Chinese J. Chem. Eng., **14**(5) 569—573 (2006)

Infeasibility Diagnosis on the Linear Programming Model of Production Planning in Refinery

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Abstract In order to effectively diagnose the infeasible linear programming (LP) model of production planning in refinery, the article proposed three stages strategy based on constraints' classification and infeasibility analysis. Generally, infeasibility sources involve structural inconsistencies and data errors, and the data errors are further classified into Ⅰ, Ⅱ and Ⅲ. The three stages strategy are: (1) Check data when they are inputted to detect data error Ⅰ and repair them; (2) Inspect data whether they are accorded with material balance before solving the LP model to identify data error Ⅱ and repair them; (3) Find irreducible inconsistent system of infeasible LP model and give diagnosis information priority-ranked to recognize data error Ⅲ and structural inconsistencies. These stages could be automatically executed by computer, and the approach has been applied to diagnose the infeasible model well in our graphic I/O petro-chemical industry modeling system.
Keywords diagnosis, infeasibility, inconsistent constraints, production planning

1 INTRODUCTION

The optimization of production planning is an important step in supply chain management of petrochemical industry. In general case, the mathematical models of production planning are linear and the optimization is linear programming (LP) problem. As the hardware and software have been very power in recent years, the solution of a very large LP problem has become routine. The bottleneck is no longer a run time on computer for solving the models of petrochemical industry, but the correct initial formulation of the model. Inadvertent errors are difficult to prevent, especially when integrating several smaller models into a larger one, or when modifying a complex model.

It is difficult to handle a large LP model which is infeasible. In order to repair the infeasible LP model, it is required to know where the problem is, but it is hard to determine which constraints are in conflict by simple inspection. Thus, some mathematical approaches have been developed to detect the inconsistent constraints(Chinneck and Dravnieks, 1991, Greenberg and Murphy,1991, Tamiz, *et al.*, 1996)[1—3]. Roodman[4] described how to eliminate an infeasibility when the phase Ⅰ LP terminates with some of the artificial variables having nonzero values. Greenberg *et al.*[5—9] introduced a set of heuristics which rely on tracing back through a series of manipulations of the model. Murty *et al.*[10,11] recommended how to use the phase Ⅰ LP solution to find a set of constraints which is preventing feasibility. Van Loon[12] presented a simplex variant and a set of necessary and sufficient conditions for recognition of a minimal infeasible set. Gleeson and Ryan[13] developed a complete localization algorithm. Leon and Liern[14] proposed the fuzzy method to repair infeasibility of LP. Timminga[15] and Huitzing[16] illustrated to analyze and solve infeasibility problems in test assembly models.

The approaches used in literatures to diagnose the infeasible LP model are in mathematics. However when we obtain the irreducibly inconsistent system (IIS) of the infeasible LP model, we still could not effectively repair the infeasible LP model by using the existing approaches, because the IIS has a number of constraints. Thus, we adopted other assistant ways based on the laws of different physical and chemical processes for further infeasibility diagnosis. The article proposed three stages strategy for infeasibility diagnosis based on constraints' classification and infeasibility analysis.

2 LP MODEL OF PRODUCTION PLANNING IN REFINERY

2.1 Objective function

The objective is to maximize the profit of refinery.

$$\max = \sum_{i=1}^{n} p_i s_i - \sum_{l=1}^{m}\sum_{i=1}^{k} q_l x_{i,l} - C \tag{1}$$

2.2 Constraints

Constraints are classified into different categories: material balance [Eq.(2)], production rate [Eq.(3)], flow rate bound [Eq.(4)], feedstock flow rate proportion [Eq.(5)], capacity [Eq.(6)], product required quality [Eq.(7)] and others. The classification is important for effectively diagnosing the model when it is infeasible.

Received 2005-08-04, accepted 2006-03-02.
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$$(1-\delta_l)\sum_{i=1}^{k} x_{i,l}=\sum_{j=1}^{h} y_{j,l} \tag{2}$$

$$y_{j,l} \geqslant K_{j,l}^{\mathrm{L}}\sum_{i=1}^{k} x_{i,l} \tag{3a}$$

$$y_{j,l} \leqslant K_{j,l}^{\mathrm{U}}\sum_{i=1}^{k} x_{i,l} \tag{3b}$$

$$\sum_{i=1}^{k} x_{i,l} \geqslant V_l^{\mathrm{L}} \tag{4a}$$

$$\sum_{i=1}^{k} x_{i,l} \leqslant V_l^{\mathrm{U}} \tag{4b}$$

$$x_{i,l} \geqslant \phi_{i,l}^{\mathrm{L}}\sum_{j=1}^{k} x_{j,l} \tag{5a}$$

$$x_{i,l} \leqslant \phi_{i,l}^{\mathrm{U}}\sum_{j=1}^{k} x_{j,l} \tag{5b}$$

$$\sum_{i=1}^{k} \omega_{i,l}x_{i,l} \geqslant \omega'_{j,l}y_{j,l} \tag{6}$$

$$s_i \geqslant F_i^{\mathrm{L}} \tag{7a}$$

$$s_i \leqslant F_i^{\mathrm{U}} \tag{7b}$$

The model with formulations (1)—(7) is automatically generated by our graphic I/O petro-chemical industry modeling system (GIOPIMS). For infeasibility diagnosis, we need to mark each constraint with three parameters. For example, we mark a constraint of formulation (2) with “A” to show the type of constraint is material balance, “1” to denote the constraint belongs to unit 1 and “0” to figure the constraint does not belong to any stream, written as [A, 1, 0].

3 INFEASIBILITY OF THE LP MODEL

Generally, infeasibility sources are partitioned into two categories: data errors and structural inconsistencies.

3.1 Data errors

The LP model of production planning in refinery refers to a great deal of data. These data perhaps contain errors when they are inputted. The errors can be classified into three types: Ⅰ, Ⅱ and III. Data error Ⅰ is that data disobeys its property such as a positive number has been written as a negative number and/or a couple of data only belonging to one unit whose upper and lower bound are conflicting. Data error Ⅱ is that a group data only belonging to one unit disobey the mass conservation.

Data error Ⅲ is that data of different units and streams are inconsistent. These data refer to production rate, capacity, feedstock flow rate proportion and flow rate bound. These errors are the bottleneck to diagnose the infeasible LP model of production planning in refinery.

3.2 Structural inconsistencies

Structural inconsistencies denote that some streams of a unit in the flow sheet do not exit really, which should destroy mass conservation. These errors less arise than data errors, and they are only considered if no data errors have been found during infeasibility diagnosis.

4 INFEASIBILITY DIAGNOSIS ON THE LP MODEL

Data error Ⅰ, Ⅱ, Ⅲ and structural inconsistencies are interacted for infeasibility diagnosis. Data error Ⅰ will have an impact on detecting data error Ⅱ, while data error Ⅱ will cause interference during identifying data error Ⅲ. As the same, data errors will disturb the identification of structural inconsistencies. Thus to identify the errors effectually, three stages strategy could be carried through the infeasibility diagnosis (Fig.1): (1) Check data when they are inputted to detect data error Ⅰ and repair them; (2) Inspect data whether they are accorded with material balance, if not, identify data error Ⅱ and repair them; (3) Find IIS of infeasible LP model and give diagnosis information priority-ranked to recognize data error Ⅲ and structural inconsistencies.

4.1 Stage one: Check inputted data to detect data error Ⅰ and repair them

Data error Ⅰ could be detected when data are

Table 1 Data error Ⅰ and Ⅱ in the LP model

Data error Ⅰ	Data error Ⅱ
$0 > K_{j,1}^{\mathrm{U}} > 1,\ 0 > \phi_{i,l}^{\mathrm{U}} > 1, 0 > \phi_{i,l}^{\mathrm{L}} > 1$	$\sum_{j=1}^{h} K_{j,l}^{\mathrm{L}} + \delta_l > 1,\quad \sum_{j=1}^{h} K_{j,l}^{\mathrm{U}} + \delta_l < 1$
$V_l^{\mathrm{U}} < 0,\ F_i^{\mathrm{U}} < 0$	
$K_{j,l}^{\mathrm{U}} < K_{j,l}^{\mathrm{L}}, \phi_{i,l}^{\mathrm{U}} < \phi_{i,l}^{\mathrm{L}}$	$\sum_{i=1}^{k} \phi_{i,l}^{\mathrm{L}} > 1,\quad \sum_{i=1}^{k} \phi_{i,l}^{\mathrm{U}} < 1$
$V_l^{\mathrm{U}} < V_l^{\mathrm{L}},\ F_i^{\mathrm{U}} < F_i^{\mathrm{L}}$	$\omega'_{j,l} > \max\{\omega_{i,l} \mid i = 1, 2, \cdots, k\}$

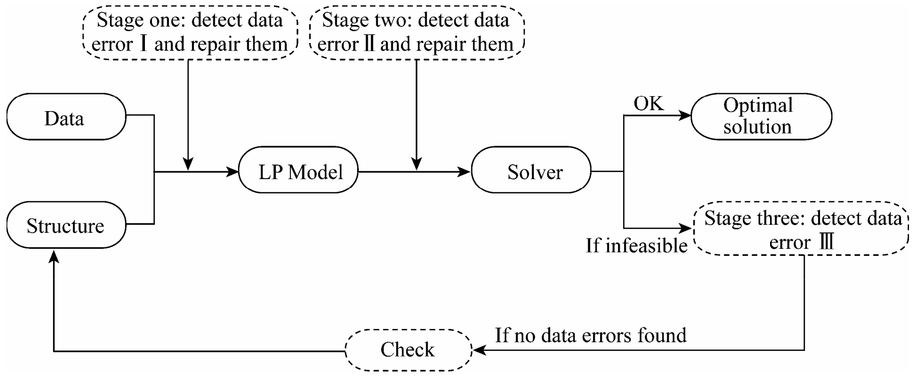

Figure 1 Three stages strategy for infeasibility diagnosis

inputted. This stage is simple to be completed through the parameter input interface-dialog box of our GIOPIMS (Fig.2). If some data contain error Ⅰ, we could identify the error data and repair them immediately.

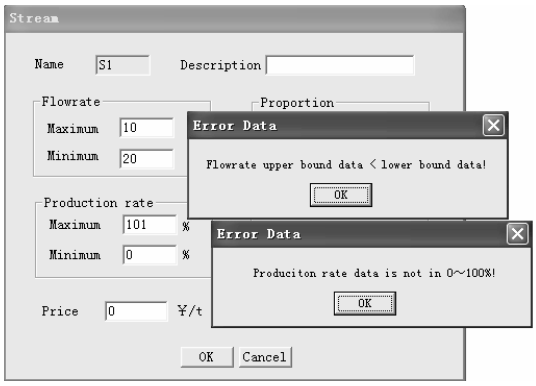

Figure 2 Stage one: Check inputted data to detect data error Ⅰ

4.2 Stage two: Identify data error Ⅱ and repair them

Data error Ⅱ could be detected by inspecting material balance before solving the LP model. Data without error Ⅰ still may disobey material balance. If data contain the error Ⅱ, we should give a good diagnosis immediately. For example, a LP model with a group data (Table 2) is infeasible, because its total product rate lower bound is more than 100%. The error data is detected before solving the LP model automatically, thus we repair the model at once.

Table 2 A group data with error Ⅱ of production rate in a FCC unit

Products	Rate lower bound, %	Rate upper bound, %
dry gas	5.0	5.0
liquid hydrocarbon	14.0	14.0
gasoline	43.5	43.5
diesel oil	21.4	21.4
slurry oil	7.0	7.0
coke	9.5	9.5
loss	0.6	0.6
total	101.0	101.0

4.3 Stage three: Recognize data error Ⅲ and structural inconsistencies

After the stage one and two have been accomplished well, if the LP model is still infeasible, we need to analyze the IIS obtained from the solver and identify the inconsistent constraints. The constraints in the IIS are further classified into two disjoint sets, namely the necessary set and the sufficient set. The sufficient set refers to a crucial subset of the IIS in the sense that removing any one of its members from the entire model renders the model feasible. Note that not all infeasible models have sufficient sets. The necessary set contains those constraints and bounds that are likely to contribute to the overall infeasibility of the entire model. Thus, the necessary set requires a correction in at least one member to make the original model feasible.

Sometimes the sufficient set and necessary set have large numbers of constraints, thus it is not easy to repair the infeasible LP model. Therefore, we further classify the sufficient and necessary sets into three disjoint subsets respectively according to the type of the constraints. Capacity and flow rate constraints are in subset 1, while production rate, proportion, product required quality and other constraints are in subset 2, and only material balance constraints are in subset 3, seeing Fig.3. The classification could be completed automatically through the mark parameters of each constraint.

The constraints in subset 1 most possibly contains error or unreasonable data, while the constraints in subset 2 have less opportunity to be inconsistent because the data in these constraints have been checked in the stage one and two , and the constraints in subset 3 have least chance to be inconsistent except the flow sheet covers structural inconsistencies. Based on the classification, we should firstly check the constraints in subset 1 and then subset 2 of sufficient set. If no error has been found, we should inspect the constraints in subset 1 and then subset 2 of necessary set, and if also no error has been detected, we should examine the structure of flow sheet through the constraints in subset 3 of the sufficient set and then necessary set, seeing Fig.3.

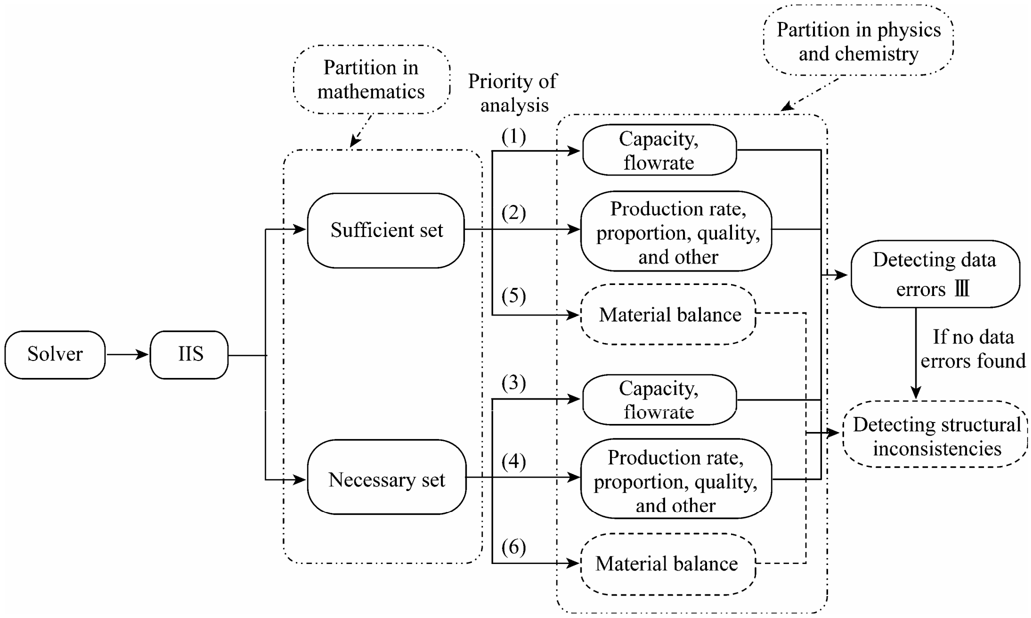

Figure 3 The classification of IIS

Table 3 Infeasibility diagnosis report using three stages strategy

No.	Constraints	Number	Type	Stream/Unit
	sufficient set	17		
(1)	capacity, flowrate	4		
	S406=36.00		flowrate	S406
	S358=0		flowrate	S358
	S344=27.00		flowrate	S344
	S298⩾100. 00		flowrate	S298
(2)	production rate, proportion, quality, other	0		
(5)	material balance	13		
	S2+S3−S405−S406+S419+S420+S431+S432=0		material balance	C165
	…			
	S67−S301−S336−S337+S358−next 8=0		material balance	T124
	necessary set	138		
(3)	capacity, flowrate	21		
	S559+S560+S561+S562+S563+S564=240.00		capacity	MD176
	…			
	S372+S386⩽6.00		capacity	D157
(4)	production rate, proportion, quality, other	101		
	−0.11×S305−0.11×S307−0.11×S310+0.89×S312=0		proportion	S312
	…			
	x85=0		other	
(6)	material balance	16		
	S345−S450−S451−S564−next 23=0		material balance	T148
	…			
	S105+S248−S282−S359−S360−next 1=0		material balance	T114

5 CASE STUDY

A refinery involving 30 primary units with 3 atmosphere and vacuum distillation columns and 3 FCC units processes 850×10^4 tons of crude oil per year and its primary products are gasoline, kerosene, diesel oil and lube. The LP model of production planning involves about one thousand constraints and two thousand variables. The optimal solution is facile to gain by solver, but it is more difficult how to repair the model without feasible region. For an infeasible LP model of production planning in the refinery, in the IIS obtained from the solver, involves 153 constraints with 17 ones in sufficient set and 138 ones in necessary set. It is not facile to analyze the infeasibility. Further classification of the sufficient and necessary set as described in Table 3. The sign (1), (2),(3),(4),(5) and (6) denote the priority of infeasibility. Constraints in subset (1) most possibly contain error or unreasonable data while constraints in subset (6) have least chance of error. Constraints in subset (1), (2), (3) and (4) are used to identify data errors Ⅲ, as constraints in section (5) and (6) are used to confirm the structural inconsistencies. By examining the 4 constraints in subset (1), we immediately identify the 4th constraint (S298⩾100.00) contains error data and repair the infeasible model.

6 CONCLUSIONS

The article described the LP model of production planning in refinery. The constraints in model are classified into several categories according to their type, and the classification is important for providing a good diagnosis when the model is infeasible. The infeasibility sources involve data errors and structural inconsistencies. The data errors are partitioned into three types: Ⅰ, Ⅱ and Ⅲ. Data error Ⅰ, Ⅱ, Ⅲ and structural inconsistencies are interacted for infeasibility diagnosis. Data error Ⅰ will have an impact on detecting data error Ⅱ, while data error Ⅱ will cause interference during identifying data errors Ⅲ. As the same, data errors will disturb the identification of structural inconsistencies.

Based on constraints' classification and infeasibility analysis, three stages strategy is proposed to diagnose the infeasible LP model. The three stages are: (1) Check data when they are input to detect data error Ⅰ and repair them; (2) Inspect data whether they are accorded with material balance before solving the LP model to identify data error Ⅱ and repair them; (3) Find IIS of infeasible LP model and give diagnosis information priority-ranked to recognize data error Ⅲ and structural inconsistencies. These stages could be executed automatically and a case study has been demonstrated to show the effectiveness of the proposed approach.

NOMENCLATURE

C	fixed costs
F	stream flow rate, $t \cdot h^{-1}$
h	total number of out streams of a unit
K	production rate
k	total number of feed streams of a unit
m	total number of units
n	total number of streams
p	price (product, p>0; raw material, p<0; else p=0)
q	operating cost per unit
s	stream flow rate, $t \cdot h^{-1}$
V	capacity of unit, $t \cdot h^{-1}$
x	feed stream flow rate, $t \cdot h^{-1}$
y	out stream flow rate, $t \cdot h^{-1}$
δ	loss proportion of unit
ϕ	feedstock flow rate proportion
ω	quality value of feedstock
ω'	quality required of product

Superscripts

L	lower bound
U	upper bound

Subscripts

i	stream index
j	stream index
l	unit index

REFERENCES

1 Chinneck, J.W., Dravnieks, E.W., "Locating minimal infeasible constraint sets in linear programs", *ORSA J. Comput.*, **3**(2), 157—168(1991).
2 Greenberg, H.J., Murphy, F.H., "Approaches to diagnosing infeasible linear programs", *ORSA J. Comput.*, **3**(3), 253—261(1991).
3 Tamiz, M., Mardle, S.J., Jones, D.F., "Detecting IIS in infeasible linear programmes using techniques from goal programming", *Comput. Oper. Res.*, **23**(2), 113—119(1996).
4 Roodman, G.M., "Post-infeasibility analysis in linear programming"., *Manag. Sci.*, **25**(9), 916—922(1979).
5 Greenberg, H.J., A functional description of analyze: A computer-assisted analysis system for linear programming models", *ACM Trans. Math. Software*, **9**(1), 18—56(1983).
6 Greenberg, H.J., "Diagnosing infeasibility for min-cost network flow models, Part I: Dual infeasibility". *IMA J. Math. Manag.*, **1**, 99—109(1987).
7 Greenberg, H.J., "Computer-assisted analysis for diagnosing infeasible or unbounded linear programs". *Math. Program. Stud.*, **31**, 79—97(1987).
8 Greenberg, H.J., "Analyze: A computer-assisted analysis system for linear programming models". *Oper. Res. Lett.*, **6**(5), 249—255(1987).
9 Greenberg, H.J., "Diagnosing infeasibility for min-cost network flow models, Part II: Primal infeasibility". *IMA J. Math. Appl. Business Ind.*, **2**, 1—12(1988).
10 Murty, K.G., Linear Programming, New York (1983).
11 Murty, K.G., Kabadi, S.N., Chandrasekaran, R., "Infeasibility analysis for linear systems", *Arab. J. Sci. Eng.*, **25**(1C), 3—18(2000).
12 Van Loon, J., "Irreducibly inconsistent systems of linear inequalities", *Eur. J. Oper. Res.*, **8**, 283—288(1981).
13 Gleeson, J., Ryan, J., "Identifying minimally infeasible subsystems of inequalities", *ORSA J. Comput.*, **2**(1), 61—63(1990).
14 Leon, T., Liern, V., "A fuzzy method to repair infeasibility in linearly constrained problems", *Fuzzy Sets Syst.*, **122**, 237—243(2001).
15 Timminga, E., "Solving infeasibility problems in computerized test assembly", *Appl. Psychol. Measur.*, **22**(3), 280—291(1998).
16 Huitzing, H.A., "Using set covering with item sampling to analyze the infeasibility of linear programming test assembly models", *Appl. Psychol. Measur.*, **28**(5), 355—375(2004).