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THE CONCEPT OF FRACTAL COSMOS: II. MODERN COSMOLOGY

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SUMMARY: Development of the concept of fractal cosmos after Anaxagoras has been followed up to the present. It is shown how the concept reappeared in the early Renaissance as a vague idea and subsequently took up a concrete formulation at the beginning of the 20-eth century. The modern cosmology state of affairs has been considered in view of the fractal paradigm and the current disputes and controversies discussed. It is argued that the concept of the hierarchical cosmos is still alive and might become an essential ingredient within the modern view of the universe.

1. INTRODUCTION

In the previous article (Grujić 2001, to be referred to as I) we examined Anaxagoras' worldview and compared his hierarchical cosmos with that of Democritus. We analyzed a number of possible interpretations of his cosmology and argued that his ideas could be put in terms of the modern concept of fractal structuring of the material objects.

The idea of hierarchical structuring of the material world is an elaborate concept of a simple general principle that has underlined almost all cosmologies in the ancient world, not only European one, the assertion that *microcosmos is equivalent to macrocosmos*. This postulate, on its part, stems from the principle of economy, that has been best formulated by William Occam (Occam's razor). In practical social sphere all ages have witnessed various realizations of this "equivalence principle", which has been best epitomized by temples conceived as miniatures of the entire cosmos, as the most conspicuous case of the gothic cathedrals shows.

In the next chapter we shall see how a number of European thinkers took up the idea in the most primitive form. Then, in the following chapters we consider the fractal concept within the last century cosmology. Next, we present some modern concepts of the hierarchical universe, and discuss some aspects of the observational evidence and its possible interpretations, as appears the subject of the current controversies. Finally, we discuss the fractal paradigm from the epistemological point of view and outline future possible developments of the subject.

2. RENAISSANCE AND POST-RENAISSANCE EUROPE

Anaxagoras had no direct following, either in Antiquity, or in Medieval Europe. But a line of his thought continued *via* Aristotelian and Platonic tradition as a concept of hierarchical world of reality (Copleston 1976). The first prominent representative of this concept was Nicolaus of Cusanus (b. 1401), who held that in each particular object the whole universe is reflected. The universe exists in every finite thing, as *contracté*, and cosmos consists of a multitude of single entities, each related to the other and to the whole in such way that one can speak of "unity in a multitude" (Copleston 1976).

In particular, the human is microcosmos, and comprises in itself material and intellectual reality, and hence appears as a small universe, or world. Giordano Bruno (1548-1600) conceived the universe as an infinite reality, with a multitude of (equivalent) worlds, in both Democritus and democratic manners. But at the same time he adopted Cusanus' idea of material accidental minimum constituents, which he dubbed *monas*, as counterparts of mathematical unities. This idea was further developed by Franciscus Mercurius van Helmont (1618-1699) in Belgium, who conceived the material world as a collection of monadas, which may form complex structures.

All these doctrines were mixed with religious and sometimes mystic ingredients and could be considered a part of a speculative philosophy. The crown of this monadic approach was the doctrine of Godfried W. Leibniz (1646-1716). Though he met van Helmont, he presumably developed his doctrine of monadas independently. Leibniz's concept of monad appears neither simple nor clear. As Hartmann noticed (Hartmann 1946) the construct is full of apparent contradictions, many of which arise due to an inherent mind *versus* matter dichotomy. It seems that Leibniz incorporated both Abderian atomistic point of view and Anaxagoras' world picture, as can be seen from the following passages from *Monadologie* (Leibniz, 1914; we retain the original transcription).

> *3. ...Et ces monades sont les véritable atomes de la nature et en un mot les éléments des choses....*
>
> *65. ..., parce que chaque portion de la matiére n'est pas seulement divisible a l'infini, comme les anciens ont reconnu, mais encore sous-divisée actuellement sans fin chaque partie, dont chacune a quelque meuvement propre: autrement il serait impossible, que chaque portion dela matiére pût exprimer l'univers ...*

This dichotomy is resolved, however, if one observes that the atomic aspect stems from the Leibniz interpretation of monad as a spiritual entity, which reflects the totality of the world, in the informational sense. The hierarchical aspect is based, on the other hand, on a biological *ansatz*, reflecting the properties of the living bodies, as revealed by then recently discovered microscope. This was exactly what Anaxagoras inferred (see I), without a microscope. On a wider scale, however, one might argue that Leibniz's monadic affinities stem from his general adherence to *continuum* and belief that *Natura non facit saltus*. Indeed, he used to play with geometric structures that fill in the space as much as possible, like that *Leibniz packing* of a circle, as Mandelbrot (1983) dubbed it. He even ventured to meditate that the exponent k in $\left(d/dx\right)^k F(x)$, in his newly developed *calculus*, need not necessarily assume integer values only, as he mentioned it in a letter to l'Hospital (see, e.g. Mandelbrot 1983).

3. NEWTONIAN ERA

The idea that monad is coupled to the entire cosmos, albeit in a spiritual sense, may be regarded as a particular aspect of a concept of universal interaction between (material) cosmic objects, or at least mutual correlations. The physical basis of this idea was the Newton's concept of the universal gravitation (Newton 1687), which Leibniz, understandably, failed to invoke. But the law of the universal attraction was not compatible with the assumption of an infinite universe, as advocated by Giordano Bruno, for instance. Newton accepted the same hypothesis, but was warned by a number of people, like a young theologian Richard Bently in 1692, who raised the question of the stability of the stellar systems. This was a precursor to the later conundrum known as Seeliger-Neuman's paradox. Another objection was put by a physician and antiquarian William Stuckley (1720) concerning the luminosity of the night sky (which should appear to our eyes like the Milky Way, he argued). The latter question was subsequently discussed by Newton, Stuckley and Halley (during the breakfast at Newton's home) as the latter reported at the Royal Society: "Another Argument I have heard of urged that if the number of Fixed Stars were more than finite, the whole superficies of their apparent Sphere would be luminous" (see, e.g., Redhead 1998). The question seems to be raised by other people even before that occasion and might be traced as far back as to Kepler (see, e. g., Mandelbrot 1983). This was the beginning of the puzzle known today as Olbers' paradox (see, e.g. Martinov 1965), though some authors prefer the term *Blazing Sky Effect*.

We see, hence, that the concept of an infinite world is not that simple as might have appeared to his proponents. Obviously, from the point of view of those who were concerned with the stability and luminosity of the apparent cosmos, a paradigm of a static, infinite and homogeneous solution (model) would not do. Newton himself became aware of these problems and planned to tackle them in a new edition of *Principia* (which he never accomplished), playing, for instance, with the stars of various "magnitudes", an idea along the hierarchical model line of thought. The important thing to note is, nevertheless, that with Newton a qualitatively new approach to the cosmology as such has been adopted, that based on astronomical observations and mathematical analysis. This was a direct outcome of Galileo's new methodology, which was a quantitative analysis, as opposed to the speculative and qualitative scholastic considerations in his time. The newly introduced force of universal attraction made the universe a physical system, instead of a mere collection of celestial bodies. But, the same interaction, being both universal and essentially attractive, raised serious question as to the stability of the universe. Coupled with the assumption of an infinite universe, both in spatial and temporal sense, this new aspect of the physical reality made the cosmology a science, but not an easy one.

Kant's picture of the universe (Kant 1925, 1968) was essentially Abderian one. Kant boldly inferred from the available observational evidence that the so-called nebulae were extragalactic systems, like our own Galaxy. He also considered the conspicuous lack of homogeneity in the observable universe (Kant 1755, 1925, 1968):

> *"This part of my theory which gives it its greatest charm ... consists of the following ideas ... It is ... natural ... to regard [the nebulous] stars as being ... systems of many stars ... [They] are just universes and, so to speak, Milky Ways ... It might further be conjectured that these higher universes are not without relation to one another, and that by this mutual relationship they constitute again a still more immense system ... which perhaps, like the former, is yet again but one member in a new combination of numbers! We see the first members of a progressive relationship of worlds and systems; and the first part of this infinite progression enables us already to recognize what must be conjectured of the whole. There is no end but an abyss ... without bound."*

Kant adopted, also the idea of a changing universe, with cosmoses arising and disappearing, in a repetitive manner (1755, see Kant 1925), thus promoting Anaxagoras' and Democritus' ideas of the plurality of worlds. Another important contribution of Kant to the subject was the first, albeit qualitative, conception of the mechanism of the creation of our Solar system, which meant Anaxagoras'cosmogony (we use the term *cosmogony* in a wider sense, referring to the universe, see I) put into more concrete and realistic physical terms. When Laplace put forward, independently, his cosmogonical model (1797, see Laplace 1925), based on the atomistic hypothesis, the revival of the Presocratic cosmology was almost complete.

Rugiero Boscovich. In an attempt to reconcile Newton's and Leibnitz's ideological backgrounds concerning the nature of space and time and the perennial questions regarding the divisibility of matter Boscovich (1711-1787) discussed the latter in his treatise from 1758, 1763 *Theoria Philosophiae Naturalis* (Boscovich 1922). Boscovich's solution to the problem of an infinite divisibility of matter is purely geometrical one. Making use of his primary construct of material points, he defines particles of the first, second, *etc* order. Thus, we read (III.395):

> *A given mass, however small, distributed over a given space, however large, so that there remains no small and empty space larger than a given, no matter how much small space, without any particle of that mass.... We understand that this small mass is divided into as many particles and that each of them is placed in a small volume. They can further be divided at wish, so that new parts of each particle cover wall of this small volume... "*

Although no direct reference is made concerning Kant hypothesis expounded above (Kant's work appeared 3 years before Boscovich's), it resembles the former, at least formally. As for Boscovich's solution of the problem of maximum filling the space with a finite amount of matter, will reappear in the cosmological work of Fournier, as we shall see later on.

XIX century witnessed no significant advance concerning our inference into the structure and dynamics of cosmos. The principal concern was to resolve the paradoxes related to an infinite world assumption, as raised by Newton's contemporaries. In 1826 Heinrich Olbers (1758-1840) made the luminosity paradox even more astounding, by noting that the sky should be as luminous as the Sun surface. His argument runs like this. Since in an infinite universe our line of sight should encounter a star wherever we look at the sky, the latter should shine like a surface of any star, and like our Sun, for that matter. Moreover, if one integrates the total electromagnetic radiation coming from all stars in the universe, one obtains an infinitely bright sky. Of course, when one accounts for the screening effect, the net brightness is reduced to a finite value (Martinov 1965).

Equally disturbing was the gravitational paradox, known as Seeliger-Neuman paradox, as first formulated by Seeliger in Astr. Nachr. No 3273 (1895). If a celestial object, like a star, is surrounded by an innumerable like objects in an infinitely extended cosmos, the net gravitational force is undetermined both in direction and magnitude, being of the form $\infty - \infty$, and may, therefore assume any value within an interval $(0, \infty)$. One might be tempted to circumvent this inconvenience by assuming a spherical symmetric distribution of surrounding masses, but this would violate the assumption of an homogeneous cosmos, which in its turn is just one of aspects of the general Copernican principle. One notes that there would be no screening effect in the case of gravitational interaction, since any object is transparent to the force of universal attraction. This difference with respect to the luminosity paradox stems from the essentially different nature of the physical quantities involved. The light (electromagnetic radiation) is a dynamic physical field, which propagates through the empty space with a finite velocity c, and interacts with matter in various ways, including absorption, reflection, etc. Gravitational field is a static quantity, ever and everywhere present, not propagating and thus never absorbed by the matter of any form and nature. (We disregard for the moment the hypothetical existence of gravitational waves, predicted by General Relativity).

It is with these dilemmas that the cosmology entered the twentieth century.

4. EARLY TWENTIETH CENTURY

4.1 Some theoretical preliminaries

Since the appearance of Einstein's General Relativity in 1915, cosmology gained the ground for becoming a scientific theory. Further development of cosmology relied on two premises. First, there exists a fundamental theory, Einstein's or otherwise, which can provide a mathematical tool to formulate a selfconsistent picture of our universe. Second, formal solutions and cosmological models should satisfy a number of fundamental postulates. The latter are the so-called *cosmological principles* (see, e. g. Narlikar 1979, Barrow and Tipler 1986). The first (strong) cosmological principle asserts that the universe at every instant of the (global) cosmic time appears *isotropic* and *homogeneous*. These postulates are of a geometric nature and are satisfied, separately or both, by a number of cosmological paradigms. The second of these appears more general, for if a medium is homogeneous, it is isotropic too, but the opposite does not hold. If only the first of the two is satisfied, a number of Newtonian models can be constructed (Thatcher 1982). In particular, Lemaitre's model belongs to this class, too (Narlikar 1979). These models constitute a class of *monocentric* models, with a singular, space point singled out as a centre of the universe. From this point, universe looks the same in all directions. If this property of the universe is satisfied for any arbitrary point (or observer), the universe is homogeneous, and the second principle is satisfied, the postulate of *homogeneity*. Finally, the so-called *Strong cosmological principle* requires that the universe remains the same (identical to itself), regardless of the flow of (cosmic) time. This *time homogeneity* principle is usually called *The perfect cosmological principle*, for it comprises all three symmetries - rotational and translational in space and time.

What is the role of these principles? Their epistemological status appears at least twofold. First, since they invoke a particular symmetry within an abstract space, they simplify the mathematical problem of solving corresponding equations. But their meaning does not exhaust itself by these technical benefits. They carry, albeit implicitly, strong motivations from outside of the purely scientific sphere, e. g. esthetic, even religious. To some cosmologists, these principles play a role of a cosmological dogma, and carry thus a flavour of faith (see, e.g. Ribeiro and Videira 1998, on dogmatism in cosmology). Considering that an observational evidence may be vague and indecisive, as the case with observational cosmology often is, these prejudices may play crucial role in interpreting the data and are usually behind the ensuing disputes, as ideological background.

4.2 The observational evidence

Until the advent of large telescopes the cosmology was to a large extent a speculative subject, though the Kant-Laplace hypothesis could be considered scientific, albeit qualitative, approach. Despite Giordano Bruno's and Kant's concepts of an immense, if not infinite, universe, cosmos consisted of the visible (naked eye) sky, the largest stellar system being our Milky Way. Stars remained the elementary constituents of the cosmos, though a number of sky objects of obscure nature, like nebulae could be discerned on the night sky (but see I for Kant's inference on the matter). In the absence of a reliable method to determine distances of objects seen on the sky, the cosmos remained within the realm of our Galaxy. But even within this limited picture, the similarity principle was operative, with a clear analogy between our planetary system and the galactic disk. Both systems possess an axial symmetry, pointing toward the same physical mechanism of forming these rotational structures. It was this analogy that inspired Lambert (see Charlier 1922) to conceive the world as a generalized planetary system.

The breakthrough was made first in 1912 by Henrietta Leavitt (1868-1921) (see, e. g. Aczel 1999, for a popular account), who established a relation between Cepheid variable average apparent brightness and its period of apparent magnitude variation (period-luminosity relation). This enabled her to define a standard astronomical means for estimating distance from systems containing Cepheids. In 1917 Vesto Slipher from Lowell observatory published a paper where he showed that the spiral nebulae were receding from us with an immense velocity, according to his measurements of Doppler red-shift of these objects, which he considered to be intragalactic. But it will take another 12 years for Hubble to establish that these nebulae were extragalactic replicas of our own Galaxy and that they recede with the speed proportional to their distances from us (Hubble's law, see e.g. Peebles 1993 for a more detailed account of the matter). These findings will have a dramatic effect on our picture of the universe, as we shall see later on. But before proceeding along these lines, we first turn to the early attempts to reconcile the idea of an infinite universe with observational evidence, as required by two cosmological paradoxes. And we thus turn to the concept of fractal cosmos.

4.3 Charlier's fractal model

The first hint at the possible hierarchical cosmos was made by Fournier d'Albe (1907), who devised a curious geometrical structure obeying the so-called octahedral principle (see, e.g. Mandelbrot 1983). His cosmos had the property that the content of matter within each sphere was proportional to its radius. This condition was sufficient to protect the universe against both cosmic paradoxes. (Such a model yields the fractal dimension $D = 1$, *i.e.* one-dimensional cosmos, see later).

Following an idea due to Kant's contemporary Lambert, Charlier conceived the fractal universe with galaxies as elementary constituents (Charlier 1908, 1922), each containing N_1 stars (see, e.g. Ribeiro 1994 for a more detailed account of the contributors to the concept of a fractal cosmos in the last century). He then defines a cluster of galaxies G_1 as a generalized galaxy, with N_2 constituent members, then a cluster of clusters G_2 with N_2 members, etc. Then to each of consecutive constituents G_i radius $R_i, i = 0, 1, 2, ...$ is ascribed, as well as the corresponding masses M_i. Assuming, for simplicity, a spherical shape of all (sub)units G_i Charlier takes that N_i units fill up G_i in such a manner that an effective radius ρ_i is defined as

$$\rho_i = \frac{R_i}{N_i^{1/3}} \tag{1}$$

so that an average distance between two nearest neighbours in G_i is $2\rho_i$. One has for the masses the following relation (M_0 is the mass of a star)

$$M_i = N_i M_{i-1} = N_i M_{i-1} N_{i-2} \cdots N_2 N_1 M_0 \tag{2}$$

With these definitions Charlier shows that both paradoxes disappear.

Seeliger's paradox. The aim is to demonstrate that the net Newtonian force on a system G_i is finite. If one assumes the most unfavourable situation of G_i situated at the edge of G_{i+1}, the net gravitational force on G_i will be the sum of all forces from the cosmic matter

$$F_N = \sum_{i=1}^{\infty} \frac{M_i}{R_i^2} \tag{3}$$

From the requirement that the sum converges (finite value) one has from (2) and (3)

$$\frac{R_i}{R_{i-1}} > \sqrt{N_i} \tag{4}$$

If this condition is met, the overall force exerted by all cosmic gravitational sources on a single subunit will remain finite.

Olbers' paradox. If the luminosity of a galaxy G_i at a distance ρ_i from the observer (at Earth, more precisely at the Sun) is h_i and the total luminosity of G_i (counted from its centre) is L_i, and if one assumes that G_i is situated at the centre of G_{i+1}, the total electromagnetic energy influx at Earth would be

$$L = \sum_{i=1}^{\infty} L_i \tag{5}$$

The number of G_i in G_{i+1} within the sphere of radius r is then

$$n_i = \frac{r^3}{\rho} \tag{6}$$

and the apparent luminosity is

$$h_i^{ap} = h_i \frac{\rho_i^2}{r^2} \tag{7}$$

The total apparent luminosity stemming from all G_i is (after integrating over all layers from 0 to R_i)

$$L_i = 3 h_i N_i^{1/3} \tag{8}$$

The next step now is to find out the relationship between h_i and h_{i+1}. It is interesting to note here that in his original derivation (Charlier 1908) Charlier made a mistake and the following treatment was due to Seeliger, who sent to Charlier his derivation in a letter (Franz Selety derived the same relation independently, and communicated it to Charlier, after the paper by the latter was submitted).

One starts from the luminosity h_2 of G_1 at the distance ρ_2

$$h_2 = N_1 h_1 \frac{\rho_1^2}{\rho_2^2} \tag{9}$$

which gives a general relationship

$$h_i = h_{i-1} N_{i-1} \frac{\rho_{i-1}^2}{\rho_i^2} \tag{10}$$

From (8), (9) and (10) one obtains

$$\frac{L_i}{L_{i-1}} = N_i \frac{R_{i-1}^2}{R_i^2} \tag{11}$$

The sum (5) to remain finite one has

$$N_i \frac{R_{i-1}^2}{R_i^2} < 1 \tag{12}$$

which is equivalent to (4). Thus, one arrives at the remarkable property of Charlier's universe that it solves both Seeliger-Neuman's and Olbers' paradoxes.

This result might have been anticipated on the general grounds, considering that both gravitational force and apparent luminosity follow the same fall-off behaviour having regard to the distance from the observer. The latter property, in its turn, stems from the fact that the gravitational force can be expressed in terms of lines of force, whereas the electromagnetic radiation may be represented by an emission of particles (photons), and both emanations obey the Euclidian geometry.

Charlier showed also that the same conclusion holds if the assumption of the central galaxy nesting is relaxed. He also derived the relationship between the distances between members of each galaxy

$$\frac{\rho_i}{\rho_{i-1}} > \sqrt{N_{i-1} N_i} \tag{13}$$

What happens if a body (star, for instance) falls from the galaxy of the next higher order into a galaxy, with a zero initial velocity? Charlier considered that problem, too and found the relationship between final velocities attained

$$\frac{v_i}{v_{i-1}} < N_i^{1/4} \tag{14}$$

Finally, Charlier derived another remarkable property of the galaxies (conceived as subunits within his hierarchical scheme), namely that all subunits have the same period of (Keplerian) motion

$$T = \frac{\sqrt{\frac{3\pi}{\delta}}}{G} \tag{15}$$

where G is the gravitational (Newton's) constant and δ the mean density of a galaxy. Hence, each galaxy is characterized by its own unique period. This property, which appears a specific realization of Poincaré's cycle theorem (see, e.g. Pars 1965), makes the whole fractal model even more appealing.

The most remarkable property of Charlier's universe is that the overall (mean) density of his hierarchical cosmos is zero. It is this feature that stands behind, albeit implicitly, the above resistance to the gravitational and luminosity paradoxes. This feature will prove significant for further development of the modern cosmology, as we shall see below.

Before we proceed further, two points must be stressed here. First, at the time Charlier contrived his hierarchical model no evidence for the fractal structure was available. Second, Charlier did not address the question as to in which way the structure he proposed might have formed. Both questions turned out tricky ones, up to the present time.

4.4 The expanding universe

The year 1922, when Charlier's paper appeared, witnessed several remarkable events, which proved to be crucial for the further development of science. While working with Sommerfeld, Werner Heisenberg conceived semiclassical (in modern parlance) models of Hydrogen and Helium, with half-integer quantum numbers (see, e.g. Briggs 1999). The models reproduced the experimental data on the Zeeman effect and the ground-state Helium energy remarkably well, but Heisenberg gave up publishing his results after a fierce Bohr's opposition to the idea of non-integer quantum numbers. This marked the end of the pursuit for atomic models based on the concept of the electron trajectory, and led eventually to Heisenberg's discovery of the Matrix (quantum) mechanics. In the same year Einstein was awarded Nobel prize for his theoretical explanation of the photo-effect. The idea behind the theory was the much disputed concept of a quantum of (electromagnetic) energy, later to be dubbed *photon*, that was a sort of resurrection of Newton's corpuscular concept of light. Encouraged by these events, two years later De Broglie published his formula that related the momentum of a microscopic particle and an associated wavelength, which turned out instrumental for developing the Wave mechanics by Schrödinger. (Heisenberg's non-integers will be rediscovered in modern semiclassical theory as the so-called Maslov's index, see, e.g. Percival 1977).

Russian mathematician Friedmann published in 1922 his dynamical solution of Einstein's equation of General Relativity, where he demonstrated that the universe could have started from a singular point-like state of infinite density and then expand. This dynamical model proved not only realistic, as the subsequent observations corroborated, but solved at he same time several problems related to Einstein's (and Newton's, for that matter) static cosmological model. We enumerate them here.

Stability problem. As is known from the science of mechanics, no stable structure could be maintained in a static system where constituents interact by attractive and/or repulsive forces. It holds for all attractive force, like Newton's gravitational interaction, and for Coulombic systems, like a plasma of ions and electrons. To overcome this shortcoming of his static model, Einstein introduced the concept of mixed interaction, combining the attractive force with a hypothetical long-distance repulsive interaction. As is well known this remedy, which is essentially the concept of a molecular force, was introduced via so-called *cosmological constant* Λ (see, e.g. Collins *et al* 1989), as appearing in the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \frac{\Lambda}{c^2}g_{\mu\nu} = \frac{-8\pi G_N}{c^4}T_{\mu\nu} \qquad (16)$$

where $R_{\mu\nu}$ is the so-called Ricci tensor, R is Ricci scalar, $g_{\mu\nu}$ is the metric tensor and $T_{\mu\nu}$ is the energy-momentum tensor, to save the phenomena. Generally, one may establish a dynamical equilibrium in a system with monotonous forces, like Newton and Coulomb ones, either by rotation, or expansion of the system. While the first approach appears legitimate for microscopic systems, like atoms (see, e.g. Grujic 1999), the concept of a rotating cosmos raises a number of conceptual difficulties, which do not appear, however, unsurmontable (see, e.g. Reboucas and de Lima 1981, for the theoretical aspects, and Birch 1982, for the observational evidence).

Much simpler solution to the stability of cosmos is the expansion hypothesis, which asserts that all galaxies, as the principal cosmic constituents (building blocs), recede from each other. Formally, this represents *a homothetic transformation*, which makes all distances in the universe increase in time. This assumption removes the stability problem by *tour de force*, since the possibility of two bodies, e.g., to fall into each other is simply excluded by making them separating apart all the time. The overall system appears stable being put into a collective motion, which by itself prevents gravitational collapse. But this assumption has a weak points too, as we shall see now.

Thus, the concept of an expanding universe proved to be fatal for all static models, including Charlier's one, just as the concept of microphysics without classical trajectories turned out lethal to the classical models of atomic and subatomic systems.

The overall expansion can not be a universal process in our universe. Galactic and subgalactic systems do not obey this rule, as we know. Whatever the mechanism of galaxy formation is invoked, the overall rotation appears a dominant feature of the collective dynamics. Of course, when one goes to the lower physical levels, other forces enter the game, like chemical, Coulombic, *etc*, and the problem of stability takes on various forms as one goes from one system to another, or from one physical level to the other. Hence, when talking about the universal expansion, one must keep in mind that it refers to a particular (possibly undetermined) cosmic domain, starting from galaxies to a possibly upper level of the cosmological reality. In particular, it has been shown that atomic dimensions have not been noticeably changed since the early cosmic era, when atoms formed.

The problem of stability of (sub)galactic structures in the context of a particular cosmological model appears as an important aspect of the cosmology in the most general term, but we shall not dwell on it here.

As it often turns out, an idea in natural science has a precursor in an accidental mathematical result. Modern cosmology is no exception. The first expanding "universe" was treated by de Sitter, as early as 1917, a year after Einstein set up his General Relativity equation (16) without Λ). Cosmology, like physics, for example, is an exact science, and the latter is an art of approximations, i.e. of inexactness. De Sitter found the solution of Einstein's equation by reducing the content of matter (and radiation altogether) to zero. By doing so, he found the solution for the *scaling parameter* S of the form

$$S(t) = e^{Ht} \tag{17}$$

where H is Hubble (time independent) constant, which can be defined by (Narlikar 1977)

$$H^2 = \frac{1}{2}\Lambda \tag{18}$$

Note that the zero-content universe is not *no−universe*. In particular, *the empty space* is not an Euclidean one, but is endowed with the structure (Minkowsky space). Though trivial at the first glance, de Sitter's model proved very important from the heuristic point of view. First, it gave an idea of a dynamic cosmos and second, the later paradigm of an inflatory universe (see, e.g. Collins *et al* 1989) was inspired directly by de Sitter's solution. What is of a particular importance to us here is that, as we mentioned above, Charlier's cosmos had a zero-average-density property, too.

Cosmological paradoxes. When writing his famous paper in 1922 Charlier was well aware of the (preliminary) observational evidence for galaxies receding from us (in fact, his own observatory at Lund was much engaged in this sort of astrophysical investigations, as Charlier himself mentioned). He seemed to feel that that "small cloud" on the otherwise bright "fractal sky" might be announcing a storm. The latter did materialize in solving automatically both paradoxes, within the expanding cosmos paradigm. We first consider the luminosity paradox.

The explanation runs in two ways. The first refers to the finite age of cosmos, which is implied by the one-way collective motion (eventually as a part of a cyclic dynamics, at the worst). According to (17) the estimated age would be $t_0 = \frac{1}{2H}$. Accordingly, the amount of the electromagnetic (or of any other type) energy that can reach us must be finite, since only a finite portion of the (observable) cosmos is an available source of light. The second type of explanation relies on the red shift as Doppler effect (see, however, Voraček 1985, 1986, for other interpretations). From Hubble law

$$z \equiv \frac{\Delta\lambda}{\lambda} = \frac{Hr}{c} \tag{19}$$

where r is the distance from a galaxy, the net flux from a spherical layer through a unit area plane is (Martinov 1963)

$$\phi = 2\pi NLdr \qquad (20$$

where N is the number of stars in a unit volume, and L the average luminosity of a star. If one assumes that the star spectre is Planckian one, after integrating over distance and frequency

$$\Phi = \frac{2h}{c^2}\int_0^\infty dr \int_0^\infty \frac{\nu^3\gamma^4}{e^{\frac{h\nu}{kT}\gamma} - 1} d\nu, \ \gamma = 1 + \frac{Hr}{c} \tag{21}$$

where h is Planck constant and T temperature, one arrives at the finite flux at any point in the space

$$\Phi = \frac{2\pi cNL}{3H} \tag{22}$$

We note that the same conclusion is reached if a rigorous relativistic expression for the red shift

$$z = \frac{(1+\frac{v}{c})^{1/2}}{(1-\frac{v}{c})^{1/2}} \tag{23}$$

where v is the velocity of the source, is used. We note, also, that more realistic upper integral boundaries in (21) (instead of ∞) would further reduce the value of Φ.

We note that in order to replace a static Newtonian model, as Charlier's one, the model of an expanding universe had to invoke the "most heavy artillery" of the century, the quantum physics and Relativity theory. Finally, we mention here the well known fact that an expanding universe is not a unique cosmological paradigm, though it has been widely accepted and considered a standard model. For the alternative paradigm, Steady state hypothesis, see, e.g. Narlikar (1974) (see Arp *et al* 1990, for a more recent account of the relevant observational evidence).

5. THE FRACTAL PARADIGM

In a preceding section we enumerated some cosmological principles that govern a number of cosmological paradigms. Where the fractal, self-similar cosmos stands in this context? In particular, is a fractal cosmic structure endowed with isotropy and homogeneity? It possesses the so-called *local isotropy*, that is all cosmic points are equivalent concerning the isotropy (no special cosmic points), but these points are not uniformly distributed (see, e. g. Saar 1988, Einasto *et al* 1988, Sylos Labini *et al* 1998). Hence, fractal cosmos *is not* homogeneous.

5.1 Some cosmogonic remarks

As we mentioned above, Charlier did not address the problem of structure formation, while setting up his hierarchical model, as an *a priori* concept. The question arises, however, whether his model contradicts the concept of ever expanding universe. We address this question briefly, before reviewing the present day state of affairs.

It is a common evidence that the cosmos in our surrounding is neither homogeneous nor uniformly expanding. These facts are not independent, of course, and can be used to explain the present state of the cosmic inhomogeneous distribution of matter, in particular of galaxies. We know that not all galaxies are receding from us (as the case with Andromeda shows). Moreover, a large scale cosmic contraction towards Virgo cluster (the Great Attractor) convinces us that perturbations with regard to the overall expansion are possible and may be of considerable magnitude. All these phenomena are connected with the problem of the (gravitational) stability of the cosmic matter, or with the concept of gravitational collapse. It is this instability that might be responsible for the large-scale structures, as observed today, and which might be operative in forming a fractal distribution, at least up to certain cosmic scales (but see later). We shall now consider this phenomenon within the modern cosmological perspective.

5.2 Fractal or nonfractal, the question is now

Hence, is the universe fractal or not? As we have seen, the observed nonuniform distribution of galaxies (clusters, superclusters, etc) does not contradict the principal assumption of the majority of modern cosmological models (at least those based on General relativity) that the universe is homogeneous and isotropic (cosmological principles). The question is now could the two concepts, uniform universe and fractal cosmos, be reconciled. To put it differently, the question is, if a hierarchical structure does appear, up to which cosmic scale it persists. The next question is if an eventual fractal distribution is observed, could it be described by a simple fractal model, or one must contrive a multifractal cosmos (see I)?

A proper approach to the fractal issue should encompass at least the following points (questions):

(i) If at least a part of the universe has a (multi)fractal structure, what would be the mechanism of its hierarchical formation?

(ii) How should this (multi)fractal structure be conceived from the point of view of an external (metaphysical, see later) observer?

(iii) What would our observational evidence look like that should corroborate such a particular form of the observable universe?

5.2.1 Fractal formation

As we saw in I, Anaxagoras was content with the solution of the type Νοῦς από μεχανές (*Deus ex machina*). We need, of course, a more convincing explanation. It must be said rightaway here that there is no a general model for fractal formation in nature (or any other structure, for that matter), and each particular physical system requires a special mechanism, if conceived at all (see, e.g. Gouyet 1996). The situation is different in the purely mathematical domain, of course, as we showed in I (see, e.g. Ribeiro 1994). In the case of mathematical physics (e.g. nonlinear dynamics) fractals do appear, but as phenomenological objects, that has, however, a restricted heuristic value (see, e.g. Handke 1994).

One can distinguish two possible scenarios for structuring the universe. One proceeds from a unique structureless universe, which then falls apart in a specified manner. This "scenario from above" comprises more than unique initial state, like a uniform universe, a unique (cosmic) superstring, *etc.* The other scenario "from below" assumes the existence of initial small agglomerations, which in due time merge to form larger and larger clusters. Both scenarios are relevant to the issue of the fractal cosmos (see, e.g. Hogan 1980).

Without claiming to be exhuastive on the subject, we quote a number of possible mechanisms of forming a hierarchical cosmic structures. Within the "from below" scenario, one possible way to concentrate cosmic matter would be based on the gravitational instability, that might force objects like galaxies (conceived in general manner, as Charlier did) to make agglomerations, as transient units on the way of gravitational collapse. The peculiar velocities of celestial systems, which divert from the overall Hubble flow, might be considered as evidence for such a tendency. If the fractal paradigm is accepted, this picture appears to be in conformity with the general idea of the cosmological collapse, within the concept of a closed universe, followed by Big Bang (cyclic models).

Self-gravity as a structuring force

Fractal structure arising from self-gravity may form in various cosmic media, from the interstellar matter to galaxy clustering. Methods developed are based on the renormalization group, used extensively in many areas, like the quantum field theory, statistical physics, condense matter *etc.* If the system with an hamiltonian H possesses scale invariance, represented by an operator $\mathbf{R}$, a series of successive hamiltonians are generated by

$$H_{n+1} = \mathbf{R} H_n \tag{24}$$

All hamiltonians have the same structure, but different values for the parameters they include. Within Zel'dovich model, which belongs to the top-down scenario, the collapse of matter starts from large scale, and subsequently one-dimensional (filaments) and two-dimensional (sheets) structures are formed (Zel'dovich 1978; for other approaches see, e.g. Combes 1999 and references therein).

In a recent series of papers a statistical method, exploiting the scale-invariance of the gravitational force, has been developed, within down-top scenario (see, e.g. Vega *et al* 1998, Combes 1999), which comprises both interstellar matter and galaxies. The latter arise from the collapse of smaller structures, and the structures at higher levels are formed, in a non-linear manner, with various effects, like the turbulence, self-criticality *etc* accounted for. This thermodynamic approach appears general enough to promise a unifying picture of fractal structuring phenomena.

Strings and the cosmic structuring process

If the former mechanism is ascribed to the later phase of the universe evolution, the cosmic strings hypothesis pertains to the cosmogonic era. According to some authors, the cosmic strings, which are not to be confused with the strings from the quantum field theory, but are defined as topological defects of the early cosmic space-time manifold (see, e.g. Collins *et al* 1989), might be instrumental to the galaxies formation. Two different mechanisms are envisaged within the context. First, matter was accumulated around the early formed cosmic strings, giving rise to the today observed galaxy distribution. Another possibility is that the strings, endowed with angular momenta, were torn apart by the centrifugal forces, and the process proceeding in a number of consecutive phases. At the first stage the string broke into large pieces, from which superclusters arose. Then at consecutive stages the pieces broke themselves into smaller parts, from which clusters and galaxies formed.

Presumably, two classes of the cosmic strings, as primordial topologically stable objects, formed at the time of phase transition in the very early universe, are defined, the open and closed (loop) strings. Both may be considered candidates for precursors of the observed cosmic structures. Here we mention a hybrid model, which starts with open strings that break and form string loops, which themselves break into smaller loops, *etc*, then the matter is gradually accreted around them, and the Abell clusters are invoked as an observational support for the model (Turok and Brandenberger 1985). Here, we shall expose in some detail the model suggested by Tassie (1986) in a number of papers, which appears particularly interesting to the subject, as we shall see below.

The model proceeds from an observation that there is a general relation between the angular momentum J and mass M (see Tassie 1986, and references therein)

$$J_a = \kappa M^{\beta} \tag{25}$$

with κ as a universal constant, estimated as

$$\kappa \approx 4 \cdot 10^2 \frac{G_N}{c} \tag{26}$$

where G_N is Newton's gravitational constant, and c velocity of light. As for β, for a large class of celestial objects the estimate is $\beta \approx 2$. However, a more stringent analysis of the observational data shows that β varies from class to class, within an interval $(1\frac{2}{3}, 1\frac{3}{4})$, whereas Carrasco *et al* (1982) found for the overall classes $\beta = 1.94 \pm 0.09$. The explanation was that these regularities appear remnants of primordial rotational motions of the cosmic strings, conceived as rigid rotators. As for their origin the idea due to Kibble (1985) was that, in analogy with magnetic vortices in superconductors, these vortex strings arose after a sort of phase transition at the early stage of the Universe. Even more seducing analogy is that with superfluids (like that of He^4), which resembles much the ancient Pelasgian mythic picture of the world creation from Chaos, with Eurinome and Ophion (Graves 1963; also, Grujic 1996, unpublished). The beauty of the string hypothesis is that it can be related to the quantum field world, where one finds a number of relations similar to those mentioned above. Thus, in the case of Regge trajectories, one has for the hadrons

$$J_h = \kappa_h M^2 + J(0), \; \kappa_h = \hbar/(GeV/c^2)^2, \tag{27}$$

The constant κ_h differs much from its counterpart in cosmology, but the essence of the model remains. Since the superstring theory claims that it can explain the host of microworld phenomena and can be considered to be the basis for an ultimate theory of matter, like GUT *etc* (see, e.g. Collins *et al* 1989), this formal similarity gains in attractiveness. It lends support, also, to a number of bold hypothesis, like that of the self-similar cosmos due to Oldershaw (1989), who argues that the whole material world, from the elementary particles to the entire universe, has been designed after a fractal pattern. (See, also, Oldershaw 2001)

The actual decay mechanism for the primordial strings is not a matter of consensus among those who hold on to this model. Some argue that the fragments stemming from a unique, primordial cosmic string, should be massless, and therefore not candidates for the subsequent structuring of the universe. Other authors consider that finite mass fragments could have been the outcome of this disintegration, forming a successive hierarchical structures. Within each of class of objects (that is, at a particular hierarchical level) a number of processes might have contributed to the observed variations of the constants κ and β in (25), like the so-called tidal interactions, collisions, *etc*.

As we mentioned earlier, two possible modes of galaxy formation out of strings might be conceived. According to one model, strings comprise but a small part of the overall cosmic matter, otherwise uniformly distributed, and galaxies form by accretion of the surrounding matter around these "cosmic seeds". The other alternative assumes that the cosmic matter is contained in the strings, from the very beginning of the string decays. These distinctions are important for the observational cosmology. For if the former alternative holds, the universe must have been more uniform in the past and by looking into deep cosmic space, one should see more and more even distribution of the cosmic matter. On the other hand, if the string structure persisted up to the remote past, the distant parts of the universe should be more inhomogeneous, resembling string configurations more than the nearby cosmic objects.

Before leaving this string paradigm, let us mention that the string hypothesis goes beyond the fractal structure concept, and other sorts of universes may arise from a collection of primordial strings. The case of one unique string appears but an extreme one, out of many other possibilities (see, e.g. Tassie 1986, and references therein). But we shall not dwell on it here. Also, we note that all these similarities have more heuristic significance, rather than theoretical value. In a sense, these attempts to present a unifying picture of structuring material world resemble the approach of some authors to quantify celestial systems just like atomic ones, by introducing a generalized Planck constant (see, e.g. Grujić 1993, and references therein).

6. THE FRACTAL PATTERN

Whoever designed the Cosmos, (S)He evidently did not consult Alphonso X, and simple realizations of a project in ideal geometrical terms, as considered by Charlier, for example, are out of question. In the following, we shall adopt the convention due to Ribeiro and Videira (1998), and designate ideal (true) cosmic objects by capital initials, like Cosmos, as metaphysical entities. Our models, which seek to approach real structures, as tentative descriptions of the reality, we take simply as (the) cosmos, for example, and the same for our observational inference. Further, we shall distinguish three kinds of relations between different celestial objects. First, we have analogies, as between binary, ternary etc stars and the analogous binary, ternary etc galactic groups. Similarly, one observes similarity between galactic (intrinsic) rotation and the corresponding dynamics of galactic (super)clusters. Second, we notice, for example, a striking similarity between globular (stellar) clusters and some galactic clusters, as examplified by that galactic cluster in *Coma Berenices* (see, e.g. Bakulin *et al* 1977). Finally, one might have a strong similarity between (adjacent) hierarchical levels of the cosmic reality, to which a truly self-similarity transformation may be applied. It is important to stress here that one should define in advance which kind of relationship he or she is referring to, when seeking, or claiming, the fractal (or any other) kind of cosmic structure.

6.1 Fractality, self-similarity, scale-invariance and hierarchical ordering

We employ the three first terms as synonyms, though the last one conveys best the essence of the meaning behind these terms. As for the hierarchical ordering, it stands somewhat apart, and we shall elucidate its relationship with the first three notions, before going on with the issue of the cosmic structure (cf, e.g. Mandelbrot 1983, Ribeiro 1994).

Hierarchy may, but need not necessarily imply scale-invariance. This is best illustrated by enumerating the basic elements of geometry, as defined by Euclid, for instance - point, line, area, volume, each being a part of the next one. In this context, one should ask what are the basic structures observed in the surrounding universe, as a kind of building blocks of our cosmos? Starting from the elementary "point" unit, a galaxy (0D), we have further filaments (1D), sheets (2D), and finally voluminous galactic clusters (3D). This series of the cosmic elements, which follows the geometric pattern we just mentioned, should be completed by the cosmic voids. Origin of the first four cosmic ingredients is still a matter of research, not to say controversies, while for the cosmic voids see, e. g. Collins *et al* (1989). What is of importance to us here is, first, that such a hierarchical series is not scale-invariant, and second, a truly self-similar global cosmic structure may consist of all of these elements, as ingredients of a complex (multy)fractal pattern.

6.2 Fractal properties

Strictly speaking, fractality is a wider notion than self-similarity (see, e.g. Mandelbrot 1983). In Chapter 2 of I we quoted a number of general features of fractal objects. In the following we shall need a more detailed description of these mathematical objects, before proceeding with attempts to identify them on the sky. Since these geometrical structures are embedded into Euclidian space (just as their physical representatives are immersed into the real physical space), we need a measure that tells us how compactly these objects fill the host space. One of the best quantitative indicators is the (proper) dimension ascribed to a fractal, just as one speaks of proper length in relativistic kinematics, for instance. One first defines the so-called Hausdorff β-dimensional outer measure of a set $\mathcal{A}$, by introducing all possible coverings of $\mathcal{A}$, with sets with diameters $\epsilon_i \leq \epsilon$, for a given value of ϵ. If one designates this family of coverings $\Gamma^{\epsilon}_{\mathcal{A}}$, then the Hausdorff measure is (see, e.g., Mandelbrot 1983; Martinez and Jones 1990, and references therein)

$$H^{\beta}(\mathcal{A}) = \lim_{\epsilon \to 0} (\inf)_{\Gamma^{\epsilon}_{\mathcal{A}}} \sum_i \epsilon_i^{\beta}, \ \beta \geq 0, \qquad (28)$$

where *inf* stands for *infimum*, the greatest lower bound of a set of numbers. Then the Hausdorf dimension of $\mathcal{A}$, $D_H(\mathcal{A})$, is defined by the relations

$$H^{\beta}(\mathcal{A}) = \infty, \ \beta < D_H(\mathcal{A}), \qquad (29)$$

$$H^{\beta}(\mathcal{A}) = 0, \ \beta > D_H(\mathcal{A}), \qquad (30)$$

One important property of the above defined measure is that it is identically zero for a countable sets. Mandelbrot 's definition of a fractal is that it is an object whose Hausdorff dimension is strictly larger than its topological dimension. Thus, for the Koch's curve shown in Fig. 2 in I, one finds $D = 1.262$, instead of $D = 1$ (as we would expect from a decent line on a surface, or imbedded into $3D$ space.

If Cosmos is endowed with a hierarchical structure, how can an observer from Earth notice it?

Generally, in the absence of an Observer, one must resort to an indirect evidence. Before we proceed with description of some methods employed, a preliminary consideration of formal and practical possibilities seems in order.

An (intelligent) observer living in a ND-space can infer by direct observations objects belonging to $(ND-1)$-dimensional space. Since we live in a three-dimensional physical space (except, possibly, for the Flatlanders), we observe surfaces of (three-dimensional) objects. It is only through further processing of the (visual, dactylic, etc) data that we experience the full voluminosity of the real world around us. If we are restricted to the visual inference, as is the case with astronomical observation, we are left to the projection of the celestial systems onto a surface. It is due to this sort of restriction that the Ancients conceived the Cosmos as fixed on a sphere, (which sphere in due course multiplied), but the surface-like picture (spherical models) dominated modern cosmology up to Kepler's time. It is from this "projection restriction" position that one has to extend his experience in order to get a realistic, many-dimensional picture of the Cosmos. We shall enumerate the principal steps towards this goal.

(i) Moving from the projected to the cosmic depths, by estimating distances from us to the celestial objects (static approach).

(ii) Leaving the picture of a static, geometric structure, for a dynamic, time varying universe (kinematic description).

(iii) Accounting for the relativistic effects, of both Special and General Relativity, because of the finite speed of light, which is the principal means of observation (relativistic picture).

The third point means one has to abandon, in principle, the Euclidian space and resort to an abstract, four-dimensional manifold. This point becomes more and more relevant as one moves to more distant objects, and after some (not necessarily strictly determined) limit, all three points become not only relevant, but inseparably entangled. For instance, it is the kinematic of the universe that allows us to estimate cosmic distances, *via* red shift, for example (whatever interpretation of the latter would be).

The principal advances in elucidating the (phase space) structure of the cosmos have followed this methodological distinction hronologically, as it could have been expected. Thus, we start with the static universe.

The main sources of information for inferring a possible regular structuring of the universe are galactic catalogues (see, for instance, the seminal paper by Sylos Labini *et al* 1998). But since one does not expect an easy discernable cosmic structure, a suitable methodology of extracting relevant features from the accumulated data must be decided upon, prior to any data processing. And it is here that controversies arise.

As stressed by Ribeira (1994), there are two methodological approaches that may be adopted. One is to start from a particular cosmological paradigm (even model) and try to recognize in the astronomical records whether this can be ascribed to the observable universe. The other approach would be to avoid any preconceived paradigm, or model, and see whether a distinct structure emerges from the data processing. These approaches could be considered as belonging to deductive and inductive methods, respectively.

But which are principal cosmological paradigms available to us? The first is that based on the cosmological principles we mentioned before, that is on the assumptions of an homogeneous and isotropic universe. Both the Standard (Big Bang) and Steady State theories belong to this paradigm. The other paradigm is the concept of the fractal, hierarchical cosmos. The question arises – are these cosmological pictures possible (not necessarily the only) alternatives, or could each be a part of a more general situation. To put it in another way, could the universe be homogeneous at the very large (not necessarily yet specified) scale, but possessing some discernable structure at smaller scale? For we know that our observable part of the universe appears inhomogeneous (what makes it termed cosmos, after all). This question points towards the old Boltzmannian thermodynamic conundrum, why we happen to live in a structured part of the universe, which otherwise should be at the thermodynamic equilibrium. But our issue is of somewhat different nature. It is not the issue of a possible gigantic fluctuation that brings about self-similar cosmos, but the question -could, if confirmed by observational evidence, such a cosmos coexist with an overall homogeneous universe? This is a tricky question, in particular considering that the universe appears structured as far as we (better to say - catalogues) see. We shall return to this epistemological question later on.

6.2.1 Static (statistical) approach

First attempts to discerns a fractal, selfsimilar structure from the catalogues available aimed at recognizing scale invariance as one moves to ever remoter galaxies, by making use of the standard statistical methods. It turned out, however, that the latter were not quite appropriate for the problem at hand (Pietronero 1987). The fractal cosmological model appears extraordinary one, for it is conceived to fulfill an extraordinary task - to distribute a finite amount of cosmic matter over an infinite universe, with a regular structure (which turns out to be isotropic, but not homogeneous).

The primitive data from the catalogues are the angular positions and red shifts of the galaxies. Two principal statistical means for processing these data are (i) an estimate of the average density, (ii) the correlation function. Since it is the gravitational interaction that is the driving force of the cosmic structuring, one should start from the mass density function

$$\rho(\mathbf{r}) = \sum_{i}^{N} m_i \, \delta\left(\mathbf{r} - \mathbf{r}_i\right), \tag{31}$$

with δ as Dirac's *delta function*, and galaxies situated at $\mathbf{r}_i$. Since catalogues provide only positions, one passes from the mass density over to the number density

$$n(\mathbf{r}) = \sum_{i}^{N} \delta(\mathbf{r} - \mathbf{r}_i), \tag{32}$$

Then one calculates the correlation function, after averaging over angular variable,

$$\zeta(r) = \frac{\langle n(\mathbf{r}_0) n(\mathbf{r}_0 + r)\rangle_{\mathbf{r_0}} - \langle n\rangle^2}{\langle n\rangle^2}, \tag{33}$$

where the average $\langle \cdots \rangle$ is made over all referent points (points of origin) within the given volume V. For random distributions the average of product is equal to the product of averages and ζ is identically zero. The joint (conditional) probability to find an object in the volume δV_1 and another object at δV_2 is

$$P = \langle n\rangle^2 \delta V_1 \delta V_2 [1 + \zeta(r_{12})], \tag{34}$$

where r_{12} is the distance between two volumes. The function ζ is a measure of fluctuations within the volume V, and its average value is zero. If any structure appears, however, it is these fluctuations which should reveal it, by forming a regular pattern. This approach, however, assumes the existence of the average density $\langle n\rangle$, as an intrinsic property, independent of V (provided the latter is large enough to make the statistics meaningful). The problem with the fractal structures is that they have no such a (volume independent) property. In particular, fractal cosmos has been designed exactly to make the density tend to zero, as the volume increases to infinity.

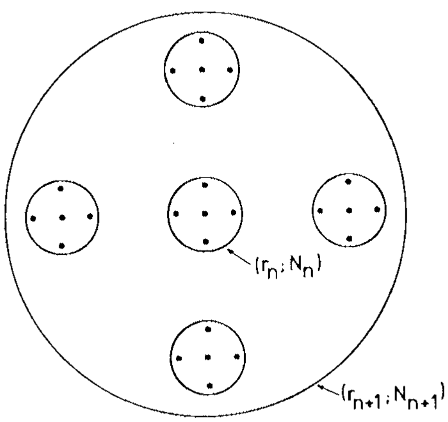

Fig. 1. *Two consecutive hierachical levels of a fractal cosmological model (schematically). (From Pietronero 1987)*

In order to overcome this problem Pietronero (1987) has proposed a more appropriate approach, suitable for systems where an average density need not exist. Suppose we have a self-similar pattern, as illustrated in Fig. 1.

If one starts from a referent sphere with radius r_0 which contains N_0 objects, then within sphere with $r_1 = kr_0$ one finds $N_1 = \tilde{k}N_0$ objects, *etc.* Generally, we have the relation

$$r_n = k^n r_0, \tag{35}$$

for the $n-th$ radius, and the number of objects within r would be

$$N(r) = Ar^D, \qquad A = \frac{N_0}{r_0^D}, \tag{36}$$

$$D = \frac{log\tilde{k}}{logk}, \tag{37}$$

where D is another, more practical definition of the fractal dimension. If a sample of the cosmic objects lies within a sphere of radius R_s, the average density is

$$\langle n\rangle = \frac{N(R_s)}{V(R_s)} = \frac{3}{4\pi} A R_s^{-\gamma}, \quad \gamma = D - 3, \tag{38}$$

An early estimate of γ was $\gamma = 1.8$ (Vaucouleurs 1970; see, also, Giavalisco *et al* 1989), implying $D = 1.2$. Since $\langle n\rangle$ in (38) depends explicitly of the sample radius R_s, a more appropriate quantity, the *conditional density*, was introduced by Pietronero (Pietronero 1987)

$$\tilde{n}(r) = \frac{1}{S(r)} \frac{dN(r)}{dr} = \frac{D}{4\pi} A r^{-\gamma}, \tag{39}$$

where $S(r)$ is the area of the spherical shell with radius r. Likewise, a more appropriate correlation function is introduced by the same author

$$\Gamma(r) = \frac{\langle n(\mathbf{r}_0) n(\mathbf{r}_0 + r)\rangle_{\mathbf{r_0}}}{\langle n\rangle}, \tag{40}$$

which can be written as

$$\Gamma(r) = \frac{1}{N} \sum_{i}^{N} \tilde{n}_i(r) = \tilde{n}(r) = \frac{D}{2\pi} A\, r^{-\gamma}, \tag{41}$$

and depending only on the intrinsic properties of the sample, not on its dimensions. Another useful quantity is the volume integral

$$I(r) = \int_0^r \Gamma(r') r'^2 dr', \tag{42}$$

Before we proceed with further elaboration of the statistical tools, we mention that another quantity, the so-called radial distribution function is often used

$$g(r) = 1 + \zeta(r), \tag{43}$$

which may be represented as a power-law function

$$g(r) = Ar^{D_2 - 3}, \qquad D_2 \approx 3 - \gamma, \tag{44}$$

where D_2 is the so-called *correlation dimension*. Generally one has $D_2 < D$, except for homogeneous fractals, when $D_2 = D$ (see, e.g., Martinez and Jones 1990).

In attempting to reveal a structure of Cosmos, one could expect three possibilities with respect to the fractal model. Either Cosmos is (i) scale- invariant, or (ii) it is such only up to some characteristic distance λ_0, the so-called *homogeneity scale*, or (iii) the fractal structure has nothing to do with the Universe. In the mixed case of a fractal embedded into otherwise homogeneous Universe, one has for the conditional number density

$$\tilde{n}(r) = \frac{D}{4\pi} A r^{-\gamma}, \qquad r < \lambda_0, \tag{45}$$

$$\tilde{n}(r) = n_0 = \frac{D}{4\pi} A \lambda_0^{-\gamma}, \qquad r \geq \lambda_0, \tag{46}$$

In such a case one would have a simple power-law behaviour of the volume integral

$$I(r) = A r^D, \qquad r < \lambda_0, \tag{47}$$

and

$$I(r) = AD\lambda_0^D \left[\frac{1}{D} - \frac{1}{3} \left(\frac{r^3}{\lambda_0^3} - 1 \right) \right], \qquad r \geq \lambda_0, \tag{48}$$

The advantage of using these quantities will show up when considering the observational evidence, as we shall see later on. The length-scale λ_0 is related to a typical dimension of the largest voids observed. Another important quantity is the so-called *correlation length* r_c, which separates the regions where there exist correlations of the density fluctuations (with respect to the average density), from the region where these correlations are absent. Obviously, the existence of λ_0 implies r_c, otherwise the latter is meaningless. Finally, we mention a statistical property described by the correlation length r_0, defined by putting $\zeta = 1$. Since it turns out that r_0 depends linearly on the sample radius R_s, it has no physical meaning in the cosmological studies.

6.2.2 Mass distribution and multifractality

If one accounts for the galactic masses then a more general fractal pattern may be expected. One first introduces a normalized distribution that can be turned into probability by dividing the mass distribution (31) by the total mass within the sample (Pietronero 1987, Sylos Labini *et al* 1998)

$$\mu(r) = \frac{1}{M_T} \rho(r), \qquad M_T = \sum_i^N m_i, \tag{49}$$

If the total volume is divided into boxes of linear size l, one defines the function

$$\mu_i(\epsilon) = \int_{i-th\,box} \mu(r) dr, \tag{50}$$

for ith box, where $\epsilon = l/L$ and $0 < \mu_i < 1$. One defines then the box-counting fractal dimension as

$$\lim_{\epsilon \to 0} \mu_i(\epsilon) \sim \epsilon^{\alpha(\mathbf{x})}, \tag{51}$$

where $\mathbf{x}$ is the box position. In the case of a simple fractal one recovers the fractal dimension $\alpha(\mathbf{x}) = D$, but generally $\alpha(\mathbf{x})$ may fluctuate considerably. If we have a number of boxes with the same measure scaling with α, these form a subset with dimension $f(\alpha)$. For a simple fractal $f(\alpha) = \alpha = D$. From $f(\alpha)$ one can estimate α_{min} and α_{max} that correspond to the largest clusters and voids respectively.

Self-similarity and fractality

The concept of self-similarity is wider than the fractal morphology. Consider, for example, a function $V(r)$, which may represent a spherically symmetric potential. If this function appears in a power-law form, $\lambda\, r^\kappa$, it is homogeneous with respect to the similarity transformations

$$V(\theta r) = \theta^\kappa V(r), \tag{52}$$

where the real number κ is the order of the scaling transformation, which multiplies the function by a constant factor, but leaves the shape intact. Generally, if a physical system has the potential function of the pair-additive form

$$V([\mathbf{r}_{ij}]) = \sum_{i<j}^N c_{ij} V_{ij}(\mathbf{r}_{ij}), \tag{53}$$

where $[\mathbf{r}_{ij}]$ stands for the set of all coordinates of the system constituents, it will be scale-invariant if all pair terms have the same order of scaling κ. Clearly, a system of gravitating bodies possesses this self-similarity property (as every Coulombic system does, see, e.g. Grujić 1993). These so-called *homogeneous systems*, when subjected to the full kinematic scale transformations exhibit a number of important properties (see, e.g. Landau and Lifshitz 1976), which make them a special class of physical systems. Among all systems with interaction function of the form (53) two distinguish themselves further, those with $\kappa = -1, 2$. The first exponent defines Newtonian and Coulombic (long- range interaction) systems, the second the *harmonic potential* system (like the harmonic oscillator). For these two potentials, every (classical) orbit is closed. It is due to this particular property of the gravitating systems that Charlier found that all bodies within a cluster have closed orbits, as we mentioned above. Moreover, as a generalization of Kepler laws, they all possess the same period.

This scaling property was noticed by Laplace (1925), who wrote in his *System of the World* (see, e.g. Mandelbrot 1983)

"One of [the] remarkable properties [of Newtonian attraction] is, that if the dimension of all the bodies in the universe, their mutual distances and their velocities were to increase or diminish proportionately, they would describe curves entirely similar to those which they at present describe; so that the universe reduced to the smallest imaginable space would always present the same appearance to observers. The laws of nature therefore only permit us to observe relative dimensions ... Geometers' attempts to prove Euclid's axiom about parallel lines have been hitherto unsuccessful ... The notion of .... circle does not involve anything which depends on its absolute magnitude. But if we diminish its radius, we are forced do diminish also in the same proportion its circumference, and the sides of all inscribed figures. This proportionality seems to be much more natural an axiom than that of Euclid. It is curious to observe this property in the result of universal gravitation"

An important consequence of these scaling-properties is that such a system has no characteristic length (and no characteristic time-period, too). All relations above, like that in (47), have no characteristic lengths, and it is their exponents that matter (like D, not the prefactor A).

Lacunarity

The above statistical quantities are necessary prerequisites for identifying eventual structuring of the observable part of Universe, but they are not sufficient. Namely, they describe a number of structural features of a general class of physical systems, but these features need not specify cosmologically relevant self-similar forms. In particular, fractal dimension D does not determine topology of a fractal, as illustrated in Fig 2.

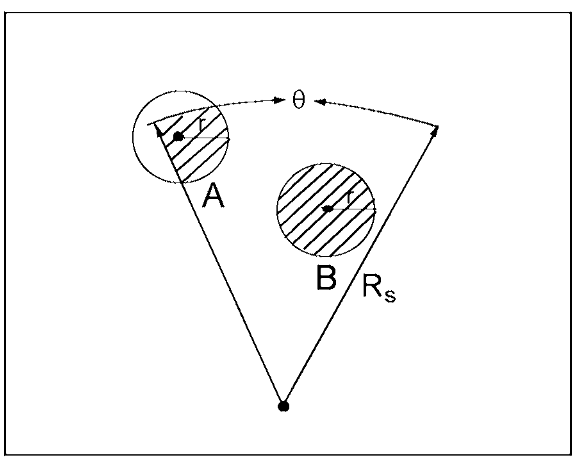

Fig. 2. *Two deterministic (upper part) and two random (stochastic), (lower part) fractal structures (Mandelbrot 1983). First three fractals have the same lacunarities, whereas the fourth has different one.*

All fractals in Fig. 2 have the same dimension D, but clearly very distinct topology. For that reason Mandelbrot (1983) has defined the so-called *lacunarity* F, by considering voids within the structure, as

$$Nr(\lambda > \Lambda) = F\Lambda^{-D}, \qquad (54)$$

where the lefthand side of (54) is the number of voids with the size $\lambda > \Lambda$. Nr scales in the same way for both deterministic (Cantor) sets, but the prefactor F has different values for two sets. As for the stochastic cases, a generalization of (54) is necessary, by introducing the conditional probability $P(\lambda)$

$$P(\lambda > \Lambda) = F\Lambda^{-D}, \qquad (55)$$

which gives probability that, if a box of the size ϵ contains points of the set, this box has a neighbouring void of the size $\lambda > \Lambda$.

Orthogonal projection

As discussed previously, we observe directly angular distributions of celestial objects, that is a projection of three-dimensional structure onto a plane. What such a projection preserves of the original structure embedded in three-dimensional real space is the size of objects. If a fractal of dimension D is projected from $d = 3$ to $d' = 2$ subspace, the projected structure has dimension D' such that (see, e.g. Sylos Labini *et al* 1998)

$$\begin{aligned} D' &= D, \qquad if \qquad D < d' = 2; \\ D' &= d', \qquad if \qquad D > d' = 2, \end{aligned} \qquad (56)$$

Thus, clouds, with $D \approx 2.5$ shed a compact shadow with $D' = 2$. Likewise, a fractal cosmos with $D \geq 2$ will exhibit a uniform angular galaxy distribution.

6.2.3 Dynamic effects

The universe is a dynamical system. Not only it is subjected to the universal Hubble flow (whatever its interpretation might be), but a number of correlated or erratic motions are superimposed upon the overall expansion, as observed from the co-moving reference system. If a self-similar Cosmos does exist, the question arises as to its time evolution, apart from the overall expansion. As far as we are aware, this question has not yet been addressed fully. We shall content ourselves here with mentioning a few relevant topics.

We first notice that erratic movements of galaxies should not contribute to forming or destroying eventual self-similar (or any other, for that matter) structure. As for the collective movements of galaxies and clusters, they might be part of global structuring process. Clearly, as we move from the nearby surrounding and local cluster to the deep space, the erratic motion looses its significance and the remote parts of the universe appear evermore stationary (though not static).

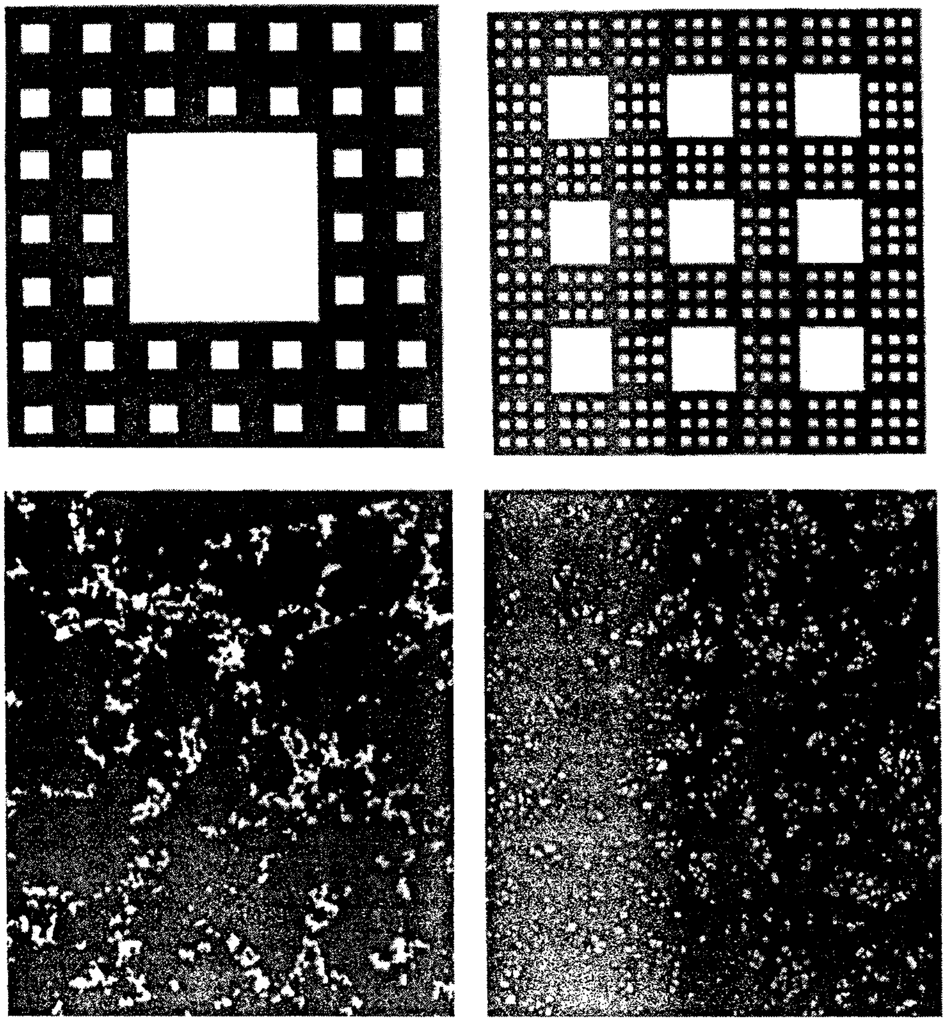

Fig. 3. *A survey sample (schematically). Inclusion of point A yields a spurious contribution to the average density.*

Now, one might ask the question whether there is a more general self-similarity, within a phase-space of the universe. Whether there is a collective expansion of a part of the cosmos, similar to Hubble flow? Or a global infall of galaxies, or clusters, resembling the Big Crunch? After all, it is this mechanism that gives rise to the assumed galactic black hole formation.

Surely, a more complete dynamic picture of the universe will clarify the issue as to the constructive or destructive role of the overall cosmic expansion regarding cosmic structuring, including fractal one. As pointed out by Ribeiro (1994), a deeper study of the (nonlinear) field equations might provide a clue to the puzzle of fractal pattern. From numerical calculations one may infer that the systems under studies exhibit structural fragility. Moreover, it is well known that strange attractors of dynamical nonlinear systems have fractal patterns in phase space. The question arises as to the possibility that tracing the fractal structure along the past null geodesic one might encounter such an attractor. And what would be the connection between Lyapunov exponents and fractal dimensions within this context? These are the questions that might be waiting for answers in the near future of cosmological studies that go beyond the geometrical, cosmographical scope.

7. THE OBSERVATIONAL EVIDENCE

In the previous section we discussed a number of properties that Cosmos might possess, if it is at least partially fractal. We consider now the question as to what would be the observational evidence of such a self-similar structure. In a sense, this issue is related to the problem of theory of measurement, as raised in other sciences of Nature, notably in Quantum mechanics. In the case of cosmology, it might be called theory of observation. We first discuss the boundary problem.

7.1 The boundary effects

If the self-similar structure is limited to a final cosmic volume, as determined by the homogeneity parameter λ_0, one would expect that the conditional average density $\Gamma(r)$ becomes constant as one passes the region $r \approx \lambda_0$. As we shall see, the galaxy surveys do exhibit this sort of behaviour, but it turns out to be spurious effect, due to a number of improper procedures employed (see, e.g. Sylos Labini *et al* 1998). One of these misleading effects is due to only partial inclusion of the spheres where an average density is to be estimated. In Fig. 3 we show schematically the situation one encounters while processing observational data from the deep sky surveys.

As shown by Pietronero and coworkers (see, e.g. Sylos Labini *et al* 1998), the observed flattering of the correlation function is due to the final size samples, and not a genuine effect that would point towards a uniform distribution of galaxies at large scale. Only spheres which lie completely within the survey volume, as indicated by shaded areas in Fig. 3, should be included in the relevant data.

We saw earlier that the correlation parameter r_c is defined by the condition $\zeta(r_c) = 1$ (see (33)). Three cases are to be considered here (Pietronero and Sylos Labini 2000).

(i) $R_s \lesssim \lambda_0 < r_c$ (fractal distribution)

Then the correlation function (41) behaves as

$$\Gamma(r) \sim r^{D-3}. \tag{57}$$

If there is a crossover to homogeneity ($R_s \gg \lambda_0$) one distinguishes two cases.

(ii) The sample behaves as a fractal up to a certain distance λ_0 (that is $\Gamma(r)$ behaves as a power law), but becomes homogeneous at scales $R_s > r \gg \lambda_0$. If there is a lower cut-off of the fractal pattern r_l, then one has

$$\Gamma(r) \sim r^{D-3}, \qquad r_l \leq r \lesssim \lambda_0, \tag{58}$$

$$\Gamma(r) \approx \langle n \rangle_{R_s}, \qquad \lambda_0 \lesssim r \leq R_s, \tag{59}$$

If the sample is large enough, one has

(iii) $\lambda_0 \lesssim R_s < r_c$ Then an average density becomes meaningful and the (standard) correlation function may be used

$$\zeta(r) = \left(\frac{r}{\lambda_0}\right)^{-\gamma} \eta(r), \tag{60}$$

with $\eta(r)$ an oscillating function, which describes fluctuations with respect to the average density $\langle n \rangle_{R_s}$.

7.2 Global effects

These fall generally into two categories, which we attribute, somewhat arbitrarily, to the cosmogonic and relativistic effects, respectively.

7.2.1 Cosmogonic effects

If the standard, Big Bang scenario is adopted as a realistic description of the birth of Cosmos, then it follows that the early Universe started with approximately homogeneous distribution of matter, in the very general sense of last term. Hence, while looking at the very distant objects, say 10^{10} light years away, one looks into the epoch when the present day structure has not yet been formed. Consequently, deep sky surveys should reveal a homogeneous universe, whatever the subsequent evolution of the cosmic material content would be. (See, however, chapter 5.2.1.). Moreover, since we can not have a snapshot picture of the Universe, there is no way to infer directly an eventual structure of the Cosmos at a constant time (Cauchy) surface.

7.2.2 Relativistic effects

These are discussed by Ribeiro (2001a,b), within the context of the so-called Fractal Debate, led between the (orthodox) supporters of the standard, Friedmannian paradigm and those who argue for a (possibly infinite) fractal matter distribution. By examining carefully the meaning and use of a number of astrophysical constructs, like the mean density, cosmological distances, *etc*, Ribeiro argues that the apparent conflicts between two schools might not be so severe, as the case might seem. With a full account of the relativistic effects, a number of the observational data should be reassessed and reinterpreted. The issue at stake is the question, though not explicitly stated, whether Friedmannian Cosmos could yield an inhomogeneous observational factography and *vice versa*, whether a fractal Cosmos might look like a homogeneous system according to our observational evidence.

7.3 Fractal or nonfractal, what the sky says

We give here a short overview of the present observational situation concerning the galaxy distribution and an eventual evidence for the (multi)fractal pattern. For a more detailed account we direct interested readers to the seminal article by Sylos Labini *et al* (1998).

Generally, it turns out that most of the observed galaxies do not belong to clusters (c. 70 %). Isolated galaxies (not in groups or clusters) are very rare, too. As for the galaxy distribution with regard to a particular morphology of a subpopulation (Hubble classification, galaxy size, mass, luminosity *etc*), which complicates greatly an overview, we shall restrict ourselves to the most general features of the large-scale distribution. We mention here only that the fact that the most of the giant galaxies are mainly clustered (*Luminosity segregation*), unlike the dwarf galaxies. This fact was interpreted previously as a consequence of the larger amplitude of the correlation function, but has been reinterpreted by the supporters of the fractal cosmos as the feature of the *exponent* parameter of the distribution, independent of the sample size, as discussed earlier.

An analysis of 11 catalogues available has revealed that there is a strong correlation between the distance and density function, as shown in Fig. 4 from Sylos Labini *et al* (1998).

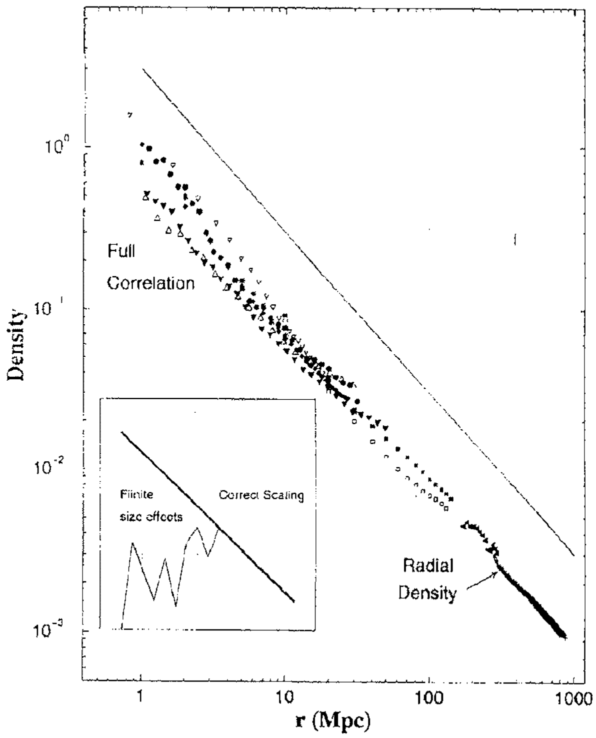

Fig. 4. *Correlation analysis of relevant samples taken from various red-shift surveys. The reference straight line with the slope* -1*, which corresponds to the fractal dimension* $D = 2$*, is shown. In the inset, results for insufficiently large distances are shown, affording a spurious (approximately) constant density (from Sylos Labini et al 1998).*

As can be seen from Fig. 4, the overall picture, with a wide span of distance $(0.5-1000 Mpc/h)$, with Hubble constant h in units 100 Mpc/km/s (with measured value $h \approx 0.65$), fits well the fractal pattern. It turns out that all surveys are mutually consistent and point to the $D = 2 \pm 0.2$ fractal dimension of the observable part of the Cosmos, with a clear fractal structure within $(0.5 - 150 Mpc/h)$ region. As argued by Pietronero and coworkers, the proper use of the statistical analysis removes a number of inconsistencies and difficulties encountered in earlier interpretations of the available data. By eliminating all quantities depending of the size of the sample considered, the so-called galaxy-cluster mismatch (paradox) is resolved. This consists in the discrepancy between conclusions regarding the correlation length. Namely, it turns out that when the clusters are considered only, the fractal pattern is detected up to a particular λ_0, but it differs greatly from the corresponding estimate for the supercluster analysis, *etc.* Once the generalized correlation function $\Gamma(r)$ is used, instead of the standard one $\xi(r)$, the discrepancy disappears. This case demonstrates the fact that the analysis based on a particular premise is bound to yield a result that might differ greatly from another interpretation which relaxes the premise adopted in the previous one. The present estimate is that $\lambda_0 \geq 50\, Mpc/h$ (Pietronero and Sylos Labini 2000).

The most recent analysis of the $CfA2$-south redshift survey data confirms the fractal structure at least up to $20\, Mpc/h$ (Joyce *et al* 1999a; see, also, Joyce *et al* 1999b). The fact that the observed fluctuations around the average counts of galaxies are of the same magnitude as the counts themselves argues in favour of the fractal pattern at all scales available (Gabrielli and Sylos Labini 2001).

In fact, a more scrutinized analysis reveals that the data follow a multifractal distribution rather than a clear fractal pattern (Martinez and Jones 1990; see, also, Sylos Labini *et al* 1998). These authors argued, however, that structures on scales less than $5\, h^{-1}\, Mpc$ are more complex than the homogeneous fractal structure, and the Housdorff dimension found is sheet-like $D \approx 2.1 \pm 0.1$).

In the most recent analysis Bak and Chen (2001) argue that the luminous matter in the universe is distributed according to a multifractal pattern up to a certain distance, and then a crossover to homogeneity is predicted. They applied a "forest-fire" model, developed previously for the reaction-diffusion process in turbulent systems, and defined an apparent dimension $D(\ell)$, which depends on the scale ℓ. According to their findings, this generalized dimension rises linearly with logarithm of the scale and at $\lambda_0 \approx 300\, Mpc$ a crossover to the homogeneous distribution, characterized by $D = 3$, is predicted, as shown in Fig. 5.

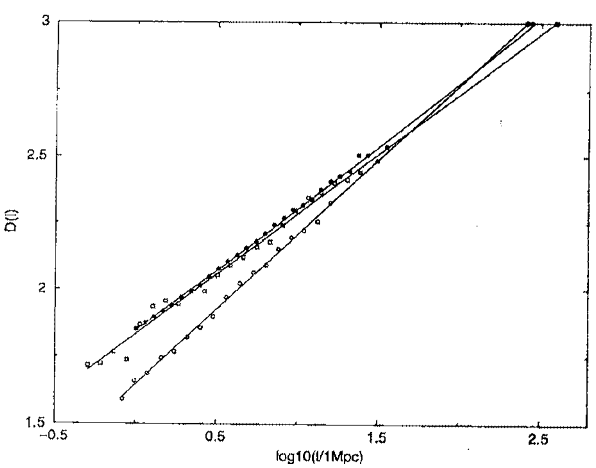

Fig. 5. *Scale dependent dimension* $D(\ell)$*, derived from three different samples (Bak and Chen 2001).*

The authors conclude that:

> *The geometry of the luminous set is not fractal when viewed over the entire range of scales, since there is no self-similarity for different scales. It is not homogeneous either. The scale dependent dimension has a clear geometrical interpretation: At small distances, the Universe is zero-dimensional and pointlike. Indeed, energy dissipation takes place on individual pointlike objects, such as stars and galaxies. At distances of the order of 1 Mpc the dimension is unity, indicating a filamentary, stringlike structure; when viewed at larger scales it gradually becomes 2-dimensional wall like, and finally at correlation length, ξ, it becomes uniform.*

Hence, the issue about the global structure of the Universe is far from being settled. The current debate moves around the questions, as put by Pietronero and Sylos Labini (2000), of (i) the proper statistical methods employed in analyzing the observational data, of (ii) the implications of the observed fractal structure up to a certain scale λ_0 and (iii) what would be the reliable estimate of this homogeneity scale? The exact value of the fractal dimension D is still uncertain, as well as the role of the cosmic dark matter, as discussed by various authors (see, e.g. Sylos Labini 2000). The question of the fractal structure genesis is still lacking a reliable approach, though methods developed in the statistical physics, such as self-organized criticality, have been invoked in order to understand the evolution of the self-gravitating systems (see, e.g. Sylos Labini and Pietronero 2001).

The search for the fractal properties of the observable universe continues, both observationally and theoretically. In a recent paper by Gaite and Manrubia (2002) scaling of the cosmic voids has been examined within the model of random fractals, with nonconclusive results. The concept of a fractal universe has been also tested against the more exotic cosmological paradigms, such as Linde's selfreproducing, eternal, stochastic inflation (see, e.g. Winitzki 2001). On a more epistemological than technical basis an interesting parallel with the anthropological aspect of the fractal paradigm has been made by Zabierowski 1988), who compares the latter with Boltzmannian solutions to the cosmic time arrow conundrum of the time.

8. SUMMARY AND CONCLUSIONS

We have shown how the concept of a fractal cosmos was gradually developed from Renaissance to the present time. It occupied some of the most powerful minds in Europe at the time, from Leibniz, Kant, to Laplace, but these ideas were formulated in such terms that one could hardly speak about definite cosmological models. The latter arose at the beginning of 20-ieth century, first paralleling the ruling relativistic cosmological paradigms, and then starting interfering with them. In the last period of the cosmological studies this self-similarity pattern begin competing with the standard relativistic paradigm, and the issuing controversy may be put in the following terms.

(i) Can the Cosmos be conceived as a fractal, self-similar structure, without violating the standard Cosmological principles, to which almost all contemporary models adhere?

(ii) What should be observational evidence that could decide which paradigm is realized?

(iii) Could both paradigms coexist, or put in the following terms - are these two paradigms just different aspects of the unique reality, or are they rival, if not antagonistic alternatives?

These are the current dilemmas, that run parallel with the growing observational evidence that cosmos is far from being smooth and homogeneous, within our present observational limits. We have seen how much the *a priori* concepts have influenced not only the interpretations of the existing data, but the very gathering of the latter. As pointed out by Pietronero and others, a particular prejudice will yield its own observational evidence. That the observation is as much recognizing a preconceived model as discovering a new fact is not bound to cosmology only, and is just a part of a more general epistemological issue. Relaxing any principle, or cosmological postulate, would surely make the cosmology more a science about Nature, and less a theoretical game.

The fractal concept does not explain how this structure has arisen, but the cosmogony appears a weak point of any other approach, though the standard Hot Big Bang paradigm claims a scientific status. As pointed out by many authors (Alfvén 1976, Lurcat 1978, Disney 2000) the reliable observational evidence is still too weak to support farfatching theoretical speculations (but see Ćirković 2002). Not only the current evidence based on the cosmic sources of the electromagnetic radiation is still insufficient to ensure a convincing choice between the current competing alternatives, but other hypothetical entities, like the dark matter in various forms (e.g. Grujić 2002) enter the game. The question whether this new cosmic matter/energy component, if detected, would alter radically the current models, including the fractal paradigm, is surely the next topic to be considered seriously by cosmologists (see, e.g. Sylos Labini *et al* 1998).

The concept of a hierarchical, self-similar cosmic structuring possesses a number of remarkable properties that make it an ingenious solution of the principal conundrum in cosmology, namely how to conceive an infinite world different from the trivial, Abderian extensive model. This problem has been tackled successfully within the Einsteinian relativistic paradigm, with space-time coupled intrinsically with the content of the universe. The fractal paradigm answers the same question in an essentially different, but equally selfconsistent way. The concept of selfsimilarity tackles the cosmological puzzle of an infinite universe in a similar way as Cantor's set theory deals with infinities within the number theory. The matter distribution in the universe appears diluted in a sense exponentially, just as the field expansion occurs exponentially fast within the inflatory paradigm. That the Copernican principle is preserved within the paradigm of scale-invariant cosmos appears a remarkable property of the fractal model too.

Going back to the Presocratic era and to Anaxagoras' concept of the selfsimilar cosmos, two things are to be observed. First, though one might speculate about the outward self-similar cosmic pattern, that is from the mesocosmos to megacosmos, Anaxagoras was primarily concerned with the structuring towards the infinitely small. Modern cosmological thoughts are mainly oriented towards larger and larger cosmic dimensions, possibly to infinite ones (but see Oldershaw 1989). The principal reasons for this are the quantum mechanical restrictions, imposed to the ultimate dimensions in the microcosmic world. It turns out that the competition between Abderian and Klazomenian solutions has finished in draw. The atomic paradigm has set a barrier on the road towards infinitely small, whereas the selfsimilar pattern has enabled us to conceive an infinite universe with a limited content of matter.

There appears a remarkable aspect of the history of the fractality in nature, that the principal contributors to the hierarchical cosmos failed to refer to their predecessors. Leibniz did not mention Anaxagoras, Kant neither Anaxagora nor Leibniz, *etc.* It would be surely an interesting issue to pursue an answer as to the question whether it was a deliberate neglect or just the matter of ignorance. In the latter case one might argue that the concept of selfsimilarity appears an inherent content of our mind, and hence less one of the speculative outcomes of our endeavors to conceive the World in its totality. As for Anaxagoras it seems that his contribution to the concept of the selfsimilar Cosmos has been almost totally forgotten, with rare exceptions (see, e.g. Markov 1990).

The modern cosmology has extended its domain well beyond the borders that might be considered purely scientific. It includes at the very front line such disciplines like philosophy (see, e.g. Ellis 1999), religion, esthetic, political ideology (see, e.g. Naddaf 1998, for the case of Anaximander; see, also, Ćirković 2002), *etc.* In a sense, modern cosmology turns out equally parascience at its outermost borders, as it was during the whole period of European culture development, from the archaic Greece, to the "Pre-Big-Bang" theoretical speculations today. There is nothing wrong with this, of course, as long as one is aware of these distinctions and does not proclaim speculative thought a scientific theory. The role of the fractal paradigm, besides its positive contribution to the cosmology as such, is just to put more weight on the physical, observational, and epistemological aspects of our search for a definite picture of the Universe, *i.e.* to the classical cosmology in general.

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КОНЦЕПТ ФРАКТАЛНОГ КОСМОСА: II. МОДЕРНА КОСМОЛОГИЈА

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Развој концепта фракталног космоса после Анаксагоре праћен је до данашњег времена. Показано је како се концепт појавио поново у раној Ренесанси као магловита идеја, да би почетком 20. века добио конкретну формулацију. Стање модерне космологије размататрано је с' тачке гледишта фракталне парадигме и дискутоване су текуће контроверзе и полемике. Показано је да је концепт хијерархијског космоса још увек жив и да може да буде битан елеменат модерне слике свемира.