

## Correlation of $^1\text{H}$ NMR Chemical Shift for Aqueous Solutions by Statistical Associating Fluid Theory Association Model\*

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**Abstract**  $^1\text{H}$  NMR chemical shifts of binary aqueous mixtures of acylamide, alcohol, dimethyl sulphoxide (DMSO), and acetone are correlated by statistical associating fluid theory (SAFT) association model. The comparison between SAFT association model and Wilson equation shows that the former is better for dealing with aqueous solutions. Finally, the specialties of both models are discussed.

**Keywords** Statistical associating fluid theory (SAFT), association model, NMR, chemical shift, aqueous solution

### 1 INTRODUCTION

The interaction between water and solvents is always an interesting problem to researchers. However, it is difficult to make clear the solution structure and intermolecular interaction in aqueous solutions due to the complex hydrogen bond. In recent years, Nuclear magnetic resonance (NMR) became one of the most important methods to study such solutions because of its high sensitivity. Some useful information can be gained from the chemical shift at different concentrations and temperatures<sup>[1–8]</sup>.

In 1953, Gutowsky and Saika<sup>[9]</sup> first investigated thermodynamic properties of associating systems by NMR. It was assumed that the chemical shift observed in associating systems is the sum of contributions by free and associated protons. The weighting factors were simply considered to be the mole fraction of free or associated protons at equilibrium. Based on this assumption, a series of chemical association models were proposed to fit the composition-dependent chemical shifts for various associating mixtures<sup>[10–15]</sup>. However, in all chemical association models, it has to assume that some aggregates are formed, and then determine if the assumption is reasonable according to the result of correlation. Furthermore, when a large number of different aggregates are formed, many model parameters are needed, which limits to wide application of chemical association models, especially for complex associating systems such as aqueous solutions.

Deng *et al.*<sup>[16]</sup> proposed a semi-empirical physical model to correlate NMR chemical shift based on Wilson's local composition theory. Fifty sets data for 14 systems were correlated, including

alcohol-hydrocarbon systems, alcohol-*N*, *N*-dimethylformamide systems and alcohol-chloroform systems. However the  $^1\text{H}$  NMR chemical shifts of aqueous solutions were not correlated.

In this paper, SAFT association model is used to correlate  $^1\text{H}$  NMR chemical shift of aqueous solutions. The comparison between SAFT association model and Wilson equation are also given.

### 2 THEORY

#### 2.1 Calculation of mole fraction of association

Statistical associating fluid theory (SAFT)<sup>[17–18]</sup> is based on Wertheim's thermodynamic perturbation theory of first order<sup>[19]</sup>. Modified SAFT equation of state is widely applied in calculations of thermodynamic property and phase equilibrium for associating and non-associating fluid mixtures<sup>[20–23]</sup>. SAFT treats real molecules as chains composed of equi-sized spherical segments. It requires three model parameters for non-associating components: *m* (segment number),  $\sigma$  (segment diameter) and  $\varepsilon/k$  (segment-segment interaction energy). SAFT attributes the association between molecules to their associating sites. Therefore, it needs to add two association parameters for associating components:  $\varepsilon^{AB}/k$  (site-site association energy) and  $\kappa^{AB}$  (site-site association volume).

With these parameters, the mole fraction of non-associated sites is derived.  $X_i^A$ , which is defined as the mole fraction of molecule *i* not bonded at site *A*, can be obtained as follows

$$X_i^A = \left[ 1 + \sum_j \sum_B \rho_j X_j^B \Delta_{AiBj} \right]^{-1} \quad (1)$$

where  $\rho_j$  is the number density of molecule *j* and

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$\Delta_{A_i B_j}$  characterizes the association strength

$$\Delta_{A_i B_j} = g(d)_{ij}^{\text{seg}} [\exp(\varepsilon_{ij}^{AB}/kT) - 1] (\sigma_{ij}^3 \kappa_{ij}^{AB}) \quad (2)$$

where  $g(d)_{ij}^{\text{seg}}$  is the segment radical distribution function, which can be approximated with the expression derived for mixtures of hard spheres

$$g(d)_{ij}^{\text{seg}} \approx g(d)_{ij}^{\text{hs}} = \frac{1}{1 - \xi_3} + \frac{d_i d_j}{d_i + d_j} \frac{3\xi_2}{(1 - \xi_3)^2} + \frac{d_i^2 d_j^2}{(d_i + d_j)^2} \frac{2\xi_2^2}{(1 - \xi_3)^3} \quad (3)$$

where  $d_i$  is the temperature-dependent effective sphere diameter of component  $i$

$$d_i = \sigma_i \left[ 1 - 0.12 \exp\left(-\frac{3\varepsilon_i}{kT}\right) \right] \quad (4)$$

and  $\xi_{n=0,1,2,3}$  is a function of density

$$\xi_n = \frac{\pi}{6} \sum_i m_i \rho_i d_i^n, (n = 0, 1, 2, 3) \quad (5)$$

## 2.2 Correlation of chemical shift of mixtures

The variations of NMR chemical shift in mixtures are usually considered to be caused by two factors: solvation and association. According to Gutowsky and Saika's assumption, the chemical shift observed in associating systems is the sum of contributions by free and associated protons. That is to say, the chemical shift of proton should be regarded as the weighted average of the chemical shifts of protons in different associating states

$$\delta_H = \delta_{\text{free}} x_{\text{free}} + \sum_i \delta_{a,i} x_{a,i} \quad (6)$$

where  $\delta_H$  is the observed chemical shift of protons,  $\delta_{\text{free}}$  and  $x_{\text{free}}$  denotes the characteristic chemical shift and mole fraction of non-associated protons respectively,  $\delta_{a,i}$  and  $x_{a,i}$  denotes the characteristic chemical shift and mole fraction of various associated protons respectively.

In aqueous solutions, the situation of association is very complicated. There are four associating sites on a water molecule. Thus water molecules not only form self-association, but also form network. Alcohol or acylamide molecules may also form self-association. Acetone or dimethyl sulfoxide (DMSO) molecules do not form self-association but may form cross-association with water molecules. There are cross-association between water molecules and alcohol or acylamide molecules too.

The NMR chemical shifts of protons are assumed only related with associating states of themselves. Hence there are three associating states for water

protons: self-association, cross-association, and non-association.

$$\delta_H = \delta_{a,1} x_{a,1} + \delta_{a,2} x_{a,2} + \delta_{\text{free}} x_{\text{free}} \quad (7)$$

where  $\delta_{a,1}$  and  $x_{a,1}$  denotes the characteristic chemical shift and mole fraction of self-associated water protons respectively,  $\delta_{a,2}$  and  $x_{a,2}$  denotes the characteristic chemical shift and mole fraction of cross-associated water protons respectively.

The mole fractions  $x_{a,1}$ ,  $x_{a,2}$  and  $x_{\text{free}}$  are obtained from Eq. (8)

$$\begin{aligned} x_{a,1} &= \rho_{B1} X_1^A X_1^B \Delta_{A1B1} \\ x_{a,2} &= \rho_{B2} X_1^A X_2^B \Delta_{A1B2} \\ x_{\text{free}} &= 1 - x_{a,1} - x_{a,2} \end{aligned} \quad (8)$$

where the variables  $X_1^A$ ,  $X_1^B$ ,  $X_2^B$ ,  $\rho_{B1}$ ,  $\rho_{B2}$ ,  $\Delta_{A1B1}$ ,  $\Delta_{A1B2}$  are defined in Eqs. (1) and (2).

## 3 RESULTS AND DISCUSSION

### 3.1 Model parameters for pure materials

Table 1 lists the model parameters for pure materials, which are obtained by fitting the data of saturated vapor pressure and liquid density of pure materials with PC (perturbed Chain)-SAFT equation<sup>[23]</sup>. These pure materials are water, *N*, *N*-dimethylformamide (DMF), *N*-methylacetamide (NMA), ethanol (EtOH), tert-butyl alcohol (TBA), dimethyl sulfoxide (DMSO), and acetone.

Table 1 Model parameters for pure materials

Materials	$m$	$\sigma \times 10^{10}$ , m	$\varepsilon/k$ , K	$\varepsilon^{AB}/k$ , K	$\kappa^{AB}$
Water	1.081	3.036	393.3	1358	0.0251
DMF	2.441	3.560	354.1	2215	1.51 E-4
NMA	2.953	3.336	274.9	2527	0.1489
EtOH	2.712	3.021	187.4	2569	0.0471
TBA	3.805	3.075	153.5	2415	0.1287
DMSO	3.000	3.317	348.6	2326	0.0000
Acetone	2.728	3.296	255.2	994	0.0000

### 3.2 Correlations

<sup>1</sup>H NMR chemical shift of water proton in binary aqueous mixtures of DMF<sup>[7]</sup>, NMA<sup>[8]</sup>, EtOH<sup>[1]</sup>, TBA<sup>[2]</sup>, DMSO<sup>[3]</sup>, and acetone<sup>[4]</sup> were correlated by SAFT association model. The results of correlation for <sup>1</sup>H NMR chemical shift of H<sub>2</sub>O-DMF and H<sub>2</sub>O-NMA mixtures are presented in Figs. 1 and 2. The deviations of correlation and characteristic chemical shifts of water proton in different associating states are obtained and listed in Table 2, in which RMSD is the root mean square deviation, defined as

$$RMSD = \left[ \sum_n \left( \frac{\delta_{H,\text{cal}} - \delta_{H,\text{exp}}}{\delta_{H,\text{exp}}} \right)^2 / n \right]^{1/2} \times 100\% \quad (9)$$

Table 2 The results of correlation for  $^1\text{H}$  NMR chemical shift of water proton in aqueous solutions

System	$T$ , K	$\delta_{a,1}$ , ppm	$\delta_{a,2}$ , ppm	$\delta_{free}$ , ppm	RMSD, %	Reference
H <sub>2</sub> O-DMF	298.15	4.663	3.017	5.541	0.57	[7]
H <sub>2</sub> O-DMF	313.15	4.519	2.810	5.161	0.49	[7]
H <sub>2</sub> O-DMF	328.15	4.405	2.630	4.749	0.46	[7]
H <sub>2</sub> O-DMF	343.15	4.261	2.378	4.507	0.30	[7]
H <sub>2</sub> O-DMF	358.15	4.139	2.135	4.245	0.25	[7]
H <sub>2</sub> O-NMA	308.15	4.364	3.271	6.625	0.33	[8]
H <sub>2</sub> O-NMA	323.15	4.207	3.185	6.094	0.26	[8]
H <sub>2</sub> O-NMA	338.15	4.099	3.250	5.496	0.15	[8]
H <sub>2</sub> O-EtOH	275.15	4.465	2.132	9.732	0.57	[1]
H <sub>2</sub> O-EtOH	283.15	4.264	1.748	9.690	0.52	[1]
H <sub>2</sub> O-EtOH	298.15	3.932	1.159	9.376	0.43	[1]
H <sub>2</sub> O-EtOH	313.15	3.760	1.143	8.474	0.39	[1]
H <sub>2</sub> O-EtOH	323.15	3.652	1.012	8.045	0.38	[1]
H <sub>2</sub> O-TBA	274.15	5.129	3.519	5.890	0.89	[2]
H <sub>2</sub> O-TBA	296.45	4.771	2.835	5.803	0.66	[2]
H <sub>2</sub> O-TBA	321.65	4.552	2.396	5.326	0.60	[2]
H <sub>2</sub> O-DMSO	274.15	5.843	3.962	1.122	1.17	[3]
H <sub>2</sub> O-DMSO	296.45	4.885	3.263	5.263	0.74	[3]
H <sub>2</sub> O-DMSO	321.65	4.606	2.970	5.235	0.61	[3]
H <sub>2</sub> O-acetone	274.15	5.544	2.889	2.475	1.06	[4]
H <sub>2</sub> O-acetone	296.45	5.305	2.263	2.877	0.71	[4]
H <sub>2</sub> O-acetone	321.65	5.177	1.620	2.988	0.65	[4]

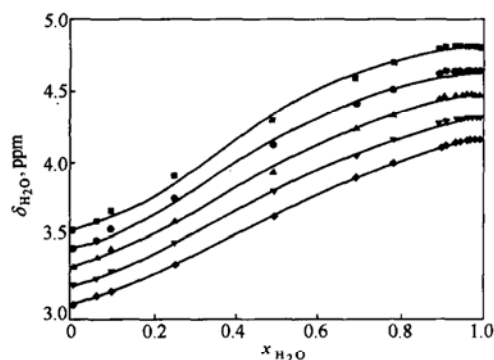


Figure 1 The results of correlation for  $^1\text{H}$  NMR chemical shift of H<sub>2</sub>O-DMF mixture. Points are experimental data<sup>[7]</sup> and lines were calculated using the SAFT association model.  $T$ , K: ■ 298.15; ● 313.15; ▲ 328.15; ▼ 343.15; ◆ 358.15

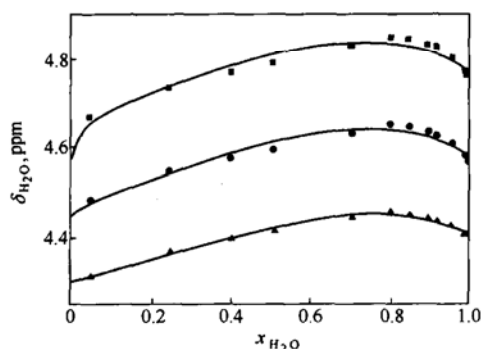


Figure 2 The results of correlation for  $^1\text{H}$  NMR chemical shift of H<sub>2</sub>O-NMA mixture. Points are experimental data<sup>[8]</sup> and lines were calculated using the SAFT association model.  $T$ , K: ■ 308.15; ● 323.15; ▲ 338.15

As shown in Figs. 1 and 2, the variation of chemical shift of water proton with concentration of water increases in majority of the concentration region and falls in the water-rich region. This interesting phenomenon indicates the complexity of solution structure and intermolecular interaction in aqueous solutions. However, the correlated curve of SAFT association model fits the experimental points very well. This result indicates that SAFT association model is suitable to deal with complex associating systems such as aqueous solutions.

### 3.3 Comparison and discussion

The above-mentioned result of correlation has proved the applicability of SAFT association model for dealing with aqueous solutions. However, other models are also used to investigate properties of associating systems. For example, Wilson equation can correlate chemical shift data of some associating systems including alcohol-hydrocarbon systems and alcohol-acylamide systems very well<sup>[16]</sup>. In order to find the most suitable model for aqueous solutions,  $^1\text{H}$  NMR chemical shift of aqueous solutions are also correlated by Wilson equation. The deviations of correlations are listed in Table 3. The results indicate that SAFT associating model fit the chemical shift of water proton in aqueous solutions very well, but Wilson equation does not.

Wilson equation is a semi-empirical physical model so that it is suitable for monotonous data only. SAFT association model can deal with complex systems in which multiple associating states coexist. Thus SAFT association model has great advantage in dealing with

this type of systems.

**Table 3** The comparison of deviation of correlation with different models

System	$T$ , K	$x_{\text{SAFT}}$ , %	$x_{\text{Wilson}}$ , %
H <sub>2</sub> O-DMF	298.15	0.57	0.99
H <sub>2</sub> O-DMF	313.15	0.49	0.91
H <sub>2</sub> O-DMF	328.15	0.46	0.87
H <sub>2</sub> O-DMF	343.15	0.30	0.78
H <sub>2</sub> O-DMF	358.15	0.25	0.72
H <sub>2</sub> O-NMA	308.15	0.33	1.24
H <sub>2</sub> O-NMA	323.15	0.26	1.04
H <sub>2</sub> O-NMA	338.15	0.15	0.70
H <sub>2</sub> O-EtOH	275.15	0.57	1.42
H <sub>2</sub> O-EtOH	283.15	0.52	1.32
H <sub>2</sub> O-EtOH	298.15	0.43	1.17
H <sub>2</sub> O-EtOH	313.15	0.39	1.04
H <sub>2</sub> O-EtOH	323.15	0.38	1.10
H <sub>2</sub> O-TBA	274.15	0.89	1.84
H <sub>2</sub> O-TBA	296.45	0.66	1.43
H <sub>2</sub> O-TBA	321.65	0.60	1.34
H <sub>2</sub> O-DMSO	274.15	1.17	1.87
H <sub>2</sub> O-DMSO	296.45	0.74	1.65
H <sub>2</sub> O-DMSO	321.65	0.61	1.28
H <sub>2</sub> O-acetone	274.15	1.06	0.91
H <sub>2</sub> O-acetone	296.45	0.71	0.87
H <sub>2</sub> O-acetone	321.65	0.65	0.79

$^1\text{H}$  NMR chemical shifts of aqueous solutions are correlated by SAFT association model and Wilson equation. It is found that SAFT association model fits the chemical shift of water proton in aqueous solutions very well, but Wilson equation cannot be used to deal with non-monotonous data. The comparison shows that SAFT association model is more suitable for dealing with complex associating systems in which multiple associating states coexist.

## NOMENCLATURE

$d$	temperature-dependent effective sphere diameter, $10^{-10}\text{m}$
$g(d)_{ij}$	radical distribution function
$k$	Boltzmann constant, $\text{J}\cdot\text{K}^{-1}$
$m$	segment number
$T$	Kelvin temperature, K
$X_i^A$	mole fraction of molecule, $i$ not bonded at site $A$
$x_{\text{free}}$	mole fraction of non-associated proton
$x_{a,i}$	mole fraction of various associated proton
$x_{a,1}$	mole fraction of self-associated water proton
$x_{a,2}$	mole fraction of cross-associated water proton
$\Delta_{AiBj}$	association strength, $10^{-30}\text{m}^3$
$\delta_{a,i}$	characteristic chemical shift of various associated proton, ppm
$\delta_{a,1}$	characteristic chemical shift of self-associated water proton, ppm
$\delta_{a,2}$	characteristic chemical shift of cross-associated water proton, ppm
$\delta_{\text{free}}$	characteristic chemical shift of non-associated proton, ppm
$\delta_{\text{H}}$	observed chemical shift of proton, ppm

$\varepsilon/k$	segment-segment interaction energy, K
$\varepsilon^{AB}/k$	site-site association energy, K
$\kappa^{AB}$	site-site association volume
$\xi_{n=0,1,2,3}$	a function of density
$\rho$	number density of molecule, $\text{m}^{-3}$
$\sigma$	segment diameter, $10^{-10}\text{m}$

## Superscripts

$A, B$	site $A, B$
hs	hard sphere
seg	segment

## Subscripts

$i, j$	component $i, j$
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## REFERENCES

- Mizuno, K., Miyashita, Y., Shindo, Y., "NMR and FT-IR studies of hydrogen bonds in ethanol-water mixtures", *J. Phys. Chem.*, **99** (10), 3225–3228 (1995).
- Mizuno, K., Kimura, Y., Morichika, H., Nishimura, Y., Shimada, S., Maeda, S., Imafuji, S., Ochi, T., "Hydrophobic hydration of tert-butyl alcohol probed by NMR and IR", *J. Molecular Liquids*, **85**, 139–152 (2000).
- Mizuno, K., "Hydration of the CH groups in dimethyl sulfoxide probed by NMR and IR", *J. Phys. Chem. B*, **104** (47), 11001–11005 (2000).
- Mizuno, K., Ochi, T., Shindo, Y., "Hydrophobic hydration of acetone probed by nuclear magnetic resonance and infrared: Evidence for the interaction C-H...OH<sub>2</sub>", *J. Chem. Phys.*, **109** (21), 9502–9507 (1998).
- Lei, Y., Li, H.R., Pan, H.H., Han, S.J., "Structures and hydrogen bonding analysis of *N, N*-dimethylformamide and *N, N*-dimethylformamide-water mixtures by molecular dynamics simulations", *J. Phys. Chem. A*, **107** (10), 1574–1583 (2003).
- Lei, Y., Li, H.R., Han, S.J., "An all-atom simulation study on intermolecular interaction of DMSO water system", *Chem. Phys. Lett.*, **380** (5–6), 542–548 (2003).
- Lei, Y., Li, H.R., Zhu, L.H., Han, S.J., " $^1\text{H}$  NMR studies of DMF-H<sub>2</sub>O association system", *Acta Chim. Sinica (China)*, **60** (10), 1747–1753 (2002). (in Chinese)
- Zhang, R., Li, H.R., Lei, Y., Han, S.J., " $^1\text{H}$  NMR studies on complex NMA-H<sub>2</sub>O association system", *Acta Chim. Sinica (China)*, **62** (7), 667–673 (2004). (in Chinese)
- Gutowsky, H.S., Saika, A., "Dissociation, chemical exchange and the proton magnetic resonance in some aqueous electrolytes", *J. Chem. Phys.*, **21**, 1688 (1953).
- Karachewski, A.M., Mcniel, M.M., Eckert, C.A., "A study of hydrogen bonding in alcohol solutions using NMR spectroscopy", *Ind. Eng. Chem. Res.*, **28** (3), 315–324 (1989).
- Karachewski, A.M., Howell, W.J., Eckert, C.A., "Development of the AVEC model for associating mixtures using NMR spectroscopy", *AIChE J.*, **37** (1), 65–73 (1991).
- Tkadlecová, M., Havlíček, J., Dohnal, V., "Association between halothane and oxygenated solvents by  $^1\text{H}$  NMR spectroscopy", *Can. J. Chem.*, **73**, 1406–1411 (1995).
- Tkadlecová, M., Dohnal, V., Coasts, M., " $^1\text{H}$  NMR and thermodynamics study of self-association and complex formation equilibria by hydrogen bonding. Methanol with chloroform or halothane", *Phys. Chem. Chem. Phys.*, **1** (7), 1479–1486 (1999).
- Tucker, E.E., Becker, E.D., "Alcohol association Studies. II. Vapor pressure, 220-MHz proton magnetic resonance, and infrared investigation of tert-butyl alcohol association in hexadecane", *J. Phys. Chem.*, **77** (14), 1783–1795 (1973).
- Chen, J.S., Rosenberger, F., "Accurate NMR data evaluation for monomer shift, dimer shift and dimerization constant in a self-associating system", *Tetrahedron Lett.*, **31** (28), 3975–3978 (1990).

- 16 Deng, D.S., Li, H.R., Yao, J., Han, S.J., "Simple local composition model for  $^1\text{H}$  NMR chemical shift of mixtures", *Chem. Phys. Lett.*, **376** (1-2), 125—129 (2003).
- 17 Chapman, W.G., Gubbins, K.E., Jackson, G., Radosz, M., "SAFT: Equation-of-state solution model for associating fluids", *Fluid Phase Equilib.*, **52**, 31—38 (1989).
- 18 Chapman, W.G., Gubbins, K.E., Jackson, G., Radosz, M., "New reference equation of state for associating liquids", *Ind. Eng. Chem. Res.*, **29** (8), 1709—1721 (1990).
- 19 Wertheim, M.S., "Thermodynamic perturbation theory of polymerization", *J. Chem. Phys.*, **87** (12), 7323—7331 (1987).
- 20 Huang, S.H., Radosz, M., "Equation of state for small, large, polydisperse, and associating molecules", *Ind. Eng. Chem. Res.*, **29** (11), 2284—2294 (1990).
- 21 Huang, S.H., Radosz, M., "Equation of state for small, large, polydisperse, and associating molecules: Extension to fluid mixtures", *Ind. Eng. Chem. Res.*, **30** (8), 1994—2005 (1991).
- 22 Gross, J., Sadowski, G., "Perturbed-Chain SAFT: An equation of state based on a perturbation theory for chain molecules", *Ind. Eng. Chem. Res.*, **40** (4), 1244—1260 (2001).
- 23 Gross, J., Sadowski, G., "Application of the perturbed-chain SAFT equation of state to associating systems", *Ind. Eng. Chem. Res.*, **41** (22), 5510—5515 (2002).