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The IMC Structure of Multi-rate Multivariable Predictive Control Systems and An Improved Algorithm

ZHOU Lifang(周立芳)* and **QIAN Jixin(钱积新)**
Institute of Industrial Process Control, Zhejiang University, Hangzhou 310027, China

Abstract Multirate multivariable predictive control system with the sampling mechanism that adjusts the plant inputs only once but detects the plant outputs several times during a period is examined. The IMC structure of the system is derived, and its robust stability and zero steady state error characteristics are analyzed. A new control algorithm is developed by adding the variation of the outputs to the index performance. The simulation results show that the method is effective and has zeros steady-state error.
Keywords multirate, predictive control, internal model control (IMC)

1 INTRODUCTION

In synthesizing a multiple-input multiple-output digital control system, it is usually assumed that both the plant input updating and the plant output measurement are performed at the same constant rate and in a synchronous way. However, this hypothesis is often not realistic or necessary in the technological and/or economical sense. These may be due to the following fact[1]:

(1) One or more sensors take significant time to conduct the measurement of some output variables, while other outputs are easily available without time and rate limitation.

(2) The same sensor is periodically switched to measure different plant variables at different times, or all the plant outputs are measured at the same time and rate but the measurements are transmitted to the control processing unit sequentially.

(3) Technological constraints prevent one from computing and/or updating the control components at a desired sampling period, while the output measurement is available with no rate or time limitations.

It is then natural to resort to multirate control schemes[2–5]. The digital controller which incorporates periodical time-varying mechanisms is classified into the three classes: the first class employs a special type of sampling mechanism which changes the plant input several times and detect the plant output only once during a period T_0; The second employs another type of sampling mechanism which changes the plant input only once but detects the plant output several times in T_0; The third employs the ordinary sampling mechanism which changes the plant input and detects the plant output simultaneously, but also employs a periodical time-vary digital system as the controller.

The predictive control is a new control algorithm developed from industrial process control late in the 1970's. Applying the predictive control to the multirate system has received more and more attention[6–10]. In order to eliminate the effect of model mismatch and disturbances, the self-tuning mechanism has been employed[9,10], and the model parameters are modified as soon as the output is available. Most of these researches are based on the parameter model. Ling and Lim extended the state space GPC to the multirate controllers and proposed the multirate-input controller (MRIC) and the multirate output controller (MROC)[7].

The internal model control (IMC) is an effective tool to design and analyze the predictive control system, but there are few papers on the IMC structure of multirate predictive control systems. In this paper, the multirate-output predictive control for multivariable systems with the sampling mechanism changing the plant input only once but detecting the plant output several times in a period is studied. The IMC structure of the above system is deduced on this basis. Taking advantage of the quality of the IMC, the characteristics of the robust stability and the zero-steady-state are analyzed. To reduce overshoot of the control system, the weight on the variation of the predictive outputs is added to the quadratic objective function. The simulation results indicate that the method is effective.

2 MULTIRATE-OUTPUT MULTIVARIABLE PREDICTIVE CONTROL

Considering the following n inputs n outputs system:

(1) The base-sampling period is τ_0. For simplicity of notation, τ_0 is assumed to be 1. The sampling period of the output y_i is $T_i (i = 1, \ldots, n)$. The ith input

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* To whom correspondence should be addressed. E-mail: lfzhou@iipc.zju.edu.cn

action is performed at the period T_0. T_i is considered as an integer.

(2) The step response model at sampling period τ_0 is known.

(3) T_0 =l.c.m(T_1, T_2, ..., T_n) ("l.c.m" means the lease common multiple)

2.1 The predictive model

Assume the model time horizon is NT_0, the step response coefficients of the qth output y_q to the ith input u_i at the base period τ_0 are $\{\hat{a}_1^{qi}, \hat{a}_2^{qi}, \ldots, \hat{a}_{NT_0}^{qi}\}$. Choose the predictive horizon to be PT_0 and the control horizon to be MT_0, the predicted value of the qth output at instant $k+j$ can be written as

$$y_{m,q}(k+j|k) = y_{0,q}(k+j|k) + \sum_{i=1}^{n}\sum_{l=1}^{j} \hat{a}_l^{qi} \Delta u_i(k+j-l) \tag{1}$$

$$\text{for} \quad \Delta u_i(k+d) = 0 \quad (d \geqslant MT_0),$$
$$q = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, PT_0$$

where $y_{m,q}(k+j|k)$ denotes the value of the qth output y_q at time $k+j$, predicted at time k. $y_{0,q}(k+j|k)$ means the initial value of the qth output y_q at time $k+j$, which is calculated based on the information known at time k. $\Delta u_i(k+j-l)$ is the ith control increment at time $k+j-l$. Then the output equation can be rewritten in vector form

$$\boldsymbol{y}_m(k+1) = \boldsymbol{y}_0(k+1) + \boldsymbol{A}\Delta\boldsymbol{u}(k) \tag{2}$$

where

$$\boldsymbol{y}_m(k+1) = \left[\boldsymbol{y}_{m,1}^{\mathrm{T}}(k+1), \boldsymbol{y}_{m,2}^{\mathrm{T}}(k+1), \cdots, \boldsymbol{y}_{m,n}^{\mathrm{T}}(k+1)\right]^{\mathrm{T}}$$

$$y_{m,i}(k+1) = \left[y_{m,i}(k+1/k), y_{m,i}(k+2/k), \cdots, y_{m,i}(k+PT_0/k)\right]^{\mathrm{T}} \quad i = 1, \ldots, n$$

$$\boldsymbol{y}_0(k+1) = \left[\boldsymbol{y}_{0,1}^{\mathrm{T}}(k+1), \boldsymbol{y}_{0,2}^{\mathrm{T}}(k+1), \cdots, \boldsymbol{y}_{0,n}^{\mathrm{T}}(k+1)\right]^{\mathrm{T}}$$

$$\boldsymbol{y}_{0,i}(k+1) = \left[y_{0,i}(k+1/k), y_{0,i}(k+2/k), \cdots, y_{0,i}(k+PT_0/k)\right]^{\mathrm{T}} \quad i = 1, \ldots, n$$

$$\Delta\boldsymbol{u}(k) = \left[\Delta\boldsymbol{u}_1^{\mathrm{T}}(k), \Delta\boldsymbol{u}_2^{\mathrm{T}}(k), \cdots, \Delta\boldsymbol{u}_n^{\mathrm{T}}(k)\right]^{\mathrm{T}}$$

$$\Delta\boldsymbol{u}_j(k) = \left[\Delta u_j(k), \Delta u_j(k+1), \cdots, \Delta u_j(k+MT_0-1)\right]^{\mathrm{T}} \quad j = 1, \ldots, n$$

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} & \cdots & \boldsymbol{A}_{1n} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} & \cdots & \boldsymbol{A}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{A}_{n1} & \boldsymbol{A}_{n2} & \cdots & \boldsymbol{A}_{nn} \end{bmatrix}$$

$$\boldsymbol{A}_{ij} = \begin{bmatrix} \hat{a}_1^{ij} & & & \\ \hat{a}_2^{ij} & \hat{a}_1^{ij} & & \\ \vdots & \vdots & \ddots & \\ \hat{a}_{PT_0}^{ij} & \hat{a}_{PT_0-1}^{ij} & \cdots & \hat{a}_{PT_0-MT_0+1}^{ij} \end{bmatrix}$$

The matrix $\boldsymbol{A}$ consists of $\boldsymbol{A}_{ij}(i = 1\ldots n,\ j = 1\ldots n)$. In order to eliminate the effect of model mismatch, the predicted output equation should be modified using the error between the model outputs and the system outputs at current time. Then the predictive equation is described as follows

$$\boldsymbol{y}_p(k+1) = \boldsymbol{y}_0(k+1) + \boldsymbol{A}\Delta\boldsymbol{u}(k) + \boldsymbol{H}\boldsymbol{e}(k) \tag{3}$$

where

$$\boldsymbol{e}(k) = \begin{bmatrix} y_1(k) - y_{m,1}(k), & y_2(k) - y_{m,2}(k), \\ \cdots, & y_n(k) - y_{m,n}(k) \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{H} = \mathrm{diag}\{\boldsymbol{H}_i\}, \quad i = 1, \ldots, n$$

$$\boldsymbol{H}_i = \begin{bmatrix} h_{i1}, & h_{i2}, & \cdots, & h_{iPT_0} \end{bmatrix}^{\mathrm{T}}$$

$\boldsymbol{H}_i$ is the weighting vector for the $\boldsymbol{e}(k)$. Then the predictive outputs from time $k+1$ to $k+P$ can be calculated.

2.2 Optimization strategy

In the predictive control, rolling optimization is an important feature. Appropriate inputs are selected so that the predicted outputs over the predictive horizon would follow the reference trajectories as closely as possible. In other words, the manipulated variables are determined to minimize the following quadratic objective function

$$J_p = [\boldsymbol{y}_p(k+1) - \boldsymbol{y}_R(k+1)]^{\mathrm{T}}\boldsymbol{Q}[\boldsymbol{y}_p(k+1) - \boldsymbol{y}_R(k+1)] + \Delta\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{\lambda}\Delta\boldsymbol{u}(k) \tag{4}$$

$$\boldsymbol{y}_R(k+1) = \left[\boldsymbol{y}_{R,1}^{\mathrm{T}}(k+1), \boldsymbol{y}_{R,2}^{\mathrm{T}}(k+1), \cdots, \boldsymbol{y}_{R,n}^{\mathrm{T}}(k+1)\right]^{\mathrm{T}}$$

$$\boldsymbol{y}_{R,i}(k+1) = \begin{bmatrix} y_{R,i}(k+1/k), y_{R,i}(k+2/k), \\ \cdots, y_{R,i}(k+PT_0/k) \end{bmatrix}^{\mathrm{T}}$$
$$i = 1, \ldots, n$$

Substituting (3) into (4), J_p can be rewritten as

$$J_p' = \Delta\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{W}\Delta\boldsymbol{u}(k) + 2\boldsymbol{t}^{\mathrm{T}}(k)\Delta\boldsymbol{u}(k) \tag{5}$$

where

$$\boldsymbol{W} = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{A} + \boldsymbol{\lambda}$$

$$\boldsymbol{t}(k) = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}[\boldsymbol{y}_0(k+1) + \boldsymbol{H}\boldsymbol{e}(k) - \boldsymbol{y}_R(k+1)]$$

$\boldsymbol{Q}$ and $\boldsymbol{\lambda}$ are weighting matrixes for predicted error and control moves respectively. Because the input actions are performed at rate T_0, the components $\Delta u_i(j)$ of input vector $\Delta\boldsymbol{u}(k)$ should satisfy with the following condition. In other words, the following constraints should be added to the quadratic objective function:

$$\text{If} \quad j\tau_0 \neq sT_0 (s \text{ is integer}) \text{ then}$$
$$\Delta u_i(j) = 0 \quad (i = 1, \dots, n)$$

After calculating the inputs, only the first changes in the inputs $\Delta u_i(k)(i = 1, \dots, n)$ are performed. The next optimization will be done at the time $(k+1)T_0$.

3 IMC STRUCTURE

3.1 IMC structure of multirate predictive control

Internal model control (IMC) is often used to analysis the predictive control system. So we can also use it to analyze the multirate predictive control. Define the vector

$$\Delta\widetilde{\boldsymbol{u}}(k) = \left[\Delta\widetilde{\boldsymbol{u}}_1^{\mathrm{T}}(k) \quad \Delta\widetilde{\boldsymbol{u}}_2^{\mathrm{T}}(k) \quad \cdots \quad \Delta\widetilde{\boldsymbol{u}}_n^{\mathrm{T}}(k)\right]^{\mathrm{T}}$$

$$\Delta\widetilde{\boldsymbol{u}}_i(k) = \begin{bmatrix} \Delta u_i(kT_0) & \Delta u_i(kT_0 + T_0) & \cdots \\ \Delta u_i(kT_0 + (M-1)T_0) & & \end{bmatrix}^{\mathrm{T}}$$

$$\widetilde{\boldsymbol{u}}(k-1) = \left[\widetilde{\boldsymbol{u}}_1^{\mathrm{T}}(k-1) \ \widetilde{u}_2^{\mathrm{T}}(k-1) \ \cdots \ \widetilde{\boldsymbol{u}}_n^{\mathrm{T}}(k-1)\right]^{\mathrm{T}}$$

$$\widetilde{\boldsymbol{u}}_i(k-1) = \begin{bmatrix} u_i(kT_0 - NT_0) & u_i(k-(N-1)T_0) \\ \cdots \quad u_i(kT_0 - T_0) & \end{bmatrix}^{\mathrm{T}}$$

Then the output predictive equation can be described as

$$\boldsymbol{y_p}(k+1) = \widetilde{\boldsymbol{A}}_0\widetilde{\boldsymbol{u}}(k-1) + \widetilde{\boldsymbol{A}}\Delta\widetilde{\boldsymbol{u}}(k) + \boldsymbol{He}(k) \quad (6)$$

$$\widetilde{\boldsymbol{A}}_0 = \left\{\widetilde{\boldsymbol{A}}_0^{ij}\right\}, i = 1, \dots, n; \ j = 1, \dots, n$$

$$\widetilde{\boldsymbol{A}} = \left\{\widetilde{\boldsymbol{A}}^{ij}\right\}, i = 1, \cdots, n; \ j = 1, \cdots, n$$

$$\widetilde{\boldsymbol{A}}^{ij} = \begin{bmatrix} \widetilde{\boldsymbol{a}}_1^{ij} & & & \\ \widetilde{\boldsymbol{a}}_2^{ij} & \widetilde{\boldsymbol{a}}_1^{ij} & & \\ \vdots & \vdots & \ddots & \\ \widetilde{\boldsymbol{a}}_P^{ij} & \widetilde{\boldsymbol{a}}_{P-1}^{ij} & \cdots & \widetilde{\boldsymbol{a}}_{P-M+1}^{ij} \end{bmatrix}$$

$$\widetilde{\boldsymbol{a}}_1^{ij} = \begin{bmatrix} \hat{a}_{(l-1)T_0+1}^{ij} & \hat{a}_{(l-1)T_0+2}^{ij} & \cdots & \hat{a}_{lT_0}^{ij} \end{bmatrix}^{\mathrm{T}}$$

Using the same method described in this paper, the control variables can be obtained

$$\Delta\widetilde{\boldsymbol{u}}(k) = \left(\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Q}\widetilde{\boldsymbol{A}} + \boldsymbol{\lambda}\right)^{-1}\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Q}[\boldsymbol{y_R}(k+1) - \widetilde{\boldsymbol{A}}_0\widetilde{\boldsymbol{u}}(k-1) - \boldsymbol{He}(k)] \quad (7)$$

Then the ith-input increment $\Delta u_i(kT_0)$ is

$$\Delta u_i(kT_0) = \boldsymbol{d}_i^{\mathrm{T}}[\boldsymbol{y_R}(k+1) - \widetilde{\boldsymbol{A}}_0\widetilde{\boldsymbol{u}}(k-1) - \boldsymbol{He}(k)]$$
$$i = 1, \dots, n \quad (8)$$

where $\boldsymbol{d}_i^{\mathrm{T}}$ is the $[(i-1)M+1]$th line vector of the matrix $(\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Q}\widetilde{\boldsymbol{A}} + \boldsymbol{\lambda})^{-1}\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Q}$. In the multirate predictive control proposed in this paper, the control variables are not changed during the period T_0, so the following equation should be satisfied

$$\begin{aligned} \Delta u_i(kT_0) &= u_i(kT_0) - u_i(kT_0 - 1) \\ &= u_i(kT_0) - u_i(kT_0 - T_0) \\ &= (1 - z^{-T_0})u_i(kT_0) \end{aligned} \quad (9)$$

Define: $\overline{\boldsymbol{u}}(k) = [u_1(kT_0) \ u_2(kT_0) \ \cdots \ u_n(kT_0)]^{\mathrm{T}}$, then Eq. (8) can be rewritten as

$$\begin{aligned} &\left(1 - z^{-T_0}\right)\overline{\boldsymbol{u}}(k) \\ &= \begin{bmatrix} \boldsymbol{d}_1^{\mathrm{T}} \\ \boldsymbol{d}_2^{\mathrm{T}} \\ \vdots \\ \boldsymbol{d}_n^{\mathrm{T}} \end{bmatrix} [\boldsymbol{y_R}(k+1) - \widetilde{\boldsymbol{A}}_0\widetilde{\boldsymbol{u}}(k-1) - \boldsymbol{He}(k)] \\ &= D\left[y_R(k+1) - \widetilde{A}_0\widetilde{u}(k-1) - He(k)\right] \end{aligned} \quad (10)$$

in which

$$\begin{aligned} &\widetilde{\boldsymbol{A}}_0\widetilde{\boldsymbol{u}}(k-1) \\ &= \widetilde{\boldsymbol{A}}_0 \begin{bmatrix} \boldsymbol{z}(z^{-T_0}) & & & \\ & \boldsymbol{z}(z^{-T_0}) & & \\ & & \ddots & \\ & & & \boldsymbol{z}(z^{-T_0}) \end{bmatrix} \overline{\boldsymbol{u}}(k) \\ &= \begin{bmatrix} F_{11}(z^{-T_0}) & F_{12}(z^{-T_0}) & & F_{1n}(z^{-T_0}) \\ F_{21}(z^{-T_0}) & F_{22}(z^{-T_0}) & & F_{2n}(z^{-T_0}) \\ \vdots & \vdots & \ddots & \vdots \\ F_{(n*PT_0)1}(z^{-T_0}) & F_{(n*PT_0)2}(z^{-T_0}) & & F_{(n*PT_0)n}(z^{-T_0}) \end{bmatrix}. \end{aligned}$$

$$\overline{\boldsymbol{u}}(k) = \overline{\boldsymbol{F}}(z^{-T_0})\overline{\boldsymbol{u}}(k) \quad (11)$$

$$\widetilde{\boldsymbol{A}}_0^{ij} = \begin{bmatrix} \hat{a}_{NT_0}^{ij} - \hat{a}_{NT_0-T_0+1}^{ij} & \hat{a}_{NT_0-T_0+1}^{ij} - \hat{a}_{NT_0-2T_0+1}^{ij} & \cdots & \hat{a}_{2T_0+1}^{ij} - \hat{a}_{T_0+1}^{ij} & \hat{a}_{T_0+1}^{ij} \\ & \hat{a}_{NT_0}^{ij} - \hat{a}_{NT_0-T_0+2}^{ij} & \cdots & \hat{a}_{2T_0+2}^{ij} - \hat{a}_{T_0+2}^{ij} & \hat{a}_{T_0+2}^{ij} \\ 0 & & \ddots & \vdots & \vdots \\ & \hat{a}_{NT_0}^{ij} - \hat{a}_{NT_0-T_0+PT_0}^{ij} & \cdots & \hat{a}_{2T_0+PT_0}^{ij} - \hat{a}_{T_0+PT_0}^{ij} & \hat{a}_{T_0+PT_0}^{ij} \end{bmatrix}$$

$$\boldsymbol{z}(z^{-T_0}) = \begin{bmatrix} z^{-NT_0} & z^{-(N-1)T_0} & \cdots & z^{-T_0} \end{bmatrix}^{\mathrm{T}}$$

$$\begin{aligned} F_{lj}(z^{-T_0}) = & (\hat{a}^{lj}_{NT_0} - \hat{a}^{lj}_{NT_0-T_0+t})z^{-NT_0} + \\ & (\hat{a}^{lj}_{NT_0-T_0+t} - \hat{a}^{lj}_{NT_0-2T_0+t})z^{-(N-1)T_0} + \\ & \cdots + (\hat{a}^{lj}_{2T_0+t} - \hat{a}^{lj}_{T_0+t})z^{-2T_0} + \hat{a}^{lj}_{T_0+t}z^{-T_0} \end{aligned}$$

$$i = (l-1)PT_0 + t, \ t \in [1, \ PT_0], \ l \in [1, \ n]$$

If we assume the setpoints of the controlled variables are invariant during the period T_0 substituting Eq. (11) to Eq. (10), we obtain

$$\overline{\boldsymbol{u}}(kT_0) = [\boldsymbol{F}(z^{-T_0})]^{-1}[\widetilde{\boldsymbol{D}}(z^{-T_0})\boldsymbol{y_r}(k+P) - \boldsymbol{DHe}(k)] \tag{12}$$

where

$$\boldsymbol{y_r}(k+P) = \begin{bmatrix} y_{r1}(kT_0+PT_0) \ y_{r2}(kT_0+PT_0) \ \cdots \\ y_{rn}(kT_0+PT_0) \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{F}(z^{-T_0}) = \left[(1-z^{-T_0})\boldsymbol{I_n} + \boldsymbol{D}\overline{\boldsymbol{F}}(z^{-T_0})\right],$$

(assume the inverse of $\boldsymbol{F}(z^{-T_0})$ existing)

$$\widetilde{\boldsymbol{D}}(z^{-T_0}) = \left[\widetilde{\boldsymbol{d}}_1(z^{-T_0}) \ \ \widetilde{\boldsymbol{d}}_2(z^{-T_0}) \ \ \cdots \ \ \widetilde{\boldsymbol{d}}_n(z^{-T_0})\right]$$

From Eq. (12), it can be said that the multirate predictive expressed in this paper has the same structure as the IMC system. Now the method used to analysis the IMC system can be applied to the multirate predictive control system, such as the robust stability and the zero steady state features.

3.2 The robust stability

According to the characteristics of IMC systems, when the model is accurate, the close-loop stability only depends on the stability of the controller and the actual system. When the model is mismatched, we can regulate the parameters of the feedback filter to assure the stability of the close loop system. From Eq. (12), we can obtain that if the roots of det $\boldsymbol{F}(\boldsymbol{z}^{-T_0}) = 0$ are located at the inside of unit circle, the control system is stable.

3.3 Zeros steady-state errors characteristic

The error equation of the closed system is

$$\begin{aligned} \boldsymbol{y}(kT_0) = & \boldsymbol{G}[\boldsymbol{I} + \boldsymbol{F}^{-1}\boldsymbol{DH}(\boldsymbol{G} - \hat{\boldsymbol{G}})]^{-1}F^{-1}\tilde{D}\boldsymbol{y_r} + \\ & \left\{\boldsymbol{I} - \boldsymbol{G}[\boldsymbol{I} + \boldsymbol{F}^{-1}\boldsymbol{DH}(\boldsymbol{G} - \hat{\boldsymbol{G}})]^{-1}\boldsymbol{F}^{-1}\boldsymbol{DH}\right\}\xi \end{aligned} \tag{13}$$

when the system is close to steady-state, $k \to \infty$, $z^{-1} \to 1$, so we can obtain

$$\begin{aligned} \boldsymbol{F}(1) &= \boldsymbol{D}\overline{\boldsymbol{F}}(1) \\ &= \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1(n^*PT_0)} \\ d_{21} & d_{22} & \cdots & d_{2(n^*PT_0)} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \cdots & d_{n(n^*PT_0)} \end{bmatrix} \begin{bmatrix} F_{11}(1) & F_{12}(1) & \cdots & F_{1n}(1) \\ F_{21}(1) & F_{22}(1) & \cdots & F_{2n}(1) \\ \vdots & \vdots & \ddots & \vdots \\ F_{(n^*PT_0)1}(1) & F_{(n^*PT_0)2}(1) & \cdots & F_{(n^*PT_0)n}(1) \end{bmatrix} \\ &= \begin{bmatrix} d^{11} & d^{12} & \cdots & d^{1n} \\ d^{21} & d^{22} & \cdots & d^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d^{n1} & d^{n2} & \cdots & d^{nn} \end{bmatrix} \begin{bmatrix} \hat{a}^{11}_{NT_0} & \hat{a}^{12}_{NT_0} & \cdots & \hat{a}^{1n}_{NT_0} \\ \hat{a}^{21}_{NT_0} & \hat{a}^{22}_{NT_0} & \cdots & \hat{a}^{2n}_{NT_0} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}^{n1}_{NT_0} & \hat{a}^{n2}_{NT_0} & \cdots & \hat{a}^{nn}_{NT_0} \end{bmatrix} \\ &= \overline{\boldsymbol{D}}\boldsymbol{G}(1) \end{aligned}$$

$$d^{ij} = d_{i[(j-1)PT_0+1]} + d_{i[(j-1)PT_0+2]} + \cdots + d_{i[jPT_0]}$$

$$\widetilde{\boldsymbol{D}}(1) = \overline{\boldsymbol{D}}$$

If set $\boldsymbol{H} = \mathrm{diag}(\boldsymbol{e_{PT_0}}, \boldsymbol{e_{PT_0}}, \cdots, \boldsymbol{e_{PT_0}})$, $\boldsymbol{e_{PT_0}} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^{\mathrm{T}}$, then $\boldsymbol{DH} = \overline{\boldsymbol{D}}$. Therefore

$$\boldsymbol{y}(\infty) \to \boldsymbol{y_r}(\infty)$$

that is the steady-state error is zero to the step response and disturbance.

$$\widetilde{\boldsymbol{d}}_1(z^{-T_0}) = \begin{bmatrix} (d_{11} + d_{12} + \cdots + d_{1(T_0-1)})z^{-PT_0} + (d_{1T_0} + d_{1(T_0+1)} + \cdots + d_{1(2T_0-1)})z^{-(P-1)T_0} + \\ \cdots + (d_{1[(P-1)T_0]} + d_{1[(P-1)T_0+1]} + \cdots + d_{1(PT_0-1)})z^{-T_0} + d_{1(PT_0)} \\ (d_{21} + d_{22} + \cdots + d_{2(T_0-1)})z^{-PT_0} + (d_{2T_0} + d_{2(T_0+1)} + \cdots + d_{2(2T_0-1)})z^{-(P-1)T_0} + \\ \cdots + (d_{2[(P-1)T_0]} + d_{2[(P-1)T_0+1]} + \cdots + d_{2(PT_0-1)})z^{-T_0} + d_{2(PT_0)} \\ \vdots \\ (d_{n1} + d_{n2} + \cdots + d_{n(T_0-1)})z^{-PT_0} + (d_{nT_0} + d_{n(T_0+1)} + \cdots + d_{n(2T_0-1)})z^{-(P-1)T_0} + \\ \cdots + (d_{n[(P-1)T_0]} + d_{n[(P-1)T_0+1]} + \cdots + d_{n(PT_0-1)})z^{-T_0} + d_{n(PT_0)} \end{bmatrix}$$

4 AN IMPROVED ALGORITHM

In a general way, the performance index is dependent on the weight on the error between the setpoints and the predictive outputs and the weight on the future variation of the control variables. By this strategy, only the tracking problem and the future variations of control variables are considered, and the variation of controlled variables is not taken into account. System overshooting and oscillating occur frequently. So, the weight on the variations of controlled variables is added to the quadratic performance index in this paper. That is

$$J_p = \boldsymbol{e_p}(k+1)^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{e_p}(k+1) + \Delta\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{\lambda}\Delta\boldsymbol{u}(k) + \boldsymbol{e_y}(k+1)^{\mathrm{T}}\boldsymbol{\Phi}\boldsymbol{e_y}(k+1) \tag{14}$$

$$\boldsymbol{e_p}(k+1) = \boldsymbol{y_p}(k+1) - \boldsymbol{y_R}(k+1)$$
$$\boldsymbol{e_y}(k+1) = \boldsymbol{y_p}(k+1) - \boldsymbol{y_a}(k+1)$$

$$\boldsymbol{y_a}(k+1) = \begin{bmatrix} y_1(k), y_1(k+1/k), \\ \cdots, y_1(k+P-1/k), y_2(k), \\ \cdots, y_n(k+P-1/k) \end{bmatrix}^{\mathrm{T}}$$

$\boldsymbol{\Phi}$ is the weighting matrix on the variations of the controlled variables. To minimize the performance index (14) with respect to $\Delta\boldsymbol{u}(k)$, the future control variables can be calculated.

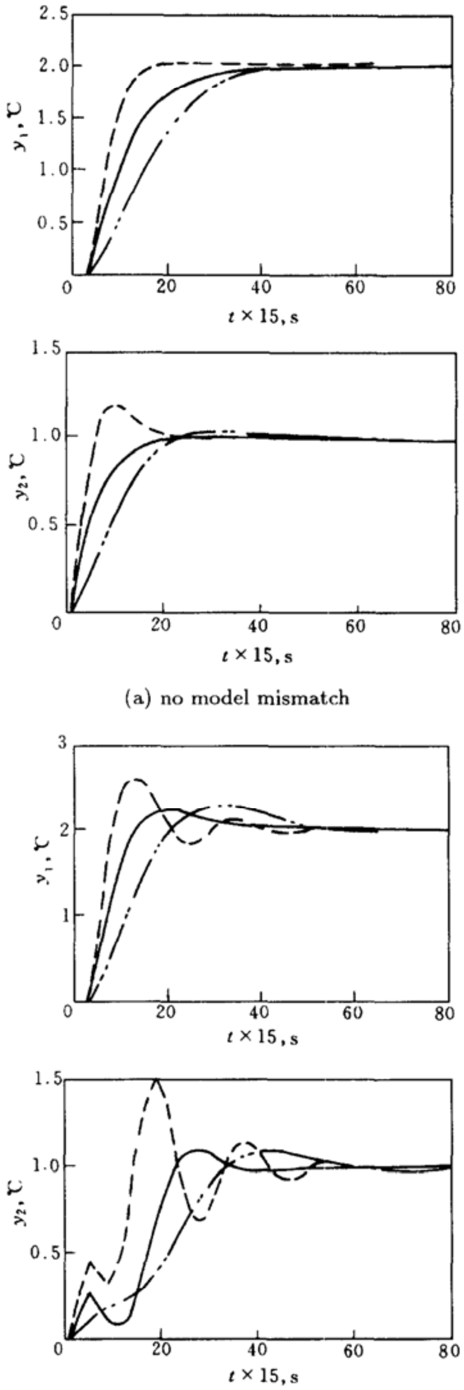
4.1 Demonstration 1

Consider the following 2 inputs and 2 outputs multirate system

$$\boldsymbol{G_m}(s) = \begin{bmatrix} \dfrac{e^{-8s}}{15s+1} & \dfrac{e^{-2s}}{8s+1} \\ -\dfrac{e^{-4s}}{5s+1} & \dfrac{1}{10s+1} \end{bmatrix}$$

$$\boldsymbol{G_p}(s) = \begin{bmatrix} \dfrac{0.5e^{-8s}}{15s+1} & \dfrac{1.5e^{-2s}}{8s+1} \\ -\dfrac{2e^{-4s}}{5s+1} & \dfrac{0.5}{10s+1} \end{bmatrix}$$

$\boldsymbol{G_m}(s)$, $\boldsymbol{G_p}(s)$ are the transfer function of the model and the system respectively. The sampling periods of the output y_1 and y_2 are 1, 2, the input actions u_1 and u_2 are performed at the period of 2 and 2. The initial values of the outputs are 0, and the setpoints of the outputs are 2, 1. The control horizon is 6 and the predictive horizon is 24. The performance index is showed as Eq. (5), where the weighting matrixes are: $\boldsymbol{Q}$ = diag(1,1...,1), $\boldsymbol{\lambda}$ = diag(1,1,...,1), $\boldsymbol{\Phi}$=diag(50,50,... 50).



(a) no model mismatch

(b) Model mismatched

Figure 1 The step response curves for multirate DMC system

—— using the improved algorithm, $\boldsymbol{\lambda}$=diag(1,1,···,1)

···· using the conventional quadratic performance index, $\boldsymbol{\lambda}$=diag(1,1,···,1)

– · – using the conventional quadratic performance index, $\boldsymbol{\lambda}$=diag(50,50,···,50)

Two cases are studied in the simulation: one is no model mismatch [$\boldsymbol{G_p}(s) = \boldsymbol{G_m}(s)$], the other is mismatched model. The simulation results are described in Fig. 1. The dotted line express the response curves, which are obtained by using the conventional quadratic performance index. The solid lines express the response curves using the improved algorithm. The dashdot lines are obtained by using the conventional quadratic performance index, but the weighting matrix $\boldsymbol{\lambda}$ is diag(50,50,...,50).

From the Fig. 1, we observe that the responses resorting to the method presented in this paper show small overshoot and short transient time. When the model mismatch is existing, the control system can also track the change of the setpoints. Increasing the weight on the increments of manipulated variables can also reduce the overshoot, but the response speed will become slower and the transient time will increase.

4.2 Demonstration 2

We consider another practical example, an atmospheric distillation column process with the product of 2.5 million ton per year (see Fig. 2).

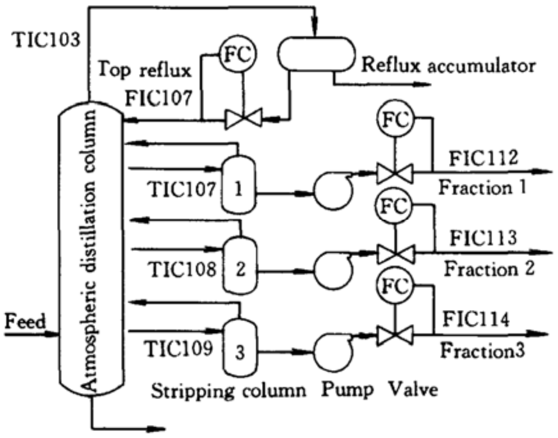


Figure 2 The flow chart of atmospheric distillation column

The atmospheric distillation column deals with the crude oil directly. It separates crude oil into several products. The top product is naphtha and the other products become the feeds of the following workshop section. The qualities of products are controlled indirectly by means of controlling their temperatures for some reasons. The top temperature is controlled by the method of soft measure. So, we adopt the multivariable advance control system where temperature TIC107 and temperature TIC108 are regarded as controlled variables (CVs), the setpoints of the flow-rate FIC112 and the flow-rate FIC113 are regarded as manipulated variables (MVs). The model of the above system is the following

$$\boldsymbol{G_m}(s) = \begin{bmatrix} \dfrac{0.524e^{-120s}}{313s+1} & 0 \\ \dfrac{0.867e^{-120s}}{367s+1} & \dfrac{0.652e^{-120s}}{677s+1} \end{bmatrix}$$

The sampling periods of the output y_1 and y_2 are 15 s, 30 s, the input actions u_1 and u_2 are performed at the periods of 30 s and 30 s. The initial values of the outputs are 0℃, and the setpoints of the outputs are 2℃, 1℃. The control horizon is 60 s and the predictive horizon is 450 s. The performance index is showed as Eq. (5), where the weighting matrixes are: $\boldsymbol{Q}$=diag (1,1,...,1), $\boldsymbol{\lambda}$ =diag(1,1,...,1), $\boldsymbol{\Phi}$=diag(100,100,...,100).

Considering the case of model match $[\boldsymbol{G_p}(s) = \boldsymbol{G_m}(s)]$, the simulation results are described in Fig. 3. The dotted line express the response curves which are

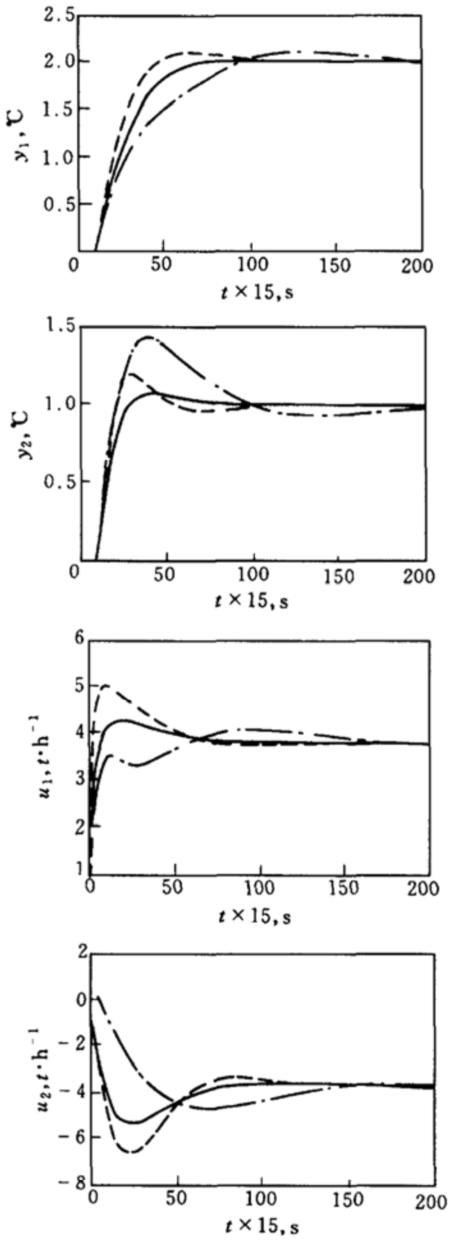


Figure 3 The response curves of the atmospheric distillation column

—— using the improved algorithm, $\boldsymbol{\lambda}$=diag(1,1,···,1)

······ using the conventional quadratic performance index, $\boldsymbol{\lambda}$=diag(1,1,···,1)

·—·— using the conventional quadratic performance index, $\boldsymbol{\lambda}$=diag(50,50,···,50) obtained by using the conventional quadratic performance index with $\boldsymbol{\lambda}$=diag(1,1,...,1). The solid lines express the response curves using the improved algorithm. The dashdot lines are obtained by using the conventional quadratic performance index, but the weighting matrix $\boldsymbol{\lambda}$ is diag(5,5,...,5).

Figure 3 shows that the control performance resorting to the method presented in this paper is better than other methods. The transient process time is shortened and the overshoot is reduced. The steady-state errors are zeros to the step response.

NOMENCLATURE

$\{\hat{a}_1^{qi}, \hat{a}_2^{qi}, \cdots, \hat{a}_{NT_0}^{qi}\}$	the step response coefficients of the qth output y_q to the ith input u_i at the base period τ_0
$\boldsymbol{d}_i^{\mathrm{T}}$	the $[(i-1)M+1]$th line vector of the matrix $(\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Q}\widetilde{\boldsymbol{A}}+\boldsymbol{\lambda})^{-1}\widetilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Q}$
$\boldsymbol{H}_i$	the weighting vector for the predicted error
J_p, J_p'	the quadratic objective function
MT_0	the control horizon
PT_0	predictive horizon
$\boldsymbol{Q}$	the weighting matrixes for predicted error
T_i	sampling period of the output y_i
T_0	sampling period of the input, T_0=l.c.m(T_1, T_2, $\cdots$, T_n)
$\boldsymbol{y}_m(k+1)$	the predictive output vector at time k
$y_{m,q}(k+j\|k)$	the predictive value of the qth output y_q at time $k+j$, predicted at time k
$\boldsymbol{y_R}(k+1)$	the setpoint vector at time k
$y_{0,q}(k+j\|k)$	the predictive initial value of the qth output y_q at time $k+j$, predicted at time k
$y_0(k+1)$	the predictive output initial vector at time k
λ	the weighting matrixes for control moves respectively
τ_0	the base-sampling period
$\boldsymbol{\Phi}$	the weighting matrix on the variations of the controlled variables

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