

## Direct Measurements of Turbulence in the Pacific Equatorial Undercurrent

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### ABSTRACT

Measurements of small-scale fluctuations of temperature are used to estimate turbulent parameters such as viscous dissipation rate ( $\epsilon$ ), dissipation rate of temperature variance ( $\chi$ ), and turbulent diffusion coefficients of momentum ( $\nu_T$ ) and temperature ( $D_T$ ). Results from two locations are reported: one in the center of the undercurrent (0N 150W) and one toward the northern edge (1N 150W); both at depths of about 100 m where high vertical shear and high vertical stability are found. Universal similarity and local isotropy assumptions were used to determine the dissipation rates from measured spectra. While  $\epsilon$  ( $\sim 0.08 \text{ cm}^2 \text{ sec}^{-3}$ ) was about the same at both locations,  $\chi$  at 0N [ $7 \times 10^{-5} (\text{C})^2 \text{ sec}^{-1}$ ] was larger by a factor of 9. Even greater differences were found in  $D_T$ :  $27 \text{ cm}^2 \text{ sec}^{-1}$  at 0N vs  $0.52 \text{ cm}^2 \text{ sec}^{-1}$  at 1N indicating large vertical mixing at the equator. From two independent methods  $\nu_T$  yielded about the same results within 15%:  $12 \text{ cm}^2 \text{ sec}^{-1}$  at 1N and  $25 \text{ cm}^2 \text{ sec}^{-1}$  at 0N.

### 1. Introduction

The Pacific Equatorial Undercurrent or "Cromwell Current" system is characterized by a large transport of water (comparable to the Gulf Stream) from the western Pacific toward the east at depths to about 150 m beneath the surface producing a large vertical shear, typically 2–3 kt (100 m)<sup>-1</sup>. An important factor in understanding the development and structure of this current system is the role of turbulence, turbulent friction and turbulent mixing. However, no direct measurements of turbulence parameters have been reported up to this time. Direct measurements are difficult. Strong thermal stratification at the equator confines the turbulence to relatively small length scales. Therefore, it is necessary to use sensors with rapid response and high spatial resolution. Signal levels are low and noise levels are high. Large samples of data must be recorded with wide bandwidth, and statistical data processing is mandatory. The purpose of this paper is to describe the results of some measurements of small-scale fluctuations of temperature in the Cromwell Current, and by assuming local isotropy and universal similarity of the measured spectra, to infer parameters of turbulence and mixing. Further details of the measurements and technique may be found in Gibson and Williams (1973).

### 2. Implications of universal similarity in turbulent fields

An important consequence of scalar similarity hypotheses (Kolmogoroff, 1941; Obukhoff, 1949; Corrsin,

1951; Batchelor, 1959) for high Reynolds number turbulent mixing is that the spectral function has the form

$$\varphi_T = \beta \chi \epsilon^{-1/3} k^{-5/3}; \quad k_0 > k > k_K = L_K^{-1} = (\epsilon/\nu^3)^{1/4}, \quad \nu > D, \quad (1)$$

where  $\varphi_T$  is the power spectrum of the temperature fluctuations [ $(\text{C})^2 \text{ cm}$ ];  $\beta$  a universal constant;  $\epsilon$  the rate of dissipation of turbulent kinetic energy ( $\text{cm}^2 \text{ sec}^{-3}$ );  $k$  the one-dimensional radian wavenumber;  $k_0^{-1}$  either an energy scale or buoyancy length scale, whichever is smaller;  $\nu$  the kinematic viscosity;  $D$  the thermal diffusivity; and  $L_K$  the Kolmogoroff length scale. The rate of dissipation of temperature variance,  $\chi$ , is given by

$$\chi = 2D \langle (\nabla T)^2 \rangle, \quad (2)$$

where  $T$  is temperature. Further, if the temperature gradient field is isotropic for scales at which dissipation occurs, then  $\chi$  may be calculated by

$$\begin{aligned} \chi &= 6D \left\langle \left( \frac{dT}{dx} \right)^2 \right\rangle = 6D \int_0^\infty k^2 \varphi_T(k) dk \\ &= \frac{24\pi^2 D}{\bar{u}^2} \int_0^\infty f^2 \varphi_T(f) df, \quad (3) \end{aligned}$$

where  $\bar{u}$  is the velocity and  $f$  is the frequency.

Since the Prandtl number of water  $\nu/D \approx 7$ , the temperature field should extend to much smaller scales than the velocity.

Measurements in a water tunnel by Gibson *et al.* (1970) show that at

$$k = \frac{k_K}{30}$$

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the spectrum of weakly diffusive scalar fields such as temperature shows an inflection to a slope  $-1$ , i.e.,

$$\left. \begin{aligned} \varphi_T &= \beta' \left( \frac{\chi}{\gamma} \right) k^{-1}; & \frac{k_K}{30} < k < k_K \text{Pr}^{\frac{1}{2}} \\ \gamma &\equiv (\epsilon/\nu)^{\frac{1}{2}} \end{aligned} \right\} \quad (4)$$

as predicted by Batchelor (1959), where  $\beta'$  is another universal constant ( $\beta$  is about  $\frac{1}{2}$ ,  $\beta'$  about 2). If the temperature spectrum can be measured at the inflection point  $k = k_K/30$ ,  $\epsilon$  can be inferred using Eq. (4). In addition,  $\epsilon$  can be calculated from (1).

### 3. Diffusion coefficients and heat flux

Starting with the equation for entropy for a multi-component fluid derived by Eckart (1940), Osborn and Cox (1972) develop an equation relating small-scale temperature fluctuations to the turbulent heat flow. From this a thermal eddy diffusion coefficient can be extracted:

$$D_T = \frac{D \langle (\nabla T)^2 \rangle}{\left\langle \frac{dT}{dz} \right\rangle^2} \quad (5)$$

The averaging process,  $\langle \rangle$ , is over a volume assumed to be in steady state and laterally homogeneous with respect to temperature statistics.

Making use of (2),

$$D_T = \frac{\chi}{2 \left\langle \frac{dT}{dz} \right\rangle^2} \quad (6)$$

Measurements of particle dispersion in the ocean by Richardson and Stommel (1948), Stommel (1949) and more extensively by Ozmidov (1960) have shown that the eddy viscosity has a "4/3 law" dependence on scale:

$$\nu_T = C_1 \epsilon^{\frac{1}{3}} L^{\frac{2}{3}}, \quad L_k < L < L_{\max}, \quad (7)$$

for length scales  $L$ , where  $C_1$  is a universal constant. For vertical diffusion,  $L_{\max}$  has been proposed by Ozmidov (1965) to be

$$L_{\max} = L_R \equiv \left( \frac{\epsilon}{N^3} \right)^{\frac{1}{2}}, \quad (8)$$

where  $N$  (Väisälä-Brunt frequency) =  $[(g/\rho)(d\rho/dz)]^{\frac{1}{2}}$  and  $\rho$  the density. Furthermore, for vertical scales longer than  $L_R$ ,  $\nu_T$  becomes independent of scale, i.e.,

$$\nu_T = C_1 \epsilon^{\frac{1}{3}} L_R^{\frac{2}{3}} = \frac{C_1 \epsilon}{N^2}, \quad (9)$$

and only dependent on external parameters  $\epsilon$  and  $N$ . Ozmidov (1960) estimates  $C_1$  to be 0.1 from estimates of  $\epsilon$  and Eq. (7).

There exists another method, perhaps rougher, for calculating  $\nu_T$ . Munk and Anderson (1948) combine theories of Rossby and Montgomery (1935) and Taylor (1931) to predict the ratios of turbulent eddy viscosity to turbulent thermal diffusion:

$$\frac{\nu_T}{D_T} \approx 2 \text{ Ri} \quad \text{for } \text{Ri} > 3, \quad (10)$$

where Ri (the Richardson number)  $\equiv N^2/(d\bar{u}/dz)^2$ ,  $\bar{u}$  being the horizontal velocity at depth  $z$ . Data exists which roughly support Eq. (10). Values of  $\nu_T/D_T = 20-30$  were calculated by Proudman (1953) from oceanographic observation for  $4 < \text{Ri} < 10$ . More direct measurements by Kolesnikov (1960) in water under ice in Lake Baikal yield  $\nu_T/D_T \approx 20$  for moderate stability to values as high as 50 for higher stability. Thus, for  $\text{Ri} > 3$ , Eq. (10) provides a rough estimate of  $\nu_T$ , knowing  $D_T$  and Ri.

From a knowledge of the value of  $D_T$ , a vertical heat flux can be calculated (Monin and Yaglom, 1971):

$$Q = C_p \rho \overline{T'w'} = -C_p \rho D_T \frac{\partial T}{\partial z}, \quad (11)$$

where  $Q$  is the vertical heat flux (cal  $\text{cm}^{-2} \text{sec}^{-1}$ ), the specific heat,  $C_p = 0.93 \text{ cal } [\text{gm } (^\circ\text{C})^{-1}]$ , and  $\overline{T'w'}$  is the correlation of temperature and vertical velocity fluctuations.

### 4. Work done on buoyancy forces

We note that for a given mass of fluid, the potential energy of a stably stratified fluid is less than one of neutral stability. Turbulent motions in a stable medium such as the ocean will do work on buoyancy forces and therefore raise the ocean's potential energy. For thermal stratification, this results in reducing the magnitude of the vertical temperature gradient. In the turbulent energy equation (Monin and Yaglom, 1971), this transfer from kinetic to potential energy is a sink term

$$B = -\overline{\rho'w'g/\rho}, \quad (12)$$

where  $B$  is the rate of turbulent energy consumed per unit mass in performing work against buoyancy forces ( $\text{cm}^2 \text{sec}^{-3}$ ),  $\rho'$  is the fluctuating component of density,  $g$  the acceleration due to gravity, and  $\bar{\rho}$  the mean density. If stratification is due mainly to temperature, then  $\rho'/\rho = gT'$ ,  $\alpha$  being the thermal expansion coefficient [ $= -2.5 \times 10^{-4} (^\circ\text{C})^{-1}$ ] for our conditions), and

$$B = -g\alpha \overline{T'w'} = -g\alpha Q (\bar{\rho} C_p). \quad (13)$$

### 5. Experimental arrangement

The towed body was a 5x5 inch steel pipe filled with 600 lb of lead. A 24-inch wide plate on the back served as a fin. Instruments were mounted on a 36-inch up-

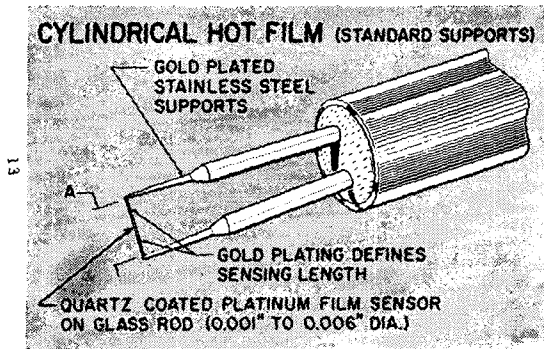


FIG. 1a. Cylindrical hot film sensor used as resistance thermometer.

stream strut. The body was supported by a 1500 ft,  $\frac{3}{16}$ -inch diameter, four-conductor armored cable on a winch. Depth variations of less than 1 m peak to peak were observed during towing tests at 4 kt with a vibratron mounted on the body. Signals from instruments were generally brought up the cable and recorded in the ship laboratory. Cross talk between channels made it necessary to disable certain instruments for some recordings, and for some measurements a separate low impedance cable was attached by electrical tape to the support cable, in order to reduce noise level, cross talk and cable losses.

High-frequency temperature fluctuations were sensed by a Thermo-Systems hot film anemometer probe shown in Fig. 1a. The probe was a  $10^{-3}$  inch diameter quartz-insulated, platinum-coated quartz fiber with active element length of 0.5 mm. The probe was used as a resistance thermometer in an ac bridge, using a Princeton Applied Research HR 124 lock-in amplifier as phase sensitive detector.

A rapid response Marine Advisors ducted current meter shown in Fig. 1b was used to measure the stream-wise velocity fluctuations to frequencies of 1–2 Hz. Magnets in the five-bladed plastic impeller induced a

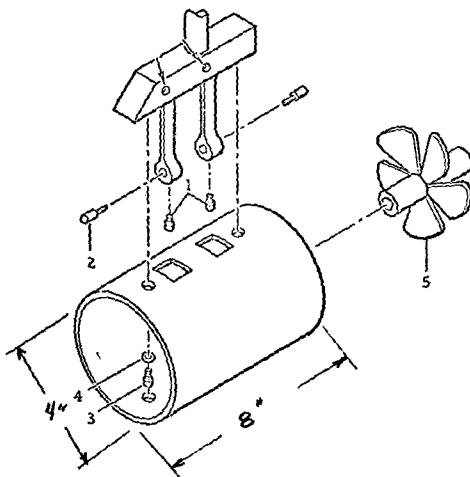


FIG. 1b. Ducted current meter used to measure velocity spectra to 1 Hz.

16 Hz per knot pulse signal in a reed switch outside the 4-inch diameter cylindrical tube containing the impeller. It was necessary to reinforce the base of the impeller blades with epoxy and fiberglass, since the first two (and only) spare impellers lost their blades within an hour of service by fatigue failure.

Signals were recorded by a Sangamo 3500 FM analog instrumentation tape recorder, with about 50 dB signal-to-noise ratio. A tape servo control was used to reduce wow induced by the ship motion. Digital signals were recorded on disc using the IBM 1800 on-board computer. Statistical analysis such as fast-Fourier-transform spectral calculation and plotting was also carried out directly, or the digital samples were preserved on tape. A time mark generator signal was recorded and synchronized to 1 sec with the computer clock, so that the turbulence data can be coordinated with a wide variety of ship motion, navigation and environmental data logged routinely by the computer.

## 6. Results and discussion

Plotted in Fig. 2 is  $f^2\varphi_T(f)$ , where noise spikes in the spectrum due to pick-up of ship's electrical system (principally 60 Hz and harmonics) have been ignored (see Gibson and Williams, 1973);  $\varphi_T(f)$  has been averaged over 200 records of 2048 samples per record, representing a total horizontal averaging length of 350 m. The values of  $\chi$ , obtained by integration of such curves [Eq. (3)], are reported in the summary Table 1. Hacker's (1973) measurements in the Pacific near San Diego for depths of 100–365 m yield values of  $\chi$  from  $1.3 \times 10^{-9}$  to  $1.2 \times 10^{-7}$  ( $^{\circ}\text{C})^2 \text{sec}^{-1}$  which, although quite variable, are two to five orders of magnitude smaller than those reported here. Grant *et al.* (1968) report values of  $\chi$  in the thermocline ranging from  $2 \times 10^{-7}$  to  $9 \times 10^{-6}$  ( $^{\circ}\text{C})^2 \text{sec}^{-1}$ , which, although closer to our values, are still lower. The higher values reflect the larger turbulent activity in the equatorial shear layer.

Estimates of  $\epsilon$  from the two independent methods described above agree within expected error due to uncertainties in universal constants. Although the two methods differ by only 25%, accumulated errors due to uncertainties in  $\beta$  and the inflection point ( $k = k_k/30$ )

TABLE 1. Summary of turbulence parameter computations at 1N, 150W and 0N, 150W.

	1N	0N
$\epsilon$ ( $\text{cm}^2 \text{sec}^{-3}$ )	0.08	0.08
$\chi$ [ $(^{\circ}\text{C})^2 \text{sec}^{-1}$ ]	$8 \times 10^{-6}$	$7 \times 10^{-5}$
$N^2$ ( $\text{sec}^{-2}$ )	$6.7 \times 10^{-4}$	$2.75 \times 10^{-4}$
$dT/dz$ ( $^{\circ}\text{C cm}^{-1}$ )	$2.75 \times 10^{-3}$	$1.13 \times 10^{-3}$
$v_T$ ( $\text{cm}^2 \text{sec}^{-1}$ )	12	25
$D_T$ ( $\text{cm}^2 \text{sec}^{-1}$ )	0.52	27
$Pr_T$	22	0.91
$L_R$ (m)	0.7	1.4
Ri	10	1
$Q$ ( $\text{cal cm}^{-2} \text{sec}^{-1}$ )	$-1.4 \times 10^{-3}$	$-28 \times 10^{-3}$
$B$ ( $\text{cm}^2 \text{sec}^{-3}$ )	$-3.6 \times 10^{-4}$	$-7.5 \times 10^{-3}$
$\tau$ ( $\text{dyn cm}^{-2}$ )	0.1	0.3

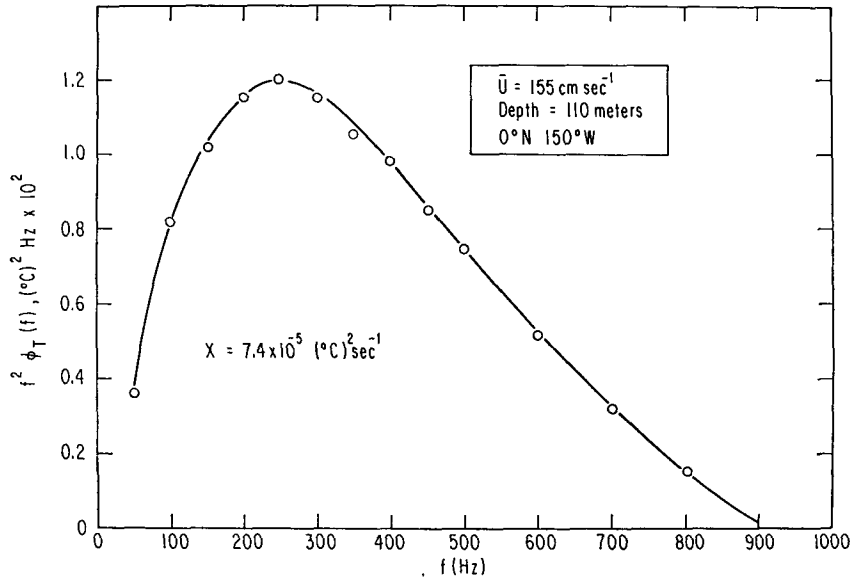


FIG. 2. Temperature dissipation spectrum at 0N 150W at 110 m depth. The curve represents 200 records of 2048 samples per record. The value of  $\chi$  is obtained by the integration of the spectrum.

would increase the uncertainty in  $\epsilon$  to perhaps as much as 50%. Furthermore, verification of the inflection point method on Nasmyth's (1970) data yielded errors of about this magnitude. The values for  $\epsilon$  ( $8 \times 10^{-2} \text{ cm}^2 \text{ sec}^{-3}$ ) obtained from our measurements are higher than for more normal ocean conditions (Grant *et al.*, 1968; Nasmyth, 1970), but lie at the upper limit of the range of estimates by Ozmidov (1960) for open ocean conditions in the upper water.

From a knowledge of the important dissipation parameters  $\epsilon$  and  $\chi$ , turbulent diffusion coefficients can be obtained. From Eq. (9),  $\nu_T = 12 \text{ cm}^2 \text{ sec}^{-1}$  (at 1N) and  $25 \text{ cm}^2 \text{ sec}^{-1}$  (at 0N). The second (rougher) method of calculating  $\nu_T$  can be applied to the measurements at 1N where  $Ri \approx 10$ , yielding  $\nu_T = 10 \text{ cm}^2 \text{ sec}^{-1}$ . This agreement is surprisingly close, since the allotted error in  $\epsilon$  is about 50%. Of particular note is that this second method does not involve  $\epsilon$  so that this agreement can be interpreted as another support for the value of  $\epsilon$ .

Turbulent (or Reynolds') stresses can be calculated from  $\nu_T$  using the relation

$$\tau = \rho \nu_T \frac{d\bar{u}}{dz}, \quad (14)$$

where  $\tau$  is the turbulent stress ( $\text{dyn cm}^{-2}$ ). A physical interpretation can be made that one layer exerts a stress on the layer below it, with the top-most layer receiving stresses from the wind. Taft *et al.* (1973) have estimated the wind stress to be about  $0.4 \text{ dyn cm}^{-2}$  on the equator, which can be compared to 0.1 (at 1N) and 0.3 (at 0N)  $\text{dyn cm}^{-2}$  for the stresses at depths of about 100 m from our measurements. These values indicate that our measurements are consistent with a constant

stress (vs depth) model. Further confirmation of this would have to come from more measurements at different depths. Unfortunately, these two stations are the only ones that yielded signals worthy of analysis. If it is possible to assume a constant stress model, one

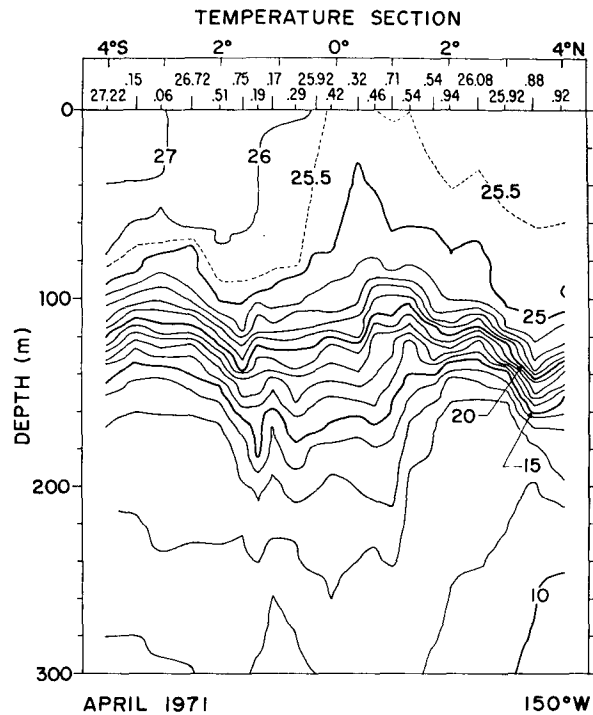


FIG. 3. Temperature section at 150W during April 1971. Note the vertical spreading of isotherms at the equator believed due to turbulent mixing. Numerical values at top are surface temperature.

can obtain a  $\nu_T$  profile from a measured shear profile and a wind stress estimate.

Although  $\chi$  values reported are two to five orders of magnitude larger than Hacker's (1973), turbulent thermal diffusion coefficients did not have such a large difference. Hacker reports values of  $D_T = 0.2\text{--}15\text{ cm}^2\text{ sec}^{-1}$  off San Diego while values from 0.52 to 27  $\text{cm}^2\text{ sec}^{-1}$  were obtained from our study.

An interesting characteristic of the Pacific Equatorial Undercurrent system is the reduction of the vertical thermal gradient at the equator from gradients found  $1^\circ$  to the north or south (see Fig. 3). Knauss (1960) has suggested this feature can be explained by either convective processes or by more vigorous turbulent mixing at the equator. From Table 1 we see that  $B$  is approximately 20 times that at  $1N$ , so that turbulent mixing causes a greater reduction in the thermal gradient on the equator than at  $1N$ . For this cruise, the flow pattern was not consistent with this feature being generated by convection.

## 7. Summary and conclusions

Fine-scale temperature fluctuations were made in the Pacific Equatorial Undercurrent at depths of about 100 m at two locations;  $1N$ ,  $150W$  and  $0N$ ,  $150W$ . These measurements were used to infer the dissipation parameters  $\epsilon$  and  $\chi$ . These parameters, together with estimates of turbulent diffusion coefficients, heat flux, turbulent stresses, and work done on buoyancy forces characterize the turbulence of this flow system. It is concluded that turbulence is more vigorous in this region than in more typical ocean conditions, even though vertical stability is very high. The larger value of  $B$  found on the equator as compared to that at  $1N$  strongly suggests that it is turbulent mixing that creates the feature of thermal gradient reduction on the equator. These measurements hint at the possibility of a roughly constant stress layer to depths of about 100 m, although a more dense sampling would be needed to support this. The uncertainty in  $\epsilon$  was large (50%), and could be reduced by measuring small-scale velocity fluctuations as well as using the present less precise methods based on temperature spectra.

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