is a justification for using Novosyadlyj's data as a prototype for the values of cosmological parameters: firstly, the assumption $\Omega_o = 1$ is well corroborated by the Boomerang experiment and the prediction of the inflationary models. Secondly, the analysis is based on a comprehensive set of observational data. Thirdly, the most probable value of Ω_m which ensues from the analysis lies within the range of variation of Ω_m as given by equation (1). Fourthly, the average value of Ω_m which results from equation (1) lies within the range of variation of Ω_m as given by Novosyadlyj. Finally, Novosyadlyj's value for h_o is consistent with the value newly obtained by the Hubble Space Telescope: $h_o = 0.71 \pm 0.06$ kms^{-1}Mpc^{-1} (Mould *et al.* 2000).

3. Astrophysical estimates of the age of the universe

It was first suggested by Peebles & Dicke (1968) that the globular clusters have about the same mass as the Jeans's mass at recombination. They are thus considered as the successors of the first generation of gravitationally bound systems formed in the early universe. On this basis, they are reckoned as the oldest structures in the universe, the age of which gives the most stringent lower limit on the age of the universe. The most comprehensive statistical analysis of the effect of systematic errors included in the age estimation of globular clusters is the Monte Carlo analysis of Chaboyer *et al.* (1996a, 1996b, 1996c). Owing to the essential dependence of the analysis on the techniques of distance estimation, their analysis underwent a revision with the release of the Hipparcos catalogue for field subdwarf parallaxes, which via the main sequence fitting method, leads to a higher estimate for the distance of globular clusters and thus, a lower estimate for their age. Chaboyer *et al.*'s (1998) final result for the age of seventeen of the oldest clusters in our galaxy is

$$\tau_{gc} = 11.5 \pm 1.3 \text{ Gyr}. \tag{12}$$

Chaboyer *et al.* (1998) are among five different research groups which used the Hipparcos database for this purpose: Reid (1997, 1998); Gratton *et al.* (1997); Grundahl *et al.* (1998); Pont *et al.* (1998); and Chaboyer *et al.* (1998). Based on a sample of five globular clusters, Reid's analysis of 1998 leads to "age estimates of no more than 11 Gyr for any of the clusters included in the sample". Pont *et al.* (1998) report an age near 14 Gyr for M92 but their method mainly depends on the agreement between the shapes of the theoretical isochrones and the data near the turnoff and it overlooks the uncertainties due to convection and helium diffusion. From a fit to the main sequence of metal poor subdwarfs with Hipparcos parallaxes, Grandahl *et al.* (1998) derive an age near 12 Gyr for M13, assuming [Fe/H] $= -1.61$ and $[\alpha/\text{Fe}] = 0.3$. Gratton *et al.* (1997) based their former analysis on main sequence fitting for main clusters, six of which are included among Reid's (1997) analysis. In a revision of this work, Carretta, Gratton, and their co-workers (2000) have recently reported the age

$$\tau_{gc} = 12.9 \pm 2.9 \text{ Gyr}, \tag{13}$$

for galactic globular clusters, by a reconciliation of short and long distance scales to the Large Magellanic Cloud (Carretta *et al.* 2000). To make our comparison exhaustive, we must also mention the analysis of Salaris & Weiss (1997), who applied a combination of Iben's method and a method based on the color difference between the turnoff and the base of the red giant branch to a sample of twenty five clusters. They concluded

$$\tau_{gc} = 11.8 \pm 0.9 \text{ Gyr}, \tag{14}$$

or,

$$\tau_{gc} = 12.3 \pm 2.9 \text{ Gyr} \tag{15}$$

if the clusters Arp 2 and Rup 106 are not included in the sample. We must further point, in passing, to the idiosyncratic method of Jimenez (1996), who by using the horizontal branch morphology for a sample containing eight clusters, has come up with the final result

$$\tau_{gc} = 13.5 \pm 2 \text{ Gyr}. \tag{16}$$

According to Peebles & Dicke (1968), "the globular clusters formed (as gas clouds) before the galaxies appeared". However, what is meant by "age" in the above result is the time elapsed since the stars in the cluster became mature enough to start their main sequence life. Multiple observational evidence (Sandage 1993), together with the theories of galaxy formation (Coles & Lucchin 1997), point to the fact that the time of maturity of galaxy components corresponds to a redshift as high as $z_{pg} \simeq 10$, where pg denotes a protogalaxy. This leads, by a simple calculation similar to the one in section 2, to the "age" of a protogalaxy elapsed since the big bang singularity

$$\tau_{pg} = 0.4 \pm 0.07 \text{ Gyr}. \tag{17}$$

Combining this with the astrophysical estimates of globular cluster ages gives the astrophysical estimates of the age of the universe

$$\tau_A = \tau_{pg} + \tau_{gc}. \tag{18}$$

A problem now arises: How can we combine different astrophysical estimates of the age of the universe? Since the error of τ_A encompasses both statistical and systematic errors, the statistical combination of different astrophysical estimates of the age of the universe seems to be illegitimate. The only path open is to consider each single τ_A in its own right.

4. Nucleocosmochronology

Nucleocosmochronology derives the time-scales for the nucleosynthesis of solar system elements from the abundance and production ratios of radioactive nuclides and from a model for the chemical evolution of the Galaxy. Both of these sources of knowledge, are nevertheless also sources of ignorance, for they give rise to considerable uncertainties in the age determinations. Fowler & Hoyle (1960) first derived an age of the order of 15 Gyr for the Galaxy, by using an exponentially decaying model for the chemical evolution. Schramm & Wassenberg's (1970) research was original, in that it involved a calculation of the mean age of r-process elements independently of the model for the chemical evolution of the Galaxy. This mean age provides a lower limit for the Galactic age. Meyer & Schramm's (1986) quest for a lower and upper limit on the age of the galaxy which are independent of both the evolution function of the galaxy and the uncertainties in cosmochronological input data culminated in

$$8.9 \text{ Gyr} \leq \tau_g \leq 28.1 \text{ Gyr} \tag{19}$$

which, in view of some questions about β-delayed fission, some uncertainties in the decay rate of ^{187}Re caused by thermal resonance, and discussions about the Th/Nd abundance in stars, was changed into (Schramm *et al.* 1996):

$$9.6 \text{ Gyr} \leq \tau_g \leq 28.1 \text{ Gyr}. \tag{20}$$

The zero of time in the above estimates of the age of the Galaxy is the time when the first generation stars in the Galaxy became mature enough to start their nucleosynthesis. By combining (20) and (17) we thus arrive at the nucleocosmochronological estimate of the age of the universe

$$\tau_N = 19.3 \pm 9.3 \text{ Gyr}. \tag{21}$$

Recent spectral studies of the star CS31082-001 in the Galactic halo, in which an ionized uranium transition was detected, led to the simultaneous determination of the abundances of thorium and uranium, enabling the authors to obtain more accurate age estimates (Cayrel *et al.* 2001).

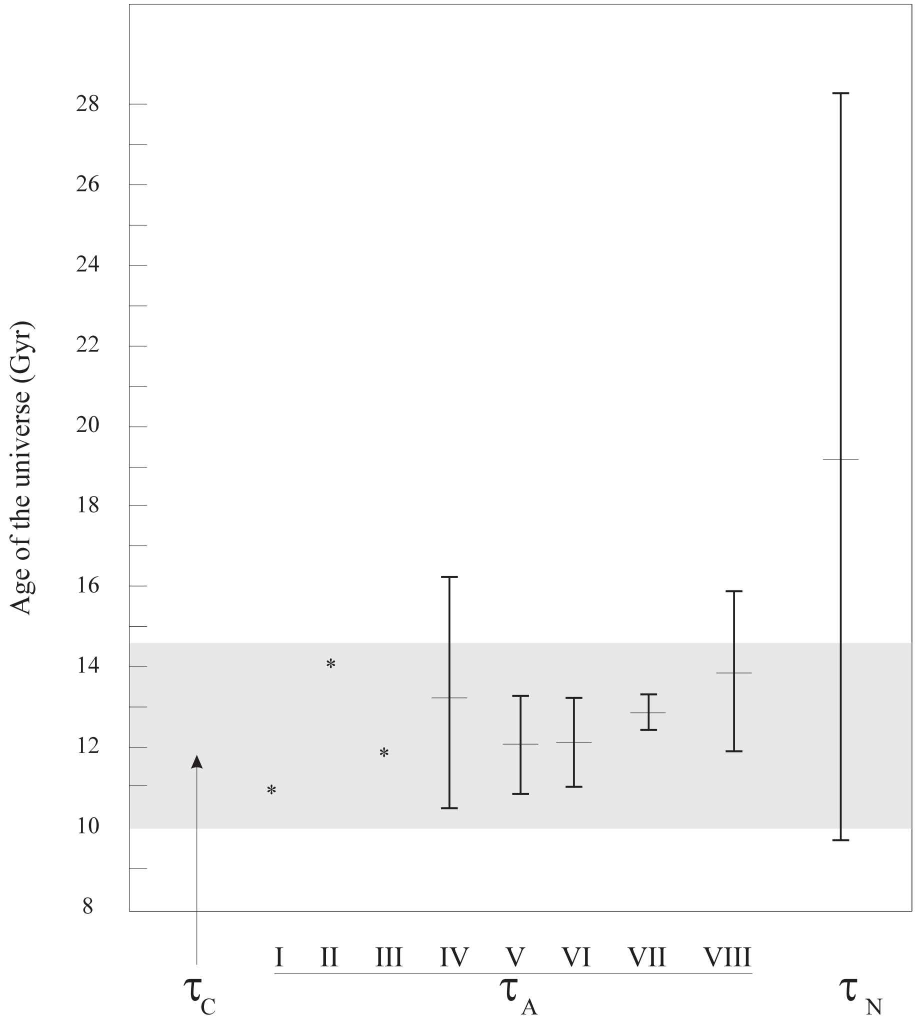

Figure 1. Comparison between different estimates of the age of the universe. References: **I:** Reid (1998), **II:** Pont *et al.* (1998), **III:** Grundahl *et al.* (1998), **IV:** Gratton *et al.* (1997), **V:** Chaboyer *et al.* (1998), **VI:** Salaris & Weiss (1997), Rup 106 and Arp 2 excluded, **VII:** Salaris & Weiss (1997), Rup 106 and Arp 2 included, **VIII:** Jimenez *et al.* (1996).

5. Discussion and conclusion

The results mentioned in the previous sections may be brought together using the diagram illustrated in Fig. 1. We refrain from a statistical combination of astrophysical estimates, the justification for which was given in section 3. The shaded area in the diagram corresponds to the plausible range of the cosmological age estimates based on Novosyadlyj's data as a prototype. An argument in favor of choosing Novasyadlyj's data as a prototype was given in section 2. Finally, the nucleocosmochronological result of Meyer and Schramm given by equation (21) is depicted in the diagram, as an archetype of a host of nucleocosmochronological ages with broad error ranges (Clayton 1996). Due to these broad ranges of error, the nucleocosmochronological ages defy any useful comparison with the astrophysical and cosmological age, but a plausible comparison between the astrophysical estimates and the cosmological estimate seems to be possible. Let us define the range of variation of τ_A and τ_c as

$$R_{\tau_A} \equiv \left[\tau_{A,O} - E_{\tau_A}, \tau_{A,O} + E(\tau_A)\right], \tag{22}$$

and

$$R_{\tau_C} \equiv \left[\tau_{C,O} - E_{\tau_C}, \tau_{C,O} + E(\tau_C)\right], \tag{23}$$

where O and E denote the most probable value and the corresponding error respectively. The diagram in Fig. 1 then shows clearly that

$$\tau_{A,O} \in R_{\tau_C}, \tag{24}$$

and

$$\tau_{C,O} \in R_{\tau_A}. \tag{25}$$

The above two relations authenticate the consistency of the cosmological and astrophysical estimates of the age of the universe within the range of the corresponding errors and substantiate the claim that modern observational and theoretical techniques are leading to the gradual disappearance of the longstanding age problem from the scene of cosmological debates. However, in spite of its plausible appearance, we should take this final result with some scruples as to the fundamentals of the cosmological theory within which this final conclusion is arrived at. To take the cosmological principle first, the wonderful discovery of the Great Wall in the late eighties has cast a shadow of doubt on the validity of this principle. It is possible to redeem the principle by posing the existence of non-baryonic dark matter, which may remedy the inhomogeneity betrayed by the Great Wall or other very large-scale structures, by a disguised homogeneity. Nevertheless, the existence of non-baryonic dark matter in itself is amenable to criticism, for the simple reason that, except for the massive neutrinos in the recent Super-Kamiokande and Sudbury experiments (Fukuda *et al.* 1998) which is still open to doubt, none of the exotic elementary particles proposed as candidates for non-baryonic matter have been detected yet. Secondly, our analysis was entirely based on the validity of the general theory of relativity. It may turn out that a more fundamental theory (such as the string theory or non-commutative geometry) will provide a better framework for doing cosmological calculations. The solution of the age problem has thus been accomplished within the context of the big bang model and not by a critical analysis of the model itself. The big bang model, whatever it may be, is not the final theory of the universe, and like every scientific theory, is susceptible of a fundamental criticism.

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