PREFACE

Game Theory studies the behavior of decision-makers ("players") whose decisions affect each other. As in one-person decision theory, the analysis is from a rational, rather than a psychological or sociological viewpoint. The term "game" stems from the formal resemblance of these interactive decision problems to parlour games such as Chess, Bridge, Poker, Monopoly, Diplomacy, or Battleship. To date, the largest single area of application has been economics; other important connections are with political science (on both the national and international levels), evolutionary biology, computer science, the foundations of mathematics, statistics, accounting, social psychology, law, and branches of philosophy such as epistemology and ethics. The applications are supported by a sizeable body of pure theory that is significant and important in its own right. Needless to say, the relation is two-sided: the theory influences – and is influenced by – the applications, both in the questions asked and in the answers provided.

There is an important distinction between multi-person and one-person decision problems. In the one-person context, we are usually led to a well-defined optimization problem, like maximizing an objective function subject to some constraints. While this problem may be difficult to solve in practice, it involves no conceptual issue. The *meaning* of "optimal decision" is clear; we must only *find* one. But in the interactive multi-person context, the very meaning of "optimal decision" is unclear, since in general, no one player completely controls the final outcome. One must address the conceptual issue of *defining* the problem before one can start *solving* it. Game Theory is concerned with both matters: defining "solution concepts", and then investigating their properties, in general as well as in specific models coming from the various areas of application. This leads to mathematical theories that ultimately yield important and novel insights, quantitative as well as qualitative.

Game Theory may be viewed as a sort of umbrella or "unified field" theory for the rational side of social science, where "social" is interpreted broadly to include human individuals as well as other kinds of players (collectives such as corporations and nations, animals and plants, computers, etc.). Unlike other approaches to disciplines like economics or political science, Game Theory does not use different, ad-hoc constructs to deal with various specific issues, such as perfect competition, monopoly, oligopoly, international trade, taxation, voting, deterrence, animal behavior, and so on. Rather, it develops methodologies that apply in principle to *all* interactive situations, then sees where these methodologies lead in each specific application.

One may distinguish two approaches to Game Theory: the non-cooperative and the cooperative. A game is *cooperative* if commitments – agreements, promises, threats – are fully binding and enforceable.[1] It is *non-cooperative* if commitments are not enforceable. (Note that pre-play communication between the players does not imply that any agreements that may have been reached are enforceable.) Though this may not look like a basic distinction, it turns out that the two theories have quite different characters. The non-cooperative theory concentrates on the strategic choices of the individual – how each player plays the game, what strategies he chooses to achieve his goals. The cooperative theory, on the other hand, deals with the options available to the group – what coalitions form, how the available payoff is divided. It follows that the non-cooperative theory is intimately concerned with the details of the processes and rules defining a game; the cooperative theory usually abstracts away from such rules, and looks only at more general descriptions that specify only *what* each coalition can get, without saying *how*. A very rough analogy – not to be taken too literally – is the distinction between micro and macro, in economics as well as in biology and physics. Micro concerns minute details of process, whereas macro is concerned with how things look "on the whole". Needless to say, there is a close relation between the two approaches; they complement and strengthen one another.

This is the first volume of the *Handbook of Game Theory with Economic Applications*, to be followed by two additional volumes. Game Theory has burgeoned greatly in the last decade, and today it is an essential tool in much of economic theory. The vision laid out by the founding fathers, John von Neumann and Oskar Morgenstern, in their 1944 book *Theory of Games and Economic Behavior* has become a reality.

While it is no longer possible in three volumes even to survey Game Theory adequately, we have made an attempt to present the main features of the subject as they appear today. The three volumes will cover the fundamental theoretical aspects, a wide range of applications to economics, several chapters on applications to political science, and individual chapters on relations with other disciplines.

A list of the chapters planned for all the volumes is appended to this

[1]This definition is due to John C. Harsanyi ('A general theory of rational behavior in game situations', *Econometrica*, 34: 616 (1966)).

Preface.[2] We have organized this list roughly into "non-cooperative" and "cooperative"; there are also some "general" chapters. The boundary is often difficult to draw, as there are important connections between the categories; chapters may well contain aspects of both approaches. Within each category, some chapters are more theoretical, others more applicative; here again, the distinction is often hazy. It is to be noted that the division of the chapters of the Handbook into the three volumes was dictated only partly by considerations of substantive relationships; another, more mundane consideration was which chapters were available when the volume went to press.

We now provide a short overview of the organization of this volume. Chapters 1 through 11 may be viewed as "non-cooperative" and Chapters 12 through 18 as "cooperative". The final chapter, Chapter 19, is in the "general" category. Most of the chapters belong to conceptually well-defined groups, and require little further introduction. Others are not so clearly related to their neighbors, so a few more words are needed to put them in context. (Thus the space that this Preface devotes to a chapter is no indication of its importance.)

Historically, the first contribution to Game Theory was Zermelo's 1913 paper on chess, so it is fitting that the "overture" to the Handbook deals with this grand-daddy of all games. The chapter covers chess-playing computers. Though this is not mainstream game theory, the ability of modern computers to beat some of the best human chess players in the world constitutes a remarkable intellectual and technological achievement, which deserves to be recorded in this Handbook.

Chapter 2 provides an introduction to the non-cooperative theory. It describes the "tree" representation of extensive games, the fact that for many purposes one can limit oneself to consideration of strategies, and the related classical results. Unlike in most of the other chapters, there is no attempt here at adequate coverage (which is provided in later chapters); it only provides some basic tools.

Conceptually, the simplest games are those of perfect information: games like chess, in which all moves are open and "above board", in which there is no question of guessing what the other players have done or are doing. The fundamental fact in this area is the 1913 theorem of Zermelo (mentioned above), according to which each zero-sum game of perfect information has optimal pure strategies. In 1953 Gale and Stewart showed that this result does not always extend to infinite games of perfect information, and identified conditions under which it does. Chapter 3 deals with these results, and with the literature in the foundations of mathematics (set theory) that has grown from them.

[2] A fairly detailed historical survey of game theory, with cross-references to the chapters of the *Handbook*, is planned for a subsequent volume.

Repeated games model ongoing relationships; the theory "predicts" phenomena such as cooperation, communication, altruism, trust, threats, punishment, revenge, rewards, secrecy, signalling, transmission of information, and so on. Chapters 4, 5, and 6 are devoted to repeated games. Though this theory is basically "non-cooperative", it brings us to the interface with the cooperative theory; it may be viewed as a non-cooperative model that "justifies" the assumption of binding agreements that underlies cooperative theory.

Another such "bridge" between the non-cooperative and the cooperative is bargaining theory. Until the early eighties, most of bargaining theory belonged to the cooperative area. After the publication, in 1982, of Rubinstein's seminal paper on the subject, much of the emphasis shifted to the relation of non-cooperative models of bargaining to the older cooperative models. These and related developments are covered in Chapter 7.

Chapter 7 is also the first of five chapters in this volume dealing with economic applications of the non-cooperative theory. Chapters 8 through 11 are about auctions, location, entry deterrence, and patents. In each case, equilibrium analysis leads to important qualitative insights.

Starting with Chapter 12, we turn to the cooperative theory and its applications. Chapters 12 through 16 offer a thorough coverage of what is perhaps the best known solution concept in cooperative game theory, the core. Chapters 12 and 13 provide theoretical foundations, while Chapters 14, 15, and 16 cover the best known economic applications.

Though the definition of the core is straightforward enough, it is perhaps somewhat simplistic; a careful consideration leads to some difficulties. Several solution concepts have been constructed to deal with these difficulties. One – historically the first cooperative solution concept – is the von Neumann–Morgenstern stable set; it is studied, together with some of its applications to economic and political models, in Chapter 17. Chapter 18 covers the extensive literature dealing with another class of "core-like" solutions: the bargaining set and the related concepts of kernel and nucleolus.

Though Game Theory makes no ethical recommendations – is ethically neutral – game-theoretic ideas nevertheless do play a role in ethics. A fitting conclusion to this first volume is Chapter 19, which deals with the relation between Game Theory and ethics.

ROBERT J. AUMANN and SERGIU HART

List of Chapters Planned for all the Volumes[3]

Non-Cooperative

The game of chess (I, 1)
Games in extensive and strategic forms (I, 2)
Games of perfect information (I, 3)
Games of incomplete information
Two-player games
Conceptual foundations of strategic equilibrium
Strategic equilibrium
Correlated and communication equilibria
Stochastic games
Repeated games of complete information (I, 4)
Repeated games of incomplete information: zero-sum (I, 5)
Repeated games of incomplete information: non-zero-sum (I, 6)
Non-cooperative games with many players
Differential games
Economic applications of differential games
Non-cooperative models of bargaining (I, 7)
Bargaining with incomplete information
Oligopoly
Implementation
Auctions (I, 8)
Principal-agent models
Signalling
Search
Location (I, 9)
Entry and exit (I, 10)
Patent licensing (I, 11)
Biological games
International conflict

Cooperative

Taxonomy of cooperative games
Cooperative models of bargaining

[3]"I, *n*" means that this is chapter *n* of volume I.

General