Study on Transfer Mechanism of Quantum Entangled Information in Coherent Coupling Cavity-field*

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Abstract: The physical model composed of coherent multimode cavity-field interacting with coherent coupling-atom is proposed. The evolution properties for the process of multi-photon interaction in the system mentioned above are studied by utilizing complete quantum theory. It shows that multiphoton entangled states and multi-atom entangled states can be swapped into each other periodically in the process of coupling multi-atoms interacting with multimode cavity-fields. Meanwhile the entanglement information transfer can be realized in the above process. The results of the entanglement transfer are thereby illustrated by means of calculating the fidelity of atoms numerically. And the general characteristics of the entangled information transfer are further revealed. It is pointed out that the results reported up to now are only specific examples of the research under different conditions.

Key words: Quantum informatics; Quantum optics; Quantum entanglement; Coherent coupling cavity-field; Coherent coupling atoms; Swapping transfer

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0 Introduction

Since Einstein, Podolsky and Rosen suggested EPR[1], quantum entanglement phenomena has attracted attention of both theorists and experimentalists[2-6]. It is one of the most important features of quantum mechanics. Entanglement indicates a system composed of two or more particles, can exhibit the novel property that results of measurement on one particle cannot be specified independently of the parameters of the measurements on other particles. It has been well known that quantum entanglement plays a key role in many such applications as quantum teleportation[7-12], super-dense coding[13], quantum error correction[14], quantum key distribution[15], and quantum computational speedups[16]. The resource of entanglement has a number of useful applications in quantum communication and light-quanta communication.

Since Bennett[17], et al have proposed the scheme of quantum teleportation via quantum entanglement states of separation variation in 1993, the field of quantum teleportation, a way to transfer unknown state of a quantum system from one location to another, is focused on quantum communication and plays an important role in a number of quantum computation schemes. The significant experimental and theoretical advances have been made in teleportation. In 1994, Vaidman[18] extended theoretically the quantum teleportation into infinite-dimensional Hilbert space. Braunstein and Kimble[19] presented the method (B-K method) of quantum teleportation with two-mode squeezed vacuum states as quantum channel in 1998. Many schemes are offered by Chinese up to now. Based on the proposal of Braunstein and Kimble, Xia Yun-jie[20] and co-workers studied the continuous variables quantum teleportation via minimum-correlation mixed state. The results show that the mixed entangled state as a generalized type of Einstein-Podolsky- Rosen entangled states is a good quantum channel. They analyzed in detail teleporting a coherent state by means of this channel as an example. When the parameters of minimum-correlation mixed state are chosen properly, the fidelity will almost reach to unity. The minimum-correlation mixed state as a quantum channel is better than the two-mode squeezed vacuum state. Zhou Xiao-qing[21], et al made three-particle entanglement using the source of three-particle W states. Put this particle entanglement down as quantum channel, and

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transfer information of Bell states measurement and von Neumann measurement using classical channel, then the quantum teleportation net may be realized. Based on this idea , they investigated the physical principle for building the quantum teleportation net using three-particle W states, find out the unitary transformation matrix for quantum teleportation of three-particle W states, and design a quantum teleportation net. They propose a scheme for quantum communications net and its protocol. If all stations work as schemed, quantum communications between any two stations may be realized.

However, the studies of quantum information transfer via the interaction of atoms with light-fields is less than the above mentioned. Recently, Lai, et al[22-23] have suggested a scheme for entanglement swapping in the process of two-level atoms interacting with cavity-fields of coherent sate. But up to now, the general system consisting of coherent cavity-field and coherent-atom has never been reported. We shall discuss it further in this paper.

1 The theoretical model and exact solution

Based on Ref. [22-23], now we consider the physical system made up atoms interacting with cavity-field as shown in Fig. 1. Let the relative phase of corresponding atomic transition in each neighboring cavity-field is a constant ξ, and the M pairs of atoms coupled are coherent. The dotted lines and the arrows represent their coherence

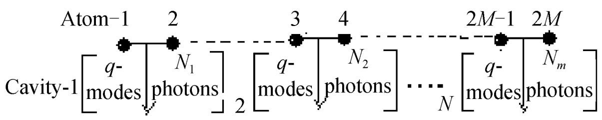


Fig. 1 The model of the M pairs coupled atoms interacting with the M coherent multimod light fields

among atoms and the direction of atomic motion respectively in Fig. 1. The relative phase of light-field corresponding mode in each neighboring cavity-field is always η. There is also coherence among M cavity-fields. In the interacting picture and under the rotating-wave approximation, the effective Hamiltonian for the system can be (here, $\hbar=1$)

$$H_{\mathrm{eff}}=\sum_{k=1}^{M}H_{I,k}=\sum_{k=1}^{M}\{g_k\sum_{l=1}^{2}[(\prod_{j=1}^{q}a_{jk}^{+N_{jk}}(t))\sigma_{lk}^{-}(t)+\sigma_{lk}^{+}(t)(\prod_{j=1}^{q}a_{jk}^{N_{jk}}(t))]+\lambda_k[\sigma_{1k}^{+}(t)\sigma_{2k}^{-}(t)+\sigma_{2k}^{-}(t)\sigma_{1k}^{+}(t)]\} \tag{1}$$

where, all of $k(k=1,2,3,\cdots,M)$, $j(j=1,2,3,\cdots,q)$, and $l\,(l=1,2)$ in both superscript and subscript stand k-th cavity-field, j-th mode light-field, and l-th atom respectively (Ditto). Since conserving the relation between interaction picture and Schrödinger picture, the coherence among atoms, and among cavity-fields during the interaction, then $\sigma_{lk}^{-}(t)=A_{\xi,l,k}\sigma_l^{-}\exp[-\mathrm{i}(k-1)\xi]\cdot\exp(-\mathrm{i}\omega_{a,k}t)$, $a_{j,k}(t)=A_{\eta,l,k}a_j\exp[-\mathrm{i}(k-1)\eta]\cdot\exp(-\mathrm{i}\omega_{f,k}t)$, $k=1,2,3,\cdots,M$. where $\omega_{f,k}(\omega_{a,k})$ is the light field(atomic transition) frequency, and $A_{\xi,l,k}$ ($A_{\eta,l,k}$) is complex coefficient of atom (photon) operator, $\sigma_l^{\pm}$ are the atomic operators of l-th atom, a_j^{+} (a_j) denotes photon creation (annihilation) operator of j-th mode, g and λ mean atom-field and atom-atom coupling constants, while we assume $g_k=g$, $\lambda_k=\lambda(k=1,2,3,\cdots,M)$ in order to simple. The state evolution of the system can be described by $|\psi(t)\rangle=U(t)|\psi(0)\rangle$, where $|\psi(0)\rangle$ is the initial state of the system, $U(t)$ is time evolution operator for the system. The evolution matrix in the k-th subsystem can be obtained by lots of complicated and repeated calculation, as following

$$U_k(t)=\begin{pmatrix} 1+A_k\dfrac{\cos\beta_k-1}{B_k}A_k^{+} & -\mathrm{i}gtA_{\xi,k}^{*}A_{\eta,k}A_k\dfrac{\sin\beta_k}{\beta_k}\cdot e_1^{+}e_2^{-}\mathrm{e}^{-\mathrm{i}\gamma_k} & -\mathrm{i}gtA_{\xi,k}^{*}A_{\eta,k}A_k\dfrac{\sin\beta_k}{\beta_k}\cdot e_1^{+}e_2^{-}\mathrm{e}^{-\mathrm{i}\gamma_k} & 2g^2t^2A_{\xi,k}^{*2}A_{\eta,k}^{2}A_k\dfrac{\cos\beta_k-1}{\beta_k^2}\cdot A_k(e_1^{+})^2(e_2^{-})^2 \\ -\mathrm{i}gtA_{\xi,k}A_{\eta,k}^{*}\dfrac{\sin\beta_k}{\beta_k}A_k^{+}e_1^{-}e_2^{+} & \dfrac{1}{2}[\cos\beta_k\mathrm{e}^{-\mathrm{i}\gamma_k}+\mathrm{e}^{\mathrm{i}\gamma_k}] & \dfrac{1}{2}[\cos\beta_k\mathrm{e}^{-\mathrm{i}\gamma_k}-\mathrm{e}^{\mathrm{i}\gamma_k}] & -\mathrm{i}gtA_{\xi,k}^{*}A_{\eta,k}\dfrac{\sin\beta_k}{\beta_k}A_ke_1^{+}e_2^{-} \\ -\mathrm{i}gtA_{\xi,k}A_{\eta,k}^{*}\dfrac{\sin\beta_k}{\beta_k}A_k^{+}e_1^{-}e_2^{+} & \dfrac{1}{2}[\cos\beta_k\mathrm{e}^{-\mathrm{i}\gamma_k}-\mathrm{e}^{\mathrm{i}\gamma_k}] & \dfrac{1}{2}[\cos\beta_k\mathrm{e}^{-\mathrm{i}\gamma_k}+\mathrm{e}^{\mathrm{i}\gamma_k}] & -\mathrm{i}gtA_{\xi,k}^{*}A_{\eta,k}\dfrac{\sin\beta_k}{\beta_k}A_ke_1^{+}e_2^{-} \\ 2g^2t^2A_{\xi,k}^{2}A_{\eta,k}^{*2}A_k^{+}\dfrac{\cos\beta_k-1}{\beta_k^2}\cdot A_k^{+}(e_1^{-})^2(e_2^{+})^2 & -\mathrm{i}gtA_{\xi,k}A_{\eta,k}^{*}A_k^{+}\dfrac{\sin\beta_k}{\beta_k}\cdot e_1^{-}e_2^{+}\mathrm{e}^{-\mathrm{i}\gamma_k} & -\mathrm{i}gtA_{\xi,k}A_{\eta,k}^{*}A_k^{+}\dfrac{\sin\beta_k}{\beta_k}\cdot e_1^{-}e_2^{+}\mathrm{e}^{-\mathrm{i}\gamma_k} & 1+A_k^{+}\dfrac{\cos\beta_k-1}{B_k}A_k \end{pmatrix} \tag{2}$$

Where

$$A_{\xi,k}=A_{\xi,l,k}(l=1,2);\ A_{\eta,k}=\prod_{j=1}^{q}A_{\eta,j,k}^{N_{jk}};$$

$$A_k=\prod_{j=1}^{q}a_j^{N_{jk}};\ A_k^{+}=\prod_{j=1}^{q}a_j^{+N_{jk}};$$

$$B_k=\prod_{j=1}^{q}(a_j^{+N_{jk}}a_j^{N_{jk}}+a_j^{N_{jk}}a_j^{+N_{jk}});$$

$e_1^+ = \mathrm{e}^{\mathrm{i}(k-1)\xi}\mathrm{e}^{\mathrm{i}\omega_{a,k}t}$；$e_1^- = \mathrm{e}^{-\mathrm{i}(k-1)\xi}\mathrm{e}^{-\mathrm{i}\omega_{a,k}t}$；

$\gamma_k = \lambda t|A_{\xi,k}|^2$；$e_2^+ = \prod_{j=1}^{q}(\mathrm{e}^{\mathrm{i}N_{jk}(k-1)\eta}\mathrm{e}^{\mathrm{i}N_{jk}\omega_{a,k}t})$；

$e_2^- = \prod_{j=1}^{q}(\mathrm{e}^{-\mathrm{i}N_{jk}(k-1)\eta}\mathrm{e}^{-\mathrm{i}N_{jk}\omega_{a,k}t})$；

$\beta_k = \sqrt{2B_k}gt|A_{\xi,k}||A_{\eta,k}|$.

2 The evolution property of state vector for the system

As a specific example, let $M=2$, namely $k=1,2$, assuming that the light-field are in initial common number state at $t=0$

$$|\psi_f(0)\rangle = \sum_{\{n_{j1}\},\{n_{j2}\}=0}^{\infty}\prod_{j=1}^{q}F_{n_{j1}}\prod_{j=1}^{q}F_{n_{j2}}\bigotimes_{j=1}^{q}|n_{j1}\rangle\cdot\bigotimes_{j=1}^{q}|n_{j2}\rangle = \sum_{\{n_{j1}\},\{n_{j2}\}=0}^{\infty}F_{n_1}F_{n_2}\bigotimes_{j=1}^{q}|n_{j1},n_{j2}\rangle$$

where

$$F_{n_k} = \prod_{j=1}^{q}F_{n_{jk}};\ \bigotimes_{j=1}^{q}|n_{jk}\rangle = |n_{1k}\rangle|n_{2k}\rangle|n_{3k}\rangle\cdots|n_{qk}\rangle\quad(k=1,2)$$

and all the atoms coupled are in the excited state $|\psi_a(0)\rangle = |e_{11}e_{21},e_{12}e_{22}\rangle$, where $|e_{lk}\rangle$ $(l=1,2)$ denoted that l-th atom in k-th cavity-field is in excited state. According to $|\psi(t)\rangle = U(t)|\psi(0)\rangle$, the evolution formulation of the state vector describing this system at arbitrary time $t>0$ can be given as follows

$$\begin{aligned}
|\psi_2^{(ee)}(t)\rangle = &\sum_{\{n_{j1}\},\{n_{j2}\}=0}^{\infty}F_{n_1}F_{n_2}\Big\{\Big[1+\frac{\cos\alpha_{ee1}-1}{N1_1}(N2_1)^2\Big]\Big[1+\frac{\cos\alpha_{ee2}-1}{N1_2}(N2_2)^2\Big]\bigotimes_{j=1}^{q}|n_{j1},n_{j2}\rangle|e_{11}e_{21},\\
&e_{12}e_{22}\rangle - \mathrm{i}A_{\xi,2}A_{\eta,2}^{*}gt\Big[1+\frac{\cos\alpha_{ee1}-1}{N1_1}(N2_1)^2\Big]\frac{\sin\alpha_{ee2}}{\alpha_{ee2}}N2_2e_1^-e_2^+\bigotimes_{j=1}^{q}|n_{j1},n_{j2}+N_{j2}\rangle|e_{11}e_{21},e_{12}g_{22}\rangle-\\
&\mathrm{i}A_{\xi,2}A_{\eta,2}^{*}gt\Big[1+\frac{\cos\alpha_{ee1}-1}{N1_1}(N2_1)^2\Big]\frac{\sin\alpha_{ee2}}{\alpha_{ee2}}N2_2e_1^-e_2^+\bigotimes_{j=1}^{q}|n_{j1},n_{j2}+N_{j2}\rangle|e_{11}e_{21},g_{12}e_{22}\rangle+\\
&2A_{\xi,2}^2A_{\eta,2}^{*2}g^2t^2\Big[1+\frac{\cos\alpha_{ee1}-1}{N1_1}(N2_1)^2\Big]\frac{\cos\alpha_{ee2}-1}{\alpha_{ee2}^2}N3_2(e_1^-)^2(e_2^+)^2\bigotimes_{j=1}^{q}|n_{j1},n_{j2}+2N_{j2}\rangle|e_{11}e_{21},\\
&g_{12}g_{22}\rangle - iA_{\xi,1}A_{\eta,1}^{*}gt\frac{\sin\alpha_{ee1}}{\alpha_{ee1}}N2_1e_1^-e_2^+\Big[1+\frac{\cos\alpha_{ee2}-1}{N1_2}(N2_2)^2\Big]\bigotimes_{j=1}^{q}|n_{j1}+N_{j1},n_{j2}\rangle|e_{11}g_{21},e_{12}e_{22}\rangle-\\
&A_{\xi,1}A_{\eta,1}^{*}A_{\xi,2}A_{\eta,2}^{*}g^2t^2\frac{\sin\alpha_{ee1}\sin\alpha_{ee2}}{\alpha_{ee1}\alpha_{ee2}}N2_1N2_2(e_1^-)^2(e_2^+)^2\bigotimes_{j=1}^{q}|n_{j1}+N_{j1},n_{j2}+N_{j2}\rangle|e_{11}g_{21},e_{12}g_{22}\rangle-\\
&A_{\xi,1}A_{\eta,1}^{*}A_{\xi,2}A_{\eta,2}^{*}g^2t^2\frac{\sin\alpha_{ee1}\sin\alpha_{ee2}}{\alpha_{ee1}\alpha_{ee2}}N2_1N2_2(e_1^-)^2(e_2^+)^2\bigotimes_{j=1}^{q}|n_{j1}+N_{j1},n_{j2}+N_{j2}\rangle|e_{11}g_{21},g_{12}e_{22}\rangle-\\
&2\mathrm{i}A_{\xi,1}A_{\eta,1}^{*}A_{\xi,2}^2A_{\eta,2}^{*2}g^3t^3\frac{\sin\alpha_{ee1}(\cos\alpha_{ee2}-1)}{\alpha_{ee1}\alpha_{ee2}^2}N2_1N3_2(e_1^-)^3(e_2^+)^3\bigotimes_{j=1}^{q}|n_{j1}+N_{j1},n_{j2}+2N_{j2}\rangle|e_{11}g_{21},\\
&g_{12}g_{22}\rangle - iA_{\xi,1}A_{\eta,1}^{*}gt\frac{\sin\alpha_{ee1}}{\alpha_{ee1}}N2_1e_1^-e_2^+\Big[1+\frac{\cos\alpha_{ee2}-1}{N1_2}(N2_2)^2\Big]\bigotimes_{j=1}^{q}|n_{j1}+N_{j1},n_{j2}\rangle|g_{11}e_{21},e_{12}e_{22}\rangle-\\
&A_{\xi,1}A_{\eta,1}^{*}A_{\xi,2}A_{\eta,2}^{*}g^2t^2\frac{\sin\alpha_{ee1}\sin\alpha_{ee2}}{\alpha_{ee1}\alpha_{ee2}}N2_1N2_2(e_1^-)^2(e_2^+)^2\bigotimes_{j=1}^{q}|n_{j1}+N_{j1},n_{j2}+N_{j2}\rangle|g_{11}e_{21},e_{12}g_{22}\rangle-\\
&A_{\xi,1}A_{\eta,1}^{*}A_{\xi,2}A_{\eta,2}^{*}g^2t^2\frac{\sin\alpha_{ee1}\sin\alpha_{ee2}}{\alpha_{ee1}\alpha_{ee2}}N2_1N2_2(e_1^-)^2(e_2^+)^2\bigotimes_{j=1}^{q}|n_{j1}+N_{j1},n_{j2}+N_{j2}\rangle|g_{11}e_{21},g_{12}e_{22}\rangle-\\
&2iA_{\xi,1}A_{\eta,1}^{*}A_{\xi,2}^2A_{\eta,2}^{*2}g^3t^3\frac{\sin\alpha_{ee1}(\cos\alpha_{ee2}-1)}{\alpha_{ee1}\alpha_{ee2}^2}N2_1N3_2(e_1^-)^3(e_2^+)^3\bigotimes_{j=1}^{q}|n_{j1}+N_{j1},n_{j2}+2N_{j2}\rangle|g_{11}e_{21},\\
&g_{12}g_{22}\rangle + 2A_{\xi,1}^2A_{\eta,1}^{*2}g^2t^2\frac{\cos\alpha_{ee1}-1}{\alpha_{ee1}^2}N3_1(e_1^-)^2(e_2^+)^2\Big[1+\frac{\cos\alpha_{ee2}-1}{N1_2}(N2_2)^2\Big]\bigotimes_{j=1}^{q}|n_{j1}+2N_{j1},n_{j2}\rangle|\cdot\\
&g_{11}g_{21},e_{12}e_{22}\rangle - 2iA_{\xi,1}^2A_{\eta,1}^{*2}A_{\xi,2}A_{\eta,2}^{*}g^3t^3\frac{(\cos\alpha_{ee1}-1)\sin\alpha_{ee2}}{\alpha_{ee1}^2\alpha_{ee2}}N3_1N2_2(e_1^-)^3(e_2^+)^3\bigotimes_{j=1}^{q}|n_{j1}+2N_{j1},\\
&n_{j2}+N_{j2}\rangle|g_{11}g_{21},e_{12}g_{22}\rangle - 2iA_{\xi,1}^2A_{\eta,1}^{*2}A_{\xi,2}A_{\eta,2}^{*}g^3t^3\frac{(\cos\alpha_{ee1}-1)\sin\alpha_{ee2}}{\alpha_{ee1}^2\alpha_{ee2}}N3_1N2_2(e_1^-)^3(e_2^+)^3\cdot\\
&\bigotimes_{j=1}^{q}|n_{j1}+2N_{j1},n_{j2}+N_{j2}\rangle|g_{11}g_{21},g_{12}e_{22}\rangle + 4A_{\xi,1}^2A_{\eta,1}^{*2}A_{\xi,2}^2A_{\eta,2}^{*2}g^4t^4\frac{(\cos\alpha_{ee1}-1)(\cos\alpha_{ee2}-1)}{\alpha_{ee1}^2\alpha_{ee2}^2}\cdot\\
&N3_1N3_2(e_1^-)^4(e_2^+)^4\bigotimes_{j=1}^{q}|n_{j1}+2N_{j1},n_{j2}+2N_{j2}\rangle|g_{11}g_{21},g_{12}g_{22}\rangle\Big\}
\end{aligned}\tag{3}$$

Where

$$N1_k = \prod_{j=1}^{q}[2(n_{jk}+N_{jk})+1];$$

$$N2_k = \prod_{j=1}^{q}\sqrt{\frac{(n_{jk}+N_{jk})!}{n_{jk}!}};$$

$$N3_k = \prod_{j=1}^{q}\sqrt{\frac{(n_{jk}+2N_{jk})!}{n_{jk}!}};$$

$\alpha_{eek} = \sqrt{2}gt|A_{\xi,k}||A_{\eta,k}|[N1_k]^2$, (here $k=1,2$)

Obviously, Eq. (3) includes interference factor sin

$\alpha_{eek}/\alpha_{eek}$, $(k=1,2)$. It embodies the coherence of atoms and light-field. If the initial atoms are respectively in $|\psi_a(0)\rangle=|e_{11}g_{21}, e_{12}g_{22}\rangle$, and $|\psi_a(0)\rangle=|g_{11}e_{21}, g_{12}e_{22}\rangle$; $|\psi_a(0)\rangle=|g_{11}g_{21}, g_{12}g_{22}\rangle$, we can also obtain evolution equations by this method, marking respectively $|\psi_2^{(eg)}(t)\rangle$、$|\psi_2^{(ge)}(t)\rangle$、$|\psi_2^{(gg)}(t)\rangle$. The Eq. (3) and these can be simply extended into the situation of $k=1,2,\cdots,M$ (here M is arbitrary positive integer). Each of them includes 2^{2M} items for the system composed of arbitrary M subsystem.

3 Periodical swapping of entangled information

3.1 Initial atoms in entanglement state

For Initial atoms in entanglement state, if $M=2$, assuming that initial atoms are in maximum quantum entangled state

$$|\psi_a(0)\rangle=1/\sqrt{2}(|e_{11}e_{21}, e_{12}e_{22}\rangle+|g_{11}g_{21}, g_{12}g_{22}\rangle) \tag{4}$$

and light-field are in state vacuum

$$|\psi_f(0)\rangle=\sum_{\{n_{j1}\},\{n_{j2}\}=0}^{\infty}F_{n_1}F_{n_2}\bigotimes_{j=1}^{q}|0_{j1},0_{j2}\rangle \tag{5}$$

Form $|\psi_2^{(ee)}(t)\rangle$ and $|\psi_2^{(gg)}(t)\rangle$, in the special case when

$$t=(2p+1)\pi/[\sqrt{2}g|A_{\xi,k}||A_{\eta,k}|(N1_k)^{\frac{1}{2}}] \quad (p=0,1,2,3,\cdots)\ (k=1,2) \tag{6}$$

The system vector state become

$$|\psi(t)\rangle=1/\sqrt{2}(\bigotimes_{j=1}^{q}|0_{j1},0_{j2}\rangle+C_1\bigotimes_{j=1}^{q}|2N_{j1}, 2N_{j2}\rangle)|g_{11}g_{21}, g_{12}g_{22}\rangle+C_2\bigotimes_{j=1}^{q}|0_{j1},0_{j2}\rangle|e_{11}e_{21}, e_{12}e_{22}\rangle+C_3\bigotimes_{j=1}^{q}|0_{j1},2N_{j2}\rangle|e_{11}e_{21}, g_{12}g_{22}\rangle+C_4\bigotimes_{j=1}^{q}|2N_{j1},0_{j2}\rangle|g_{11}g_{21}, g_{12}g_{22}\rangle \tag{7}$$

where

$$C_1=16A_{\xi,1}^2A_{\eta,1}^{*2}A_{\xi,2}^2A_{\eta,2}^{*2}g^4t^4\prod_{j=1}^{q}\sqrt{(2N_{j1})!\,(2N_{j2})!}\cdot(e_1^-)^4(e_2^+)^4/[(2p+1)\pi]^4;$$

$$C_2=1/\sqrt{2}[1-2\prod_{j=1}^{q}\frac{(N_{j1})!}{(2N_{j1}+1)}]\cdot[1-2\prod_{j=1}^{q}\frac{(N_{j2})!}{(2N_{j2}+1)}];$$

$$C_3=-2\sqrt{2}A_{\xi,2}^2A_{\eta,2}^{*2}g^2t^2[1-2\prod_{j=1}^{q}\frac{(N_{j1})!}{2N_{j1}+1}]\cdot\prod_{j=1}^{q}\sqrt{(2N_{j2})!}(e_1^-)^2(e_2^+)^2/[(2p+1)\pi]^2;$$

$$C_4=-2\sqrt{2}A_{\xi,1}^2A_{\eta,1}^{*2}g^2t^2\prod_{j=1}^{q}\sqrt{(2N_{j1})!}(e_1^-)^2\cdot(e_2^+)^2[1-2\prod_{j=1}^{q}\frac{(N_{j2})!}{2N_{j2}+1}]/[(2p+1)\pi]^2.$$

The evolution of the state vector for the system made of arbitrary M subsystem can be rewritten as following

$$|\psi(t)\rangle=1/\sqrt{2}(\bigotimes_{j=1}^{q}|0_{j1},0_{j2},\cdots 0_{jM}\rangle+C'_1\bigotimes_{j=1}^{q}|2N_{j1}, 2N_{j2},\cdots,2N_{jM}\rangle)|g_{11}g_{21}, g_{12}g_{22},\cdots g_{1M}g_{2M}\rangle+C'_2\bigotimes_{j=1}^{q}|0_{j1},0_{j2},\cdots,0_{j(M-1)}0_{jM}\rangle|e_{11}e_{21}, e_{12}e_{22},\cdots, e_{1(M-1)}e_{2(M-1)}, e_{1M}e_{2M}\rangle+C'_3\bigotimes_{j=1}^{q}|0_{j1},0_{j2},\cdots, 0_{j(M-1)},2N_{jM}\rangle|e_{11}e_{21}, e_{12}e_{22},\cdots, e_{1(M-1)}e_{2(M-1)}, g_{1M}g_{2M}\rangle+\cdots+C'_{2^M}\bigotimes_{j=1}^{q}|2N_{j1},2N_{j2},\cdots, 2N_{j(M-1)},0_{jM}\rangle|g_{11}g_{21}, g_{12}g_{22},\cdots, g_{1(M-1)}g_{2(M-1)}, e_{1M}e_{2M}\rangle \tag{8}$$

It is found that atomic entangled state can be transfer into light-field entangled state under the time of their interaction satisfied the condition shown Eq. (6). The desired photon entangled state must be purified from mixed entangled state. However, when the following condition is fulfilled

$$t=2p\pi/[\sqrt{2}g|A_{\xi,k}||A_{\eta,k}|(N1_k)^{\frac{1}{2}}] \tag{9}$$

The Eq. (8) is simplified as

$$|\psi_a(t)\rangle=1/\sqrt{2}\sum_{\{n_{j1},\cdots,n_{jM}\}=0}^{\infty}F_{n_1}F_{n_2}\cdots F_{n_M}(|e_{11}e_{21}, e_{12}e_{22},\cdots,e_{1M}e_{2M}\rangle+|g_{11}g_{21}, g_{12}g_{22},\cdots, g_{1M}g_{2M}\rangle)\bigotimes_{j=1}^{q}|0_{j1},0_{j2},\cdots,0_{jM}\rangle \tag{10}$$

At this instance, the atoms are maximal quantum entangled state while light-field resume into ground state. Analyzing the procedure expressed by Eqs (8) and (10), the results shown that with time running, i. e. with the p value gradually increasing, the atoms and light-field are periodically in entangled and disentangled state respectively. And the atoms reach the maximal entangled degree while the light-field is disentanglement namely complete separate state and vice verse. So that two kinds of entangled states for photon and atoms are changed periodically and alternately. That is to say, They can be transformed periodically into each other. Its period is $T=\pi/[\sqrt{2}g|A_{\xi,k}||A_{\eta,k}|(N1_k)^{\frac{1}{2}}]$. It is closely related to the coupling constants between the atom and light-field, the complex factors of the atoms and photon operator, the photon number of initial light-field, and the photon number of interaction. But the coupling intensity λ between atoms never influence on its period.

It is important problem of fidelity in process of quantum states transfer because quantum information transformation with high fidelity has only practical significance. So according to the results obtained above, the fidelity of atoms in our model can be illustrated by means of numerical calculating. In order to describe the deviation between input and output quantum states, the important physical concept, that is fidelity, was proposed. The fidelity of mixed quantum states is firstly defined as[26]

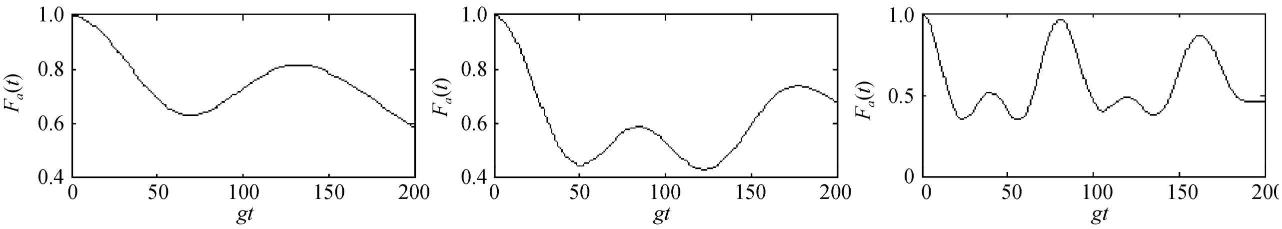
$$F(\rho_1,\rho_2)=\left[\operatorname{tr}\left(\sqrt{\rho_1}\rho_2\sqrt{\rho_1}\right)^{\frac{1}{2}}\right]^2 \tag{11}$$

where ρ_1 and ρ_2 indicate the density operator of two different states respectively, and $0\leqslant F(\rho_1,\rho_2)\leqslant 1$. In the special case that ρ_1 and ρ_2 are both pure quantum states, $\operatorname{tr}\rho_1^2=\operatorname{tr}\rho_2^2=1$, $F(\rho_1,\rho_2)=\operatorname{tr}\rho_1\rho_2$, when $F(\rho_1,\rho_2)=0$, quantum information (quantum states) are completely distorted in the procedure of transfer, namely, the initial state is perpendicular to the final state; when $F(\rho_1,\rho_2)=1$, it correspond to $||\rho_1-\rho_2||$ being close to zero.

If the atoms and light-fields are initially satisfied Eqs. (4) and (5) respectively, according to Eq (7), the system density matrix can be written as $\rho_s(t)=|\psi(t)\rangle\langle\psi(t)|$, and reducible density operators of atoms and light-fields denote as $\rho_a(t)=tr_f(\rho_s(t))$ and $\rho_f(t)=\operatorname{tr}_a(\rho_s(t))$ respectively. For simply, in Eq (1), let $q=1$, $N_{jk}=1$, we will consider the situation of the single-mode light-field interacting with single-photon in each subsystem. The fidelity of the atoms can be written as by Eqs (7) and (11)

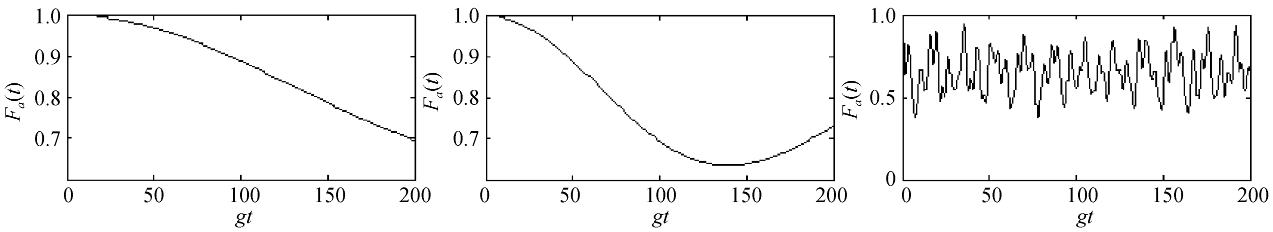
$$\begin{aligned}F_a(\rho_a(0),\rho_a(t))=&\left[tr\left(\sqrt{\rho_a(0)}\rho_a(t)\sqrt{\rho_a(0)}\right)^{\frac{1}{2}}\right]^2=\frac{1}{4}\sum_{n=0}^{\infty}|Fn|^2\left\{\left|1+\frac{n-1}{2n+3}(\cos\theta_{2n+3}-1)\right|^2+\right.\\&\left|\frac{A_\xi^{*2}A_\eta^2\sqrt{(n+1)(n+2)}(\cos\theta_{2n+1}-1)}{|A_\xi|^2|A_\eta|^2(2n+1)}\exp(\mathrm{i}2\Delta_{k,t})\right|^2+2\left[1+\frac{n}{2n-1}(\cos\theta_{2n-1}-1)\right]\left[1+\frac{n+1}{2n+3}(\cos\theta_{2n+3}-1)\right]+\\&\left.\left|\frac{A_\xi^2A_\eta^{*2}\sqrt{(n+1)(n+2)}(\cos\theta_{2n+3}-1)}{|A_\xi|^2|A_\eta|^2(2n+3)}\exp(-\mathrm{i}2\Delta_{k,t})\right|^2+\left|1+\frac{n}{2n-1}(\cos\theta_{2n-1}-1)\right|^2\right\}\end{aligned} \tag{12}$$

where, $\theta_m=\sqrt{2}|A_\xi||A_\eta|gt\sqrt{m}$. We can perform the fidelities of the system and the light-filed by the same method, writing as $F_s(t)$ and $F_f(t)$. The problems of $F_s(t)$ and $F_f(t)$ will discuss in other paper due to space is limited only. The evolution law of $F_a(t)$ may be directly displayed utilizing numerical calculating as showed in Fig. 2 and 3, where A_1 and A_2 are A_ξ and A_η respectively, n_0 is the initial mean photon number. From Fig. 2 and Fig. 3, we can see that the atoms fidelity evolution can present the oscillation property. Comparing with (a), (b) and (c) in two figures, we can also see that the oscillation period of F_a decrease with the increasing of n_0, A_ξ and A_η. It is revealed that more and more quick transforming from the atoms entanglement states into the cavity-field entanglement states. So the entanglement information appeared periodically in atoms and light-fields. And the transfer process has no correlation with atom-atom coupling constant λ. This completely agrees with the results discussed in former.



(a)A_1=0.1;A_2=0.1;n_0=1 (b)A_1=0.1;A_2=0.1;n_0=10 (c)A_1=0.1;A_2=0.1;n_0=100

Fig. 2 Time evolution characteristic of fidelity for atoms in different initial mean photon-number n_0



(a)A_1=0.05;A_2=0.05;n_0=1 (b)A_1=0.1;A_2=0.05;n_0=1 (c)A_1=0.6;A_2=0.6;n_0=1

Fig. 3 Time evolution characteristic of fidelity for atoms in different coefficient A_1 and A_2

3.2 The case of initial light-field in entangled state

At $t=0$, the field modes are in maximal photon entangled state given by $|\psi_f(0)\rangle = 1/\sqrt{2}\cdot(\bigotimes_{j=1}^{q}|2N_{j1},2N_{j2}\rangle + \bigotimes_{j=1}^{q}|0_{j1},0_{j2}\rangle)$ when the atoms are in ground state $|\psi_a(0)\rangle = |g_{11}g_{21},g_{12}g_{22}\rangle$. Using $|\psi_2^{(gg)}(t)\rangle$, Eq. (6) can be satisfied by adjusting the interaction time t properly. Then we have

$$|\psi(t)\rangle = 1/\sqrt{2}(D_1|e_{11}e_{21},e_{12}e_{22}\rangle + |g_{11}g_{21},g_{12}g_{22}\rangle)\bigotimes_{j=1}^{q}|0_{j1},0_{j2}\rangle + D_2\bigotimes_{j=1}^{q}|0_{j1},2N_{j2}\rangle|e_{11}e_{21},g_{12}g_{22}\rangle + D_3\bigotimes_{j=1}^{q}|2N_{j1},0_{j2}\rangle|g_{11}g_{21},e_{12}e_{22}\rangle + D_4\bigotimes_{j=1}^{q}|2N_{j1},2N_{j2}\rangle|g_{11}g_{21},g_{12}g_{22}\rangle \quad (13)$$

Where

$$D_1 = 16A_{\xi,1}^{*2}A_{\eta,1}^2A_{\xi,2}^{*2}A_{\eta,2}^2g^4t^4\prod_{j=1}^{q}\sqrt{(2N_{j1})!\,(2N_{j2})!}\cdot(e_1^+)^4(e_2^-)^4/[(2p+1)\pi]^4;$$

$$D_2 = -2\sqrt{2}A_{\xi,1}^{*2}A_{\eta,1}^2g^2t^2\prod_{j=1}^{q}\sqrt{(2N_{j1})!}(e_1^+)^2(e_2^-)^2\cdot[1-2\prod_{j=1}^{q}\frac{(2N_{j1})!}{(N_{j2})!\,(2N_{j2}+1)}]/[(2p+1)\pi]^2;$$

$$D_3 = -2\sqrt{2}A_{\xi,2}^{*2}A_{\eta,2}^2g^2t^2[1-2\prod_{j=1}^{q}\frac{(2N_{j1})!}{(N_{j1})!(2N_{j1}+1)}]\cdot\prod_{j=1}^{q}\sqrt{(2N_{j2})!}(e_1^+)^2(e_2^-)^2/[(2p+1)\pi]^2;$$

$$D_4 = 1/\sqrt{2}[1-2\prod_{j=1}^{q}\frac{(2N_{j1})!}{(N_{j1})!\,(2N_{j1}+1)}]\cdot[1-2\prod_{j=1}^{q}\frac{(2N_{j2})!}{(N_{j2})!\,(2N_{j2}+1)}].$$

Also from Eq. (13) we can know that entangled information stored initially in light-field can be transfered into the atoms which are initially in ground state and which light-field can back to vacuum state. There are also disturbs in Eq. (13). We can obtain desired entangled state by purifying or distilling. There is no loss of generality here since the formulas in Eq. (13) can be also extended to the following manifestation while the Eq. (9) is satisfied

$$|\psi(t)\rangle = 1/\sqrt{2}\sum_{\{n_{j1},\cdots,n_{jM}\}=0}^{\infty}F_{n_1}F_{n_2}\cdots F_{n_M}(\bigotimes_{j=1}^{q}|2N_{j1},2N_{j2},\cdots,2N_{jM}\rangle + \bigotimes_{j=1}^{q}|0_{j1},0_{j2},\cdots,0_{jM}\rangle)\cdot|g_{11}g_{21},g_{12}g_{22},\cdots,g_{1M}g_{2M}\rangle \quad (14)$$

Similarly, the atoms disentangle completely but light-field is in maximal photon entangled state. These evidences show that entangled information can be periodically swapped between atoms and cavity-field too, when either of them is initially entangled state.

Otherwise, if Eqs. (6) and (9) aren't satisfied, both of light-field and atoms are mixed entangled state during most of a period. If we want to obtain entanglement state the interaction time must be controlled by adjusting many elements properly. In this case, the fidelities of the system, the atoms and the light-fields can be calculated by using the same method as shown in Fig. 2 and Fig. 3. They may show directly the distortion of quantum information transfer process. These contents aren't given due to space on a printed page is limited only.

On the other hand, the coupling intensity λ between atoms has never influence on the transfer of entangled information. It should be noted that the λ have directly controlled reserving or swapping for W-type multi-atom entangled state by another calculating.

3.3 Discussion

The outcome mentioned above indicate that when $A_{\xi,k}=A_{\eta,k}=1$, $\xi=\eta=0$, and $q=1$, in this special case, there is not coherence among the atoms and among the cavity-field, that is, considering only the multi-photon interaction of coupling atoms with single-mode light-field. These are the same as Ref. [24]. As $A_{\xi,k}=A_{\eta,k}=1$, $\xi=\eta=0$, and $N_{jk}=1$, this result agrees with the entangled information transfer using the interaction of multimode cavity-field with the coupling atoms, reported in the Ref. [25]. The course of the single atom interacting with one mode light-field, that is, not considering coupling interaction between atoms, can be occurred under $A_{\xi,k}=A_{\eta,k}=1$, $\xi=\eta=0, \lambda=0, q=1$ and $N_{jk}=1$. The results should be exhibited in Refs. [22] and [23]. In contrast to the special case studied in Refs. [22-25] and this paper, one can immediately see that the results obtained under different conditions and this paper possess certain generalization.

4 Conclusions

We have shown that general evolution equations of the system via studying multi-photon interaction of the multimode coherent light-field with coherent coupling multi-atom. By analyzing the process of evolution, it is found that the phenomenon of the atom and the photon entangled state swapping each other periodically under either the atoms or the light-fields are initially in the entanglement states. The entangled information swapping transfer will be realized under the certain time condition. Its period is clearly related to many elements. The above results have simply proved via numerical calculating the fidelity of atoms.

In principle, the entangled information can be transferred using any photon interaction of any mode coherent cavity-field with any coherent atoms coupled by this method.

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相干耦合腔场中量子纠缠信息交换传递机理研究

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摘　要：提出了相干多模腔场-相干耦合原子相互作用系统的物理模型.利用全量子理论，研究了该系统中多光子相互作用过程的演化性质，结果表明：在多模腔场与相干耦合多原子相互作用过程中，多光子纠缠态与多原子纠缠态可以周期性的相互转换，在此过程中，同时可以实现纠缠信息的交换传递.通过对原子保真度的数值计算，给出了纠缠信息交换传递的图示说明.并进一步揭示出纠缠信息交换传递的一般特征.目前所报道的研究结果仅是本文所得普遍性结果在各种不同情况下的特例.

关键词：量子信息学；量子光学；量子纠缠；相干耦合腔场；相干耦合原子；交换传递

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