On the co-existence of spot and contract markets: the delivery requirement as contract externality

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Summary

A contract between an upstream and a downstream party consists of a contract price and a delivery requirement. Contract formation entails an externality. It changes the probability distribution of the spot market price by removing high reservation price buyers and various sellers from the spot market. The first effect decreases the expected spot market price when the number of contracts is small, whereas the decrease in the number of sellers and additional residual contract demand increase the expected spot market price beyond a certain number of contracts. It implies an endogenous upper bound on the number of contracts. Contract prices are positively related to the number of contracts. Finally, additional contract formation reduces the variance of the spot market price when the number of contracts is sufficiently large.

Keywords: spot market, contract externality, co-existence, delivery requirement

JEL classification: L14, Q11, Q13

1. Introduction

Co-existence of various governance structures is common in many agricultural markets. First, the markets for corn, soybeans, wheat and cattle are characterised by the co-existence of spot and contract markets (Carriquiry and Babcock, 2004, unpublished data). Menard and Klein (2004: 752) reported that 'In France, over 80 per cent of the growers in the poultry industry operated under contracts in 1994. In the US pork industry, about 72 per cent of total hogs were sold through marketing contracts in 2001'.[1] Second, partial vertical integration regarding internal shipments to manufacturing establishments is a widespread phenomenon. McDonald (1985) reported incidence varying from 10 per cent in industries like tobacco, furniture and leather to more than 50 per cent in the transportation equipment and petroleum-refining industries. Finally, the co-existence of investor-owned firms and producer co-operatives

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1 This phenomenon is not limited to agricultural markets. Electricity markets are an example (Newbery, 1998).

can be found in agricultural markets throughout the world (Hendrikse, 1998; Sexton and Lavoie, 2001).

There are at least two reasons for analysing the co-existence of spot and contract markets. First, the empirical significance of the co-existence of various governance structures is obvious, but it is not well understood. For example, the models in the field of industrial organisation (Perry, 1989)[2] gene-rate extreme predictions: either all exchanges via contracts in the industry are privately optimal or there is no contracting at all. However, there are a few exceptions. Perry (1978) modelled a dominant manufacturer adopting a policy of price discrimination. This firm contracts only with the stages having more elastic derived demands in order to be able to charge the remaining stages higher prices. Carlton (1979a) modelled the assurance of supply argument. Contracts arise from a desire to avoid input rationing and to transfer risk from one sector of the economy to another. Firms contract partially in equilibrium in order to assure a sufficient probability of sale. Carlton (1979b) addressed the costs of the variability of spot market. Uncertainty and transaction costs create incentives for firms to use both markets for the sale of their output. Products exchanged via contracts sell at a lower price than those exchanged via the spot market, because the marginal cost of satis-fying contract demand is lower than that of spot demand. Both the spot and contract markets clear in equilibrium. Hendrikse and Peters (1989) established co-existence in a world characterised by rationing, differences in the reser-vation prices of buyers and differences in the risk attitudes of buyers and sellers. Finally, Xia and Sexton (2004) and Carriquiry and Babcock (2004) investigated various aspects of 'top-of-the-market' contract clauses in agricul-tural markets. Xia and Sexton (2004) analysed the impact of the adoption of these contracts on the intensity of competition in the market in a duopsony model, whereas Carriquiry and Babcock analysed the impact of many buyers and sellers on a market equilibrium characterised by co-existence.

Second, an increasing share of agricultural production is governed by con-tracts (Cook and Chaddad, 2000). This raises questions about the viability of spot markets, or the viability of co-existence of governance structures in general. The classic formulation regarding the choice of governance structure is the 'make or buy' question (Coase, 1937). This article is in line with the 'make or buy' question at the level of the enterprise, but this does not imply that only one governance structure prevails at the level of the industry.[3] It will establish that the co-existence of various governance structures is quite natural.

The co-existence of spot and contract markets raises a number of questions: What conditions determine the co-existence of these two markets? What

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2 Perry distinguished three broad determinants of contracting: technological economies, transac-tional economies and market imperfections. Market imperfections entail both imperfect compe-tition and imperfect or asymmetric information.

3 Single enterprises also adopt different forms of exchange, but this is outside the scope of this article. Examples are Pearce and Stacchetti (1998) and Baker *et al.* (1994), which highlight the inter-action between explicit and implicit contracts in a repeated relationship.

determines the size of the contract market? Which sellers and buyers sign contracts? What is the relationship between the extent of contracting and the contract price, the expected spot market price and the variance of the spot market price? A model consisting of heterogeneous buyers, stochastic supply and costs associated with spot market exchange is used to address these questions. The results are driven by the endogeneity of the probability distribution of the spot market price because contract formation entails an externality. It alters the composition of buyers and sellers in the spot market and therefore the probability distribution of the spot market price.[4]

 A contract consists of a delivery requirement and a contract–benefit parameter. In our model, both features are exogenous in order to focus on the impact of (real world) contracts on market structure. First, delivery requirements are a common feature of contracts between buyers and sellers. Nilsson (1998: 42) wrote regarding agricultural co-operatives: 'The delivery obligation for members is the dominating practice everywhere; in some countries it is even an obligation by law'. Cook and Tong (1997) identified as one of the main organisational characteristics of New Generation Cooperatives that each member has the right, but also the obligation, to deliver a specific quantity of the commodity each season. If the quantity delivered is lower than initially agreed in the delivery right, the co-operative has the right to buy the commodity on behalf of the producer and charge them for the difference in price. Cook and Iliopoulos (1999: 526) cited a co-operative expert stating, 'Farmers are required to deliver according to plan regardless of the open market'. Second, exchange costs differ between a contract and the spot market. For example, risk management is an important characteristic of contract markets compared to a spot transaction.[5] More generally, according to Bogetoft and Olesen (2004: 133–134), the practice of contracting indicates that contracts facilitate co-ordination, provide risk sharing, facilitate the use of local information, regulate the total quantity and ensure a high level of food safety through the tight control of inputs. The benefits of contracting are summarised by a parameter, reflecting the net benefit associated with contract exchange compared with spot market exchange.

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4 Aghion and Bolton (1987) and Bolton and Whinston (1993) also analysed a contract formation externality. Bolton and Whinston considered the choice of governance structure in a setting with one seller and two buyers, and uncertainty about the ability of the seller to deliver. The main focus was on supply assurance concerns when several downstream firms are competing for inputs in a limited supply. This article models many sellers and buyers, focusing on the relationship between the spot and contract markets. Carriquiry and Babcock (2004: 5) are closest to this article, but they had to use numerical techniques to overcome the inability to obtain analytic solutions for some cases. This article presents a model with an analytic solution.

5 Carlton (1979b) modelled the well-known supply assurance feature of contracts. He stated that '...real costs are associated with operating in a variable market. For example, one reason for contracts is that transaction costs of finding buyers on the spot market are eliminated by a long-term contract. This transaction cost of finding buyers is likely to depend on the variability of the spot price. (More variability implies more likely initial dispersion of prices which in turn implies more search)'.

The article is organised as follows. Section 2 presents the model. Section 3 states and proves the results. Section 4 formulates the comparative static results. Section 5 formulates conclusions and directions for further research.

2. The model

The co-existence of spot and contract markets is analysed with a model consisting of σ sellers and β buyers. Denote the set of sellers by $S = \{1, \ldots, \sigma\}$ and the set of buyers by $B = \{1, \ldots, \beta\}$. All sellers are assumed to be identical. Two possible states are distinguished for each seller i, $i \in$ S. Seller i has either one unit for sale or nothing at all. The probability that seller i does not have a unit for sale is μ. Define a σ-dimensional vector e such that $e_i = 0$ when seller i does not have a unit for sale and $e_i = 1$ otherwise. Define R_j as the reservation price of buyer j for the product, and assume that $R_1 > R_2 > \cdots > R_\beta$. Only one state is considered for each buyer j, i.e. buyer j always wants to buy one unit. (Notice that with this specification, no assumption is made with respect to the slope of the demand function.) Figure 1 presents these features of the market.

A market structure M is a partition of the set of buyers and sellers. The set of buyers and sellers in contracts is defined to be C_M. The alternative to a contract is to be in the spot market. The spot market price $p(M,e)$ is defined to clear the market, i.e. the spot market price allocates the available goods in the spot market to the buyers with the highest reservation price. The level of the market clearing price may be anywhere between the reservation price of the marginal buyer and the highest reservation price of the buyers who do not get a unit. The price formation in markets is the result of a bargaining process, where the price rule $p(M,e)$ summarises the distribution of bargaining power in the spot market. Vincent (1992) showed with a number of simple many-person bargaining games that many subtleties are involved in the modelling of competitive forces in a strategic setting. The equilibrium price turns out to be quite sensitive to the model and to the equilibrium concept used. However, in our model, the most demanding equilibrium requirements point towards the buyers having all the bargaining power. We assume, therefore, that the spot market price $p(M,e)$ is equal to the reservation price of the buyer with the highest reservation price among buyers who do not get a unit of the product. If supply is equal to, or larger than, demand, then the spot market price is equal to zero.
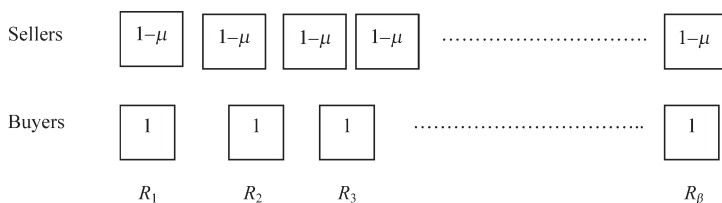

Figure 1. Sellers and buyers.

A contract between a buyer and a seller is a binding agreement, i.e. each party abides by the rules of the contract. Contracts are allowed only between one buyer and one seller. This captures the belief that there are limits to the extent of contracting. The assumption of efficient rationing (Tirole, 1988) is used to break ties in situations where the demand for contracts exceeds supply. The contract has to address all possible states of the economic environment. We adopt the following rules for a contract between seller i and buyer j: (i) if seller i has a unit for sale, then this unit has to be delivered to buyer j, (ii) buyer j receives a contract–benefit α when seller i delivers the unit, (iii) if seller i does not have a unit for sale, then buyer j will try to buy a unit from another seller not in a contract and (iv) seller i receives a contract price c_i, regardless of the ex-post realisation of e.[6] The vector of contract prices is defined to be c.

The contract–benefit, assumed to be exogenous and identical across firms, is defined to be α, and summarises the difference between a contract and the spot market mode of exchange to deal with co-ordination and incentive problems. Higher values of α imply that contract exchange is more attractive than spot market exchange. The expected payoff of seller i in a market structure M is denoted as:

$$s_i(M) = \begin{cases} c_i, & i \in C_M \\ (1 - \mu)E\{p(M, e) | \ e_i = 1\}, & i \notin C_M. \end{cases}$$

Define the one-to-one correspondence φ_M: $C_M \cap S \to C_M \cap B$, i.e. each seller in a contract is assigned a particular buyer. The expected payoff of buyer j in a market structure M is

$$b_j(M) = \begin{cases} (1 - \mu)(R_j + \alpha) - c_i + \mu E\{ \max\{0, R_j - p(M, e)\} | e_i = 0\}, & j = \varphi_M(i), \ j \in C_M \\ E\{\max\{0, R_j - p(M, e)\}\}, & j \notin C_M. \end{cases}$$

The sequence of decisions is presented in Figure 2. All sellers are assumed to choose a contract price simultaneously in a non-co-operative way. Subsequently, buyers decide simultaneously which contract to be accepted (or to be in the spot market), given the vector of contract prices c. Third, nature chooses for each seller independently the value of e_i. Finally, trade takes place and the spot market price is determined to clear the market. An outcome (c, M) is defined to be an equilibrium when it is a Nash equilibrium

6 This contract rule implies that payments are not contingent on the state. This seems to be a natural way of modelling vertical integration when we think of it as the buyer making a bid for the seller. However, contract rules could easily be changed into state-contingent payments. For example, the payment c_i could be made contingent on the seller having a unit for sale by multiplying by $1/(1-\mu)$. Although this scheme is harder to accept than the interpretation used here, it does not change the qualitative nature of our results. Notice that the specification is not at all close to a complete contingent contract. Bajari and Tadelis (2001) provided evidence and explanations why these contracts are not observed in many sectors.
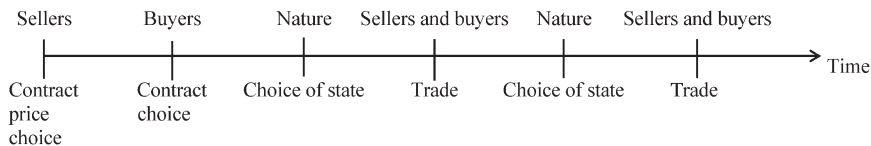
Figure 2. Decision sequence.

of the above game. We refer to the M of this equilibrium as an equilibrium market structure.

3. Equilibrium

The equilibrium features of the model will be formulated in this section. Theorem 1 establishes that the buyers with the high reservation prices are in contracts.

Theorem 1   *Suppose $R_j > R_{j+1}$. If buyer $j + 1$ is in a contract in equilibrium, then buyer j is as well.*

The proof is formulated in the Appendix. It boils down to Nash equilibrium requiring that each player chooses a payoff-maximising strategy. Theorem 2 establishes that the equilibrium contract prices are identical and equal to the equilibrium payoff of a seller in the spot market.

Theorem 2   *If (c,M) is an equilibrium, then $c_i = s_l(M)$, where i, $l \in S$, $i \in C_M$ and $l \notin C_M$.*

Proof   Assume that the set C_M of equilibrium outcome (c, M) consists of k buyers and k sellers. Theorem 1 has shown that the buyers with $R_1, \ldots, R_k$ will be in C_M. Define the ranking of contract prices of the sellers such that $c(1) \leq c(2) \leq \cdots \leq c(\sigma)$. Observe that $c_i < s_l(M)$ cannot be a part of an equilibrium outcome, because a contract price $s_l(M)$ is strictly preferred by seller i. A contract price $c_i > s_l(M)$ also cannot be a part of an equilibrium outcome, because an offer $s_l = s_l(M) + [c_i - s_l(M)]/2$ strictly improves the expected payoff of seller j and upsets (c, M) as an equilibrium outcome. The proof is completed by observing that no player can strictly improve his expected payoff by changing his contract price or mode of exchange when $c_i = s_l(M)$.

Define $\alpha(k)$ as the minimum value α for which adopting the k-th contract is advantageous for buyer R_k, given that the $k-1$ buyers with the highest reservation prices are in contracts. The Appendix provides the proof of the following lemma:

$$0 = \alpha(1) = \alpha(2) < \alpha(3) < \cdots < \alpha(N).$$

The difference $\alpha - \alpha(j)$ reflects the reservation price of buyer j for having a contract, i.e. $\alpha - \alpha(j)$ is the demand curve for contracts.

Theorem 3 establishes the unique equilibrium market structure for every value of α.

Theorem 3 *If $\alpha(k) \leq \alpha < \alpha(k+1)$, then the unique equilibrium market structure consists of k contracts, involving the k buyers with the highest reservation prices.*

This result follows immediately from Theorems 1 and 2, and the definition of $\alpha(k)$. It shows that the co-existence of the spot and contract markets in our model is driven by demand heterogeneity and some kind of market failure (i.e. α). There are no contracts when $\alpha < 0$. The lemma specifies the demand curve for contracts, whereas the supply curve of contracts is perfectly elastic in our set-up because there are no costs involved in starting or carrying out contract exchange. Figure 3 shows the demand and supply curve for contracts.

Notice that the above result is independent of the reservation prices of the buyers. It is robust with respect to any demand schedule. The spot market offers the high reservation price buyers an additional opportunity to satisfy their unfulfilled demand. The role of the low reservation price buyers is to support the existence of the spot market. These buyers are not able to compensate the remaining sellers in the spot market for giving up the option of supplying high reservation value buyers in states of low overall supply. However, they survive because it is sufficient for them to get the product at least once
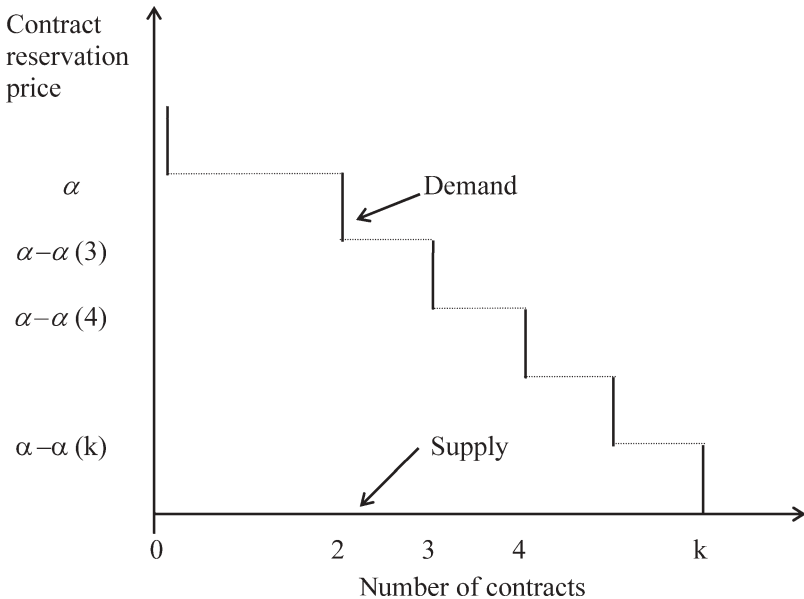

Figure 3. The contract market.

in a while. Sellers in the spot market earn the same profits as their counterparts in contracts because they have the opportunity of making an exchange with a high reservation price buyer, once in a while. Notice that our model has uncertainty on the supply side and differences between buyers. This is qualitatively equivalent to differences between sellers and uncertainty on the demand side.

We have taken the number of sellers to be equal to the number of buyers. This is a short-run situation. Suppose that the number of buyers is also fixed in the long run, but that the number of sellers is determined by market conditions. There are three kinds of sellers in the long run: (i) sellers with contracts having positive average output, (ii) sellers without contract having positive average output and (iii) sellers having zero average output and in the spot market (potential entrants). Buyers can contract with any seller who has not already done so. Suppose that sellers have to pay fee each period in order to participate in this market. The expected spot market price will be equal to this fee in the long-run equilibrium, because the zero profit condition of entrants determines the expected spot market price. Entry has a negative effect on the expected payoff of a seller. This will limit the extent of entry. A decrease in the expected payoff of sellers will increase the number of contracts, because more buyers are now able to afford a contract. The formation of additional contracts increases the expected payoff of a seller and will (partially) offset the decrease in the expected payoff due to entry. This will limit the entry less.

Observe that the equilibrium is only efficient when the contract–benefit parameter is negative or zero. If the contract–benefit parameter is positive, then there are at least two contracts. Situations will occur where buyer R_1 does not receive a product, while the seller in the second contract is producing. This is inefficient. However, this is inevitable due to the limits of contracting (Bajari and Tadelis, 2001). Another welfare observation is that mandatory contracting is not necessarily efficient. The advantage of mandatory contracting compared with an equilibrium characterised by the co-existence of spot and contract markets is that additional contract–benefits are generated, but surplus is lost due to the lack of a spot market for the high reservation price buyers.

4. Comparative statics

This section shows the relationship between the number of contracts, the expected payoff of a seller in the spot market, the expected spot market price and the variance of the spot market price. The proofs are provided in the Appendix. We also show numerical comparative static results with respect to the probability of having a unit for sale.

Theorem 4 *The equilibrium expected payoff of a seller in the spot market as a function of the extent of contracting k is constant when $k \in \{0,1,2\}$ and increases otherwise.*

The proof is cumbersome because the computation of the expected payoff of a seller in the spot market requires a probability distribution depending

on the number of contracts k. The Appendix section characterises the probability distribution of the spot market price as a function of the extent of contracting k from the viewpoint of a seller in the spot market, given that this seller is producing and the k buyers with the highest reservation prices have contracts. It is shown that this probability distribution is the same for $k = 0$, $k = 1$ and $k = 2$. Consider first the probability of spot market price R_2 in the cases $k = 0$ and $k = 1$. R_2 emerges only when all suppliers are not producing, except for the supplier being considered. This probability is μ^{N-1} when there are 0 or 1 contracts. The probability of R_2 continues to be μ^{N-1} when there are two contracts, because it emerges only when both contracts do not produce and one unit is available in the spot market. R_3's probability weight also does not change, because it does not matter from a combinatorial point of view whether the contract partner of the buyer with R_1, or the buyer with R_2 does not produce, or neither.

   If the number of contracts becomes larger than two, then the expected payoff of a seller in the spot market will increase. The reason is that buyers in contracts with relatively low reservation prices may receive a unit when a high reservation price buyer does not, because his contract partner does not produce and the available units in the spot market are so few that they are obtained by buyers with even higher reservation prices. More probability weight is, therefore, shifted to higher spot market prices when the number of contracts increases beyond two. This will drive up the spot market price and, therefore, the expected payoff of a seller in the spot market. When the number of contracts is larger than two, a positive relationship emerges between the expected payoff of a seller in the spot market and the number of contracts. Figure 4 illustrates Theorem 4.
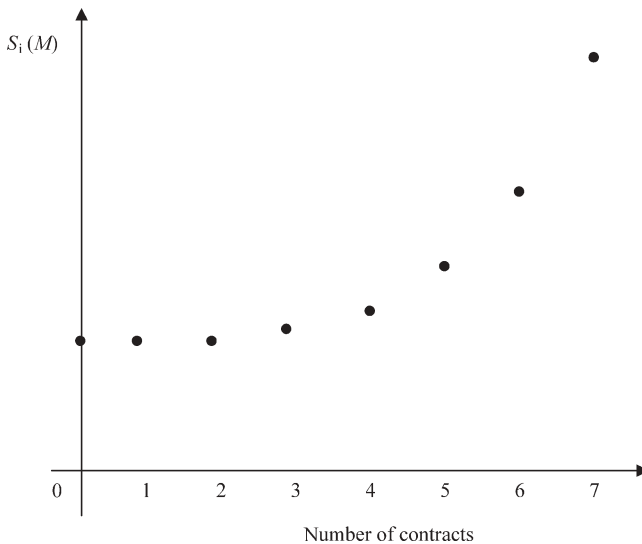

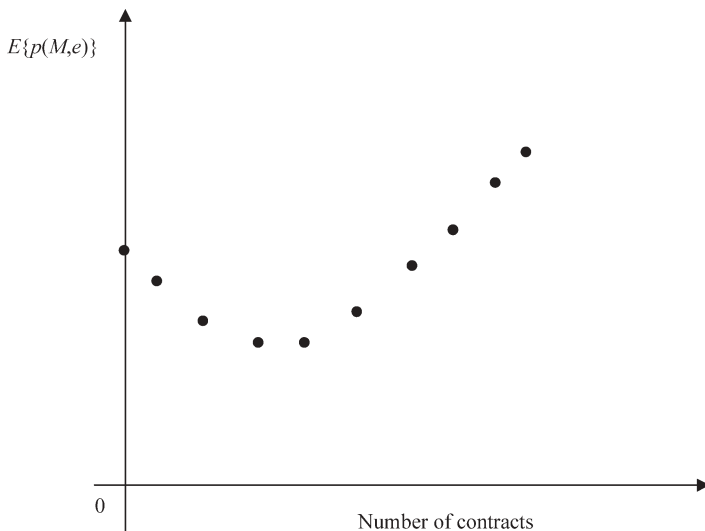**Figure 4.** The expected payoff of a seller in the spot market.

Figure 5. The expected spot market price.

Theorem 5   *The expected spot market price decreases as a function of α when α is small and increases for sufficiently large values of α, given that the market is large enough.*

The intuition for the first part is that the formation of the first contract implies that the buyer with reservation price R_1 is not in the spot market when his contract partner is producing. The probability that the spot market price is R_2 is therefore reduced. All other spot market prices continue to have the same probability weight. The formation of the second contract reinforces this effect. The probability of spot market price R_2 is further reduced, as is the probability of R_3. All other spot market prices continue to have the same probability weight.

The second part of the theorem requires that k and N are sufficiently large. This result can be made intuitive by considering the probability that the spot market price is R_2. This requires that the contract partners of the buyers with R_1 and R_2 are not delivering. The probability of this event is μ^2. R_2 emerges when exactly one seller is producing in the spot market. The probability of this event depends on the size of the spot market. It increases when the spot market becomes smaller. A similar argument holds for the probability weight of the other spot market prices.[7] The exact turning point depends on the reservation prices $R_2, \ldots, R_N$ and μ. Figure 5 depicts Theorem 5.

Three forces drive the result of Theorem 5. First, if the extent of contracting increases, then the supply of the product on the spot market will be lower on

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7 Table 1 provides an example in which the market is too small for the emergence of the second part of Theorem 5, whereas Table 2 shows a numerical example where the market is large enough.

average. This has an increasing effect on the expected spot market price. Second, contracting partially removes the buyers from the spot market. Buyers in contracts are only in the spot market when the upstream contract partner does not deliver. This has a decreasing effect on the expected spot market price. This second effect dominates when α is positive and close to zero (i.e. the extent of contracting is small), because the high reservation price buyers are in the market less often. Theorem 5 has provided the formal argument for why this is strictly so when a market with zero contracts is compared with one contract and one contract is compared with two contracts. Third, buyers in contracts will be in the spot market when the upstream contract partner does not deliver. Additional contracting entails that residual contract demand gains importance in the spot market. The increased use of the contracting mode of exchange consists of the buyers with the intermediate reservation prices. This implies that the probability of intermediate spot market prices decreases, i.e. the relative probability weight of high spot market prices increases. Additional residual contract demand therefore increases the expected spot market price. This third effect reverses the negative relationship between the spot market price and α, when α is above a certain level. However, the value of the parameters β, σ and μ may be such that the market is too small for the first and third effect to dominate the second. A numerical example with $\beta = \sigma = 2$ and $\mu = 0.5$ illustrates this market size feature in Table 1, where Var$\{p(M,e)\}$ is the variance of the spot market price $p(M,e)$.

Our probability distribution argument provides an endogenous bound on the extent of contracting in an industry. Additional contracting increases the expected spot market price and therefore reinforces the upper bound. The expected spot market price increases as a function of the extent of contracting, because the reduction in supply dominates the reduction in spot market demand. However, an increase in α, when α is either small or large, decreases the expected spot market price. The demand effect dominates the supply effect when α is small, whereas the probability that at least one unit is available in the spot market is responsible for a declining expected spot market price when α is large.

Notice the difference between Theorems 4 (Figure 3) and 5 (Figure 4). The focus of Theorem 4 is on a seller, whereas Theorem 5 represents the market point of view. The probability distribution of the spot market price associated with the first perspective is not the same as the probability distribution of the

Table 1. The impact of contract formation in a market with two buyers and two sellers, and $\mu = 0.5$

j	R_j	K	$E\{p(M,e)\}$	Var$\{p(M,e)\}$
1	2	0	0.6667	0.2222
2	1	1	0.5000	0.2500

spot market price seen from a market perspective. These probability distributions are identical only when the equilibrium consists of $N-1$ contracts.

The expressions for the expected value and the variance turn out to be analytically too cumbersome to derive additional comparative static results. The next theorem therefore presents the results from numerical analyses.

Theorem 6 *The variance of the spot market price declines, subsequently increases and finally decreases as a function of* α*, given that the market is large enough.*

Three forces determine the comparative static impact of an increasing contract–benefit parameter on the variance: reduction of high reservation price spot market demand, size of the spot market and reduction of spot market supply. An increase in α from zero to a positive number induces contracting by the high reservation price buyers. They will compete less often in the spot market against each other and therefore high spot market prices will occur less frequently; the buyers in the spot market become more similar on average. This implies a lower variance of the spot market price. An additional increase in the contract–benefit parameter will also drive buyers with intermediate reservation prices to the contracting mode of exchange. This additional reduction of the spot market supply restores the relative probability weight on high spot market prices: residual contract demand gains importance compared with the spot market demand from the non-integrated buyers. Finally, the reduction in the spot market supply determines the downward pattern in the variance when the level of the contract–benefit induces a large extent of contracting. The spot market supply is such that at a low level only the high reservation price residual contract demand can be satisfied: buyers actually getting the product are more similar on average. Spot market prices will almost always be high, which implies a low variance.

Figure 6 illustrates Theorems 5 and 6. Suppose that we have a linear reservation price schedule. The curve D_k represents the expected spot market demand, when there are k contracts. If $p(M,e) > R_{k+1}$, then D_k is a fraction $1 - \mu$ of D_0. If $p(M,e) < R_{k+1}$, then D_{k+1} is to the left of D_k, because the expected spot market demand of the firm with reservation price R_{k+1} is μ, whereas it is 1 when there are only k contracts. The expected spot market demand decreases when there are more contracts. This has a decreasing effect on the expected spot market price. However, the expected spot market supply also decreases. This is represented by the vertical lines S_k and S_{k+1}. [The expected spot market supply is $(1 - \mu)(N - k)$ when there are k contracts.] This has an increasing effect on the expected spot market price. If the number of contracts is small, then the first effect dominates. Otherwise, the second effect will dominate. As shown in Figure 6, the expected demand to the left of point A is not affected by the formation of more than k contracts, whereas the expected supply curve is. If the number of contracts is large, then the variance of the spot market price will increase with further
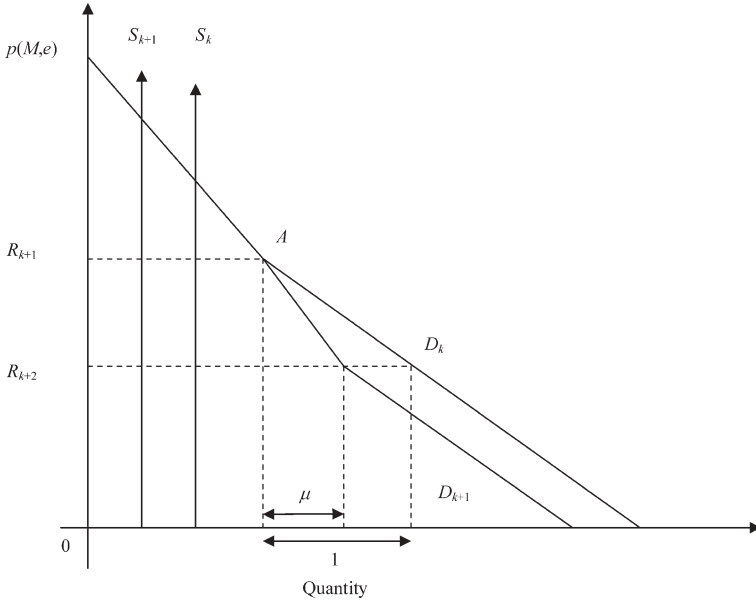
Figure 6. The spot market.

contract formation. This is caused by a decrease in the expected spot market supply, whereas demand remains almost the same.

Although contracts are not formed in our model to decrease price variability (Carlton 1979b), it should be noted that Theorem 6 would reinforce the co-existence result. The incorporation of the variance of the spot market price in the contracting benefit entails that α depends on $\text{Var}\{p(M,e)\}$. Carlton's observation implies that α is positively related to $\text{Var}\{p(M,e)\}$. The values of $\alpha(k)$, $k = 0, \ldots, N-1$ will not change when considerations regarding $\text{Var}\{p(M,e)\}$ play a role in the contracting decision. It reinforces the result of Theorem 5 that there is an endogenous upper bound on the extent of contracting, because the value of α will decrease as a result of contracting when many market participants have already integrated.

A numerical example with $\mu = 0.5$ is summarised in Table 2. The value of the exogenous parameter α does not show up in this table, because there is a one-to-one correspondence between α and the equilibrium number of contracts k, i.e. there are k contracts when $\alpha(k) \leq \alpha < \alpha(k+1)$. The column with the value of k could therefore be replaced by a column specifying intervals for α, where the endpoints are $\alpha(k)$ and $\alpha(k+1)$.

Some comparative static results regarding μ are straightforward and not explicitly stated in the form of theorems. First, the extent of contracting at which the variance of the spot market price attains its maximum declines when μ is increased. Notice that it is not claimed that the maximum is reached at $\mu = 0.5$. This might seem strange because the (stochastic) event that a seller has a unit for sale has a variance of $\mu(1 - \mu)$. However, the

Table 2. A numerical example

j	R_j	K	$E\{p(M,e)\}$	$Var\{p(M,e)\}$	$s_i(j,M)$
1	13	0	6.4992	3.2452	3.0000
2	12	1	6.4985	3.2419	3.0000
3	11	2	6.4973	3.2366	3.0000
4	10	3	6.4963	3.2402	3.0001
5	9	4	6.5022	3.3094	3.0017
6	8	5	6.5377	3.5819	3.0114
7	7	6	6.6507	4.2246	3.0479
8	6	7	6.9034	5.2356	3.1448
9	5	8	7.3373	6.2593	3.3384
10	4	9	7.9387	6.7292	3.6448
11	3	10	8.6390	6.3253	4.0479
12	2	11	9.3506	5.2348	4.5114
13	1	12	10.0034	3.9267	5.0017

variance of the spot market price consists of two parts: an endogenous component and an exogenous component. The exogenous uncertainty is captured by the probability that a seller has a unit for sale. The variance of this stochastic variable is $\mu(1 - \mu)$ for each seller. The effect of the endogenous component is described in Theorem 6. A larger value of μ implies that the buyer in a contract will be in the spot market more often and will outbid the permanent spot market buyers. This reduces the variance of the spot market price. A smaller extent of contracting will increase the variability of the spot market price. Second, if μ increases, then $E\{p(M,e)\}$ increases. The buyers with the highest reservation prices are the only ones to get a unit on the spot market, given that the spot market supply is low. If the probability of not having a unit for sale increases for every seller, then there is on average less for sale on the spot market. This tilts the distribution of buyer valuations in the spot market towards higher values in equilibrium and therefore a higher expected spot market price. Third, an increase in μ reduces the extent of contracting at which the minimum of $E\{p(M,e)\}$ is attained. The initial decrease in the expected spot market price when α is small and increased is because buyers with high reservation prices are partially removed from the spot market. An increase in μ has a stronger reverse effect on the expected spot market price, because spot market supply will be lower and residual contract demand will be higher on average. Table 2 shows that the lowest expected spot market price for $\mu = 0.5$ emerges (Figure 4) when the number of contracts is three. This minimum is reached at $k = 2$ when $\mu = 0.75$ and at $k = 4$ when $\mu = 0.1$.

5. Conclusions and further research

The co-existence of spot and contract markets is explained with a model of endogenous contract formation and endogenous uncertainty. The model

consists of heterogeneous buyers, stochastic supply and costs associated with spot market trade. Results have been established regarding the stability of the co-existence of the spot and contract markets, the composition and size of the spot market, the expected spot market price and the variance of the spot market price. It has been established that the co-existence of spot and contract markets is quite natural. Contracts arise due to the costs associated with a spot market, regardless of the heterogeneity of buyers and the uncertainty of supply. However, the extent of contracting is limited due to a contract formation externality.

Contract formation removes high reservation price buyers and various sellers from the spot market. The buyers with high reservation prices will be in contracts because they are able to compensate the seller for not being in the spot market. The role of the spot market is to provide these buyers with an additional opportunity to satisfy their unfulfilled demand. Not all firms can afford a contract. Buyers with a low reservation price are unable to compensate sellers for not being in the spot market when spot market demand is high. The role of these buyers is to support the existence of the spot market. These buyers can survive because it is sufficient for them to receive the product at least once in a while. Sellers in the spot market earn the same profit as their counterparts in a contract because they face a small probability of making an exchange with a high reservation price buyer. It entails an endogenous upper bound on the equilibrium number of contracts.

The comparative static analysis establishes that an increase in the benefit of contracting increases the number of contracts. Contract prices are positively related to the number of contracts. The impact on the expected spot market price depends on three economic forces: the change in the spot market supply, the change in the composition of buyers not vertically integrated and the change in residual contract demand in the spot market. It is shown that a switch from no contracting to some contracting will decrease the expected spot market price. The reduction in spot market demand due to con-tracting (second effect) dominates the reduction in supply effect (first effect), because the high reservation price buyers are active in the spot market less often. Additional increases in the contract–benefit parameter will increase the expected spot market price. Although high reservation price buyers will switch to the contracting mode of exchange, they will still sometimes be in the spot market when the contract partner is not able to deliver a unit of the product (third effect). This demand effect dominates the reduction in spot market supply effect beyond a certain level of the contract–benefit parameter. This result establishes that there is an endogenous upper bound on the number of contracts that can be formed.

The variance of the spot market price exhibits a similar pattern, except for high values of the contract–benefit parameter. An increase in the costs of spot market exchange takes a high reservation price buyer out of the spot market, given that the costs of spot market exchange are small. The remaining buyers in the spot market are more similar, which reduces the variance of the spot market price. Additional contracting reduces the differences between the

buyers in the spot market even further, but it increases the variety in the residual contract demand. This second effect dominates for intermediate values of the contract–benefit parameter and therefore increases the variance of the spot market price. Finally, a further increase in the contract–benefit parameter will reduce the variance when almost all exchange goes via contracts. Spot market supply is at such a low level that only the high reservation price buyers are able to buy a unit (at a high price) in the spot market when their contract partner is not delivering.

Various directions for future research are possible. First, all sellers are assumed to produce either one unit or nothing. Our qualitative results are not influenced by relaxing this assumption. However, the combinatorial difficulties associated with the calculation of the expected spot market price and the expected payoff of the market participants increase considerably. A similar remark holds for relaxing the assumption that sellers in contracts are not allowed to trade on the spot market. Qualitative results are robust with respect to this specification, because the contract–benefit would *ex post* prevent buyers with higher valuation from obtaining the product. Second, the contract–benefit α is specified as a constant parameter. The above results are robust when the contract–benefit parameter depends on the number of contracts. A more ambitious extension is to derive the value of this parameter endogenously. The current reduced form specification is chosen in order to focus on the market effects of contracts, as in Riordan and Williamson (1985).

Agricultural markets exhibit a rich variety of governance structures: spot markets, vertical integration, co-operatives, contract farming, networks and so on. A standard way of delineating a governance structure is to distinguish income and decision rights. Income rights specify the rights to receive benefits and the obligations to pay costs, whereas decision rights concern all rights and rules regarding a transaction (Hansmann, 1996).[8] This article can to be viewed as a contribution to the analysis of governance structure choice from an income rights perspective, with a focus on the co-existence of spot and contract markets. Three additional possibilities for future research are identified from a governance perspective. Third, the focus of the model has been on two income rights of the contract: the delivery requirement and the contract price. The delivery requirement of agricultural contracts is motivated by empirical observations, but a richer model has to motivate this contract clause on theoretical grounds (Schmidt and Schnitzer, 1995; Bajari and Tadelis, 2001).[9] Fourth,

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8 The analysis of income rights/incentives is the realm of complete contracting theory (Bolton and Dewatripont, 2005). It is assumed that everything that is known, can and will be incorporated into the design of optimal remuneration schemes/contracts without cost. Incomplete contracting theory addresses decision rights/authority. The starting point is that the design of contracts is costly, which results in incomplete contracts. Incomplete contracts allocate decision power in situations left open by formal (incentive) contracts. (Authority has no meaning in a complete contracting setting because everything is covered in the contract.)

9 Kvaloy (2006) provides references to efficiency explanations for the simplicity of agricultural contracts.

governance structures such as co-operatives, contract farming, franchises, joint ventures, networks and so on are usually distinguished by decision rights (Menard, 2004). One way to extend the model is therefore to incorporate decision rights into the analysis (Hendrikse and Veerman, 2001). Fifth, exchange between sellers and buyers is usually characterised as a long-term relationship, i.e. exchange occurs repeatedly and many aspects of exchange are often not specified in a formal contract. The literature on relational contracts captures these two aspects of exchange. It integrates the income rights effects of repeated interaction in terms of rewarding (punishing) good (bad) behaviour with the decision right effects of various governance structures in terms of different outside options (Baker *et al.*, 2002).[10]

Finally, empirical analyses have to show the relevance of the above model. An attractive feature of our model is that only a limited number of parameters (β, σ, μ and α) have to be determined as a result of robustness of the results with respect to the reservation prices of buyers and sellers. Empirical research has to determine the value of these parameters based on a model capturing more fundamental, structural and technological co-ordination and incentive features, as discussed in Crocker and Reynolds (1993). The European sugar industry is a promising case, because there is a considerable variation in the co-existence of co-operatives and investor-owned firms across countries.[11] Our model predicts that sugar prices will be higher in countries where co-operatives have a larger share of the market.[12] However, detailed empirical research is needed to determine whether the co-existence in these markets is driven by the features highlighted in this article.

Acknowledgements

10 Baker *et al.* (2002) compared the governance structures Relational Employment, i.e. repeated exchange in a contract, with repeated Relational Outsourcing, i.e. repeated exchange in a spot market (Kvaloy, 2006). These papers focus on a relationship consisting of one seller and one buyer.

11 A co-operative is viewed as a vertical relationship.

12 The model also applies most likely to markets outside the agricultural sector. An example is the oil tanker market. The case studies by Porter (1980) and Frankel *et al.* (1985) seem to support certain aspects of the above model. Porter observed very different approaches to doing business on the part of the main participants in the oil tanker industry (oil companies, independent ship owners, shipping brokers and the shipbuilding industry). Oil companies can be thought of as the high reservation price buyers in our model. They use continuous process operations, which are difficult as well as extremely costly to shut down. However, they do not rely completely on their own fleet, also using both chartered and spot vessels. The description of the independent ship owners is also not at odds with our model. Porter described ship owners as having a number of options in running their fleets, from operating all ships under long-term charter agreements to running their whole entire fleet on the spot market.

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Appendix

This section provides the proofs of Theorem 1, the lemma, Theorem 4 and Theorem 5. Without loss of generality, we use $\beta = \sigma = N$ in order to simplify the formulation of the proofs.

Proof of Theorem 1. Suppose, contrary to the proposition, that there exists an equilibrium market structure M in which buyer $j + 1$ is in a contract and buyer j is not. Denote an alternative market structure by M' that differs from M only by the fact that buyer $j + 1$ is not in a contract. By the assumption of a Nash equilibrium $M \geq_{j+1} M'$, for example, the market structure M is weakly preferred to market structure M' by buyer $j + 1$. Also define market structure M'' as the market structure that differs from M only by the fact that the buyer j is in a contract and the buyer $j + 1$ is not. Assuming a Nash

equilibrium in the acceptance stage implies $M \geq {}_jM''$. The efficient rationing assumption entails that buyer j can always decide to replace buyer $j+1$. This will be done, because buyer j strictly prefers $M'' > M$, while buyer $j+1$ weakly prefers $M > M'$. This is a contradiction and implies that M cannot be an equilibrium market structure. The conclusion is therefore that a contract is attractive for buyer j when it is attractive for buyer $j+1$.

Proof of the lemma. Buyer R_k faces a different probability distribution for the spot market price in the situation with a contract partner not delivering than in the situation without a contract. Define g_k as the expected payoff of buyer R_k when his contract partner is not delivering and h_k as the expected payoff of buyer R_k without a contract, given that buyers $R_1, \ldots, R_{k-1}$ have contracts. The reservation contract price $c_k(k-1)$ of buyer R_k when the buyers $R_1, \ldots, R_{k-1}$ have contracts is determined by

$$(1 - \mu)(R_k + \alpha) + \mu g_k - c_k(k-1) = h_k$$
$$\Leftrightarrow R_k + \alpha - c_k(k-1)/(1-\mu) = (h_k - \mu \, g_k)/(1-\mu).$$

Expressions g_k and h_k are determined by the probability distribution of the spot market price. Define a $(N+1, N+1)$ matrix $M(k)$ such that the probability distribution of the spot market price when buyers j with $R_j \geq R_k$ are the only buyers in contracts is

$$\text{Pr}_k \begin{pmatrix} - \\ R_2 \\ R_3 \\ \cdot \\ \cdot \\ \cdot \\ R_N \\ 0 \end{pmatrix} = M(k) \begin{pmatrix} \mu^N \\ \mu^{N-1}(1-\mu) \\ \cdot \\ \cdot \\ \cdot \\ \mu(1-\mu)^{N-1} \\ (1-\mu)^N \end{pmatrix},$$

where '$-$' denotes that there are zero units available in the spot market. The sum of the elements of a particular column of $M(k)$ is again independent of k. If $k = 0$, then $M_{jj}(0) = \binom{N}{j-1}$, $j = 1(1)N+1$. All other elements of $M(0)$ are zero. The formation of the first contract increases the probability that the spot market does not exist, i.e. no units are produced by suppliers without contracts. Fewer suppliers in the spot market are responsible for this result. Some of the probability weight is shifted from R_2 to the event that the spot market does not exist. All other elements of $M(1)$ are the same as in $M(0)$. The formation of the k-th contract involves $M_{jj}(k)$, with $M_{jj}(k-1) < 0$ and $M_{ij}(k) - M_{ij}(k-1) > 0$ for $j = 2(1)K+1$. The combinatorial difference $M_{jj}(k-1) - M_{jj}(k)$ shifts completely to the event that the spot market does not exist when $M_{jj}(k-1) - M_{jj}(k) = M_{1j}(k) - M_{1j}(k-1)$.

Otherwise, the difference $M_{jj}(k-1) - M_{jj}(k) - (M_{1j}(k) - M_{1j}(k-1))$ is distributed over $M_{ij}(k)$, where $2 \leq i \leq j-1$. The probability that the spot market does not exist is μ^{N-k} when there are k contracts, which is equal to $\mu^{N-k} \sum_{\ell=0}^{k} \binom{k}{\ell} \mu^{k-\ell}(1-\mu)^{\ell}$. The remaining part of the construction is identical to the one of $S(k)$. This procedure is formally captured by

$$M_{1j}(k) = \begin{cases} \binom{k}{j-1}, & 1 \leq j \leq k+1 \\ 0, & \text{otherwise} \end{cases}$$

$$M_{ij}(k) = \begin{cases} \left(\binom{N}{i-1} - \sum_{\ell=1}^{i-1} M_{\ell i}(k)\right)\binom{k-i}{j-i}, & i \geq 2, \ i \leq j \leq k \\ \binom{N}{i-1}, & i = j \geq k+1 \\ 0, & i \geq 2, \quad \text{otherwise.} \end{cases}$$

This completes the description of $M(k)$.

The size $(N+1, N+1)$ of matrix $M(k)$ has thus far been suppressed in order to simplify the notation. It turns out that this size is important in determining $(h_k - \mu g_k)/(1-\mu)$ and will therefore be made explicit by using the notation $M^{N+1}(k)$ and $\Pr_k^{N+1}(.)$ in the rest of this proof. The combinatorial features of h_k are completely described by $M^{N+1}(k-1)$. The definition of g_k implies that spot market price 0 will not occur due to the non-delivery of the contract partner of buyer k. Only $N-1$ sellers determine the probability weight of the spot market price in the expression for g_k, where the combinatorial aspects of the probability weight are described by $M^N(k-1)$.

The combinatorial aspects of the probabilities of the spot market prices in $(h_k - \mu g_k)/(1-\mu)$ are summarised in a (N,N) matrix $D^N(k)$. Observe that the weight attached to the event that no units are supplied in the spot market in the expression of h_k and μg_k is the same. The difference $h_k - \mu g_k$ has to be divided by $(1-\mu)$, which leaves

$$\Pr \, d_k \begin{pmatrix} R_2 \\ R_3 \\ \cdot \\ \cdot \\ \cdot \\ R_N \\ 0 \end{pmatrix} = D^N(k) \begin{pmatrix} \mu^{N-1} \\ \mu^{N-2}(1-\mu) \\ \cdot \\ \cdot \\ \cdot \\ \mu(1-\mu)^{N-2} \\ (1-\mu)^{N-1} \end{pmatrix}.$$

The elements of $D^N(k)$ are formally captured by

$$k = 1, 2 \quad D^N(k) = M^N(0)$$

$$k \geq 3 \quad D_{ij}^N(k)$$

$$= \begin{cases} M_{i+lj+1}^{N+1}(k-1) - M_{i+lj+1}^N(k-1), & 1 \leq i, \; j \leq N-1 \\ M_{i+lj+1}^{N+1}(k-1), & i = N \; \text{or} \; j = N. \end{cases}$$

We have therefore that

$$R_k + \alpha - c_k(k-1)/(1-\mu) = \sum_{l=k+1}^{N+1} \Pr \; d_k(R_l)(R_k - R_l)$$

$$\Leftrightarrow c_k(k-1)/(1-\mu) = \alpha + \sum_{l=2}^{k} \Pr \; d_k(R_l)R_k$$

$$+ \sum_{l=k+1}^{N+1} \Pr \; d_k + (R_l)R_l,$$

where $R_{N+1} \equiv 0$. Notice that $c_k(k-1)$ is negatively related to the value of k.

Define $c(k)$ as the expected payoff of a seller having a unit available in the spot market when there are k contracts. The definition of $\alpha(k)$ entails that buyer R_k can just afford a contract, i.e. the reservation contract price $c_k(k-1)$ is equal to $c(k)$. Using the expression for $c_k(k-1)$ results in

$$\alpha(k) = c(k)/(1-\mu) - \sum_{l=2}^{k} \Pr \; d_k(R_l)R_k - \sum_{l=k+1}^{N+1} \Pr \; d_k(R_l)R_l.$$

Theorem 2 states that $c(1) = c(2)$ and that $c(k)$ is an increasing function in k for $k \geq 3$. Similarly, it has been shown in this proof that $(h_1 - \mu g_1)/(1-\mu) = (h_2 - \mu g_2)/(1-\mu)$ and that $(h_k - \mu g_k)/(1-\mu)$ is decreasing in k for $k \geq 3$. It follows immediately from these observations that $\alpha(k)$ is increasing with k.

Finally, it is straightforward to calculate that $\alpha(1) = \alpha(2) = 0$ using the above expression for $\alpha(k)$. The explanation is that the move to the contract mode of exchange by buyer R_1 does not change the distribution of prices that have to be paid by this buyer when there are no contracts. Buyer R_1 acquires the product at the prevailing market price, or has to compensate his contract partner for not being in the spot market. The expected cost of acquiring a unit by buyer R_1 is the same for both modes of exchange when there is only one contract. However, the contract generates an additional benefit α for buyer R_1 when the contract partner delivers. The value of α at which vertical

integration is at least as attractive as spot market exchange is therefore zero. A similar argument applies to $\alpha(2)$. If there are two or more units produced by the sellers, then the distribution of prices is unaffected by the formation of the second contract. The only possibility for the value of $\alpha(2)$ to be different from $\alpha(1)$ is therefore the situation in which only one unit is available in the market. Buyer R_2 will not obtain this unit when it is either produced by the contract partner of buyer R_1 or by one of the sellers in the spot market. If the contract partner of R_2 is the only one producing, then it will be delivered in this contract. However, the contract partner has to be compensated for not being in the spot market, i.e. R_2 has to be paid. The surplus associated with this possibility for R_2 is therefore zero. The contract–benefit α has to be larger than zero in order to have a third contract. The reason is that buyer R_3 may receive a unit when either buyer R_1 or R_2 would have received it at spot market price R_2 and R_3, respectively, without this third contract. Prices R_2 and R_3 will occur less frequently in the spot market, because the contract partner of buyer R_3 has to be compensated for these opportunities. It does not matter for the expected payoff of buyer R_3 if spot market price R_3 occurs, but it does for buyer R_2. This results in a positive value $\alpha(3)$. The other inequalities in the lemma are explained in the same way. This completes the proof.

Proof of Theorem 4. Define an (N,N) matrix $S(k)$ such that the probability distribution of the spot market price faced from the viewpoint of a seller in the spot market when producing and the buyers $R_1, \ldots, R_k$ having contracts is

$$
\Pr \; s_k \begin{pmatrix} R_2 \\ R_3 \\ \cdot \\ \cdot \\ \cdot \\ R_N \\ 0 \end{pmatrix} = S(k) \begin{pmatrix} \mu^{N-1} \\ \mu^{N-2}(1-\mu) \\ \cdot \\ \cdot \\ \cdot \\ \mu(1-\mu)^{N-2} \\ (1-\mu)^{N-1} \end{pmatrix}.
$$

The element in the i-th row and j-th column of $S(k)$ is defined to be $S_{ij}(k)$. It is equal to the number of possibilities in which $j-1$ units can be produced by the other $N-1$ suppliers and generates a spot market price R_{i+1} when the buyers j with $R_j \geq R_k$ use the contracting mode of exchange. Notice that the sum of the elements of a particular column j of $S(k)$ reflects the number of ways in which $j-1$ units can be produced by $N-1$ suppliers. It does not depend on k. It will be shown how this number is distributed over $R_2, \ldots, R_j$ as a function of the number of contracts. This depends on k because the identities of the firms in contracts having to buy in the spot market matter for the level of the spot market price.

First, the determination of $S(k)$ will be done for the case $k = 0$. If there are no contracts, then the spot market price is completely determined by the

number of units that are produced. It does not matter which sellers are produc-
ing. The probability that the spot market price is R_i, $i = 2(1)N$ is equal to
$\binom{N-1}{i-2} \mu^{N-1-(i-2)}(1-\mu)^{i-2}$. All off-diagonal elements are therefore zero,
whereas the diagonal elements are $S_{ii}(0) = \binom{N-1}{i-1}$, $i = 1(1)N$.

$S(k)$ is computed from the viewpoint of a seller in the spot market having a
unit available. This perspective is responsible for the claim that
$S(2) = S(1) = S(0)$. The buyer with R_1 is in the first contract. $S(1)$ is always
identical to $S(0)$, because the buyer with R_1 will always get a unit when
there is one available in either the spot market or the contract. The number
of combinations at which a particular spot market price emerges does not
change. If there are two contracts, then the only way that R_2 emerges as
spot market price is that everybody else is broken down. R_3 clears the
market when one other unit is available. The number of combinations at
which this is realised does not depend on whether this one unit is produced
in the spot market or in a contract.

$S(k)$ changes with the formation of the third contract. The spot market price
is not only determined by the number of units produced, but also by which
sellers in contracts are not delivering. Suppose that only the supplier of
the third contract is producing and nobody else, except for the particular
supplier in the spot market we are considering. The market clearing price
will be R_2. The spot market price would be R_3 with fewer than three contracts,
because there are two units available in the spot market. So, some of the prob-
ability weight of R_3 is shifted to R_2. All other elements of $S(3)$ are the same as
in $S(2)$.

The probability weight of R_2 continues to increase when the number of con-
tracts is further expanded. It is equal to the probability that the first two con-
tracts do not produce and only the seller under consideration in the spot market
is producing. If there are k contracts, then this probability is equal to
$\mu^2 \times \mu^{N-k+1}$. It can be written as

$$\mu^{N-k+1} = \mu^{N-k+1} \sum_{\ell=0}^{k-2} \binom{k-2}{\ell} \mu^{k-2-\ell}(1-\mu)^{\ell}.$$

This determines the first $k-1$ elements of the first row of $S(k)$. All other
elements of the first row are zero. A similar combinatorial procedure is used
for the determination of all other rows of $S(k)$. The only difference is that
the (combinatorial) numbers of a particular row have to be multiplied by
the diagonal numbers of this row. This accounts for the number of combi-
nations that result in this market-clearing price. This number is already
obtained in the calculation of the previous rows, because the numbers of a par-
ticular column add up to a number that is independent of the number of
contracts.

This procedure is formally captured by

$$
k = 0, 1 \quad S_{ij}(k) = \begin{cases} \binom{N-1}{i-1}, & i = j \\ 0, & \text{otherwise} \end{cases}
$$

$$
k = 2 \quad S_{ij}(k) = \begin{cases} \binom{k-2}{j-1}, & 1 \le j \le k-1 \\ 0, & \text{otherwise} \end{cases}
$$

$$
S_{ij}(k) = \begin{cases} \left(\binom{N-1}{i-1} - \sum_{\ell=1}^{i-1} S_{\ell i}(k) \right) \binom{k-1-i}{j-1}, & i \ge 2, i \le j \le k-1 \\ \binom{N-1}{i-1}, & i = j \ge k \\ 0, & i \ge 2, \quad \text{otherwise.} \end{cases}
$$

This completes the description of $S(k)$. The equilibrium expected payoff of a seller in the spot market when there are k firms in contracts is equal to

$$
\sum_{i=2}^{N} \Pr \ s_k(R_i) R_i.
$$

The proof is completed by observing that $\Pr \ s_{k+1}(R_i) - \Pr \ s_k(R_i) = 0$ for every $k = 1, 2$ and $i \in \{2, \ldots, N\}$ and $\Pr \ s_{k+1}(R_i) - \Pr \ s_k(R_i) \ge 0$ for every $k \ge 3$ and $i \in \{2, \ldots, N\}$.

Proof of Theorem 5. The intuition for the first part is that the formation of the first contract implies that the buyer with the reservation price R_1 is not in the spot market when his contract partner is producing. The probability that the spot market price is R_2 is therefore reduced. All other spot market prices continue to have the same probability weight. It is therefore straightforward that

$$
E\{p| \ k = 0\} - E\{p| \ k = 1\}
$$
$$
= [M_{12}(0) - M_{12}(1)]\mu^{N-1}(1 - \mu)R_2
$$
$$
= \mu^{N-1}(1 - \mu)R_2 > 0.
$$

The formation of the second contract reinforces this effect. The probability of spot market price R_2 is further reduced and also the probability of R_3 is decreased. All other spot market prices continue to have the same probability

weight. Formally,

$$E\{p|\ k=1\} - E\{p|\ k=2\}$$
$$= [M_{12}(1) - M_{12}(2)]\mu^{N-1}(1-\mu)R_2$$
$$+ [M_{13}(1) - M_{13}(2)]\mu^{N-2}(1-\mu)^2 R_3$$
$$= \mu^{N-1}(1-\mu)R_2 + \mu^{N-2}(1-\mu)^2 R_3 > 0.$$

Theorem 6 is obtained from a numerical analysis. The matrices $S(k)$ and $M(k)$ regarding the equilibrium contract prices as a function of the extent of contracting and the expected spot market price have been programmed. The input consisted of numerical values of μ, $R_1, \ldots, R_\beta$ and σ. Table 1 serves as a first check of the numerical analysis.

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