

地面荷载下浅埋隧道围岩的粘弹性应力和变形分析

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摘要 采用解析法研究地面荷载下浅埋圆形隧道围岩的粘弹性应力和变形问题。利用对应性原理求得围岩应力和变形粘弹性解的 Laplace 变换,解表达为多个局部坐标中的级数之和,并由加法公式把它们变换为某个局部坐标中的双重级数,以满足边界条件。文末给出了围岩应力和变形的计算结果,讨论了地面荷载对围岩应力的影响。

关键词 地面荷载,浅埋隧道,围岩应力,解析解

分类号 TU451

1 引言

隧道围岩应力和变形分析是隧道设计的重要内容。深埋隧道的分析可忽略地表边界和地面局部荷载的影响,围岩视为无限岩土介质,对此类问题的研究已取得了不少的成果^[1,2]。对于城市地下工程中经常遇到的浅埋隧道的分析必须考虑地表边界和地面荷载影响,因此其围岩应力和变形分析比深埋隧道情况更为复杂和困难,相应的研究也比较少。浅埋隧道围岩为半无限多连通域,问题的求解需满足地表和洞周边界条件,一般只能采用数值方法^[3,4],通常计算量较大。解析法分析隧道围岩应力和变形目前仅限于深埋隧道,对于受地表边界和地面荷载影响的浅埋隧道围岩分析在数学处理上存在一定的困难。本文对地面荷载下浅埋圆形隧道围岩进行分析并给出了粘弹性应力和位移的解析解。文中把围岩的弹性应力和位移表示为多个局部坐标中的级数之和,并由加法公式把它们变换为某个局部坐标中的双重级数,以满足边界条件;由对应性原理和 Laplace 逆变换给出了粘弹性应力和位移解析解,问题归结为求解一组无穷型线性代数方程。文末给出了算例的应力和位移结果,并讨论了地面荷载对围岩应力的影响。

2 围岩的应力分析

本文讨论图 1 所示的浅埋圆形隧道在地面荷载下的围岩应力和位移分析问题。暂只考

1996年11月7日收到初稿,1997年1月19日收到修改稿。

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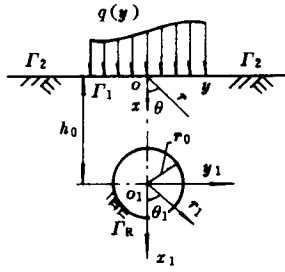


图1 地面荷载下的浅埋圆形隧道

Fig. 1 Shallow circular tunnel under ground load

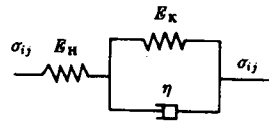


图2 Kelvin-Voigt 模型

Fig. 2 Kelvin-Voigt model

考虑毛洞情况, 对有支护情况拟于另文讨论。围岩视为 Kelvin-Voigt 粘弹性体(见图2), 假定泊松比 ν 不随时间变化^[5], 因此围岩平面应变基本方程的 Laplace 变换表示为

$$\left. \begin{aligned} \tilde{\sigma}_{ij,j} + \tilde{\gamma} &= 0, & \tilde{\varepsilon}_{ij} &= \frac{1}{2}(\tilde{w}_{i,j} + \tilde{w}_{j,i}) \\ \tilde{\varepsilon}_{ij} &= p \tilde{J}(p)(1 + \nu)(\tilde{\sigma}_{ij} - \nu \tilde{e} \delta_{ij}) \end{aligned} \right\} \quad (1)$$

式中: $\tilde{e} = \tilde{\sigma}_x + \tilde{\sigma}_y$, p 为 Laplace 变换参数, $\tilde{J}(p)$ 为柔度函数^[6]的 Laplace 变换:

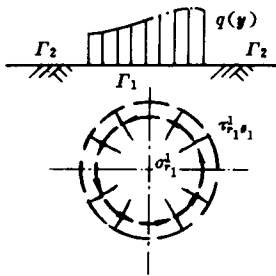
$$\tilde{J}(p) = (E_H + E_K + \eta p) / p E_H (E_K + \eta p) \quad (2)$$

边界条件表示为

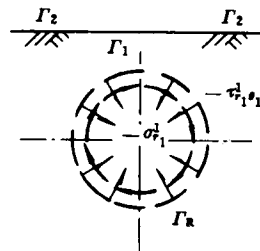
$$\tilde{\sigma}_z = \begin{cases} -\tilde{q}(y) & (x, y) \in \Gamma_1 \\ 0 & (x, y) \in \Gamma_2 \end{cases}, \quad \tilde{\tau}_{xy} = 0 \quad (x, y) \in \Gamma_1 + \Gamma_2 \quad (3a)$$

$$\tilde{\sigma}_n = \tilde{\sigma}_t = 0, \quad (x, y) \in \Gamma_R \quad (n, \tau - \Gamma_R \text{ 的法向和切向}) \quad (3b)$$

假定隧道开挖时地面荷载 $q(y)$ 及重力场的一次应力和位移已经稳定, 隧道开挖产生应力重分布后的二次围岩应力则由图3(a), 3(b)两个应力场叠加所得。



(a) 一次(初始)应力场($\sigma_{r1}^0 = \sigma_{\theta1}^0$)示意图



(b) 开挖附加应力场(σ_{r1}^e)示意图

图3 围岩的一次(初始)应力和开挖附加应力

Fig. 3 Primary and excavation induced stress field of surrounding rocks

由弹性理论的 Flamant 解与自重应力场的叠加可得 (r_1, θ_1) 坐标中的一次应力场:

$$\begin{Bmatrix} \sigma_{r_1}^1 \\ \sigma_{\theta_1}^1 \\ r_{r_1\theta_1}^1 \end{Bmatrix} = \gamma \begin{Bmatrix} f_r \\ f_\theta \\ f_{r\theta} \end{Bmatrix} - \frac{1}{2\pi} \sum_{m=0}^{\infty} \left\{ \begin{Bmatrix} A_m(r_1) \\ B_m(r_1) \\ C_m(r_1) \end{Bmatrix} \cos m\theta_1 + \begin{Bmatrix} D_m(r_1) \\ E_m(r_1) \\ F_m(r_1) \end{Bmatrix} \sin m\theta_1 \right\} \quad (4)$$

式中:

$$\left. \begin{aligned} f_r &= -\frac{1}{2} \left[(1+\lambda)h_0 + \frac{1}{2}(1+3\lambda)r_1\cos\theta_1 - (1-\lambda)h_0\cos 2\theta_1 - \frac{1}{2}(1-\lambda)r_1\cos 3\theta_1 \right] \\ f_\theta &= -\frac{1}{2} \left[(1+\lambda)h_0 + \frac{1}{2}(3+\lambda)r_1\cos\theta_1 + (1-\lambda)h_0\cos 2\theta_1 + \frac{1}{2}(1-\lambda)r_1\cos 3\theta_1 \right] \\ f_{r\theta} &= -\frac{1}{2}(1-\lambda) \left[\frac{1}{2}r_1\sin\theta_1 + h_0\sin 2\theta_1 + \frac{1}{2}r_1\sin 3\theta_1 \right], \quad \lambda = \nu/(1-\nu) \end{aligned} \right\} (5)$$

对于均布于地表 $[-b, b]$ 区域的荷载 q_0 , 系数 A_m, \dots, F_m 为

$$\left. \begin{aligned} C_m &= D_m = E_m = 0 \quad (m = 0, 1, 2, 3, \dots) \\ A_0 &= 2a_0 + b_2 - c_0, \quad A_1 = 2a_1 + b_1 + b_3 - c_0 - c_2 \\ B_0 &= 2a_0 - b_2 + c_2, \quad B_1 = 2a_1 - b_1 - b_3 + c_0 + c_2 \\ A_m &= 2a_m + b_{m+2} - c_{m+2} + b_{m-2} + c_{m-2} \quad (m \geq 2) \\ B_m &= 2a_m - b_{m+2} + c_{m+2} - b_{m-2} - c_{m-2} \quad (m \geq 2) \\ F_m &= -b_{m+2} + c_{m+2} + b_{m-2} + (1 - \delta_{2m})c_{m-2} \quad (m \geq 1), \quad F_0 = 0, \quad \delta_{2m} = \begin{cases} 1 & m = 2 \\ 0 & m \neq 2 \end{cases} \end{aligned} \right\} (6)$$

式中: a_m, b_m, c_m 的具体表达式由附录给出。

类似于电像法^[7]把图3(b)所示的开挖附加弹性应力场的应力函数取为图5所示域外各局部坐标 $(r_j, \theta_j) (j = 1, 2, 3)$ 中的应力函数之和, 即

$$\varphi = \sum_{j=1}^3 \varphi_j(r_j, \theta_j) = \sum_{j=1}^3 \sum_{m=1}^{\infty} [(A_m^j \cos m\theta_j + B_m^j \sin m\theta_j) r_j^{-m} + (C_m^j \cos m\theta_j + D_m^j \sin m\theta_j) r_j^{-m+2}] \quad (7)$$

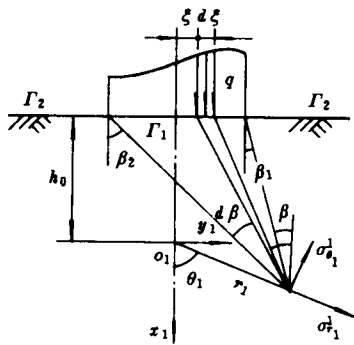


图4 地面荷载应力场叠加

Fig. 4 Superposition of stress field under ground load

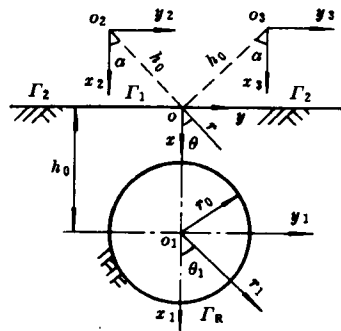


图5 多个局部坐标系

Fig. 5 Multiple local coordinate system

为利用边界条件确定上式中的常数 A_m^j, \dots, D_m^j , 由如下的加法公式(参考图6):

$$\left. \begin{aligned} \left\{ \begin{matrix} \cos m\theta_j \\ \sin m\theta_j \end{matrix} \right\} r_j^{-m} &= \sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{d_{j'}}\right)^m C_{m+n-1}^n \left\{ \begin{matrix} \cos(m+n)\theta_{j'} \cos n\theta_l + \sin(m+n)\theta_{j'} \sin n\theta_l \\ \sin(m+n)\theta_{j'} \cos n\theta_l - \cos(m+n)\theta_{j'} \sin n\theta_l \end{matrix} \right\} \rho_l^n \\ &\quad (\rho_l < 1) \\ \left\{ \begin{matrix} \cos m\theta_j \\ \sin m\theta_j \end{matrix} \right\} r_j^{-m} &= \sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{d_{j'}}\right)^m C_{m+n-1}^n \left\{ \begin{matrix} \cos n\theta_{j'} \cos(m+n)\theta_l + \sin n\theta_{j'} \sin(m+n)\theta_l \\ -\sin n\theta_{j'} \cos(m+n)\theta_l + \cos n\theta_{j'} \sin(m+n)\theta_l \end{matrix} \right\} \\ &\quad \cdot \rho_l^{-(m+n)} \quad (\rho_l > 1) \end{aligned} \right\} \quad (8)$$

和式(7)求得局部坐标 (r_1, θ_1) 中开挖附加弹性应力为

$$\left\{ \begin{matrix} \sigma_{r_1}^e \\ \sigma_{\theta_1}^e \\ \tau_{r_1\theta_1} \end{matrix} \right\} = \sum_{m=1}^{\infty} [m(m+1) \left\{ A_m^1 \left\{ \begin{matrix} -\cos m\theta_1 \\ \cos m\theta_1 \\ -\sin m\theta_1 \end{matrix} \right\} + B_m^1 \left\{ \begin{matrix} -\sin m\theta_1 \\ \sin m\theta_1 \\ \cos m\theta_1 \end{matrix} \right\} \right\} \frac{1}{r_1^{m+2}} + \\ (m-1) \left\{ C_m^1 \left\{ \begin{matrix} -(m+2)\cos m\theta_1 \\ (m-2)\cos m\theta_1 \\ -m\sin m\theta_1 \end{matrix} \right\} + D_m^1 \left\{ \begin{matrix} -(m+2)\sin m\theta_1 \\ (m-2)\sin m\theta_1 \\ m\cos m\theta_1 \end{matrix} \right\} \right\} \frac{1}{r_1^m}] + \\ \sum_{j=2}^3 \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{1}{h^2} [n(n-1) \left\{ A_{mn}^j \left\{ \begin{matrix} -\cos n\theta_1 \\ \cos n\theta_1 \\ \sin n\theta_1 \end{matrix} \right\} + B_{mn}^j \left\{ \begin{matrix} -\sin n\theta_1 \\ \sin n\theta_1 \\ -\cos n\theta_1 \end{matrix} \right\} \right\} r_1'^{n-2} + \\ (n+1) \left\{ C_{mn}^j \left\{ \begin{matrix} -(n-2)\cos n\theta_1 \\ (n+2)\cos n\theta_1 \\ n\sin n\theta_1 \end{matrix} \right\} + D_{mn}^j \left\{ \begin{matrix} -(n-2)\sin n\theta_1 \\ (n+2)\sin n\theta_1 \\ -n\cos n\theta_1 \end{matrix} \right\} \right\} r_1'^n] \quad (0 < r_1' = r_1/h < 1) \quad (9)$$

式中: $\rho_l = r_l/d_{j'}$, $C_{m+n-1}^n = \binom{m+n-1}{n}$, 且 $\binom{0}{0} = \binom{\alpha}{0} = 1 \quad (\alpha \neq 0)$,

$$\left\{ \begin{matrix} A_{mn}^j \\ B_{mn}^j \end{matrix} \right\} = (-1)^n h^{-n} C_{m+n-1}^n \left[\left(A_m^j + \frac{m-1}{m+n-1} h^2 C_m^j \pm \frac{m+n-1}{m} h^2 \delta_{1n} C_{m+2}^j \right) \cdot \right. \\ \left. \left\{ \begin{matrix} \cos(m+n)\theta_{j1} \\ \sin(m+n)\theta_{j1} \end{matrix} \right\} \pm \left(B_m^j + \frac{m-1}{m+n-1} h^2 D_m^j + \frac{m+n+1}{m} h^2 \delta_{1n} D_{m+2}^j \right) \left\{ \begin{matrix} \sin(m+n)\theta_{j1} \\ \cos(m+n)\theta_{j1} \end{matrix} \right\} \right] \quad (10a)$$

$$\left\{ \begin{matrix} C_{mn}^j \\ D_{mn}^j \end{matrix} \right\} = (-1)^n \frac{1-m}{n+1} h^{-m+2} C_{m+n-1}^n \left[C_m^j \left\{ \begin{matrix} \cos(m+n)\theta_{j1} \\ \sin(m+n)\theta_{j1} \end{matrix} \right\} \pm \right. \\ \left. D_m^j \left\{ \begin{matrix} \sin(m+n)\theta_{j1} \\ \cos(m+n)\theta_{j1} \end{matrix} \right\} \right] \quad (10b)$$

式中: $h = \sqrt{2(1 + \cos\alpha)}h_0$, $\theta_{j1} = \pm \tan^{-1} \frac{\sin\alpha}{1 + \cos\alpha}$ ($j=2$ 取正, $j=3$ 取负)。类似可求得 $r_1' > 1$ 情况及坐标 (r, θ) 中的开挖附加弹性应力表达式。利用边界条件(参考图 3(b) 情况)可得确定待定常数 A_m^j, \dots, D_m^j 的代数方程:

$$\left. \begin{aligned} & \sum_{j=1}^3 \sum_{m=1}^n [E_{kmn}^j A_m^j + F_{kmn}^j B_m^j + G_{kmn}^j C_m^j + H_{kmn}^j D_m^j] = 0 \quad (n = 1, 2, 3, \dots) \\ & \sum_{j=1}^3 \sum_{m=1}^n [I_{kmn}^j A_m^j + J_{kmn}^j B_m^j + K_{kmn}^j C_m^j + L_{kmn}^j D_m^j] = 0 \quad (n = 0, 1, 2, \dots) \\ & P_{kn} A_n^1 + Q_{kn} B_n^1 + R_{kn} C_n^1 + S_{kn} D_n^1 + \\ & \sum_{j=2}^3 \sum_{m=1}^n [P_{kmn}^j A_m^j + Q_{kmn}^j B_m^j + R_{kmn}^j C_m^j + S_{kmn}^j D_m^j] = E_{kn} \quad (n = 0, 1, 2, \dots) \end{aligned} \right\} \quad (11)$$

式中: $k = 1, 2, 3, 4$, 且

$$\left. \begin{aligned} E_{1n} &= \frac{\nu}{2} [(1 + \lambda)h_0\delta_{0n} + \frac{1}{2}(1 + 3\lambda)r_0\delta_{1n} - (1 - \lambda)h_0\delta_{2n} - \frac{1}{2}(1 - \lambda)r_0\delta_{3n}] + \frac{1}{2\pi}A_n(\tau_0) \\ E_{2n} &= \frac{1}{2\pi}D_n(\tau_0), \quad E_{4n} = \frac{1}{2\pi}C_n(\tau_0), \quad \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad (i, j = 0, 1, 2, \dots) \\ E_{3n} &= \frac{1}{2}(1 - \lambda) [\frac{1}{2}r_0\delta_{1n} + h_0\delta_{2n} + \frac{1}{2}r_0\delta_{3n}] + \frac{1}{2\pi}F_n(\tau_0) \end{aligned} \right\} \quad (12)$$

其余系数 E_{kmn}^j, \dots 由附录给出。从式(11)求得常数 A_m^j, \dots 后, 便可由式(4)和(9)的应力叠加得到围岩的二次弹性应力。由于二次弹性应力与弹性模量 E 无关, 由对应性原理^[6]和 Laplace 逆变换求得的围岩粘弹性应力与二次弹性应力相同。

3 围岩的变形分析

围岩位移也是由初始位移与隧道开挖引起的附加位移叠加所得。由于初始位移在隧道开挖前已形成, 对工程设计无实际价值, 因此围岩变形分析仅需确定隧道开挖引起的附加位移。由几何方程和物理方程可得隧道开挖引起的附加弹性位移为

$$\left. \begin{aligned} \begin{Bmatrix} u_{r_1}^* \\ u_{\theta_1}^* \end{Bmatrix} &= \frac{1 + \nu}{E} \left\{ \sum_{m=1}^{\infty} \left[\left(A_m^1 \begin{Bmatrix} \alpha_m^1 \cos m\theta_1 \\ \beta_m^1 \sin m\theta_1 \end{Bmatrix} + B_m^1 \begin{Bmatrix} \alpha_m^1 \sin m\theta_1 \\ -\beta_m^1 \cos m\theta_1 \end{Bmatrix} \right) \frac{1}{r_1^{m+1}} + \left(C_m^1 \begin{Bmatrix} \alpha_m^2 \cos m\theta_1 \\ \beta_m^2 \sin m\theta_1 \end{Bmatrix} + \right. \right. \\ D_m^1 \begin{Bmatrix} \alpha_m^2 \sin m\theta_1 \\ -\beta_m^2 \cos m\theta_1 \end{Bmatrix} \right) \frac{1}{r_1^{m-1}} \right] + \sum_{j=2}^3 \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{1}{h} \left[\left(A_{mn}^j \begin{Bmatrix} \alpha_{mn}^j \cos n\theta_1 \\ \beta_{mn}^j \sin n\theta_1 \end{Bmatrix} + B_{mn}^j \begin{Bmatrix} \alpha_{mn}^j \sin n\theta_1 \\ -\beta_{mn}^j \cos n\theta_1 \end{Bmatrix} \right) \cdot \right. \\ \left. r_1^{n-1} + \left(C_{mn}^j \begin{Bmatrix} \alpha_{mn}^2 \cos n\theta_1 \\ \beta_{mn}^2 \sin n\theta_1 \end{Bmatrix} + D_{mn}^j \begin{Bmatrix} \alpha_{mn}^2 \cos n\theta_1 \\ \beta_{mn}^2 \sin n\theta_1 \end{Bmatrix} \right) r_1^{n+1} \right] \right\} \quad (0 < r_1' < 1) \end{aligned} \right\} \quad (13)$$

$r_1' > 1$ 时, 附加弹性位移与上式类似。式中:

$$\left. \begin{aligned} \alpha_m^1 &= \beta_m^1 = m, \quad \alpha_m^2 = (1 - \delta_{1m})(m + 2 - 4\nu), \quad \alpha_{mn}^1 = (1 - \delta_{1n})n, \quad \alpha_{mn}^2 = 2 - n - 4\nu, \\ \beta_m^2 &= (m - 4 + 4\nu) + \frac{1}{m}(m + 2 - 4\nu)\delta_{1m}, \quad \beta_{mn}^1 = n - \delta_{1n}, \quad \beta_{mn}^2 = n + 4(1 - \nu) \end{aligned} \right\} \quad (14)$$

比较平面应变问题的弹性力学方程和粘弹性方程的 Laplace 变换(1)和(3), 根据对应性原理并由 Laplace 逆变换求得隧道开挖引起的附加粘弹性位移为

$$u_{r_1} = (1 + \nu)\Phi(t)\{\dots\}, \quad u_{\theta_1} = (1 + \nu)\Phi(t)\{\dots\} \quad (15)$$

上式中的“...”为式(13)中大括号内的内容, 且

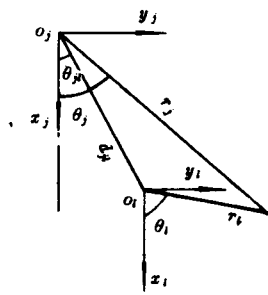


图6 坐标转换参考图
Fig. 6 Reference diagram of coordinate transformation

$$\Phi(t) = \frac{1}{E_H} \left[1 + \frac{E_H}{E_K} (1 - e^{-E_K t / \eta}) \right] \quad (16)$$

4 数值结果

对以上给出的解析解作了数值计算, 对于地面区域 $[-b, b]$ 内作用均布荷载情况给出了围岩应力和变形的数值结果。计算中所取参数 $q_0 = 100 \text{ kPa}$, $E_H = 10 \text{ GPa}$, $E_K = 2.5 \text{ GPa}$, $\eta = 3.6 \times 10^6 \text{ GPa} \cdot \text{s}$, $\nu = 0.25$, $\lambda = \frac{1}{3}$, $h_0 = 8.0 \text{ m}$, $r_0 = 1.5 \text{ m}$, $\gamma = 22 \text{ kN/m}^3$, 并取 $\alpha = 45^\circ$, $2b = 4.0 \text{ m}$, 16.0 m , 48.0 m 。从应力

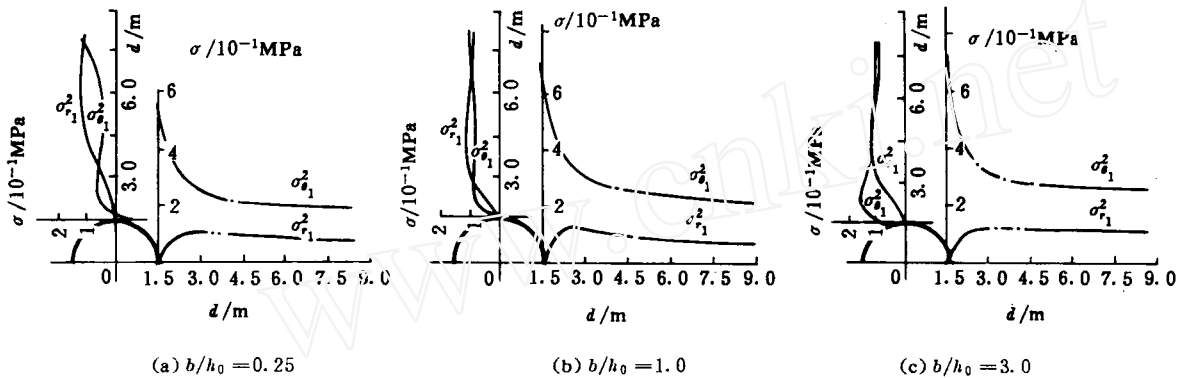
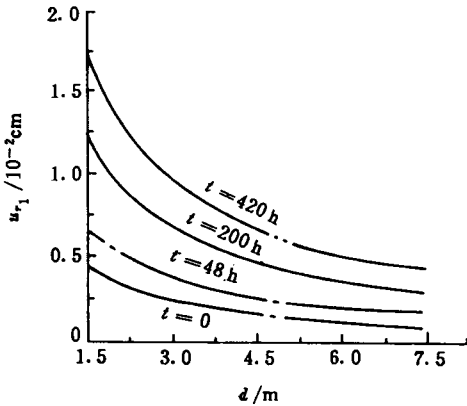


图7 围岩应力分布曲线

Fig. 7 Stress distribution of surrounding rocks



$b/h_0 = 1.0, \theta = \pi/2$

图8 围岩位移分布曲线
Fig. 8 Displacement distribution of surrounding rocks

结果图7可知围岩应力随 b/h_0 变化, 因此 b/h_0 可作为衡量地面荷载对围岩应力影响的一个参数。 b/h_0 较小时围岩垂线 ($\theta_1 = \pi$) 上应力分布较复杂, 且出现拉应力(图中以压应力为正); b/h_0 较大时应力分布与深埋隧道情况接近。水平线上应力值随 b/h_0 增大而增大, 但分布规律基本不变。由图8可知开挖附加位移一般较小, 这是由于浅埋隧道开挖释放的自重应力较小且地面荷载不大。

5 结语

(1) 本文由多个局部坐标解的叠加给出了浅埋隧道围岩粘弹性应力和位移解析解, 可考虑地面荷载和地表边界的影响, 与数值法相比计算量小而计算精度

高;分析结果可为围岩稳定分析及支护结构形式的选择和设计提供参考。

(2) 地面荷载宽度与隧道埋深的比值 b/h_0 可作为衡量地面荷载对围岩应力影响的参数; b/h_0 较小时, 围岩应力分布较复杂, b/h_0 较大时与深埋隧道情况接近。

(3) 地面荷载对围岩垂线上的应力影响较大。 $\lambda = 1/3$ 时, 深埋隧道围岩不出现切向拉应力; 浅埋隧道围岩受地面荷载影响, b/h_0 较小时却出现切向拉应力, 这不利于围岩的稳定。

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ANALYSIS OF VISCO-ELASTIC STRESS AND DEFORMATION OF SURROUNDING ROCKS FOR SHALLOW TUNNEL UNDER GROUND LOAD

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Abstract The visco-elastic analytic solution of stress and deformation is given for surrounding rock of shallow circular tunnel under ground load. The Laplace transformation of visco-elastic solution is obtained from corresponding elastic one with the aid of correspondence principle. The solution is first expressed as the sum of multiple solutions in each load coordinate and is then converted into the form of dual infinite series of a local coordinate system by addition formulas to satisfy boundary conditions. The numerical results of stress and displacement of surrounding rocks are presented and the influence of ground load on stress is discussed.

Key words ground load, shallow tunnel, stress of surrounding rocks, analytic solution

附 录

$$(E_{1m}^i, E_{2m}^i, E_{3m}^i, E_{4m}^i)^T = (F_{3m}^i, F_{4m}^i, -F_{2m}^i, -F_{1m}^i)^T = \alpha_{m}^1 \begin{Bmatrix} \cos[(n-m)\theta_{\mu} \mp \frac{n}{2}\pi] \\ \sin[(n-m)\theta_{\mu} \mp \frac{n}{2}\pi] \end{Bmatrix}$$

$$(G_{1m}^i, G_{2m}^i, -H_{1m}^i, -H_{2m}^i)^T = \frac{(1-m)\alpha_{m}^1 h_0^2}{n-m+1} \begin{Bmatrix} \cos[(n-m)\theta_{\mu} \mp \frac{n}{2}\pi] \\ \sin[(n-m)\theta_{\mu} \mp \frac{n}{2}\pi] \end{Bmatrix} + \frac{n}{n-m+2} \begin{Bmatrix} \cos[(n-m+2)\theta_{\mu} \mp \frac{n}{2}\pi] \\ \sin[(n-m+2)\theta_{\mu} \mp \frac{n}{2}\pi] \end{Bmatrix} + \begin{Bmatrix} \cos(\theta_{\mu} \pm \frac{n}{2}\pi) \\ \sin(\theta_{\mu} \mp \frac{n}{2}\pi) \end{Bmatrix}$$

$$(G_{3m}^i, G_{4m}^i, H_{3m}^i, H_{4m}^i)^T = \frac{(1-m)\alpha_{m}^1 h_0}{n-m+1} \begin{Bmatrix} \sin[(n-m)\theta_{\mu} \mp \frac{n}{2}\pi] \\ \cos[(n-m)\theta_{\mu} \mp \frac{n}{2}\pi] \end{Bmatrix} + \frac{n+2}{n-m+2} \begin{Bmatrix} \sin[(n-m+2)\theta_{\mu} \mp \frac{n}{2}\pi] \\ \cos[(n-m+2)\theta_{\mu} \mp \frac{n}{2}\pi] \end{Bmatrix} + \begin{Bmatrix} \sin(\theta_{\mu} \pm \frac{n}{2}\pi) \\ \sin(\theta_{\mu} \mp \frac{n}{2}\pi) \end{Bmatrix}$$

$$(I_{1m}^i, I_{2m}^i, J_{2m}^i, J_{1m}^i)^T = (n+2)(-J_{4m}^i, -J_{3m}^i, I_{3m}^i, I_{4m}^i)^T = \alpha_{m}^2 \begin{Bmatrix} \cos[(m+n+2)\theta_{\mu} \mp \frac{n}{2}\pi] \\ \sin[(m+n+2)\theta_{\mu} \pm \frac{n}{2}\pi] \end{Bmatrix}$$

$$(K_{1m}^i, K_{2m}^i, L_{1m}^i, L_{2m}^i)^T =$$

$$\frac{(m-1)\alpha_{m}^2 h_0^2}{n+m+1} \begin{Bmatrix} \cos[(m+n+2)\theta_{\mu} \pm \frac{n}{2}\pi] \\ \sin[(m+n+2)\theta_{\mu} \mp \frac{n}{2}\pi] \end{Bmatrix} + \frac{n}{n+m} \begin{Bmatrix} \cos[(m+n)\theta_{\mu} \mp \frac{n}{2}\pi] \\ \sin[(m+n)\theta_{\mu} \mp \frac{n}{2}\pi] \end{Bmatrix}$$

$$(K_{3m}^i, K_{4m}^i, L_{3m}^i, L_{4m}^i)^T =$$

$$\frac{(n+2)(m-1)\alpha_{m}^2 h_0}{n+m+1} \begin{Bmatrix} \sin[(m+n+2)\theta_{\mu} \pm \frac{n}{2}\pi] \\ \cos[(m+n+2)\theta_{\mu} \mp \frac{n}{2}\pi] \end{Bmatrix} - \frac{n(n+1)}{(m+n)^2} \begin{Bmatrix} \sin[(m+n)\theta_{\mu} \pm \frac{n}{2}\pi] \\ \cos[(m+n)\theta_{\mu} \pm \frac{n}{2}\pi] \end{Bmatrix}$$

$$(P_{1a}, P_{3a}, Q_{2a}, -Q_{4a})^T = -n(n+1)/\tau_0^{n+2}(1, 1, 1, 1)^T$$

$$(R_{1a}, S_{2a})^T = -(n-1)(n+2)/\tau_0^n(1, 1)^T$$

$$(S_{4a}, R_{3a})^T = n(n-1)/\tau_0^n(1, 1)^T, (P_{2a}, P_{4a}, Q_{1a}, Q_{3a}, R_{2a}, R_{4a}, S_{1a}, S_{3a})^T = 0$$

$$(P_{1m}^j, P_{3m}^j, P_{2m}^j, P_{4m}^j)^T = (Q_{4m}^j, Q_{2m}^j, Q_{1m}^j, Q_{3m}^j)^T = n(1-n)\rho_0^{n-2}\beta_{m\pm} \begin{cases} \pm \cos(m+n)\theta_{j1} \\ \pm \sin(m+n)\theta_{j1} \end{cases}$$

$$(R_{1m}^j, R_{2m}^j)^T = (-S_{2m}^j, S_{1m}^j)^T =$$

$$\frac{n(1-n)\rho_0^{n-2} + (n-2)(m+n+1)\rho_0^n}{m+n+1} (m-1)\beta_{m\pm} h^2 \begin{cases} \cos(m+n)\theta_{j1} \\ \sin(m+n)\theta_{j1} \end{cases}$$

$$(R_{3m}^j, R_{4m}^j)^T = (-S_{4m}^j, S_{3m}^j)^T =$$

$$\frac{(n-1)\rho_0^{n-2} - (m+n+1)\rho_0^n}{m+n-1} n(m-1)\beta_{m\pm} h^2 \begin{cases} \cos(m+n)\theta_{j1} \\ \sin(m+n)\theta_{j1} \end{cases}$$

$$\alpha_{m\pm} = (-1)^{n-m} h_0^{-m} C_{n-m}^{\pm}, \alpha_{m\pm}^2 = (-1)^n b_0^{-m} C_{n+m+1}^{\pm}, \beta_{m\pm} = (-1)^n h^{-m} C_{n+m-1}^{\pm}, \rho_0 = r_0/h$$

$$(\theta_{11}, \theta_{21}, \theta_{31}) = (\pi, \alpha, -\alpha), (\theta_{j1}, \theta_{j2}) = (\tan^{-1} \frac{\sin \alpha}{1 + \cos \alpha}, -\tan^{-1} \frac{\sin \alpha}{1 + \cos \alpha})$$

$$a_0 = 2q_0 \arctan \frac{b}{h_0 - r_1} - \frac{2q_0}{(h_0^2 + b^2)(1 - \rho_1^2)} \left\{ [(3h_0^2 + b^2)\rho_1^2 + (h_0 - b)^2] \arctan \frac{\rho_1 \sin \theta_0}{1 + \cos \theta_0} - \right.$$

$$\left. b h_0 (1 - \rho_1^2) \ln(1 + \rho_1^2 - 2\rho_1 \cos \theta_0) - 2h_0 b \rho_1^2 - 2b \rho_1 \sqrt{h_0^2 + b^2} \right\}$$

$$a_m = \frac{-2r_1 q_0}{(h_0^2 + b^2)(1 - \rho_1^2)} \left\{ b \delta_{1m} + (-1)^m \rho_1^n [2r_1 \sin m \theta_0 + h_0 \rho_1 \sin(m+1)\theta_0 + b \rho_1 \cos(m+1)\theta_0] \right\} \quad (m \geq 1)$$

$$b_m = \frac{-2r_1 q_0}{(h_0^2 + b^2)(1 - \rho_1^2)} \delta_m \left\{ [2h_0 b + \frac{1}{2} r_1^2 \rho_1^2 \sin 2\theta_0 - h_0 r_1 \rho_1 \sin \theta_0 - b r_1 \rho_1 \cos \theta_0] \delta_{0m} + \right.$$

$$\left. (b r_1 - \frac{1}{2} r_1^2 \rho_1^2 \sin \theta_0) \delta_{1m} + (-1)^m \rho_1^n [h_0 b \cos m \theta_0 - b r_1 \rho_1 \cos(m+1)\theta_0 - h_0 r_1 \rho_1 \sin(m+1)\theta_0 + \right.$$

$$\left. \frac{1}{2} r_1^2 \rho_1^2 \sin(m+2)\theta_0 \right\} \quad (m \geq 0)$$

$$c_m = \frac{q_0}{(h_0^2 + b^2)(1 - \rho_1^2)} \left\{ (r_1^2 \rho_1^2 \sin \theta_0 - 2b r_1) \delta_{1m} + (-1)^m \rho_1^n [2h_0 r_1 \rho_1 \sin(m+1)\theta_0 - 2b r_1 \rho_1 \cos(m+1)\theta_0 - r_1^2 \rho_1 \sin(m+2)\theta_0 - 2(h_0^2 + b^2) \sin m \theta_0] \right\} \quad (m \geq 0)$$

$$\delta_m = \begin{cases} 2 & m = 0 \\ 1 & m \neq 0 \end{cases}, \quad \delta_{m\pm} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \quad (m, n = 0, 1, 2)$$

$$\rho_1 = r_1 / \sqrt{h_0^2 + b^2}, \quad \theta_0 = \arctan \frac{b}{h_0}$$